Haskelleene: A Very Haskell Implementation of Regular Expressions and Finite Automata Equivalence

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Introduction

"Automata are pretty cool."

- Liam Chung

Motivation: What Do We Do These Things That We Do?

this is some test that is gonna go here to talk about why we do be doing things that we do be doing

Motivation: more stuff?

look at this this is more stuff that we do
wanna see a trick?
check this out:

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Deterministic Automata

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For instance, the following represents a NA:

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1 \arrow[r, shift left, "\epsilon"] \arrow[d, "b"] & 2 \arrow[l, shift left, "c"] \arrow[loop above, "b"] \arrow[dl, "a"] \arrow[d, "\epsilon"] \\
3 & 4
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Our Haskell implementation of the type of NA:

```
data NDetAut l s = NA { nstates :: [s]
                      , naccept :: [s]
                      , ndelta  :: Maybe l -> s -> [s] }
```

Semantics of Non-Deterministic Automata

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A NA accepts an input string u if there is a *possible* path that terminates at an accepting state.

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Suppose the initial state is 1, and the only accepting state is 4:

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1 \arrow[r, shift left, "\epsilon"] \arrow[d, "b"] & 2 \arrow[l, shift left, "c"] \arrow[dl, "a"] \arrow[d, "\epsilon"] \arrow[loop above, "b"] \\
3 & \color{red}{4}
\end{tikzcd}$$

Then this NA accepts

$$\epsilon, b, c, bc, \cdots$$

Algorithm for Running NA

General Idea

Searching paths in a finite graph in general requires a lot of computational resources. However, we do not need to output the whole path, thus we only keep track of how states transform.

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Active states: $([ba], 1)$.

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Active states: $([a], 3), ([ba], 2)$.

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Searching paths in a finite graph in general requires a lot of computational resources. However, we do not need to output the whole path, thus we only keep track of how states transform.

Say we run the input *bca* on the previous example:

$$\begin{tikzcd}
1 \arrow[r, shift left, "\epsilon"] \arrow[d, "b"] & 2 \arrow[l, shift left, "c"] \arrow[dl, "a"] \arrow[d, "\epsilon"] \arrow[loop above, "b"] \\
3 & 4
\end{tikzcd}$$

Active states: $([a], 3), ([], 3), ([a], 4), ([ba], 4)$.

Haskell Implementation of Semantics for NA

The function `runNA` is defined as follows:

```
runNA :: (Alphabet l, Ord s) =>
         NDetAut l s  -> s -> [l] -> [([l], s)]
runNA na st input =
  case input of
    [] -> ([],) <$> epReachable (ndelta na) st
    (w:ws) -> concatMap (\s -> runNA na s input) nsucc ++
              case wsucc of
                [] -> [(input,st)]
                ls -> concatMap (\s -> runNA na s ws) ls
      where wsucc = ndelta na (Just w) st
    where   nsucc = ndelta na Nothing  st
```

Here the function `epReachable` calculates all the states that is reachable from the current state via ϵ-transitions.

Equivalence between DA and NA

Evidently, any DA is a NA. On the other hand, we can simulate running NA deterministically, basically via the same idea as `runNA`:

- States are subsets of states of a NA.
- A subset is accepting iff it contains some accepting state.
- Under an input I, a subset transforms to those states reachable from some state via I (with ϵ-transitions).

Haskell Implementation

```
fromNA :: (Alphabet l, Ord s) =>
          NDetAut l s -> DetAut l (Set.Set s)
fromNA nda = DA { states = Set.toList dasts
                , accept = Set.toList $ Set.filter
                  acchelp dasts
                , delta = fromTransNA ntrans
                }
  where ndasts = nstates nda
        dasts  = Set.powerSet $ Set.fromList ndasts
        ndaacc = naccept nda
        acchelp set = not $ Set.disjoint set
                          $ Set.fromList ndaacc
        ntrans = ndelta nda

fromTransNA :: (Alphabet l, Ord s) =>
               (Maybe l -> s -> [s]) -> l -> Set.Set s ->
                 Set.Set s
fromTransNA ntrans sym set = result
  where starts = listUnions (epReachable ntrans) set
        step = listUnions (ntrans $ Just sym) starts
        result = listUnions (epReachable ntrans) step
        listUnions f input = Set.unions $ Set.map Set.
            fromList $ Set.map f input
```

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The Theorem

Theorem

a theorem go here?