37011 Financial Markets Instruments

Whiteboard Tutorial 4

1. Consider a 2-year, 6% semi-annual coupon bond with a face value of \$100,000. Suppose that the semi-annually compounded market rates are

Maturity	0.5	1.0	1.5	2.0
Rate	5.0%	5.25%	5.5%	5.75%

   (a) Calculate the present value of the bond.
   (b) Apply parallel shifts of 1, 5, 10, 50 and 100 basis points to the semi-annually compounded market rates and recompute the present value of the bond in each case.
   (c) Calculate the Fisher–Weil duration of the bond.
   (d) Using the Fisher–Weil duration of the bond, determine the approximate value of the bond after a parallel shift of 1, 5, 10, 50 and 100 basis points to the yield curve.
   (e) Calculate the exact present value of the bond after a parallel shift of 1, 5, 10, 50 and 100 basis points to the continuously compounded yields.

2. Compute the Macaulay duration of a two-year, semi-annual, 6% coupon bond with face value \$100,000 and yield to maturity $y = 4.2493\%$.

3. Show that for a bond with yield to maturity y, coupon frequency k, coupon rate c and exactly n coupon periods, the Macaulay duration simplifies to

$$\mathcal{D}_{\text{Mac}} = \frac{k+y}{ky} - \frac{(k+y)+n(c-y)}{kc((1+i)^n-1)+ky}$$

4. Compute the modified duration of a two-year, semi-annual, 6% coupon bond with face value \$100,000 and yield to maturity $y = 4.2493\%$.

5. Calculate the Fisher–Weil and the Macauley duration of each of the coupon bonds in Question 2 of Whiteboard Tutorial 3.

6. On 6 March 2017, what is the Fisher–Weil duration of the bond in Question 3 of Whiteboard Tutorial 3? Assume that the yield curve is flat, i.e., yields are the same for all maturities.

7. Using loglinear interpolation where necessary, calculate the Fisher–Weil duration of each of the Australian Government Bonds in Question 8(a) of Whiteboard Tutorial 3.

8. Show that the Fisher–Weil, Macauley and modified durations of a portfolio of bonds are equal to appropriately weighted sums of the Fisher–Weil, Macauley and modified durations of the bonds in the portfolio.

9. Suppose you are holding \$1 million of face value of the Australian Government Bond AU3TB0000135 and \$2 million of face value of the Australian Government Bond AU0000097495. Based on the data used in Question 8 of Whiteboard Tutorial 3, using loglinear interpolation where necessary, calculate the Fisher–Weil, Macauley and modified durations of your portfolio.

10. Suppose a portfolio manager has a liability of \$100 million to be paid in four years' time. The manager wishes to hedge this liability by using the bonds in the following table

Bond	Maturity (years)	Coupon	Yield
$\mathcal{B}_1$	2	5%	4.6%
$\mathcal{B}_2$	10	5%	5.2%

If both bonds pay semi-annual coupons and the four-year, semi-annually compounded rate is 4.8%, then find the face value n_1 of the bond $\mathcal{B}_1$ and the face value n_2 of the bond $\mathcal{B}_2$ required to hedge the liability, in the sense that the present value of the hedge equals the present value of the liability, and the net modified duration of the hedged position is zero.

11. Show that, using the same notation as in the lecture,

(a) Fisher–Weil convexity can be written as

$$\mathcal{C}_{\text{FW}} = \sum_{j=1}^{n} w_j T_j^2$$

(b) Macaulay convexity can be written as

$$\mathcal{C}_{\text{Mac}} = \frac{\sum_{j=1}^{n} T_j^2 C_j \hat{B}(0, T_j)}{\sum_{j=1}^{n} C_j \hat{B}(0, T_j)}$$