CHAPTER-4
DETERMINANTS

EXERCISE - 4.3

1. Find the area of the triangle with vertices at the point

   (i) (1,0), (6,0), (4,3)

   (ii) (2,7), (1,1), (10,8)

   (iii) (-2,-3), (3,2), (-1,-8)

2. Show that points

$$\mathbf{A}(a, b+c), \quad \mathbf{B}(b, c+a), \quad \mathbf{C}(c, a+b) are collinear$$

3. Find values of k if the area of the triangle is 4

   (i) $(k,0),(4,0),(0,2)$

   (ii) $(-2,0),(0,4),(0,k)$

4. (i) Find the equation of joining $(1,2) and (3,6)$ using determinants.

   (ii) Find the equation of the line joining $(3,1) and (9,3)$ using determinants.

5. If the area of the triangle is 35 sq. units with vertices $(2,-6),(5,4) and (k,4)$ then k is

   (A) 12 (B) -2 (C) -12, -2 (D) 12, -2

4.5 Minors and Cofactors

In this section, we will learn to write the expansion of a determinant in compact form using minors and cofactors.

Definition 1 Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its ith row and jth column in which element a_{ij} lies. The minor of an element a_{ij} is denoted by M_{ij}.

Remark: The minor of an element of a determinant of order $n(n \geq 2)$ is a determinant of order $n-1$.

Example 19 Finding the Minor of 6 in a Determinant $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

SolurionSince 6 lies in the second row and third column, its minor M_{23} is given by:

$$M_{23} = \begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = (1 \times 8) - (2 \times 7) = 8 - 14 = -6$$

Definition 2

Cofactor of an element a_{ij}, denoted by A_{ij}, is defined by:

$A_{ij} = (-1)^{i+j} M_{ij}$

where M_{ij} is the minor of a_{ij}.

Example 20

Find minors and cofactors of all the elements of the determinan $\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$

Solution: The minor of an element a_{ij} is denoted by M_{ij}.

Here, $a_{11} = 1$, so M_{11} (minor of $a_{11} = 3$)

$M.12 =$ minor of element $a_{12} = 4$

$M.21 =$ minor of element $a_{21} = -2$

Now, the cofactors$a.ij$ is A_{ij}

- $A_{11} = (-1)^{1+1} M_{11} = (1)(3) = 3$
- $A_{12} = (-1)^{1+2} M_{12} = (-1)(4) = -4$
- $A_{21} = (-1)^{2+1} M_{21} = (-1)(-2) = 2$
- $A_{22} = (-1)^{2+2} M_{22} = (1)(1) = 1$

Example 21

Find the minors and cofactors of the elements a_{11} and a_{21} in the determinant:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Solution: By the definition of minors and cofactors, we have:

Minor of a_{11}:

$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22}a_{33} - a_{23}a_{32}$

Cofactor of a_{11}:

$A_{11} = (-1)^{1+1}M_{11} = a_{22}a_{33} - a_{23}a_{32}$

Minor of a_{21}:

$M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} = a_{12}a_{33} - a_{13}a_{32}$

Cofactor of a_{21}:

$A_{21} = (-1)^{2+1}M_{21} = (-1)(a_{12}a_{33} - a_{13}a_{32}) = -a_{12}a_{33} + a_{13}a_{32}$

RemarkExpanding the determinant Δ along row R_1,we have

$$\Delta = (-1)^{1+1}a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2}a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3}a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$= a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ where A_{ij} are the cofactors of a_{ij}.

Thus, Δ is the **sum of the product of the elements of R_1 and their corresponding cofactors**.

Similarly, Δ can be expanded along other rows (R_2, R_3) or columns (C_1, C_2, C_3). **Example 22**

Find the minors and cofactors of the elements of the determinant:

$\Delta = \begin{vmatrix} 2 & 3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & 7 \end{vmatrix}$

and verify that:

$a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 0$

Solution: The minor of an element a_{ij} is denoted by M_{ij}, and its cofactor is $A_{ij} = (-1)^{i+j}M_{ij}$.

$M_{11} = \begin{vmatrix} 0 & 4 \\ 5 & 7 \end{vmatrix} = 0 - 20 = -20;$

$A_{11} = (-1)^{1+1}(-20) = -20$

$$M_{12} = \begin{vmatrix} 6 & 4 \\ 1 & 7 \end{vmatrix} = 42 - 4 = 38;$$

$$A_{12} = (-1)^{1+2}(38) = -38$$

$$M_{13} = \begin{vmatrix} 6 & 0 \\ 1 & 5 \end{vmatrix} = (6 \times 5 - 0 \times 1) = 30$$
$$A_{13} = (-1)^{1+3}(30) = 30$$

$$M_{21} = \begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} = (3 \times 7 - 5 \times 5) = 21 - 25 = -4;$$
$$A_{21} = (-1)^{2+1}(-4) = 4$$

$$M_{22} = \begin{vmatrix} 2 & 5 \\ 1 & 7 \end{vmatrix} = (2 \times 7 - 5 \times 1) = 14 - 5 = 9;$$
$$A_{22} = (-1)^{2+2}(9) = 9$$

$$M_{23} = \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} = (2 \times 5 - 3 \times 1) = 10 - 3 = 7;\text{quad}$$
$$A_{23} = (-1)^{2+3}(7) = -7$$

$$M_{31} = \begin{vmatrix} 3 & 5 \\ 0 & 4 \end{vmatrix} = 12 - 0 = 12;$$
$$A_{31} = (-1)^{3+1}(12) = 12$$

$$M_{32} = \begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix} = 8 - 30 = -22;$$
$$A_{32} = (-1)^{3+2}(-22) = 22$$

$$M_{33} = \begin{vmatrix} 2 & 3 \\ 6 & 0 \end{vmatrix} = 0 - 18 = -18$$
$$A_{33} = (-1)^{3+3}(-18) = -18$$

Now, $a_{11} = 2, \quad a_{12} = -3, \quad a_{13} = 5$ $A_{31} = -12, \quad A_{32} = 22, \quad A_{33} = 18$
So

$$a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$$
$$= (-12) + (-3)(22) + 5(18) = -24 - 66 + 90 = 0$$