Thái Nguyễn Minh Huân - ITITIU 20202

Q1. Given the data.

t	2	2,1	2,2	2,7	3	3,4
z	6	7,752	XY,256	36,576	66	123,168

at 2,5 is between 2,1 and 2,7. (XY => 07)

$f_2(t) = b_0 + b_1(t - t_0) + b_2(t - t_0)(t - t_1)$

where

$b_0 = f(t_0) = 7,752$

$b_1 = f(t_1, t_0) = \frac{f(t_1) - f(t_0)}{t_1 - t_0} = \frac{7,256 - 7,752}{2,2 - 2,1} = -4,96$

$b_2 = \frac{f(t_2, t_1) - (f(t_1, t_0)}{t_2 - t_0} = \frac{f(t_2, t_1) - (-4,96)}{36,576 - 7,752} = \frac{58,64 + 4,96}{36,576 - 7,752}$

$= 2650/1201$

* $f(t_2, t_1) = \frac{f(t_2) - f(t_1)}{t_2 - t_1} = \frac{36,576 - 7,256}{2,7 - 2,2} = 58,64$

=> The 2nd-order of Newton interpolating is:

$f_2(t) = 7,752 + (-4,96)(2,5 - 2,1) + 2650/1201$

$(2,5 - 2,1)(2,5 - 2,2) = 6,03299$

c/ Recall: $ax^2 + bx + c = y$.

Interval [2, 2.1]

$- a + 2b + 4c + 8d = 6$.

$- a + 2.1b + c(2.1^2) + d(2.1^3) = 7.353$.

Continuity conditions:

$b + 4c + (2^2 \times 3)d = b(2.1) + c(2.1^2) + d(2.1^3)$

$\Rightarrow \begin{cases} a = 6 \\ b = 3 \\ c = 0 \end{cases} \quad d = 0. \Rightarrow S_1(t) = 6 + 3(t-2)$

Interval [2.1, 2.2]

$- a + 2.1b + 2.1^2 c + d(2.1^3) = 7.752$

$- a + 2.2b + 2.2^2 c + 2.2^3 d = 7.256$.

Continuity condition.

$b + (2 \times 2.1)c + (3 \times 2.1^2)d = b(2.2) + 2.2^2 c + 2.2^3 d$

$\Rightarrow a \approx 7.752$

$b \approx -15.36 \quad d \approx -10.24$.

$c \approx 24.512$

- Interval [2,2 , 2,7].

Repeat the process.

$\Rightarrow a = 7,256$

$b = -15,36$

$c = 24,512$

$d = -10,24.$

$\Rightarrow S_3(2,5) = 7,256 - 15,36(2,5 - 2,2) + 24,512$

$(2,5 - 2,2)^2 - 10,24(2,5 - 2,2)^3 = 4,5776.$

Q2. Given the data.

t	0	1	2.5	3	4.5	5	6
z	26	15.5	5.375	3.5	2.375	3.5	X

a/ At $t = 3.5$ ($X = 0$) 3.5 is between 2.5 and 4.5.

Recall: $f_2(t) = b_0 + b_1(t - t_0) + b_2(t - t_0)(t - t_1)$

$b_0 = 5.375$

$b_1 = \frac{3.5 - 5.375}{3 - 2.5} = -3.75.$

$f(t_2, t_1) = \frac{2.375 - 3.5}{4.5 - 3} = -0.75$

$b_2 = \frac{f(t_2, t_1) - (-3.75)}{4.5 - 2.5} = \frac{-0.75 + 3.75}{4.5 - 2.5} = 1.5.$

$\Rightarrow f_2(3.5) = 5.375 - 3.75(3.5 - 5.375) + 1.5(3.5 - 5.375)(3.5 - 3.5) = 12.40625.$

Q3 $\frac{d^2y}{dx^2} + 0.5\frac{dy}{dx} + 7y = 0.$

Reduce 2nd to 1st order.

Put $z = dy/dx \Rightarrow dz/dx = d^2y/dx^2$

$dz/dx + 0.5z + 7y = 0.$

$\Leftrightarrow dz/dx = 0.5z - 7y.$

$f(x,y,z) = z$ and $g(x,y,z) = -0.5z - 7y$

We have: $k_1 = hf(x_0, y_0, z_0)$

$k_1 = 0.5\, f(0,4,0) = 0$

$l_1 = h\, g(x_0, y_0, z_0)$

$= 0.5 \times (-28) = -14.$

$k_2 = 0.5\, f\left(0 + \frac{0.5}{2}, 4 + \frac{0}{2}, 0 + \frac{-14}{2}\right)$

$= 0.5 \times (-7) = -3.5$

$l_2 = 0.5 \times (-24.5) = 12.25.$

$k_3 = 0.5 \times 6.125 = 3.0625$

$k_3 = 0,5 \times [-0,5 \times 6,125 - 7 \times 2,25] = 9,4.$

$k_4 = 0,5 \times 9,4 = 4,703.$

There is no requirement of k_4

subtitude k_1, k_2, k_3, k_4.

$y(x) = y_0 + 1/6 [k_1 + 2k_2 + 2k_3 + k_4]$.

$\Rightarrow y(0) = 0,638$

$y(1) = 1,638$

$y(2) = 2,638$

$y(3) = 3,638$

$y(4) = 4,638$

$y(5) = 5,638.$