On the Numerical Linear Algebra exams

Dear Students,

The exams will be written exams. Each exam will contain algorithm descriptions and some simple exercises which are to be calculated. They should contain short outlines of calculations: giving the final result only is not enough. The algorithm decriptions should contain the following pieces of information:

- the aim of the algorithm: what does it do exactly?
- the details of the algorithm: how does it work?
- under what assumptions does the algorithm work?
- the theorem(s) which the algorithm is based on (however, these theorems need not be proved);
- numerical features of the algorithm etc.

Imagine that you are writing the algorithm description not for me, but for a colleague of yours, who should understand the whole algorithm on the basis of your description only.

As an example, you can see here the written exam of the year 2020. In each practice, we outlined lots of calculations; the problems that have to be solved in the exam will be not more complicated than those. Here is a short list about the calculations that have been explained in the practices and can be expected in the exams:

- inverse of triangular matrices;
- calculation of eigenvalues based on the characteristic polynomial; calculation of eigenvectors;
- solutions of systems of linear equations by Gauss and Gauss-Jordan elimination;
- LU decomposition of matrices by elimination and also by matrix multiplication;
- LDL* decomposition, Cholesky decomposition (also by matrix multiplication);
- Gram-Schmidt ortogonalization;
- QR decomposition;
- Singular Value Decomposition;
- scalar product, vector norms, matrix norms (induced by vector norms);
- fixed point theorem, solutions of systems of linear equations by fixed point iteration;

- the estimation of the number of necessary iteration steps;
- Richardson's iteration and the optimal choice of its parameter;
- Jacobi and Seidel iteration;
- localization of eigenvalues by Gershgorins's circles;
- the power iteration to approximate the dominant eigenvalue;
- the (shifted) inverse iteration;
- solution of linear systems of equations in the sense of least squares;
- linear, quadratic regression;

11. December 2020. Numerical Linear Algebra, exam

Algorithms (nothing has to be proved):
1. The QR decomposition of regular matrices.
2. The fixed point iteration method for solving linear systems of equations.
3. The gradient method.
4. The power iteration for finding the dominant eigenvalue.
5. The Singular Value Decomposition of regular matrices.

Problems (to be calculated):

6. Determine the LU decomposition of the matrix: $A = \begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}$.

7. Solve the following system of equations in the sense of least squares.

$$x_2 = 1, \quad x_1 + 2x_2 = 1, \quad 2x_1 + 4x_2 = 1, \quad 4x_1 + 8x_2 = 1$$

8. Consider the system of equations $Ax = b$ with the matrix: $A = \begin{pmatrix} 2 & -1 & -0.5 \\ -1 & 2 & -1 \\ -0.5 & 0.5 & 2 \end{pmatrix}$.
   Examine the Jacobi iteration applied to this system. Is it convergent or not?