SZE, Numerical Linear Algebra, exam, 18.01.2023.

The solutions were evaluated as follows:

1 – 5. (description of algorithms)	3 points each
Problem 6 (*QR* decomposition)	5 points
Problem 7 (an application of Gershgorin's theorem)	4 points
Problem 8 (convergence of a Jacobi iteration)	6 points

The maximal number of points is 30. The connection between the number of points and the marks:

below 8 points:	fail (1)
8 - 13 points:	pass (2)
14 - 19 points:	satisfactory (3)
20 - 25 points:	good (4)
26 points and above:	excellent (5)

Some remarks:

- The algorithm descriptions should have been written in such a way that somebody – who knowns nothing about the algorithm – will be able to understand the method, moreover, will be able to reproduce it. In particular:
  - The power iteration approximates the **dominant eigenvalue** (i.e. the eigenvalue λ_N with the largest absolute value, provided that the absolute value of the next eigenvalue is strictly less than $|\lambda_N|$) of a **normal matrix** A (i.e. $A^*A = AA^*$) as a limit of **Rayleigh quotients**. To compute them, one should consider the vector sequence $x^{(n+1)} := Ax^{(n)}$; the starting vector $x^{(0)}$ has to satisfy a special non-orthogonality condition.
  - The Singular Value Decomposition ($A = USV^*$) algorithm starts with the calculation of the **eigensystem of the self-adjoint matrix** A^*A. The (positive) square roots of the eigenvalues are the singular values; the matrix V is constructed from the corresponding normed eigenvectors. Then the matrix U is constructed from the vectors $u_k = \frac{Av_k}{\sigma_k}$. Without outlining these items, nobody will be able to understand (and reproduce) the algorithm.
- In Problem 8, the essential steps are as follows:
  - to realize that the original matrix is **not** diagonally dominant, therefore one has to calculate of the transition matrix;
  - to realize that both the row and the column norm of the transition matrix are **not** less than 1, therefore one has to calculate the eigenvalues of the transition matrix;
  - the absolute values of the eigenvalues of the transition matrix are less than 1, therefore the **Jacobi iteration is convergent**.

Results:

Neptun	Points	Mark
BBLO30	16	3
N16IAF	28	5

20.01.2023.

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