37009 Risk Management
Spring 2024
Assessment 2: Project

UTS UNIVERSITY OF TECHNOLOGY SYDNEY

1 Instructions

1. Carefully read all sections of this document before attempting the assessment task. Instructions on how to perform the assessment task and to prepare the deliverable outputs are provided in the succeeding sections.

2. Ensure that you have understood the guidelines on academic integrity in this subject (refer to the Subject Outline and the links on the Assignment Cover Sheet). *The subject teaching staff reserve the right to conduct a viva to test your understanding of your own solution and work in relation to this assessment.*

3. A penalty will apply to late submissions as follows: five per cent (5%) of the marks for the assessment task will be deducted per day for assessment tasks submitted after the due date, to a maximum of five (5) calendar days after which no marks will be awarded for any submissions.

4. Your submission will consist of three files: a `.pdf` file containing the written report, a Jupyter notebook `.ipynb` file containing your documented calculations, and an `.html` version of your Jupyter notebook.

Use the assessment cover page as the first page of your written report. Make sure your name appears at the top of your Jupyter notebook submission.

5. Submissions must be made via Canvas. Submissions made using other means will not be accepted.

6. If you have any questions about the assessment, please post these to the subject discussion forum on Canvas. If the enquiry relates to a personal matter, please contact the subject coordinator via e-mail.

2 Overview

Today is **29 July 2022**. You are exploring investment opportunities involving the Commonwealth Bank of Australia (CBA) and the Macquarie Group Ltd. (MQG) stocks. Based on your market research, you are expecting a modest change over the next six months in the CBA stock price, whereas a significant shift in price is expected over the next six months in the MQG stock. To this end, you have decided to invest today in a *butterfly spread* in the CBA stock and a *strangle* in the MQG stock.[1] The butterfly spread written on the CBA stock consists of:

1. a long position in a European call option with strike price $K_1^B = \$80$,
2. a long position in a European call option with strike price $K_3^B = \$110$, and
3. a short position in two (2) European call options, both with strike price $K_2^B = \$100$.

[1] For further details on trading strategies with European call and put options, see e.g. Hull (2022, Chapter 12).

In contrast, the strangle written on the MQG stock is comprised of:

1. a long position in a European put option with strike price $K_1^S = \$150$ and
2. a long position in a European call option with strike price $K_2^S = \$220$.

All options have a maturity of 6 months. For simplicity, you assume that the 6-month risk-free rate of interest is 2.67% continuously compounded and is constant throughout the risk management time horizons.

To determine the value of these positions, you have assumed that the assumptions of the Black-Scholes-Merton model hold. In particular, you have modelled the stock prices of CBA and MQG, denoted by S_C and S_M, respectively, using geometric Brownian motions

$$\begin{aligned}\frac{\mathrm{d}S_C(t)}{S_C(t)} &= \mu_C\,\mathrm{d}t + \sigma_C\,\mathrm{d}W_C(t)\\ \frac{\mathrm{d}S_M(t)}{S_M(t)} &= \mu_M\,\mathrm{d}t + \sigma_M\,\mathrm{d}W_M(t),\end{aligned}$$

where μ_i and σ_i are the expected return and the annualized volatility, respectively, of S_i, for $i \in \{C, M\}$, and W_C and W_M are a pair of standard Brownian motions with correlation ρ. As a result, your model for the log-return of S_i over the period $[t, t + \Delta t]$ is

$$\ln\left(\frac{S_i(t+\Delta t)}{S_i(t)}\right) = \left(\mu_i - \frac{1}{2}\sigma_i^2\right)\Delta t + \sigma_i(W_i(t+\Delta t) - W_i(t))$$

for $i \in \{C, M\}$.[2]

You have estimated the annualized volatility σ_i from the standard deviation of the daily log-returns, assuming there are 250 trading days in a year. That is, you have the estimate

$$\hat{\sigma}_i = \frac{\text{sample std. dev. of log-returns in stock } i}{\sqrt{\Delta t}},$$

for $i \in \{C, M\}$, where $\Delta t = \frac{1}{250}$. Likewise, the correlation parameter is estimated by the correlation between daily log-returns (which is equivalent to the correlation between the annualized log-returns). Your estimates are based on historical daily stock prices 2 January 2013 to 29 July 2022 (refer to the data set provided). It is assumed that the volatility of the stock prices are constant throughout the risk management time horizons.

In addition to the current value of your portfolio, you are also interested in the one-day and 10-day value-at-risk (VaR) and expected shortfall (ES) for your portfolio at the 90%, 95%, and 99% confidence levels. In the process, you would also like to investigate whether the choice of risk measurement methodology will significantly impact the risk measures that you will obtain. You are thus considering the following methods:

(M1) Analytical delta-normal approach

(M2) Historical simulation approach with a delta-gamma approximation

(M3) Weighted historical simulation approach with a delta-gamma approximation

[2] Recall the following property of the standard Brownian motion: $W_i(t + \Delta t) - W_i(t) \sim N(0, \Delta t)$.

(M4) Monte Carlo simulation approach with a delta-gamma approximation, assuming that log-returns have a joint Gaussian distribution

(M5) Monte Carlo simulation approach with a delta-gamma approximation, assuming that the joint distribution of the log-returns are modelled using a Student's t copula.

For all approaches, the log-price of the stocks are assumed to be the underlying risk factors, hence the log-returns are the risk factor changes.

Also of interest is the difference between the stand-alone risk measures for the butterfly spread and the strangle. Thus, for the one-day 95% measures of risk, you investigate the undiversified and diversified portfolio VaR and ES calculated using methods (M1), (M2), and (M4).

3 Task Description

You must implement the required calculations using a Jupyter Notebook and write a report summarizing your findings, detailing your methodology, and discussing your results and conclusions.

3.1 Python Implementation

The following shall be implemented in Python. All required data are provided in the uploaded `.csv` file.

1. Using the Black-Scholes-Merton formula, determine the mark-to-market value of the butterfly spread and the strangle.
2. Using the methods discussed in class, implement methods (M1) to (M5) to calculate the one-day and 10-day VaR and ES at the 90%, 95%, and 99% confidence levels *for the entire portfolio* (i.e. the diversified risk measures).
3. Using methods (M1), (M2), and (M4) calculate the one-day 95% VaR and ES on a standalone basis for each position (i.e. separate risk measures for the butterly spread and the strangle).

3.2 Report

Your report must, at least, contain the following sections and address the points enumerated therein. Ensure that your report is clearly written using professional language and that your results and discussions are presented in a clear and succinct manner.

Executive Summary

Provide a brief introduction of the report and a summary of your financial risk analysis. Mention key objectives, methods used, and critical insights from the analysis. The executive summary must not exceed three (3) paragraphs in length.

Methodology

This section summarizes the methods used in the portfolio valuation and the calculation of the VaR and ES. This is also where you must state assumptions, on top of the working assumptions stated in Section 2.

- Briefly discuss the valuation formulas (including relevant assumptions) used to calculate the mark-to-market value of each position and the entire portfolio.
- Derive the risk factor mappings, in the form of delta and delta-gamma approximations, of the loss in the individual position and in the overall portfolio with respect to the risk factor changes (as identified in Section 2). Discuss the extent to which the overall portfolio is exposed to changes in the risk factors.
- Discuss and justify the adjustments you need to make to your risk factor mappings to accommodate the change in the risk management time horizon from one day to 10 days.
- **(M1)** Discuss the analytical formulas for the diversified and undiversified VaR and ES under method (M1). A full derivation of the formulas is not necessary, but discuss the key assumptions and steps that allow you to conclude that these formulas are indeed correct.
- **(M2, M3)** Briefly discuss the historical and weighted historical simulation approaches and state the empirical estimators for the VaR and ES. For the weighted historical simulation method, state and justify your choice of weighting parameter and briefly explain the weight assignment.
- **(M4, M5)** Briefly discuss the Monte Carlo simulation approach for the required calculations. For method (M5), briefly explain how you estimated of the parameters of the Student's t copula.

Results

- Produce time series graphs of the CBA and MQG stock prices. Discuss qualitatively any eminent trends in the data and their implications on potential profits realized by the butterfly spread and strangle.[3]
- Discuss if the log-returns of the two stocks exhibit any dependence structures. Cite relevant statistics such as linear correlation, rank correlation, and coefficients of tail dependence.
- Present the mark-to-market value of each position in the portfolio, as well as the mark-to-market value of the entire portfolio.
- Present the risk measures you have calculated using each method, for each confidence level, and for each risk management time horizon in tabular format. You are free to choose the layout of the table/s containing your results, as long as your presentation facilitates your discussion in the next section.
- Present the undiversified and diversified one-day 95% risk measures calculated using methods (M1), (M2), and (M4) in tabular format. Again, you are free to choose the layout of the table/s containing your results, as long as your presentation facilitates your discussion in the next section.
- A label (e.g. Table 1) and a caption must be provided for all tables and figures included in the report.

[3] Refer to Hull (2022, Chapter 12) for a discussion of the financial market conditions under which butterfly spreads and strangles are appropriate trading strategies.

Discussion and Analysis

Points to tackle in this section are as follows:

- Briefly discuss the results you presented in the previous section. Stating the interpretation of the VaR and ES is a good place to start.
- Compare and contrast the measures you obtained across the different methods. Highlight the differences and discuss possible reasons for these discrepancies. What are the practical implications of choosing the risk measurement method?
- Comment on the difference between the diversified and undiversified risk measures. Why is there a difference between the two types of risk measures? What are the implications of relying on one type of risk measures over the other?
- Appraise the risk modelling methods used in this project. Discuss possible methods to improve the quality of the risk measures. Your discussion must tackle, among others, the issue of dependence modelling between the risk factors and its implications on the quality of the risk model.
- Briefly discuss how you will validate your risk projections.

Summary and Conclusions

State here the main conclusions from your discussion and analysis. Provide some insights as to how the results can be used to manage the risk that you face upon making investments.

References

Enumerate all references used and/or consulted throughout the conduct of the assignment. Refer to the Academic Integrity section of the Subject Outline for general guidance on referencing and the use of generative AI in assessment activities.

Report Format

Your report must be written and formatted in a professional manner. Use only either Microsoft Word or LaTeXto prepare your report. Using either typesetting software, use 11pt font and 1.25 line spacing. Your report (including all figures, tables, and references) must not exceed 15 pages in length.

Marking Criteria

A maximum of 100 marks can be obtained in this assessment. This assessment accounts for 30% of your final mark in this subject.

Python Implementation

Your Jupyter notebook will be assessed on the following basis:

1. **Documentation Quality [10 marks]:** Can a person (other than the programmer) familiar with the problem quickly understand what is happening in each section of the code and identify those sections where changes need to be made if the problem is varied?

2. **Correctness [20 marks]:** Does the implementation solve the problem at hand and is the output correct?

Both the `.ipynb` and `.html` versions of your Jupyter notebook enclosed in your submission must display *only the relevant outputs*. Refer to the Results section in Section 3.2 for a list of relevant outputs.

Report

Your report will be assessed on the following basis:

1. **Quality of Discussion of Methods [20 marks]:** The methodology section exhibits your mastery of the methods used through a correct technical description of the valuation and risk measurement methods. Through brief, intuitive descriptions of the methods used, non-technical readers can appreciate the contents of the report.

2. **Completeness of Results [10 marks]:** The results presented in the report address all required points enumerated in Section 3.2.

3. **Quality of Discussion of Results [20 marks]:** The discussion and analysis section addresses all required points enumerated in Section 3.2. Furthermore, key points and insights raised in this section are clearly evidenced by outputs in the results section. To this end, you must explicitly refer to your results to reinforce your discussion points.

   The discussion and analysis clearly recognizes the strengths and limitations of the methods used in the project, addresses the practical implications of these strengths and limitations, and explores potential areas to improve the risk modelling methods.

4. **Quality of Executive Summary and Conclusions [10 marks]:** The executive summary and conclusion sections of the report provide a clear and succinct description of the purpose of the report and the key findings and insights arising from the analysis.

5. **Overall Form and Organization of the Report [10 marks]:** The report is written in a manner fit for presentation in a professional setting. Tables and figures are clearly formatted, labelled, and captioned. Equations are properly typeset. All references and resources used are acknowledged.

References

Hull, J. C. (2022). *Options, Futures, and Other Derivatives*. Pearson Education Ltd., Essex, United Kingdom, 11th edition.