

- 1) Compute the number of typical operations of this code ? What is the complexity of this algo

```
int a = 0;
for (i = 0; i < N; i++) {
    for (j = N; j > i; j--) {
        a = a + i + j;
    }
}
```

```
int a = 0;                                // 1
for (i = 0; i < N; i++) {                  // 1 + 2N
    for (j = N; j > i; j--) {              // 1 + 2(N^2 - C) với C = 1+2+3....+ N
        a = a + i + j;                    // C = N(N+1)/2
    }                                     // 3(N^2 - C)
}
```

⇒ The number of typical oper = $5N^2 + 2N + 3 - 5C$

- 2) Prove that $T(n) = a_0 + a_1n + a_2n^2 + a_3n^3$ is $O(n^3)$ using the formal definition of the Big-O notation

$$G(n) = a_0 + a_1n + a_2n^2 + a_3n^3$$

Tồn tại một hằng số C_1 và n_0 để $G_1(n) = a_0 + a_1n \leq C_1n$ với $n \geq n_0$

$$\Rightarrow T_1(n) = O(G_1) = n$$

Tương tự như vậy với $G_2(n) = a_2n^2 \Rightarrow O(G_2) = n^2$; $G_3(n) = a_3n^3 \Rightarrow O(G_3) = n^3$

Quy tắc cộng Big O thì $\Rightarrow O(G) = O(G_1) + O(G_2) + O(G_3) = n^3$

- 3) Determine dominant term and O in the following table

Expression	Dominant term(s)	$O(\dots)$
$5 + 0.001n^3 + 0.025n$	n3	n3
$500n + 100n^{1.5} + 50n \log_{10} n$	n1.5	n1.5
$0.3n + 5n^{1.5} + 2.5 \cdot n^{1.75}$	n1.75	n1.75
$n^2 \log_2 n + n(\log_2 n)^2$	n2.log2(n)	n2.log(n)
$n \log_3 n + n \log_2 n$	nlog2(n)	nlog(n)
$3 \log_8 n + \log_2 \log_2 \log_2 n$	log8(n)	log(n)
$100n + 0.01n^2$	n2	n2
$0.01n + 100n^2$	n2	n2
$2n + n^{0.5} + 0.5n^{1.25}$	n1.25	n1.25
$0.01n \log_2 n + n(\log_2 n)^2$	n(log2(n))2	n(log(n))2
$100n \log_3 n + n^3 + 100n$	n3	n3
$0.003 \log_4 n + \log_2 \log_2 n$	log4(n)	log(n)

4) Fill the following table

Statement	Is it TRUE or FALSE?	If it is FALSE then write the correct formula
Rule of sums: $O(f + g) = O(f) + O(g)$	False	$O(f+g) = O(\max(O(f), O(g)))$
Rule of products: $O(f \cdot g) = O(f) \cdot O(g)$	True	
Transitivity: if $g = O(f)$ and $h = O(f)$ then $g = O(h)$	False	if $g = O(f)$ and $f = O(h)$ then $g = O(h)$
$5n + 8n^2 + 100n^3 = O(n^4)$	False	$O(n^3)$
$5n + 8n^2 + 100n^3 = O(n^2 \log n)$	False	$O(n^3)$