37009 Risk Management Assignment 2: Project

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1 Executive Summary

This report provides a comprehensive financial risk analysis for a portfolio comprising a butterfly spread on Commonwealth Bank of Australia (CBA) stock and a strangle on Macquarie Group Ltd. (MQG) stock. The primary objective was to evaluate the portfolio's risk exposure by using Value at Risk (VaR) and Expected Shortfall (ES) at various confidence levels. Considering the projected price movements for CBA (moderate) and MQG (volatile), we employed the Black-Scholes-Merton model for mark-to-market valuation of each position.

Five distinct risk assessment methods were utilized to evaluate the portfolio over one-day and ten-day time horizons: an analytical delta-normal approach, historical simulation, weighted historical simulation, and two Monte Carlo simulations (Gaussian and Student's t copula). Each method provided insights into the portfolio's risk profile by assessing loss thresholds at 90%, 95%, and 99% confidence levels, both for individual positions and the combined portfolio.

Key insights revealed that the Student's t copula simulation produced the most conservative estimates, indicating significant tail risk for the portfolio. Additionally, the diversified portfolio's risk was consistently lower than undiversified measures, emphasizing the benefits of diversification. However, the analysis highlights the importance of considering individual asset risks, as well as dynamic modeling approaches to better capture changing market conditions and manage extreme risk scenarios effectively.

2 Methodology

2.1 General assumptions and Risk factor mapping

2.1.1 General assumptions

Our total portfolio consists of the following assets:

Portfolio	Asset	Underlying stock	Position	Strike
Butterfly spread	Call option	CBA	1	80
	Call option	CBA	1	110
	Call option	CBA	-2	100
Strangle	Put option	MQG	1	150
	Call option	MQG	1	220

Table 1: Portfolio assets

With the data given only containing the price of the CBA stock and the MQG stock, we do not have the data on the term structure, which can potentially become a risk factor attributed to the portfolio. Since then, we need to have the assumption that the term structure is fixed from the day we enter to maturity of the portfolio.

The maturity of all assets in the portfolio is 6 months after we enter. Hence, we assume that the day counting convention is 30/360 for the time to maturity, which makes the time to maturity 0.5 years. According to the requirement of the project, 250 trading days are used to assume annualization of measurements, which consist of the volatility of the log-return and the mean of the log-return.

In order to estimate the volatility of the stock, equal weights will be used to facilitate the methods of estimating. However, there will be drawbacks to this method, which will ignore the more important recent volatility compared to the historical volatility (which can be held if using the EWMA methods).

As most of the estimation from the historical data will be conducted equally, it is assumed that every situation in the past may have equal probability to happen again in the future.

2.1.2 Valuation

Let denote the value of each option as a matrix $\mathbf{P}$:

$$\mathbf{P}_t = \begin{bmatrix} V_{Call_{80}}(t, S_{CBA}) \\ V_{Call_{100}}(t, S_{CBA}) \\ V_{Call_{110}}(t, S_{CBA}) \\ V_{Put_{150}}(t, S_{MQG}) \\ V_{Call_{220}}(t, S_{MQG}) \end{bmatrix}$$

Depending on the portfolio that we are considering, we are going to have the value of that portfolio depend on the positions that portfolio is going to take on each option above. Then

$$V_{total}(t) = \omega_{total}^T \cdot \mathbf{P}_t$$
$$V_{butterfly}(t) = \omega_{butterfly}^T \cdot \mathbf{P}_t$$
$$V_{strangle}(t) = \omega_{strangle}^T \cdot \mathbf{P}_t$$

$$\omega_{total} = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \omega_{butterfly} = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad \omega_{strangle} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Assume that the underlying assets follow the log-normal distribution under the risk-neutral measure, then, we have the price of the options follow the Black-Scholes formula:

$$V_{Call_K}(t, S) = S(t)\Phi(d_1) + e^{-rT} K\Phi(d_2)$$
$$V_{Put_K}(t, S) = e^{-rT} K\Phi(-d_2) - S(t)\Phi(-d_1)$$
$$d_{1,2} = \frac{\log\left(\frac{S(t)}{K}\right) + (r \pm \frac{1}{2}\sigma^2)T}{\sqrt{\sigma^2 T}}$$

2.1.3 Risk factor mapping

According to the assignment, we have the log price of the asset as the risk factors. Since then, we denote the risk factor vectors:

$$\mathbf{Z}_t := \begin{bmatrix} Z_{1,t} \\ Z_{2,t} \end{bmatrix} = \begin{bmatrix} \log S_{CBA,t} \\ \log S_{MQG,t} \end{bmatrix} \quad \mathbf{X}_{t+1} := \mathbf{Z}_{t+1} - \mathbf{Z}_t = \begin{bmatrix} \log S_{CBA,t+1} - \log S_{CBA,t} \\ \log S_{MQG,t+1} - \log S_{MQG,t} \end{bmatrix} = \begin{bmatrix} \log \frac{S_{CBA,t+1}}{S_{CBA,t}} \\ \log \frac{S_{MQG,t+1}}{S_{MQG,t}} \end{bmatrix}$$

Then:

$$\mathbf{P}(t) = \begin{bmatrix} V_{Call_{80}}(t, e^{Z_{1,t}}) \\ V_{Call_{100}}(t, e^{Z_{1,t}}) \\ V_{Call_{110}}(t, e^{Z_{1,t}}) \\ V_{Put_{150}}(t, e^{Z_{2,t}}) \\ V_{Call_{220}}(t, e^{Z_{2,t}}) \end{bmatrix}$$

So:

$$V_{port}(t) = \omega_{port}^T \cdot \mathbf{P}(t) \text{ with } port \in \{total, butterfly, strangle\}$$

Delta approximation

We have:

$$\begin{aligned}
L_{port}^{\Delta}(t+1) &= -(V_{port}(t+1) - V_{port}(t)) \\
&= -\omega_{port}^{T} \cdot (\mathbf{P}(t+1) - \mathbf{P}(t)) \\
&= \omega_{port}^{T} \cdot \begin{bmatrix} L_{Call_{80}}^{\Delta}(t+1) \\ L_{Call_{100}}^{\Delta}(t+1) \\ L_{Call_{110}}^{\Delta}(t+1) \\ L_{Put_{150}}^{\Delta}(t+1) \\ L_{Call_{220}}^{\Delta}(t+1) \end{bmatrix}
\end{aligned}$$

On the other hand:

$$\begin{aligned}
L_{Call}^{\Delta}(t+1) &= -(\frac{\partial V_{Call}}{\partial Z} X_{t+1}) \\
&= -\Delta_{Call} S(t) X_{t+1} \\
L_{Put}^{\Delta}(t+1) &= -(\frac{\partial V_{Put}}{\partial Z} X_{t+1}) \\
&= -\Delta_{Put} S(t) X_{t+1} \\
\Delta_{Expose} &= \begin{bmatrix} \Delta_{Call_{80}} S_{CBA}(t) & 0 \\ \Delta_{Call_{100}} S_{CBA}(t) & 0 \\ \Delta_{Call_{110}} S_{CBA}(t) & 0 \\ 0 & \Delta_{Put_{150}} S_{MQG}(t) \\ 0 & \Delta_{Call_{220}} S_{MQG}(t) \end{bmatrix}
\end{aligned}$$

$$\implies \begin{bmatrix} L_{Call_{80}}^{\Delta}(t+1) \\ L_{Call_{100}}^{\Delta}(t+1) \\ L_{Call_{110}}^{\Delta}(t+1) \\ L_{Put_{150}}^{\Delta}(t+1) \\ L_{Call_{220}}^{\Delta}(t+1) \end{bmatrix} = -\Delta_{Expose} \cdot \mathbf{X}_{t+1}$$

Delta-gamma approximation

We have:

$$\begin{aligned}
L_{port}^{\Gamma}(t+1) &= -(V_{port}(t+1) - V_{port}(t)) \\
&= -\omega_{port}^{T} \cdot (\mathbf{P}(t+1) - \mathbf{P}(t)) \\
&= \omega_{port}^{T} \begin{bmatrix} L_{Call_{80}}^{\Gamma}(t+1) \\ L_{Call_{100}}^{\Gamma}(t+1) \\ L_{Call_{110}}^{\Gamma}(t+1) \\ L_{Put_{150}}^{\Gamma}(t+1) \\ L_{Call_{220}}^{\Gamma}(t+1) \end{bmatrix}
\end{aligned}$$

On the other hand:

$$
\begin{aligned}
L^{\Gamma}_{Call}(t+1) &= -(\frac{\partial V_{Call}}{\partial Z} X_{i,t+1}) - \frac{1}{2}(\frac{\partial^2 V_{Call}}{\partial Z^2} X^2_{i,t+1}) \\
&= -\Delta_{Call} S(t) X_{t+1} - \frac{1}{2}(\Gamma_{Call} S(t)^2 + \Delta_{Call} S(t)) X^2_{t+1} \\
L^{\Gamma}_{Put_i}(t+1) &= -(\frac{\partial V_{Put}}{\partial Z_1} X_{i,t+1}) - \frac{1}{2}(\frac{\partial^2 V_{Put}}{\partial Z^2} X^2_{i,t+1}) \\
&= -\Delta_{Put} S(t) X_{t+1} - \frac{1}{2}(\Gamma_{Put} S(t)^2 + \Delta_{Put} S(t)) X^2_{t+1} \\
\Gamma_{Expose} &= \begin{bmatrix} \Gamma_{Call_{80}} S^2_{CBA}(t) & 0 \\ \Gamma_{Call_{100}} S^2_{CBA}(t) & 0 \\ \Gamma_{Call_{110}} S^2_{CBA}(t) & 0 \\ 0 & \Gamma_{Put_{150}} S^2_{MQG}(t) \\ 0 & \Gamma_{Call_{220}} S^2_{MQG}(t) \end{bmatrix}
\end{aligned}
$$

$$
\Longrightarrow \begin{bmatrix} L^{\Gamma}_{Call_{80}}(t+1) \\ L^{\Gamma}_{Call_{100}}(t+1) \\ L^{\Gamma}_{Call_{110}}(t+1) \\ L^{\Gamma}_{Put_{150}}(t+1) \\ L^{\Gamma}_{Call_{220}}(t+1) \end{bmatrix} = -\left[\Delta_{Expose} \cdot \mathbf{X}_{t+1} + \frac{1}{2}\left(\Delta_{Expose} + \Gamma_{Expose}\right) \cdot \mathbf{X}^2_{t+1} \right]
$$

2.1.4 Exposure and sensitivity to risk factor change

With the derivation above, we can see that the delta-gamma approximation is just having an additional gamma approximation in the exposure. Then, we can calculate the delta-gamma approximation directly and use the delta approximation only when approaching with the delta approximation approach. After performing the calculation, we got the matrix of exposure as follows:

	CBA Delta	**MQG Delta**	**CBA Gamma**	**MQG Gamma**
Total Portfolio	-12.34321	-12.408058	101.607465	-234.808803
Butterfly Portfolio	-12.34321	0.000000	101.607465	0.000000
Strangel Portfolio	0.00000	-12.408058	0.000000	-234.808803

Table 2: Portfolio Deltas and Gammas

From the table, we can see that:

- The bigger the risk factor, the less loss in portfolios in delta approximation
- The further from zero, the more loss in portfolio contains CBA stock in gamma approximation, the less loss in portfolio contains MQG stock in gamma approximation.

2.1.5 Adjustment and assumption for time scaling problems

The problem of time scaling arises when it is required to estimate the Value at Risk and Expected Shortfall for the number of days greater than 1. The reason for the scaling is that without scaling, there won't be enough data for the estimation. In order to perform the scaling, we assume that daily risk factors during those days are iid.

$$\mathbf{X}_{t+1}, \mathbf{X}_{t+2}, \cdots, \mathbf{X}_{t+n} \sim \mathbf{N}(\mu, \Sigma)$$

Then:

$$\begin{aligned}
X_{n_{day}} &\sim \mathbf{N}(n\mu, n\Sigma) \\
L_{n_{day}} &= b^T \cdot X_{n_{day}} \\
\Longrightarrow L_{n_{day}} &\sim \mathbf{N}(nb^T \cdot \mu, nb^T \cdot \Sigma) \\
\Longrightarrow VaR_\alpha(L_{n_{day}}) &= nb^T \cdot \mu + \sqrt{nb^T \cdot \Sigma}\Phi^{-1}(\alpha) \\
\Longrightarrow ES_\alpha(L_{n_{day}}) &= nb^T \cdot \mu + \sqrt{nb^T \cdot \Sigma}\frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha}
\end{aligned}$$

However, the above assumptions might not hold in practice and with the given data. Moreover, in method 5, the risk factors are assumed to come from a t-Copula, which makes the above assumption not applicable due to the fact that sum of Student-t distribution is not a Student-t distribution.

Since then, a suggested method to solve this problem is to measure the risk factor of n days directly instead of scaling the result from the daily data. There are two methods to measure it:

- Perform the overlapping (rolling window) sum over n days: By doing this, we can measure the risk factor of 10 day without the normal assumption. Furthermore, the number of historical data still good enough for performing the risk measuring. However, there will be a noticing correlation in time series data due to this practice.
- Perform the non-overlapping sum of n days: By doing this, the correlation in time series data can be reduced. However, it would lead to the decrease in the number of data, which may lead to the false in measuring the risk.

In this assignments, three above methods will all be performed for method 1, 4 and 5 due to the need of assumptions on the distribution of the risk factor. The results of each methods will be discussed in the results section.

2.2 Method explanation and additional assumption

2.2.1 Method 1: Delta-normal approach

In this method, it assumes that the risk factor are normally distributed with the mean and the volatility is measured through historical data. Depends on the method that is used for measuring risk at 10 days if it is needed, the historical of risk factor data will be gathered differently. Assume that after gathering and measuring, we have:

$$\begin{aligned}
&\mathbf{X} \sim \mathbf{N}(\mu, \Sigma) \\
\Longrightarrow\ & L^{\Delta}_{port} = \omega^T_{port} \cdot (-\Delta_{Expose}) \cdot \mathbf{X} \sim \mathbf{N}(\omega^T_{port} \cdot (-\Delta_{Expose}) \cdot \mu, \omega^T_{port} \cdot (-\Delta_{Expose}) \cdot \Sigma) \\
\Longrightarrow\ & VaR_\alpha(L_{n_{day}}) = \omega^T_{port} \cdot (-\Delta_{Expose}) \cdot \mu + \sqrt{\omega^T_{port} \cdot (-\Delta_{Expose}) \cdot \Sigma}\Phi^{-1}(\alpha) \\
\Longrightarrow\ & ES_\alpha(L_{n_{day}}) = \omega^T_{port} \cdot (-\Delta_{Expose}) \cdot \mu + \sqrt{\omega^T_{port} \cdot (-\Delta_{Expose}) \cdot \Sigma}\frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha}
\end{aligned}$$

2.2.2 Method 2: Historical simulation approach with delta-gamma approximation

With this method, it will require to gather the historical data of risk factor to compute the loss of the portfolio by using the current exposure of the portfolio with the delta gamma approach. At the end, we will have a simulation of loss from historical values of the risk factor.

$$L^{\Gamma} = -\omega^T \cdot \left[\Delta_{Expose} \cdot \mathbf{X} + \frac{1}{2} \left(\Delta_{Expose} + \Gamma_{Expose} \right) \cdot \mathbf{X}^2 \right]$$

As we assume that the weight of each loss is equal (or the probability of each historical risk factor values happening in the future is the same), the Value at Risk and Expected Shortfall are measured with emprical method as:

$$VaR_{\alpha}(L) = inf\{l \in \mathbf{R} : P(L > l) \leq 1 - \alpha\}$$
$$ES_{\alpha}(L) = \frac{1}{1-\alpha}\mathbf{E}\left[(L - VaR_{\alpha}(L))^{+}\right] + VaR_{\alpha}(L)$$

The simulation of the loss will be increasingly sorted. The weight of each simulation is

$$\omega_j = \frac{1}{n_{Loss}}$$

2.2.3 Method 3: Weighted historical simulation approach with delta-gamma approximation

This method is as same as method 2, except for the assumption that each loss simulation does not have the same weight. A decay weight will be applied for the probability of each simulation happens again in the future. In this method, the assumption is that the further the simulation value in the past, the less likely it appears in the future (lower weight compared to the more recent value).

$$\omega_j = \frac{\lambda^{j-1}(1-\lambda)}{1-\lambda^{n_{loss}}}$$

With λ is the decay factor. The decay factor is chose to be 0.94. By using this factor, the weight of the more recent value of risk factor simulation is higher.

2.2.4 Method 4: Monte Carlo simulation approach with delta-gamma approximation with log-return jointly Gaussian distributed

According to the name of the method, we assume that:

$$\mathbf{X}_{t+n} = \begin{bmatrix} \log \frac{S_{CBA,t+n}}{S_{CBA,t}} \\ \log \frac{S_{MQG,t+1}}{S_{MQG,t}} \end{bmatrix} \sim \mathbf{N}(\mu, \Sigma)$$

Since then, we just need to measure the μ and Σ from the data which is gathered based on the method that is used. After that, we can plug it into the normal generator with the mean and the variance we gather from the data to create Monte Carlo simulations of $\mathbf{X}$. At the end, we can have the simulation of loss calculated from the simulation of risk factors.

$$L^{\Gamma} = -\omega^T \cdot \left[\Delta_{Expose} \cdot \mathbf{X} + \frac{1}{2} \left(\Delta_{Expose} + \Gamma_{Expose} \right) \cdot \mathbf{X}^2 \right]$$

2.2.5 Method 5: Monte Carlo simulation approach with a delta-gamma approximation with log-return jointly Student's t copula distributed

According to the name of the method, we assume that:

$$t = \frac{\mathbf{X}_{t+n} - \mu}{\Sigma} \sim t_\nu(\rho, df)$$

Since then, we just need to measure the μ and Σ from the data which is gathered based on the method that is used. After that, we perform the standardization to get the historical value of t. Then, by using pycop, the appropriate correlation and degree of freedom can be measured from the calculated historical data by using the maximum likelihood estimation. From then, we can use the pycop.simulation to generate the value of cdf. From then, we can use the quantile function of t with the estimated correlation and degree of freedom to get the t values. From then, the risk factor can be calculated from the simulation and so does the loss.

$$\mathbf{X}_{t+n} = \mu + \Sigma t_\nu^{-1}(\rho, df, u)$$

$$L^\Gamma = -\omega^T \cdot \left[\Delta_{Expose} \cdot \mathbf{X} + \frac{1}{2}\left(\Delta_{Expose} + \Gamma_{Expose}\right) \cdot \mathbf{X}^2\right]$$

3 Results

3.1 Stock analysis

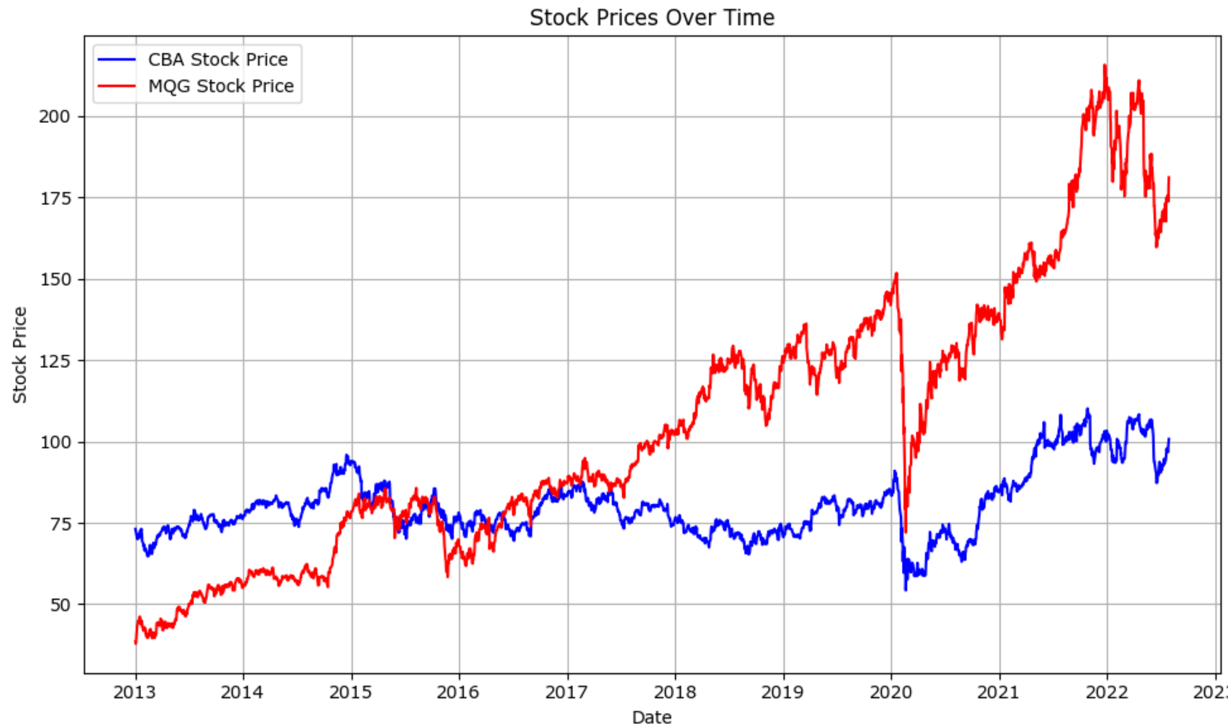

Figure 1: Time series of stock

From the time series of the stock price, we can see that these two stocks do exhibit some positive correlation.

- The MQG stock has a significant increasing trend over time, However, it decreased during 2020, which may due to the effects of COVID-19. But at the end, it has been recovered since then and fluctuated in these recent years. Due to large movement of the stock price in the near future but insecure of the direction, this strategy will efficient for the MQG stock with the required maturity.

- The CBA stock does not have a clear increasing trend over the time. Only after the COVID recovery where the stock went up higher than it used to be in previous years, this stock seems to fluctuate around. However, when taking it closer to the time series, we can see that the stock price of CBA seems to increase when it is closer to the end of the year and decrease at the beginning of the year. With the butterfly spread, if helps earning payoff with small upward movement of the stock. This one is perfectly fit with what is exhibiting in the CBA stock where it's reaching to the end of the year and most of the movement of the CBA stock seeming not to be to large.

3.2 Dependence structure

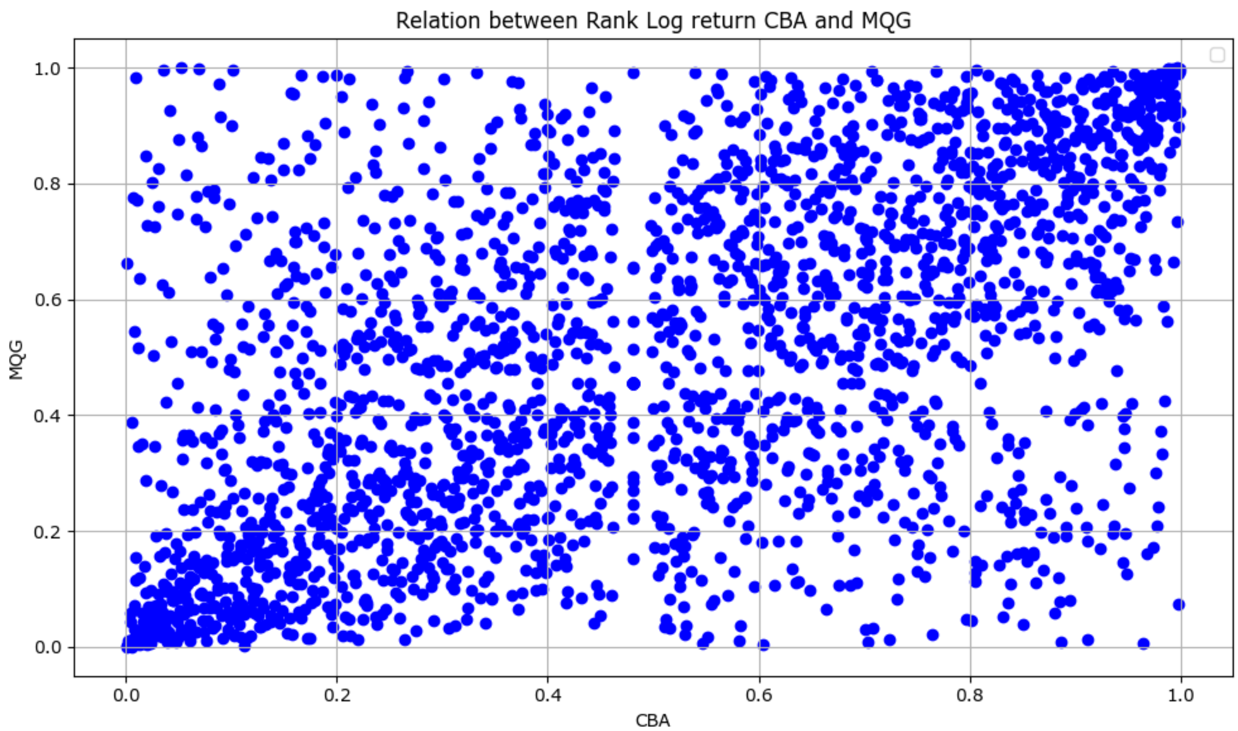

Figure 2: Rank Dependence structure

From the plot above, we can see that these two stock are heavily tail dependence. Furthermore, when performing statistical correlation, it does show that these two stock shows significantly high correlation. This helps with consolidation the probability to receive profit in the future as both stocks seems to having an increasing trend.

Metric	Value
Pearson Linear Correlation	0.5747
Spearman Correlation	0.5342
Kendall's tau	0.3883
Spearman Rank Correlation	0.5342
Lower Tail Dependence (5%)	0.4146
Upper Tail Dependence (95%)	0.3416

Table 3: Correlation and Dependence Statistics

3.3 Portfolio valuation

Portfolio	**Total Value**
CBA Portfolio	11.1068
MQG Portfolio	5.1034
Total Portfolio Value	**16.2102**

Table 4: Portfolio Values

3.4 Portfolio risk measure

3.4.1 Diversified portfolio

Risk measure	**90.0%**	**95.0%**	**99.0%**
1 day VaR	0.410077	0.529050	0.752223
10 day VaR Scale	1.231125	1.607351	2.313088
10 day VaR Overlapping	1.238867	1.615405	2.321728
10 day VaR Non-Overlapping	1.256377	1.640089	2.359867
1 day ES	0.565115	0.665889	0.863194
10 day ES Scale	1.721398	2.040074	2.664008
10 day ES Overlapping	1.729547	2.048488	2.672940
10 day ES Non-Overlapping	1.756405	2.081422	2.717770

Table 5: Method 1: Risk measurement for diversified portfolio

	Method 2			Method 3		
Risk measure	**90.0%**	**95.0%**	**99.0%**	**90.0%**	**95.0%**	**99.0%**
1 day VaR	0.275572	0.386402	0.621234	0.235908	0.412731	0.617628
10 day VaR	0.658272	0.973303	2.373378	0.695137	2.523439	3.161269
1 day ES	0.445255	0.566174	0.902753	0.432510	0.579034	0.730826
10 day ES	1.236436	1.678773	3.073507	2.210965	2.968621	3.358309

Table 6: Method 2 and 3: Risk measurement for diversified portfolio

Risk measure	Method 4 90.0%	95.0%	99.0%	Method 5 90.0%	95.0%	99.0%
1 day VaR	0.342544	0.435994	0.615092	0.392433	0.544621	0.993045
10 day VaR Scale	0.821668	1.179754	1.937240	0.909562	1.491127	3.645028
10 day VaR Overlapping	0.784553	1.116975	1.826953	0.392433	0.544621	0.993045
10 day VaR Non-Overlapping	0.805565	1.145943	1.862641	0.839188	1.299698	2.623053
1 day ES	0.465564	0.545898	0.705629	0.673422	0.888084	1.647978
10 day ES Scale	1.311487	1.642349	2.358310	2.260437	3.361677	8.107745
10 day ES Overlapping	1.242389	1.552546	2.216193	0.673422	0.888084	1.647978
10 day ES Non-Overlapping	1.270938	1.585463	2.259616	1.616946	2.192836	3.949307

Table 7: Method 4 and 5: Risk measurement for diversified portfolio

3.4.2 Undiversified portfolio

Risk measure	CBA portfolio 90.0%	95.0%	99.0%	MQG portfolio 90.0%	95.0%	99.0%
1 day VaR	0.210565	0.270722	0.383568	0.252151	0.325889	0.464210
10 day VaR Scale	0.654649	0.844883	1.201731	0.742936	0.976117	1.413524
10 day VaR Overlapping	0.635220	0.819840	1.166157	0.761058	0.997600	1.441314
10 day VaR Non-Overlapping	0.647901	0.836415	1.190035	0.757741	0.995253	1.440786
1 day ES	0.288958	0.339914	0.439679	0.348242	0.410701	0.532988
10 day ES Scale	0.902550	1.063685	1.379170	1.046802	1.244314	1.631021
10 day ES Overlapping	0.875805	1.032185	1.338359	1.069304	1.269664	1.661946
10 day ES Non-Overlapping	0.893560	1.053238	1.365870	1.067252	1.268432	1.662323

Table 8: Method 1: Risk measurement for undiversified portfolio

Risk measure	CBA portfolio 90.0%	95.0%	99.0%	MQG portfolio 90.0%	95.0%	99.0%
1 day VaR	0.187872	0.292862	0.581258	0.130495	0.153262	0.163198
10 day VaR	0.756976	1.128512	3.374953	0.149145	0.160246	0.163769
1 day ES	0.378911	0.523161	1.068029	0.150610	0.160025	0.163622
10 day ES	1.766399	2.627849	6.208862	0.158665	0.162676	0.163871

Table 9: Method 2: Risk measurement for undiversified portfolio

Risk measure	CBA portfolio 90.0%	95.0%	99.0%	MQG portfolio 90.0%	95.0%	99.0%
1 day VaR	0.239767	0.319632	0.481923	0.149994	0.160156	0.163752
10 day VaR Scale	0.938897	1.321099	2.166147	0.149072	0.160170	0.163778
10 day VaR Overlapping	0.902745	1.267549	2.072635	0.148908	0.160120	0.163757
10 day VaR Non-Overlapping	0.926351	1.306488	2.129014	0.148760	0.160086	0.163762
1 day ES	0.348173	0.420330	0.571202	0.159045	0.162638	0.163865
10 day ES Scale	1.479058	1.847924	2.670337	0.158945	0.162705	0.163873
10 day ES Overlapping	1.418766	1.770616	2.553771	0.158895	0.162628	0.163865
10 day ES Non-Overlapping	1.458601	1.822423	2.633733	0.158810	0.162620	0.163870

Table 10: Method 4: Risk measurement for undiversified portfolio

3.5 Portfolio risk measure at 95% confidence

M	Risk measure	CBA port.	MQG port.	Sum of risk	Total port.
1	1 day VaR	0.270722	0.325889	0.596611	0.529050
	10 day VaR Scale	0.844883	0.976117	1.821000	1.607351
	10 day VaR Overlapping	0.819840	0.997600	1.817440	1.615405
	10 day VaR Non-Overlapping	0.836415	0.995253	1.831668	1.640089
	1 day ES	0.339914	0.410701	0.750615	0.665889
	10 day ES Scale	1.063685	1.244314	2.307999	2.040074
	10 day ES Overlapping	1.032185	1.269664	2.301849	2.048488
	10 day ES Non-Overlapping	1.053238	1.268432	2.321670	2.081422
2	1 day VaR	0.292862	0.153262	0.446124	0.386402
	10 day VaR	1.128512	0.160246	1.288758	0.973303
	1 day ES	0.523161	0.160025	0.683186	0.566174
	10 day ES	2.627849	0.162676	2.790525	1.678773
4	1 day VaR	0.319632	0.160156	0.479788	0.435994
	10 day VaR Scale	1.321099	0.160170	1.481269	1.179754
	10 day VaR Overlapping	1.267549	0.160120	1.427669	1.116975
	10 day VaR Non-Overlapping	1.306488	0.160086	1.466574	1.145943
	1 day ES	0.420330	0.162638	0.582968	0.545898
	10 day ES Scale	1.847924	0.162705	2.010629	1.642349
	10 day ES Overlapping	1.770616	0.162628	1.933244	1.552546
	10 day ES Non-Overlapping	1.822423	0.162620	1.985043	1.585463

Table 11: Risk measure comparison

4 Discussion and Analysis

4.1 Risk measure interpretion

The Value at Risk (VaR) and Expected Shortfall (ES) calculated in the previous section represent the potential portfolio losses over specific time horizons (1-day and 10-day) at different confidence levels (90%, 95%, and 99%). VaR estimates the maximum potential loss that is not expected to be exceeded with a certain confidence, while ES provides the average loss given that the VaR threshold has been exceeded. The ES, as a tail risk measure, is generally higher than the VaR, reflecting more conservative estimates of extreme losses.

4.2 Methods comparison

4.2.1 Method 1

From the above tables, we see that this method returns the highest risk measure for the total portfolio and the MQG portfolio compared to the other methods. In this method, we can see that the risk measures for 1 day are less than 10 days, no matter the method of gathering. In the 10-day risk measure, the non-overlapping method has the highest risk measure and the scaling method has the lowest risk measure. This can be explained as the scaling underestimates the correlation of the log-return by performing the sum (which means that the assumption about the iid of daily log-return does not hold). However, the correlation of daily log-return also makes the risk measure for the overlapping method seem to be lower than the non-overlapping one. This method is easy and quick to use. However, it seems to be naive as it under-estimate the tail distribution. Since then, it will be appropriate to use this method in short-term investing.

4.2.2 Method 2 and 3

These two methods have lower risk measure compared to the first method. This can be due to the fact that the assumption of the first method makes the distribution of the log-return bigger compared to what can be achieved via historical data. The risk measure for 1-day risk is less than 10-day risk (which is appropriate). However, method 3 seems to have lower risk measurement. This can be explained as due to the method of age-weighting, it weights less for the loss further in the history (which affected the return during the COVID-19 period). These methods require an enormous amount of data so as to form conduct the measurement. However, it helps consider the historical events and value, which might be underestimated from method 1.

4.2.3 Method 4 and 5

These two methods seem to have higher risk measure compared with method 2 and 3 but still lower than the first one. Between these two method, it can be seen that except for the overlapping data gathering method, the risk measurement for by using method 5 is higher compare to method 4. This can be reasonably explained as when using the Student't copula, we tend less to underestimate the tail distribution, which can be seen heavily exhibited in the dependence structure of two log-return of the stock. The explanation for the overlapping method is the same what it has been explained from method 1, where this method will results in the correlation of the data series, which affects on the risk measurement procedure when estimating the characteristic of the distribution of the data.

Moreover, as same as method 1, these two methods have the assumption on the distribution of the log-return, which make the tail of the loss fatter than what can be achieved from the historical data. Since then, most of risk measure result from method 4 and 5 is higher than method 2 and 3. However, with method 4, the assumption of the distribution is as same as method 1, but it has a higher approximation (delta-gamma approximation). Since then, the risk measure from method 4 seems to be less than method 1. However, the result from method 5 is bigger than method 1 due to the assumption of Student't distribution of log-return. This method helps less under-estimate the tail distribution from the normal distribution, which helps considering more loss situation in the future. As previously stated, method 4 may underestimate the tail distribution, which happened in 2008 when most investors used Gaussian copula. Method 5 is sufficiently better as it consider a fatter tail distribution but requires more calculation so as to be accurate (more assumption of a long day risk measure as sum of t distribution is not Student't distribution).

4.3 Diversified vs. Undiversified Portfolio

From the comparison table of risk measure, we can see that the diversified (total) portfolio helps reduce the risk in all method. However, relying solely on diversified risk measures could mask individual asset risks, especially if correlations fluctuate under stress. In contrast, undiversified measures are useful for identifying specific exposures but may overestimate portfolio risk by ignoring potential diversification benefits.

4.4 Appraisal and Improvement of Risk Modeling Methods

Each risk modeling method used in this project has strengths and limitations as discussed above. For improvement:

- **Dependence Modeling:** Incorporating a copula-based approach, such as a Student's t copula, better captures tail dependence, which is crucial for portfolios with assets likely to experience joint extreme events.
- **Dynamic Volatility Modeling:** Applying techniques like GARCH or EWMA can enhance risk measures by capturing changing market volatility, which is particularly useful for the historical and weighted historical simulation methods.
- **Scenario Analysis and Stress Testing:** Complementing quantitative measures with scenario analysis would provide a broader view of risk, especially in non-normal conditions where traditional VaR and ES might fail.

4.5 Risk Measure Validation

To validate the risk projections, back-testing can be employed, where actual returns are compared to projected VaR breaches over historical periods. For instance, calculating the frequency of VaR exceedances will indicate the accuracy of the chosen confidence level. Additionally, ES validation could involve conditional coverage tests to assess if ES consistently exceeds VaR breaches, ensuring robustness in extreme market conditions.

5 Summary and Conclusion

This report has analyzed the financial risk of a portfolio comprising a butterfly spread on Commonwealth Bank of Australia (CBA) stock and a strangle on Macquarie Group Ltd. (MQG) stock. By employing various methodologies to calculate Value at Risk (VaR) and Expected Shortfall (ES), we have gained insights into the potential risk exposure inherent in the investment strategies.

- **Risk Measure Variability:** The analysis revealed significant differences in risk estimates across the various methods employed. The delta-normal approach tended to understate risk due to its reliance on the assumption of normality, while methods incorporating historical simulations provided more responsive estimates to market dynamics. The Student's t copula method offered the most conservative ES estimates, highlighting the importance of tail risk considerations.
- **Diversified vs. Undiversified Risks:** The results underscored the value of diversification in managing risk. The diversified portfolio risk measures were consistently lower than those for undiversified assessments, indicating that combining assets can mitigate overall risk exposure. However, reliance on diversified measures should be balanced with awareness of individual asset risks to avoid potential pitfalls during extreme market conditions.
- **Dependence Structures:** The study highlighted the significance of dependence modeling between risk factors. Assets that exhibit strong tail dependence may lead to larger-than-expected losses during market downturns, necessitating the incorporation of robust copula techniques to more accurately model these relationships.
- **Dynamic Risk Management Strategies:** The appraisal of risk modeling methods pointed to opportunities for enhancing risk assessment techniques. Employing dynamic volatility models and scenario analysis can provide a more comprehensive view of risk, allowing for timely adjustments to investment strategies based on changing market conditions.

The findings from this analysis can be effectively utilized to inform and manage investment risks in the following ways:

- **Risk Awareness and Assessment:** Investors should adopt a multi-faceted approach to risk measurement, considering both VaR and ES across different methodologies. This allows for a clearer understanding of potential losses and informs decision-making.
- **Portfolio Construction:** The importance of diversification should guide portfolio construction. By carefully selecting assets with low correlations, investors can reduce overall portfolio risk while still pursuing desired returns.
- **Dynamic Adjustments:** Continuous monitoring of market conditions and the performance of risk factors is essential. By employing advanced modeling techniques that account for changing volatility and dependence structures, investors can make more informed decisions and quickly adjust their portfolios to respond to emerging risks.
- **Regulatory Compliance and Reporting:** The insights gained from this analysis also serve to enhance compliance with regulatory requirements related to risk management. Providing robust risk assessments will not only meet regulatory standards but also improve stakeholder confidence in investment strategies.

In conclusion, the comprehensive analysis of risk presented in this report provides valuable insights for investors seeking to navigate the complexities of financial markets. By understanding and managing risk effectively, investors can enhance their potential for returns while safeguarding their capital against unforeseen market movements.