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Robust resource allocation for heterogeneous wireless network: a worst-case optimisation

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Abstract: With the development of fifth generation wireless communication technology, how to improve system capacity and spectrum efficiency is a crucial problem in resource sharing of heterogeneous networks (HetNets). Most of existing resource allocation (RA) schemes in HetNets focus on perfect channel state information, however, exact channel information is difficult to obtain under link delay and stochastic channel condition. In order to resolve the RA issues under channel uncertainties, a robust RA algorithm is proposed to maximise the sum data rate of microcell users where the users are subjected to the individual transmission power constraint and the cross-tier interference constraint of macrocell users. The multiuser RA problem in HetNets is formulated under the consideration of bounded channel gain uncertainties where both the cross-tier channel and intra-tier channel are simultaneously considered. The non-linear optimisation problem is converted into a geometric programming problem that is solved by using Lagrange dual methods in a distributed way. Simulation results show that the proposed algorithm can well restrain the effect of channel uncertainty and achieve a good robustness.

Nomenclature

U_k	user set served by microcell BS k
$p_{k,n}$	transmit power of microcell BS k over the nth subcarrier
$h_{k,n}^{m}$	interference channel gain from the nth subcarrier in microcell k to the mth subcarrier in macrocell
$g_{k,n}^{l}$	interference channel gain from microcell l to microcell k over subcarrier n
$g_{k,n}^{k}$	direct channel gain from microcell BS k to the user U_k over subcarrier n
$J_{k,n}^{m}$	interference power from macrocell user m to microcell k over subcarrier n
$r_{k,n}$	SINR from microcell BS k to the user over subcarrier n
$R_{k,n}$	data rate over subcarrier n in microcell k
$\sigma_{k,n}$	background noise over subcarrier n in microcell k
I_{th}	tolerable interference threshold by macrocell BS
$p_k^{\max}$	maximum transmit power at the kth microcell BS

1 Introduction

With the application of 4G mobile communication technology and the exponential growth of intelligent terminals, communication technologies move towards the direction of large-scale networks and multiple wireless access technologies (radio access technologies). Currently, the proposed microcell network is considered as a new communication technology with the features of low power consumption and high efficient data transmission. Additionally, this technology can eliminate the spread blind spots of traditional macrocell networks and improve the coverage of signal by intensively deploying multiple microcell networks [1–3].

Usually, microcell networks, distributed in the hotspot area of macrocell networks, have the radius range from 30 to 300 m, so that cellular networks with different kinds of cell sizes are overlapped with each other [4]. Base stations (BSs) with different transmission power in cellular networks are adjacent to each other, so that the whole communication network presents a multi-layer and heterogeneous network (HetNet) structure. Since microcell users often work in the unlicensed frequency bands, how to reduce interference power (e.g. inter-cell interference) among different cellular networks and energy consumption becomes a key technique in the research of HetNet [5].

Dynamic resource allocation (RA) is a main technology to realise spectrum sharing and network transmission in communication system, so it has been widely concerned by scholars. In [6], based on dual decomposition theory, a dynamic spectrum allocation strategy with utility maximisation was presented to solve resource sharing and spectrum access problem for a cognitive heterogeneous wireless network (HWN) with one macrocell network and single femtocell network. However, single network scenario is too ideal so that it cannot be used in practical communication system directly. To reduce the effect of cross-tier interference and improve spectrum efficiency (SE), Yang *et al* . [7] proposed an F-ALOHA-based cognitive spectrum access algorithm. The method tried to find a new degree of freedom from cross-layer spectrum access to achieve interference management and SE maximisation. In [8], to solve resource optimisation problem in cognitive HetNets, a dynamic hierarchical resource management scheme with the help of wavelet neural network, wiener process prediction and enhanced learning techniques was proposed to make the network business of communication dynamically distributing into the best access network. In [9], the authors studied the interference and energy efficiency maximisation based RA problem in cognitive HetNets with multiple cognitive small cell networks. The objective of maximising long-term discount was achieved by using the Restless Bandits model in stochastic dynamic optimisation theory. Although the above literatures have made many contributions to the RA problems of HetNets, user fairness is ignored. To reduce the influence of user fairness and spectrum sharing errors, Zhang *et al.* [10] studied the network utility maximisation based RA problem with the case of interference limitation in cognitive femtocell networks. The algorithm can ensure the quality of service (QoS) of femtocell users and restrain the cross-tier interference to the macrocell receivers. In [11], for uplink cognitive femtocell networks, the authors investigated the optimal power allocation problem with adaptive modulation to achieve throughput maximisation of system. Different from the centralised RA schemes, to reduce the overhead of mutual information exchange, the authors in [12, 13] studied the distributed power allocation and

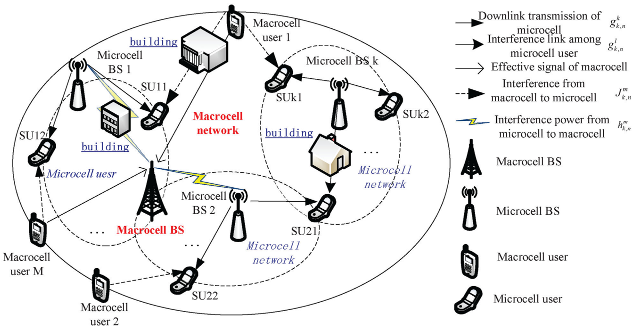

Fig. 1 *System model*

channel allocation algorithms for two-tier HWNs, where the sum rate maximisation problem was converted into the data rate of each cellular network. The optimal power allocation solution was solved by using the convex optimisation theory.

Based on state-of-the-art research, most of the above literatures only consider perfect channel state information (CSI) and ignore the impact of stochastic perturbation, channel estimation/ quantisation error and channel delays may cause certain uncertainties to system parameters. As a result, RA with consideration of partial CSI or imperfect CSI in HetNets has attracted attention gradually. By introducing game theory, Zhu *et al.* [14] studied the downlink RA problem under bounded channel uncertainties for two-tier cellular orthogonal frequency-division multiple access (OFDMA) networks. However, they considered one user in each cellular network, and ignored the cross-tier interference uncertainty from macrocell network to small cell users. To reduce energy consumption and improve operation lifetime of network, Liu *et al.* [15, 16] discussed the robust power control problem for uplink femtocell HWNs where channel uncertainty was formulated as outage probability constraint of user. However, the assumed statistical distribution models of uncertain parameters may mismatch with their actual values. Additionally, most of references lack of the analysis of parameter uncertainty in utility function, so that it results in bad performance of robustness and practicability.

In this paper, we consider the perfect CSI in both interference links and utility function are not obtained and channel uncertainties are reformulated as bounded ellipsoid uncertainty sets. To improve system performance and satisfy practical communication requirements, a robust RA problem is given to maximise total data rate of microcell network subject to maximum transmit power constraint of BS and cross-tier interference constraint of macrocell network. The original robust resource optimisation problem with infinite dimension and multiple variables is converted into a geometric programming (GP) problem which is solved by using Lagrange dual method. Finally, simulation results can demonstrate the effectiveness of robustness and protect the QoS of macrocell users.

The rest of this paper is organised as follows. Section 2 gives the system model and problem formulation. Section 3 presents the robust optimisation problem and the equivalent transformation process. Section 4 shows the simulation results. In Section 5, the conclusion is given.

2 System model

2.1 Traditional optimisation model

Consider an orthogonal frequency division downlink cognitive HetNet consisting of one macrocell network and multiple microcell networks, as shown in Fig. 1. The K microcell networks considered as cognitive users can detect the spectrum state of macrocell network by spectrum sensing technology in real time (i.e. multiple macrocell perform spectrum management and decision by detecting spectrum holes). Assuming that the total number subcarriers used to transmit data for macrocell users are N and each bandwidth is Δf Hz, the occupied frequency bands used by macrocell users can be defined as $\{B_1, \ldots, B_m, \ldots, B_M\}$. We define the set of microcell users as $\forall k \in \mathcal{K} = \{1, 2, \ldots, K\}$, and the set of subcarriers occupied by microcell users is defined as $\forall n \in \mathcal{N} = \{1, 2, \ldots, N\}$, and the set of subcarriers owned by macrocell is $\forall m \in \mathcal{M} = \{1, 2, \ldots, M\}$. To describe the problem conveniently, according to the above description and system model in Fig. 1, the symbol used in this paper is listed in the Nomenclature section.

In order to fully use the idle spectrum resource and improve spectrum utilisation, assuming that channel gain can be obtained through channel estimation or collaboration, ignoring the effect of bandwidth of each subcarrier (because it is a constant), the total rate maximisation-based RA problem for microcell users can be described as

$$
\max_{p_{k,n}} \sum_{k=1}^{K} \sum_{n=1}^{N} \log_2(1 + r_{k,n})
$$

$$
\text{s.t.} \begin{cases} C_1\colon \displaystyle\sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n} h_{k,n}^{m} \leqslant I_{\text{th}} \\ C_2\colon \displaystyle\sum_{n=1}^{N} p_{k,n} \leqslant p_k^{\max}. \end{cases} \tag{1}
$$

where $r_{k,n}$ denotes the signal-to-interference-plus-noise ratio (SINR) of each microcell user, and define

$$
r_{k,n} = \frac{p_{k,n} g_{k,n}^{k}}{\sum_{l=1, l \neq k}^{K} p_{l,n} g_{k,n}^{l} + \sum_{m=1}^{M} J_{k,n}^{m} + \sigma_{k,n}},
$$

and C_1 denotes the interference constraint which is used to protect the performance of macrocell users. C_2 is the transmit power constraint at each micro BS.

It is obvious that problem (1) is a non-convex optimisation, since the objective function is not a convex function with variable $p_{k,n}$. As a result, the global optimal solution is difficult to obtain. Therefore, the corresponding approximate optimal solution to the problem is obtained by the iterative algorithm [17, 18].

However, channel estimation error and random disturbance will bring different kinds of uncertainties which lead to certain deviation of obtained system parameter, i.e. $h_{k,n}^{m} = \bar{h}_{k,n}^{m} + \Delta h_{k,n}^{m}$. Where $\Delta h_{k,n}^{m}$ represents the deviation or residual error of channel gain which is usually decided by radio channel environment and accuracy of the used channel estimation algorithm. When estimation error $\Delta h_{k,n}^{m}$ is small, it means that the estimated channel gain $\bar{h}_{k,n}^{m}$ (known value, can be directly applied to the design of RA algorithm) is very close to the actual physical channel gain $h_{k,n}^{m}$ (unknown value, uncertain parameter). Otherwise, the estimation value of channel gain deviates from the actual value, so that the original RA algorithms via optimisation model (1) become invalid. Therefore, it is necessary to consider the robustness of RA algorithm ahead of time.

2.2 Robust optimisation model

In order to overcome the effect of uncertainty on the system performance of HetNets, this section will take channel estimation errors into consideration. The content of the section includes two parts: (i) uncertainty modelling and (ii) robust problem formulation and its transformation.

Considering channel uncertainties in optimisation problem (1), the following robust RA model is established as

$$\max_{p_{k,n}} \sum_{k=1}^{K} \sum_{n=1}^{N} \log_2\left(1 + \frac{p_{k,n} g_{k,n}^k}{\sum_{l \neq k}^{K} p_{l,n} g_{k,n}^l + \sum_{m=1}^{M} J_{k,n}^m + \sigma_{k,n}}\right)$$
$$\text{s.t.} \begin{cases} C_1:\ I_{\text{th}} - \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n} h_{k,n}^m \geqslant 0 \\ C_2:\ p_k^{\max} - \sum_{n=1}^{N} p_{k,n} \geqslant 0, \\ C_3:\ g_{k,n}^l \in R_g,\ h_{k,n}^m \in R_h \end{cases} \tag{2}$$

where C_3 denotes the sets of parameter uncertainties. R_g and R_h are the uncertainty sets of intra-tier interference links (i.e. among multiple microcells) and cross-tier interference links (i.e. between macrocell and microcell), respectively. To obtain the analytical solution of robust RA problem (2), it is necessary to build perturbation sets of uncertain parameters and then the transformation of equivalent problem.

2.2.1 Interference channel uncertainty model.: In practical system, the estimation error or the quantisation error cannot be infinite. Therefore, based on robust optimisation theory [19], define the interference channel uncertainty from microcell to macrocell as the following bounded ellipsoidal uncertainty sets.

$$R_h = \left\{ \Delta h_{k,n}^m \middle| h_{k,n}^m = \bar{h}_{k,n}^m + \Delta h_{k,n}^m : \sum_{n=1}^{N} \left(\Delta h_{k,n}^m\right)^2 \leq \left(\varepsilon_k^m\right)^2 \right\} \tag{3}$$

where the channel estimation error $\Delta h_{k,n}^m$ can be explained that the overall channel uncertainty from all users in the kth microcell network to the mth subcarrier occupied by macrocell user cannot exceed the upper boundary of uncertainty $\varepsilon_k^m \geq 0$.

Define the following interference power:

$$\begin{aligned} I^{SP} &= \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} h_{k,n}^m \\ &= \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n}\left(\bar{h}_{k,n}^m + \Delta h_{k,n}^m\right) \\ &= \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} \bar{h}_{k,n}^m + \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} \Delta h_{k,n}^m \\ &= \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} \bar{h}_{k,n}^m + \sum_{m=1}^{M} \boldsymbol{p} \Delta \boldsymbol{h}^m \end{aligned} \tag{4}$$

where transmit power vector is $\boldsymbol{p} = [\boldsymbol{p}_1, \boldsymbol{p}_2, \ldots, \boldsymbol{p}_K]$ and $\boldsymbol{p}_k = [p_{k1}, p_{k2}, \ldots, p_{kN}]$; channel uncertainty vector is $\Delta \boldsymbol{h}^m = \left[\left(\Delta \boldsymbol{h}_1^m\right)^{\mathrm{T}}, \left(\Delta \boldsymbol{h}_2^m\right)^{\mathrm{T}}, \ldots, \left(\Delta \boldsymbol{h}_K^m\right)^{\mathrm{T}}\right]^{\mathrm{T}}$ and $\Delta \boldsymbol{h}_k^m = [h_{k1}^m, h_{k2}^m, \ldots, h_{kN}^m]$. The first term of (4) is the determinate part, and the second part is the overall interference uncertainty from microcell users to macrocell users. In any case, in order to ensure the normal communication of macrocell users, the robust RA problem under the worst estimation error can be considered. Based on worst-case approach and Cauchy–Schwarz inequality [20], we have (see (5)) .

In addition, according to inequality principle, we have the following relationship:

$$\sqrt{\sum_{n=1}^{N} \left(p_{k,n}\right)^2} \leq \sum_{n=1}^{N} \sqrt{\left(p_{k,n}\right)^2} = \sum_{n=1}^{N} p_{k,n} \tag{6}$$

Based on (5) and (6), we have

$$I^{SP} \leq \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n}\left(\bar{h}_{k,n}^m + \varepsilon_k^m\right) \leq I_{\text{th}} \tag{7}$$

Thus, the robust interference constraint of C_1 is converted into a deterministic one (7). If the factor ε_k^m is big, indicating that the channel environment of kth microcell user is bad, and the different kinds of random interference and shielding effectiveness can influence the estimation accuracy of channel estimation heavily.

2.2.2 Channel uncertainty in objective function.: From the optimisation model (2), channel uncertainties not only lie in C_1 but also come into the objective function (e.g. channel gain $g_{k,n}^l$ and interference $J_{k,n}^m$). To keep the system performance under worst estimation errors, based on worst-case principle, based on constraint (7), we have

$$\max_{p_{k,n}} \min_{g_{k,n}^l} \left(\sum_{k=1}^{K} \sum_{n=1}^{N} \log_2\left(1 + \frac{p_{k,n} g_{k,n}^k}{\sum_{l=1, l \neq k}^{K} p_{l,n} g_{k,n}^l + \sum_{m=1}^{M} J_{k,n}^m + \sigma_{k,n}}\right)\right)$$
$$\text{s.t.} \begin{cases} C_2, C_3 \\ C_4: \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n}\left(\bar{h}_{k,n}^m + \varepsilon_k^m\right) \leq I_{\text{th}} \end{cases} \tag{8}$$

In order to facilitate the solution of the above non-convex optimisation problem, through introducing an auxiliary variable $q_{k,n}$ [21], the optimisation problem (8) is transformed into the following form:

$$\max_{p_{k,n}} \sum_{k=1}^{K} \sum_{n=1}^{N} \log_2\left(p_{k,n} q_{k,n}^{-1}\right)$$
$$\text{s.t.} \begin{cases} C_2,\ C_3,\ C_4 \\ C_5:\ \max\left(\dfrac{\sum_{l=1, l \neq k}^{K} p_{l,n} g_{k,n}^l + \sum_{m=1}^{M} J_{k,n}^m + \sigma_{k,n}}{g_{k,n}^k}\right) \leq q_{k,n}. \end{cases} \tag{9}$$

According to the form of optimisation problem (9), we need to design the suitable uncertainty sets so that the robust constraint C_1 can be converted into the deterministic and convex one. Similarly, define the following ellipsoidal uncertainty sets, i.e.

$$\begin{aligned} I^{SP} &= \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} \bar{h}_{k,n}^m + \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} \Delta h_{k,n}^m \\ &\leq \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} \bar{h}_{k,n}^m + \max_{\forall \Delta h_{k,n}^m \in R_h} \left(\sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} \Delta h_{k,n}^m \right) \\ &\leq \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} \bar{h}_{k,n}^m + \sum_{m=1}^{M} \sum_{k=1}^{K} \left(\sqrt{\sum_{n=1}^{N} \left(p_{k,n}\right)^2} \cdot \sqrt{\sum_{n=1}^{N} \left(\Delta h_{k,n}^m\right)^2} \right) \\ &\leq \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} \bar{h}_{k,n}^m + \sum_{m=1}^{M} \sum_{k=1}^{K} \sqrt{\left(\varepsilon_k^m\right)^2} \left(\sqrt{\sum_{n=1}^{N} \left(p_{k,n}\right)^2} \right) \\ &= \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} \bar{h}_{k,n}^m + \sum_{m=1}^{M} \sum_{k=1}^{K} \varepsilon_k^m \left(\sqrt{\sum_{n=1}^{N} \left(p_{k,n}\right)^2} \right) \end{aligned} \tag{5}$$

$$R_g = \left\{ \Delta G_{k,n}^l \middle| G_{k,n}^l = \bar{G}_{k,n}^l + \Delta G_{k,n}^l : \sum_{l \ne k}^{K} \left(\Delta G_{k,n}^l\right)^2 \le (\mu_{k,n})^2 \right\} \tag{10}$$

where $G_{k,n}^l = g_{k,n}^l / g_{k,n}^k$ denotes the normalised channel gain, $\bar{G}_{k,n}^l$ and $\Delta G_{k,n}^l$ are the corresponding channel estimation value and estimation error, respectively. The uncertainty $\Delta G_{k,n}^l$ is from the channel gain $\Delta g_{k,n}^l, \forall l$ of microcell networks.

Since the SINR of microcell users depends not only on the quality of channel environment, but also effects by the interference power of macrocell users. Considering the effect of interference power uncertainty from macrocell networks to microcell networks, we define the following uncertainty sets:

$$R_J = \left\{ \Delta D_{k,n}^m \middle| D_{k,n}^m = \bar{D}_{k,n}^m + \Delta D_{k,n}^m : \left|\Delta D_{k,n}^m\right| \le \lambda_{k,n}^m, \right\} \tag{11}$$

where $D_{k,n}^m = J_{k,n}^m / g_{k,n}^k$ denotes the normalised interference power from macrocell users, $\bar{D}_{k,n}^m$ and $\Delta D_{k,n}^m$ are the corresponding estimation value and estimation error (i.e. the magnitude of the error is determined by $\Delta J_{k,n}^m$ and $\Delta g_{k,n}^k$).

Since the background noise is small relative to the information coefficient, the influence of channel uncertainty $\Delta g_{k,n}^k$ on $\sigma_{k,n}$ can be ignored, i.e.

$$\frac{\sigma_{k,n}}{g_{k,n}^k} = \frac{\sigma_{k,n}}{\bar{g}_{k,n}^k + \Delta g_{k,n}^k} \simeq \frac{\sigma_{k,n}}{\bar{g}_{k,n}^k}$$

is established. According to the uncertainty sets (10) and (11), the constraint C_5 becomes

$$z_{k,n} \le q_{k,n}, \tag{12}$$

where $z_{k,n} = \bar{z}_{k,n} + \Delta z_{k,n}$ is the interference power at the receiver of each microcell user, $\bar{z}_{k,n} = \sum_{l=1, l \ne k}^{K} p_{l,n} \bar{G}_{k,n}^l + \sum_{m=1}^{M} \bar{D}_{k,n}^m + \sigma_{k,n} / \bar{g}_{k,n}^k$ is the corresponding estimation value, and the perturbation term is $\Delta z_{k,n} = \max\left(\sum_{l \ne k}^{K} p_{l,n} \Delta G_{k,n}^l + \sum_{m=1}^{M} \Delta D_{k,n}^m\right)$. Then, we have

$$\begin{aligned} z_{k,n} &\le \bar{z}_{k,n} + \sqrt{\sum_{l \ne k}^{K} (p_{l,n})^2} \sqrt{\sum_{l \ne k}^{K} \left(\Delta G_{k,n}^l\right)^2} + \sum_{m=1}^{M} \lambda_{k,n}^m \\ &\le \bar{z}_{k,n} + \mu_{k,n} \sqrt{\sum_{l \ne k}^{K} (p_{l,n})^2} + \sum_{m=1}^{M} \lambda_{k,n}^m \\ &\le \bar{z}_{k,n} + \sum_{m=1}^{M} \lambda_{k,n}^m + \mu_{k,n} \sum_{l \ne k}^{K} p_{l,n} \end{aligned} \tag{13}$$

Thus, constraint (12) can be formulated as

$$\bar{z}_{k,n} + \tilde{z}_{k,n} \le q_{k,n} \tag{14}$$

where $\tilde{z}_{k,n} = \sum_{m=1}^{M} \lambda_{k,n}^m + \mu_{k,n} \sum_{l \ne k}^{K} p_{l,n}$ denotes the effect of the uncertainty term on the original interference power. As a result, the constraint C_5 is transformed into a deterministic form.

2.2.3 Equivalent transformation model.: According to the transformation results of (2.2.1) and (2.2.2) and the model (9), we have

$$\begin{aligned} &\max_{p_{k,n}} \sum_{k=1}^{K} \sum_{n=1}^{N} \log_2\left(p_{k,n} q_{k,n}^{-1}\right) \\ &\text{s.t} \begin{cases} C_2: \sum_{n=1}^{N} p_{k,n} \le p_k^{\max}, \ \forall k \\ C_4: \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n}\left(\bar{h}_{k,n}^m + \varepsilon_k^m\right) \le I_{\text{th}} \\ C_6: \sum_{l \ne k}^{K} e^{x_{l,n}}\left(\bar{G}_{k,n}^l + \mu_{k,n}\right) + Y_{k,n} \le q_{k,n}, \end{cases} \end{aligned} \tag{15}$$

where $Y_{k,n} = \sum_{m=1}^{M}\left(\bar{D}_{k,n}^m + \lambda_{k,n}^m\right) + \sigma_{k,n} / \bar{g}_{k,n}^k$. As a result, we convert the robust RA problem (2) into the deterministic one (15).

3 Robust RA algorithm

Due to the constraint C_6, the problem (15) is not convex, the optimal solution is not easily achieved. Using the logarithmic transformation approach [22] to get the available form, i.e. $p_{k,n} = e^{x_{k,n}}$ and $q_{k,n} = e^{y_{k,n}}$, and the problem (15) can be converted into a convex optimisation problem. i.e.

$$\begin{aligned} &\min_{x_{k,n}, y_{k,n}} -\frac{1}{\ln 2} \sum_{k=1}^{K} \sum_{n=1}^{N} (x_{k,n} - y_{k,n}) \\ &\text{s.t.} \begin{cases} C_7: \left(p_k^{\max}\right)^{-1} \sum_{n=1}^{N} e^{x_{k,n}} - 1 \le 0, \ \forall k \\ C_8: I_{\text{th}}^{-1} \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} e^{x_{k,n}}\left(\bar{h}_{k,n}^m + \varepsilon_k^m\right) - 1 \le 0 \\ C_9: e^{-y_{k,n}}\left(\sum_{l \ne k}^{K} e^{x_{l,n}}\left(\bar{G}_{k,n}^l + \mu_{k,n}\right) + Y_{k,n}\right) - 1 \le 0. \end{cases} \end{aligned} \tag{16}$$

For the problem (15), construct the following Lagrangian function: (see (17)) where α_k, β and $\chi_{k,n}$ are the Lagrange multipliers (i.e. dual variables) for the constraints C_7, C_8 and C_9, respectively. The Lagrange function (16) has three characteristics: (i) the utility function of any microcellular user depends only on the main variables $x_{k,n}$ and $y_{k,n}$; (ii) the Lagrange function (17) can be decomposed into two sub-optimisation problems, i.e. the minimisation optimisation problem with principal variables and the maximised dual optimisation problem with Lagrangian multipliers; (iii) the Lagrange multipliers can be divided into local variables $(\alpha_k, \chi_{k,n})$ for each user and global variables (β) for the entire network.

$$\begin{aligned} L(x_{k,n}, y_{k,n}, \alpha_k, \beta, \chi_{k,n}) = &-\frac{1}{\ln 2} \sum_{k=1}^{K} \sum_{n=1}^{N} (x_{k,n} - y_{k,n}) \\ &+ \sum_{k=1}^{K} \sum_{n=1}^{N} \chi_{k,n}\left\{e^{-y_{k,n}}\left(\sum_{l \ne k}^{K} e^{x_{l,n}}(\bar{G}_{k,n}^l + \mu_{k,n}) + Y_{k,n}\right) - 1\right\} \\ &+ \beta\left\{I_{\text{th}}^{-1} \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} e^{x_{k,n}}(\bar{h}_{k,n}^m + \varepsilon_k^m) - 1\right\} \\ &+ \sum_{k=1}^{K} \alpha_k\{(p_k^{\max})^{-1} \sum_{n=1}^{N} e^{x_{k,n}} - 1\} \end{aligned} \tag{17}$$

Based on the above analysis, we use the sub-gradient update method to obtain the main variable and Lagrangian multiplier, i.e.

$$x_{k,n}(t+1) = \left[x_{k,n}(t) - \theta_{x_{k,n}} \times \frac{\partial L}{\partial x_{k,n}}\right]^+ \tag{18}$$

$$y_{k,n}(t+1) = \left[y_{k,n}(t) - \theta_{y_{k,n}} \times \frac{\partial L}{\partial y_{k,n}}\right]^+ \tag{19}$$

$$\alpha_k(t+1) = \left[\alpha_k(t) + \theta_{\alpha_k} \times \left((p_k^{\max})^{-1} \sum_{n=1}^{N} e^{x_{k,n}} - 1\right)\right]^+ \tag{20}$$

$$\beta(t+1) = \left[\beta(t) + \theta_\beta \cdot (I_{\text{th}}^{-1} \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} e^{x_{k,n}}\left(\bar{h}_{k,n}^m + \varepsilon_k^m\right) - 1)\right]^+ \tag{21}$$

$$\begin{aligned}\chi_{k,n}(t+1) = \Bigg[\chi_{k,n}(t) + \theta_{\chi_{k,n}} \cdot \Bigg\{ &\sum_{l \neq k}^{K} e^{x_{l,n} - y_{k,n}}(\bar{G}_{k,n}^l + \mu_{k,n}) \\ &+ e^{-y_{k,n}} Y_{k,n} - 1 \Bigg\}\Bigg]^+\end{aligned} \tag{22}$$

where $X^+ = \max\{0, X\}$. $\theta_{x_{k,n}}$, $\theta_{y_{k,n}}$, θ_{α_k} and $\theta_{\chi_{k,n}}$ are the corresponding update step sizes, and t represents the number of iterations. When step sizes satisfy the following relationship, the algorithm can fastly converge to the stable points [23, 24], i.e.

$$\sum_{t=0}^{\infty} \theta_{(v)}^t = \infty, \quad \lim_{t \to \infty} \theta_{(v)}^t = 0, \quad \forall v \in \{x_{k,n}, y_{k,n}, \alpha_k, \beta, \chi_{k,n}\}. \tag{23}$$

Define $\partial L / \partial x_{k,n}$ and $\partial L / \partial y_{k,n}$ as the partial derivatives of the variables $x_{k,n}$ and $y_{k,n}$, respectively, i.e.

$$\begin{aligned}\frac{\partial L}{\partial x_{k,n}} = &-\frac{1}{\ln 2} + \alpha_k (p_k^{\max})^{-1} e^{x_{k,n}} \\ &+ \beta I_{\text{th}}^{-1} e^{x_{k,n}} \Big(\sum_{m=1}^{M} (\bar{h}_{k,n}^m + \varepsilon_k^m)\Big) \\ &+ e^{x_{k,n}} \sum_{l \neq k}^{K} \chi_{k,n} e^{-y_{k,n}} \Big(\bar{G}_{k,n}^l + \mu_{k,n}\Big)\end{aligned} \tag{24}$$

$$\frac{\partial L}{\partial y_{k,n}} = \frac{1}{\ln 2} - \chi_{k,n} e^{-y_{k,n}} \Big(\sum_{l \neq k}^{K} e^{x_{l,n}} (\bar{G}_{k,n}^l + \mu_{k,n}) + Y_{k,n}\Big) \tag{25}$$

Therefore, the robust RA algorithm proposed in this paper is as follows:

1. **Initialisation**: $t = 0$, set all the main variables and the Lagrangian multiplier initial value within the feasible domain, and set the algorithm convergence to the precision threshold. Set the update step size $\theta_{x_{k,n}}$ and $\theta_{y_{k,n}}$. Set the maximum transmit power threshold $p_k^{\max}$ of the microcell BS and the maximum interference power threshold I_{th} allowed by the macrocell user. Set the upper bound of the uncertainty sets ε_k^m and $\mu_{k,n}$.
2. **The update algorithm at the *k*th microcell BS**: In each iteration $t = 1, 2, \ldots$, receive the information and parameters from the receivers, and update Lagrange multipliers of (19) and (20). Update main variables and auxiliary variables by formula (18) and (19).
3. **The update algorithm at the *k*th microcell receivers**: In each iteration $t = 1, 2, \ldots$, based on the estimated channel gains and interference power, update the interference $J_{k,n}^m(t)$ from macrocell users to microcell users and the interference $\sum_{l \neq k}^{K} p_{l,n}(t) \bar{G}_{k,n}^l(t)$ among different microcell users; update the parameter $\chi_{k,n}(t+1)$ based on formula (22), and transmit such information to microcell BS through feedback channels.
4. **End**: Repeat 2 and 3 until transmission power and Lagrange multipliers converge. Transmit out the final transmit power $p_{k,n}(t) = e^{x_{k,n}}$.

4 Simulation results and discussion

In this section, the simulation results and performance analysis of the proposed algorithm will be given. Assuming that microcell networks are randomly distributed in a macrocell network, the cell radii of microcell and macrocell network are 30 and 500 m, respectively. The total bandwidth is $B = 10$ MHz, and the bandwidth of each subcarrier is $\Delta f = 10$ kHz. The path loss between macrocell BS and its served users is $128.1 + 37.6\log(d)$ (in dB, d : km). The path loss between macrocell networks and microcell networks is $140.7 + 36.7\log(d)$ (in dB). The path loss between microcell BS and its users is $122 + 38\log(d)$ (in dB) [25]. The background noise is -110 dBm, the maximum transmit power of each macrocell user is assumed to be 23 dBm, and the maximum transmit power of each microcell BS is assumed to be 20 dBm.

In order to discuss the convenience of problem, it is assumed that all links have the same uncertainty, i.e. $\mu = \mu_{k,n}, \forall k, n$, $\lambda = \lambda_{k,n}^m, \forall k, n, m$ and $\varepsilon = \varepsilon_k^m, \forall k, m$. Next, we will consider the simulation results of different users, different degree of uncertainty and so on.

In order to evaluate the proposed algorithm, the following algorithm is introduced.

- **Without channel uncertainty consideration algorithm (WCUCA)**: the algorithm ignores the influence of channel uncertainty, only considers perfect CSI, i.e. $\Delta g_{k,n}^l = 0$, $\Delta h_{k,n}^m = 0$, the implementation of the algorithm refers to the algorithm proposed in this paper.
- **Proposed robust power control algorithm (PRPCA)**: consider the uncertainty in the interference constraint and the influence of channel uncertainties of microcell links, i.e. $\Delta g_{k,n}^l \neq 0$, $\Delta h_{k,n}^m \neq 0$.

Assuming that there is one macrocell network with four users (i.e. $M = 4$), and there is two users in each microcell network (i.e. $Uk = 2$). Only the channel gain of link 1 is uncertain. Case 1: there are one macrocell with four users and one microcell with two users (e.g. SU11, SU12). Case 2: due to the time-varying environment, another nearby microcell with two users is active (e.g. SU21, SU22). Channel uncertainties $\Delta g_{k,n}^l$, $\Delta h_{k,n}^m$ satisfy uniform distribution with the upper bound $\mu = 0.01, \varepsilon = 0.01$.

Fig. 2 shows the transmission power of microcell users under different channel uncertainties of microcell links. From Fig. 2, the transmission power of link 1 increases as the increasing uncertainty, and the transmission power of link 2 decreases as the increasing uncertainty due to the limitation of maximum transmission power of BS. Since the channel uncertainty of link 1 is considered here, the effective power allocated to the link is increased to suppress the influence of channel uncertainty. The transmit power of SU11 ($K = 2$) is greater than that of SU11 ($K = 1$). Under case 2, the increase in transmission power not only needs to overcome the impact of channel uncertainty, but also needs to make up the effect of interference power (e.g. original network users) that is caused by near new network users. In any case (e.g. $K = 1$ or $K = 2$), the transmit power of SU12 gradually reduces, the reason is that the user of SU11 is allocated more transmission power to obtain robustness while the maximum transmit power ($p_k^{\max}, \forall k$) is unchanged.

Fig. 3 shows the sum rate of microcell versus with the increasing uncertainty $\mu_{k,1}$. From Fig. 3, with the increase of upper bound of uncertain parameter, the rate of the proposed algorithm decreases, while the total rate of microcell users under traditional RA algorithm (i.e. WCUCA) without consideration of imperfect CSI remains a constant. The reason is that the traditional WCUCA does not take into account the impact of parameter uncertainty. The

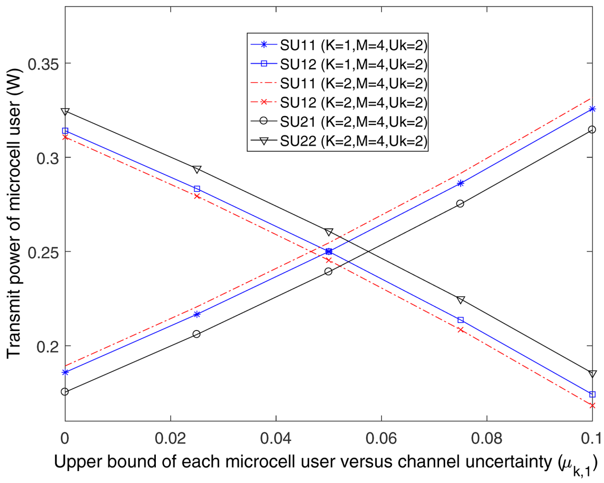

Fig. 2 *Transmit power of each microcell user versus channel uncertainty ($I_{th} = 0.006\,W$, $p_k^{max} = 0.5\,W$)*

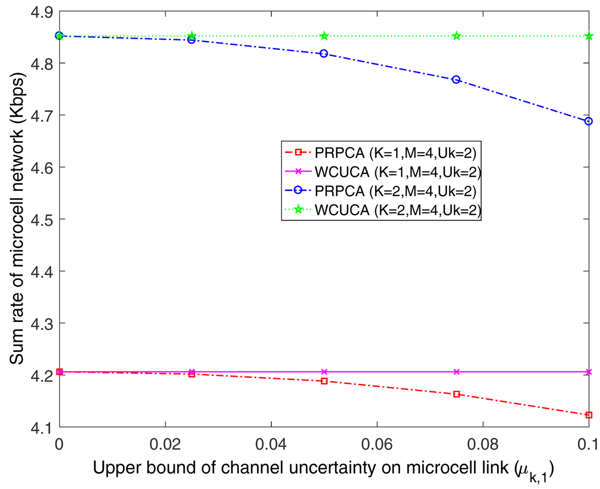

Fig. 3 *Sum rate of microcell network versus upper bound of channel uncertainty ($I_{th} = 0.006\,W$, $p_k^{max} = 0.5\,W$)*

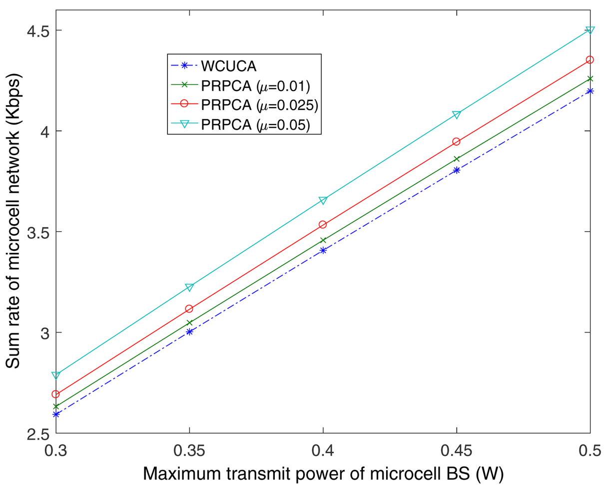

Fig. 4 *Sum rate versus maximum transmit power of microcell BS under different channel uncertainties ($I_{th} = 0.006\,W$, $K = 1, M = 2, Uk = 2$)*

objective function has nothing to do with the perturbation term, so its rate remains constant. But the objective function of the robust algorithm proposed in this paper (e.g. PRPCA) is a monotonically decreasing function of the independent variables. Therefore, under the maximum transmission power constraint, the transmission power of BS can not endlessly increase to ensure the higher data rate. So the overall data rate decreases with the increasing uncertainty, system performance has decreased. In addition, it is obvious that the total rate under the case of two microcells is significantly greater than single microcell scenario. Due to the presence of neighbouring cells, the total data rate of microcell system is improved effectively, but there is no doubling of the increase in the reason that there will be intercell interference between adjacent cells in microcell networks.

Fig. 4 presents the performance of sum data rate of the two algorithms under different maximum transmission power threshold. It is clear that the sum data rate of microcell networks increases with the maximum transmit power threshold (e.g. p_k^{max}). Under the certain interference power constraint, transmission power of each microcell user increases due to the extended range of feasible domain, so that the network resource can be used fully. Compared with the non-robust algorithm (i.e. WCUCA), the proposed algorithm (i.e. PRPCA) can obtain better performance of transmission rate under different degree of uncertainty. In addition, the data rate increases with the increase of the upper bound of uncertain parameter. The total rate difference under the high transmission power range (> 0.48) is greater than that of the low transmit power range (< 0.32), as the robust algorithm designed in this paper considers the influence of channel uncertainties in advance. The bigger upper bound of uncertain parameter means that the channel deviation ($\Delta g_{k,n}^l$) is large. In order to reduce the disruption of link, more transmission power on the link is to suppress the adverse effects of possible channel uncertainties.

Fig. 5 depicts the sum rate of microcell network under different interference power thresholds and interference uncertainties from macrocell users. It can be easily seen from the figure that the data rate of microcell network decreases as the decreasing interference power threshold (e.g. I_{th}). As the interference threshold means the protection state of macrocell user, the small interference threshold will limit the effective maximum transmission power of microcell BS. In addition, with the increasing interference uncertainty from macrocell users to microcell users, the overall transmission rate of microcell network decreases. Under the limitation of power domain, the utility function is a monotonic decreasing function with the upper bound of interference power uncertainty λ.

Fig. 6 shows the received interference power of macrocell user from microcell networks under different channel uncertainties (e.g. $\Delta h_{k,n}^m$). Assuming that the interference power threshold is $I_{th} = 0.004$ W. From Fig. 6, the PRPCA proposed in this paper can ensure that the interference power to macrocell user does not exceed the predetermined threshold under the influence of channel uncertainty with different degrees (i.e. different ε), while the traditional WCUCA exceeds the constraint of interference threshold. The greater the channel uncertainty is, the more harmful interference power to macro users is. In other words, the bad channel estimation error will increase the outage probability of users. Furthermore, since the proposed PRPCA algorithm takes into account the influence of parameter perturbation, the algorithm can effectively suppress the influence of uncertainty. On the other hand, the algorithm is effective even though random parameters are disturbed frequently (e.g. fading effect and shadowing effect are generally affected by the environment). Under the case of line of sight, anti-perturbation ability is not obvious (e.g. $\varepsilon = 0.01$). It means that the PRPCA algorithm is to sacrifice a certain rate to enhance the robustness and anti-jamming capability of the overall system.

Fig. 7 presents the service probability [26] of the macrocell user for different algorithms under parameter uncertainty. It defines the actual outage probability of macrocell user is $P_{out} = \max\,(0, (\sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n}^{*} h_{k,n}^{m} - I_{th})/I_{th})$ ($p_{k,n}^{*}$ denotes the optimal transmit power), thus satisfaction probability of the user can be expressed as $P_{sat} = 1 - P_{out}$. Define a robust power control algorithm based on outage probability constraints [15] is RPCSC (i.e. robust power control with stochastic constraint). In order to analyse the convenience of the problem, only the channel uncertainties between microcell users and macrocell users are considered. Additionally, the channel gain estimation error $\Delta h_{k,n}^m$ is assumed to a random variable (i.e. the estimation error varies

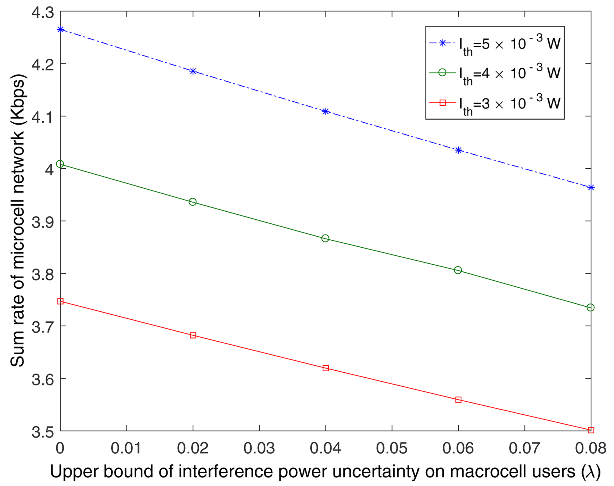

Fig. 5 *Sum rate of microcell network versus channel uncertainty under different interference power thresholds ($p_k^{\max} = 0.5\,W$, $\mu = 0.01$)*

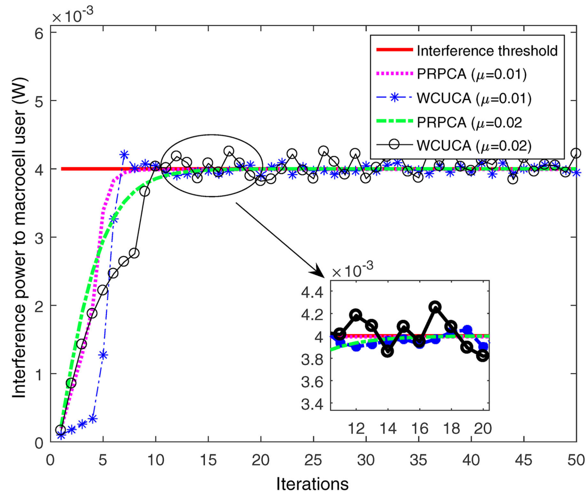

Fig. 6 *Interference power to macrocell user under different channel uncertainties*

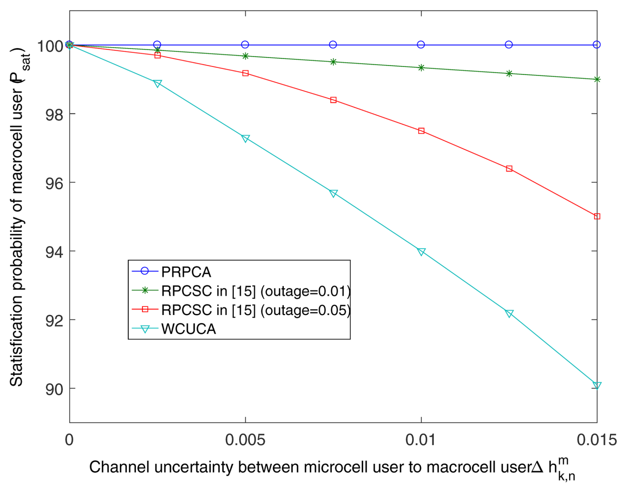

Fig. 7 *Satisfaction probability of macrocell user under different channel uncertainties of interference links*

within a certain range), and the upper bound of the total estimation error is defined as $\varepsilon = 0.01$.

From Fig. 7, we can know that the PRPCA proposed in this paper can well overcome the influence of channel gain uncertainty and will not cause the outage of user. The RPCSC algorithm based on outage probability constraint can maximum protect the QoS of user under the allowable outage threshold. The algorithm can obtain certain robustness at cost of some outage events. With the

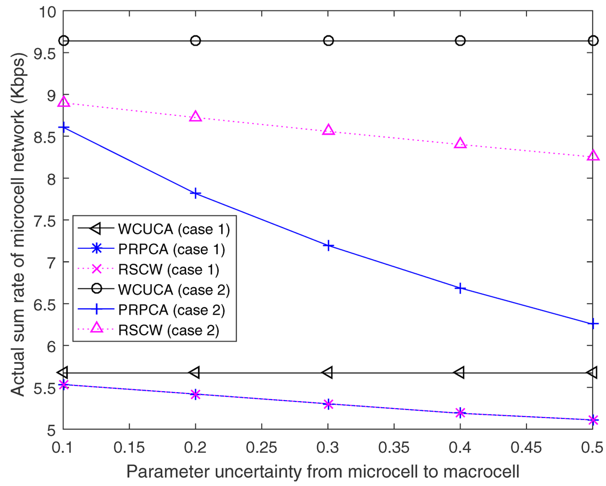

Fig. 8 *Comparison performance of total rate under the increasing uncertainty*

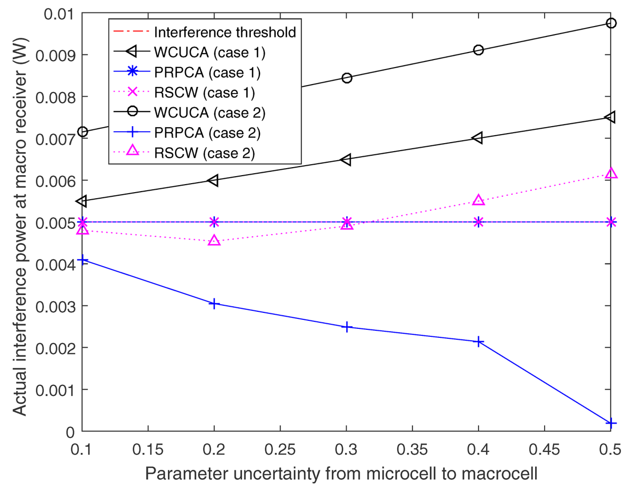

Fig. 9 *Actual received interference power at macro receiver*

interrupt threshold increases, the service probability of user is decreased gradually. In addition, the traditional WCUCA without considering the channel uncertainty $\Delta h_{k,n}^m$ has the worst performance of service probability. In addition, with the increase of uncertainty, the outage probability of user is increased.

In order to further demonstrate the performance of the PRPCA, we compare our proposed scheme with the robust power control algorithm via the column-wise uncertainty model (i.e. a bounded uniform distribution model, $|h_{k,n}^m| \le \varepsilon_{k,n}^m$) [14]. Define that the robust scheme with column-wise model is RSCW. In Figs. 8 and 9, we give the total rate of microcell users and the interference power received at the macrocell receiver, respectively.

Fig. 8 shows the comparison performance of transmission rate under two different schemes subject to bounded uncertainty models. In order to facilitate discussion, we consider the normalised channel gains, namely, the different channel gains are from the interval [0,1]. The interference threshold is $I^{\text{th}} = 5 \times 10^{-3}$ W, maximum transmit power of micro user is $p_k^{\max} = 0.5$ W. We consider the following cases, case 1: single user scenario ($K = 1$), case 2: two users scenario ($K = 2$), link 1 is assumed to be strong channel environment (e.g. small $\Delta h_{k,n}^m$), and link 2 is assumed to be week channel environment (e.g. big $\Delta h_{k,n}^m$). As shown in Fig. 8, the sum rate of the WCUCA under two cases keeps a constant without consideration of any $\Delta h_{k,n}^m$. The sum rate of two cases under the PRPCA and the RSCW decreases with the excessive parameter uncertainty, since bigger uncertainty reduces the power domain of micro user transmitter to protect the QoS of macrocell users. On the other hand, sum rate of the PRPCA and the RSCW under case 1 is same due to the same optimal power and the same uncertainty set. However, the sum rate of the RSCW under case 2 is bigger than that of the PRPCA, since the optimal power under this case is bigger than that of the PRPCA owing to $\boldsymbol{p}^*_{\text{ellipsoidal}} = I^{\text{th}}(\bar{\boldsymbol{h}} + \varepsilon)^{-1} \le \boldsymbol{p}^*_{\text{column}} = I^{\text{th}}(\bar{\boldsymbol{h}} + \varepsilon\bar{\boldsymbol{h}})^{-1}$ ($h^m_{k,n} \le 1,\ \forall k, n, m$). In other words, our PRPCA is more conservative to overcome parameter uncertainty.

Fig. 9 presents the interference power received at the macro user receiver under two schemes. From this figure, we can clearly find that the PRPCA has a good robustness to overcome the parameter uncertainty since the proposed algorithm considers overall uncertainties of links under worst-case scenario. However, when the actual uncertainties of links under case 2 mismatch the defined upper boundary in the RSCW, the total interference to the macrocell user exceeds the interference threshold under $\varepsilon > 0.3$. In addition, the interference power of the WCUCA always is bigger than the predefined threshold. Comparing Fig. 8 with Fig. 9, our PRPCA has a better robustness at cost of sum rate.

5 Conclusion

In order to improve the system capacity and stability of HetNets, this paper studies a robust RA algorithm based on orthogonal frequency division multiplexing, which is used to effectively control the interference to macrocell users. First, the RA optimisation probelm in a multiuser HetNet with on macrocell and multiple microcells is constructed. Second, the bounded channel uncertainties model of uncertain channel gains are established for formulating robust RA problem with an infinite number of constraints. Third, we utilise auxiliary variables to transform the original non-convex optimisation problem into a GP problem. A robust RA algorithm is designed via Lagrange dual theory and gradient updating method. Finally, simulation results show that the proposed algorithm can protect the performance of users under channel estimation errors, and enforce spectrum resource sharing, and provide the theoretical basis for practical application.

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