Backtracking 04
Bits

Dp with Bitmask

man ↓ , woman →

man \ woman	1	2	3	4	5
1	1	0	1	1	0
2			0		
3					
4					
5					

N x N

$a_{ij} \to \begin{cases} 1 & \to i^{th} \text{ man and } j^{th} \text{ woman are compat} \\ 0 & \to \text{not compat} \end{cases}$

Total possible ways to pair up them

$f(1, \{1, 2, 3, 4, 5\}) \Rightarrow f(2, \{1, 2, 3, 5\})$

$$f(i, W) = \sum_{c[i,x]=1} f(i+1, W - \{x\})$$

# of ways to make a valid pairing such that men [i, N] & women in the set W are available

$\forall x$ which are compatible with the i^{th} man

Dynamic Bitmask

$\{1, 2, 3\} \longrightarrow 1\ 1\ 1 \longleftarrow \longrightarrow 7$

1st ne $\longrightarrow$ 2nd wcma

$\{1, 2, 3\} \longrightarrow 1\ 0\ 1 \longrightarrow 5$

1st ne $\rightarrow$ 3rd nee

$\{1, 2, 3\} \longrightarrow 1\ 1\ 0 \longrightarrow 6$

2 variable $\Rightarrow$ 2d dp

$f(i, mask) = \sum f(i+1, mask - \{x\})$

x

rule

story

(1,2) (2,3)

(1,3) (3,1)

mask = $\boxed{0\ 1\ 0}$ → 2

$f(1, 7) \rightarrow f(2, 5) \rightarrow f(3, 4)$ - - -

$f(2, 6) \rightarrow f(3, 2)$ . . .

_ _ _ _ _ .... _ 21 bits

N bits

$N=4$ → 1111 → 15 $2^N - 1$

$f(1, \boxed{mask})$

$f(1, (1111)_{10})$

$(1 << N) - 1$

110101 &

5 4 3 2 1 0

000100

001000

≥ 0

0

$w = 1 \rightarrow n$

2-1

$((1 << (w-1)) \;\&\; \text{mask}) == 0$

X

110101 $\rightarrow$ 110001

$$
\begin{array}{rcccccc}
 & 1 & 1 & 0 & 1 & 0 & 1 \\
\wedge & 0 & 0 & 0 & 1 & 0 & 0 \\
\hline
 & 1 & 1 & 0 & 0 & 0 & 1
\end{array}
$$

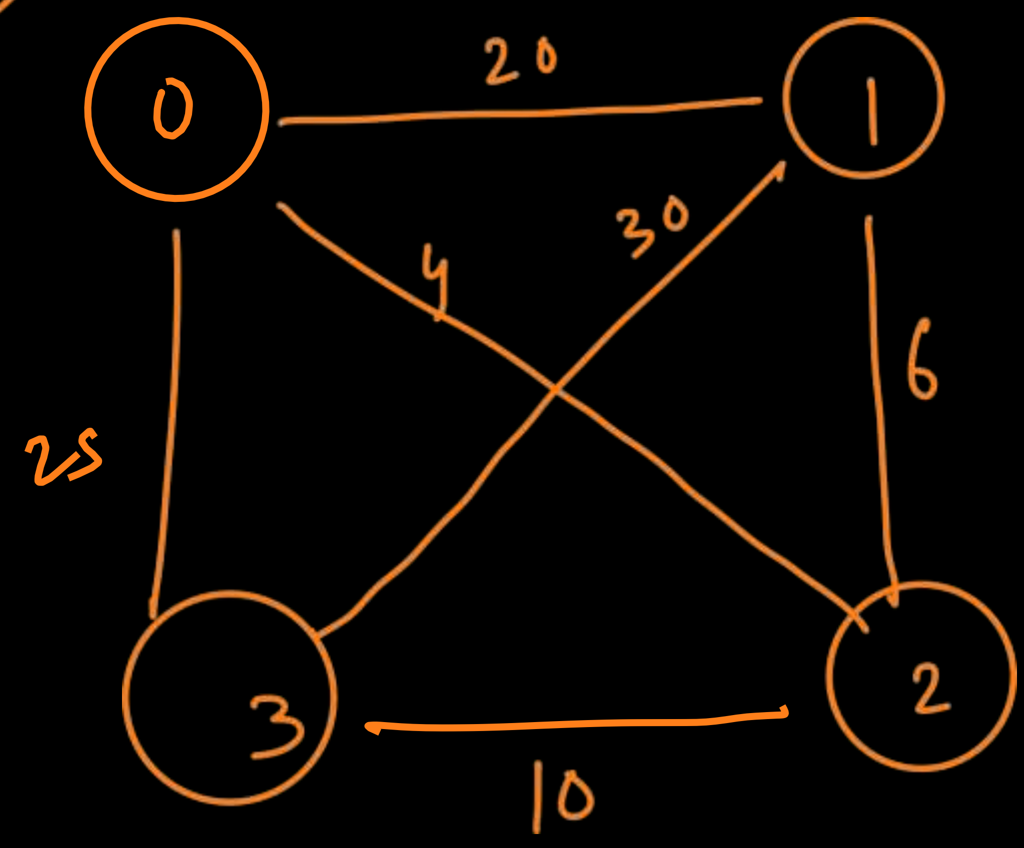
$1 << (2)$

Dp wale Bitmask ← Travelling Salesman Problem → graph

2

Calc the min wt hamiltonian cycle in the graph.

adj matrix



hamiltonian cycle is defined as the set of edges where every node once & after traversing all the nodes we come back to the source.

Brute force

0 - 1 - 2 - 3 - 0

0 - 2 - 3 - 1 - 0

↳ permutation

1 - 2 - 3 - 0 - 1

1 - 3 - 2 - 0 - 1

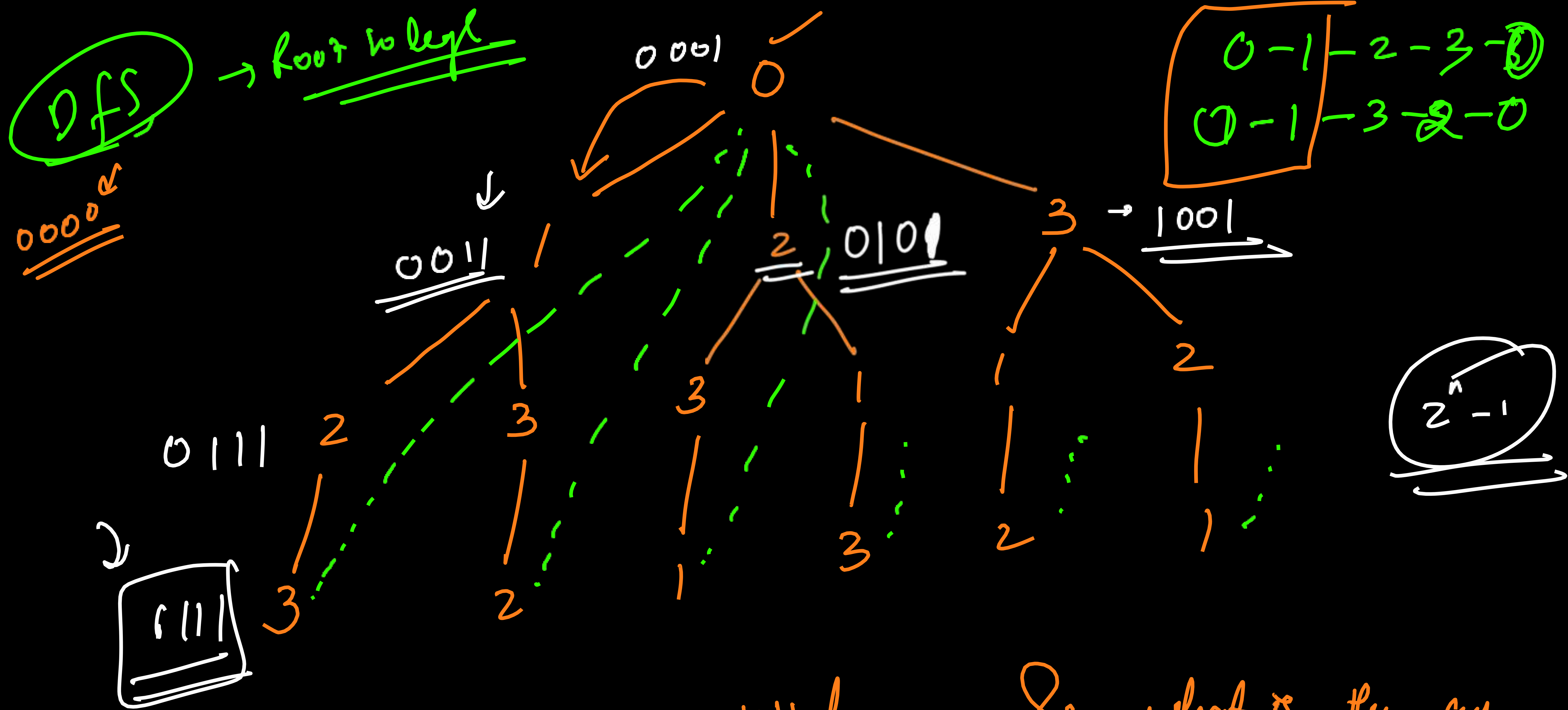
Let's pick any node as src node.

0 — 1 — 2 — 3 — 0

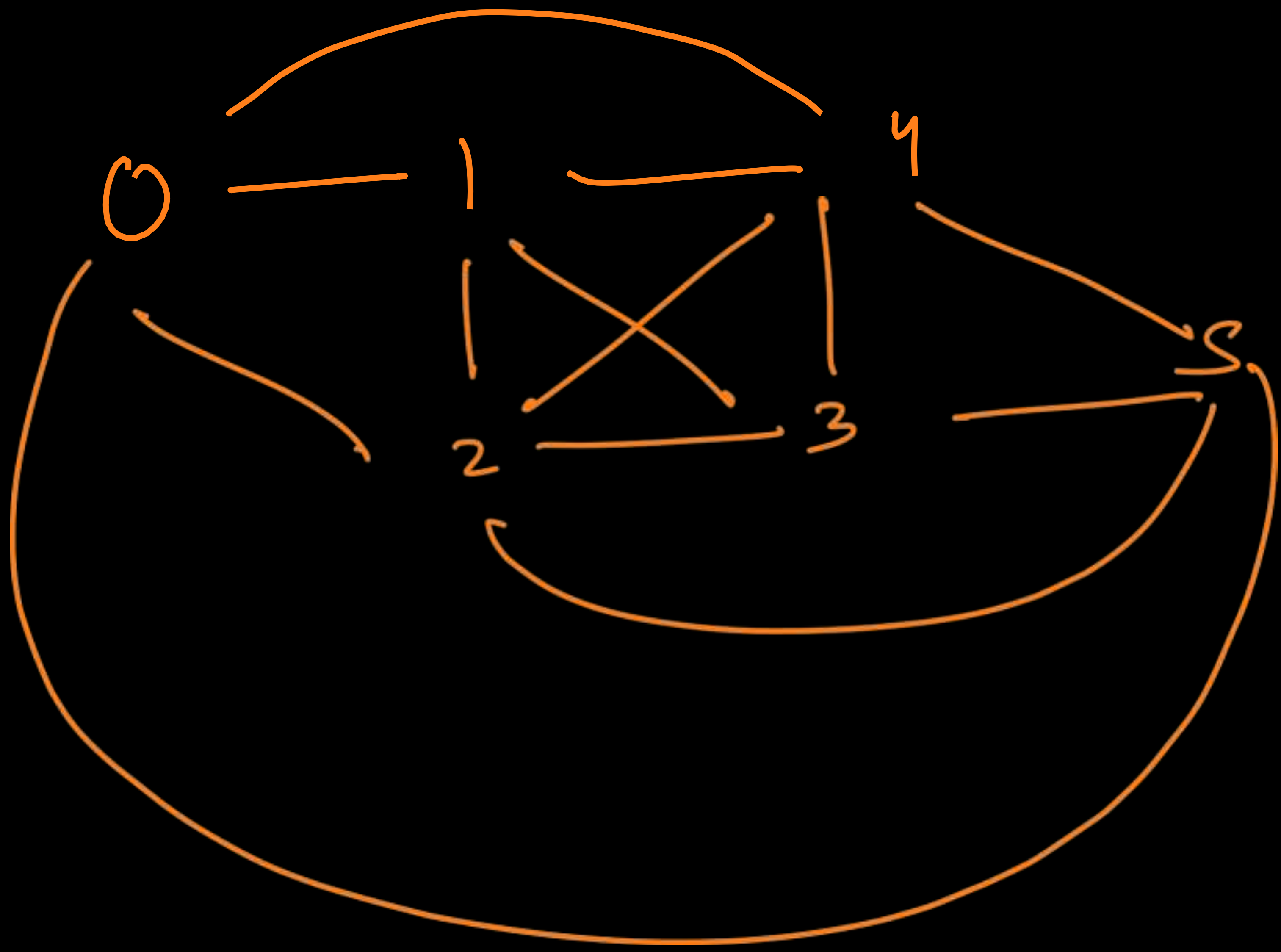
1 — 2 — 3 — 0 — 1

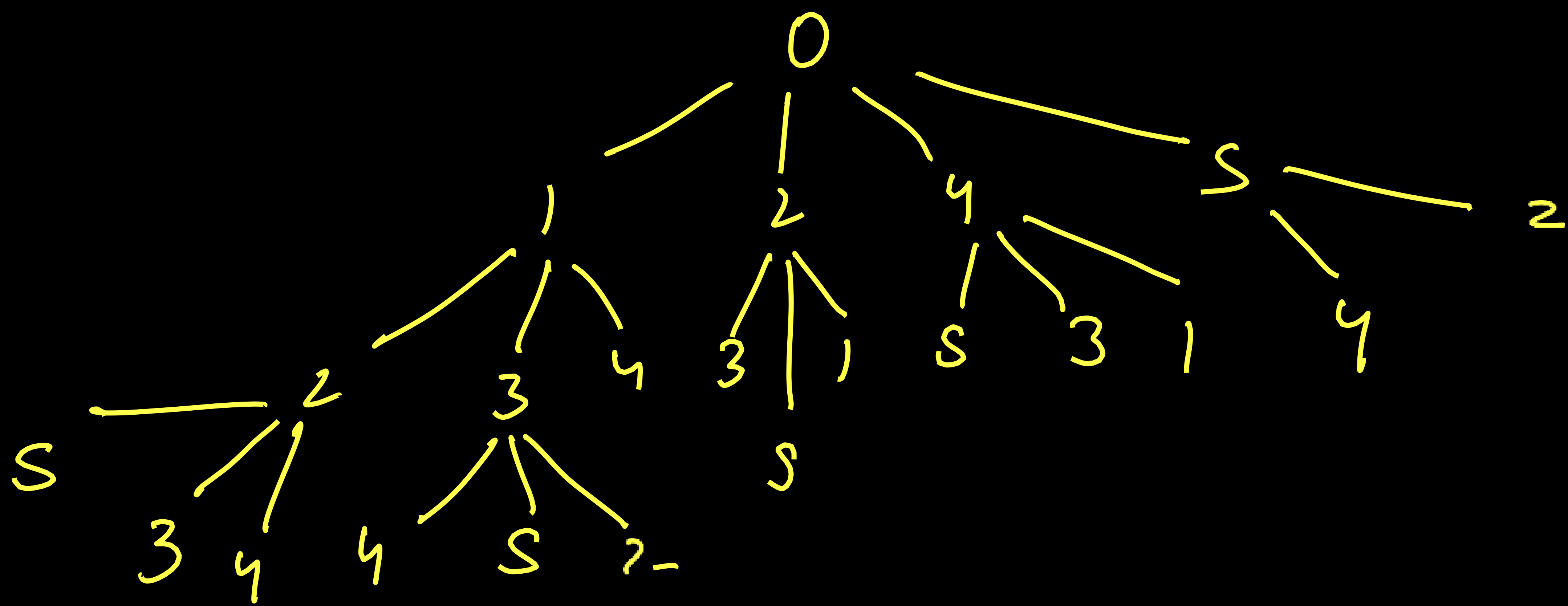
0 - 2 - 3 - 1 - 0

3 - 1 - 0 - 2 - 3



what all nodes are visited, & what is the cu noc


0
1
4
2
3
5


0
1
2
4
5
2
2
3
4
3
1
5
3
1
4
S
3
4
4
S
2
S

set

f(curr, visited { }) = min(f(neighbor, visited + curr))

?

min cost if we start from curr, with the given vis nodes

bitmask

binary → int

Backtracking with Bitmask

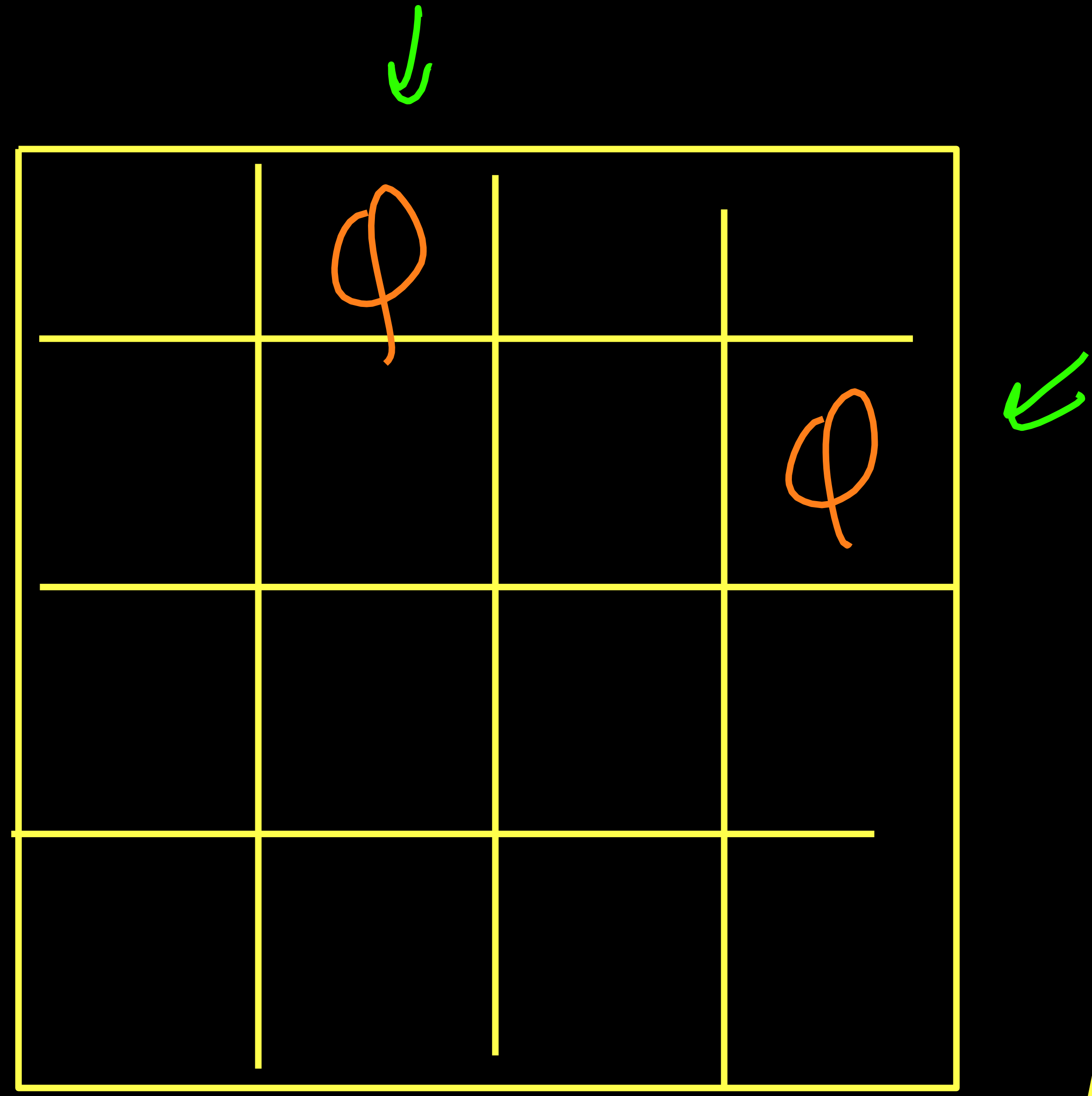
N queen

N bits

0 1 0 1

B) 0101

$$
\begin{array}{r}
0101 \\
\& \; 0100 \\
\hline
\end{array}
$$



N queens

N x N

Total l.d = $2 \times N - 1$ =

Total r.d

R.D

$\to$ R.D

$r + c$

	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	5
2	2	3	4	5	6
3	3	4	5	6	7
4	4	5	6	7	8

1 1 (9)

5 4 3 2 1 0

$r - c + n - 1$

$1 - 0 + 4$

	0	1	2	3	4
0	4	3	2	1	0
1	5	4	3	0	1
2	6	5	4	3	2
3	7	6	5	4	3
4	8	7	6	5	4

codes

_ _ _ _ 1 _ _ _ _

4 3 2 1 0

$x = r + c$

$r \cdot d \cdot mask = rdmask \mid (1 << x)$

$\boxed{x = r - c + n - 1}$

$ldmask = ldmask \mid (1 << x)$

JOIN THE DARKSIDE

N rabbits

group

$g_1 \rightarrow$ ____

$g_2 \rightarrow$ ____

$g_3 \rightarrow$ ____

$N=3 \rightarrow$

$\{1\}\ \{2\}\ \{3\} \rightarrow$ 1 way

$\{1,2\}\ \{3\} \rightarrow$ 1 way

$\{1,3\}\ \{2\}$

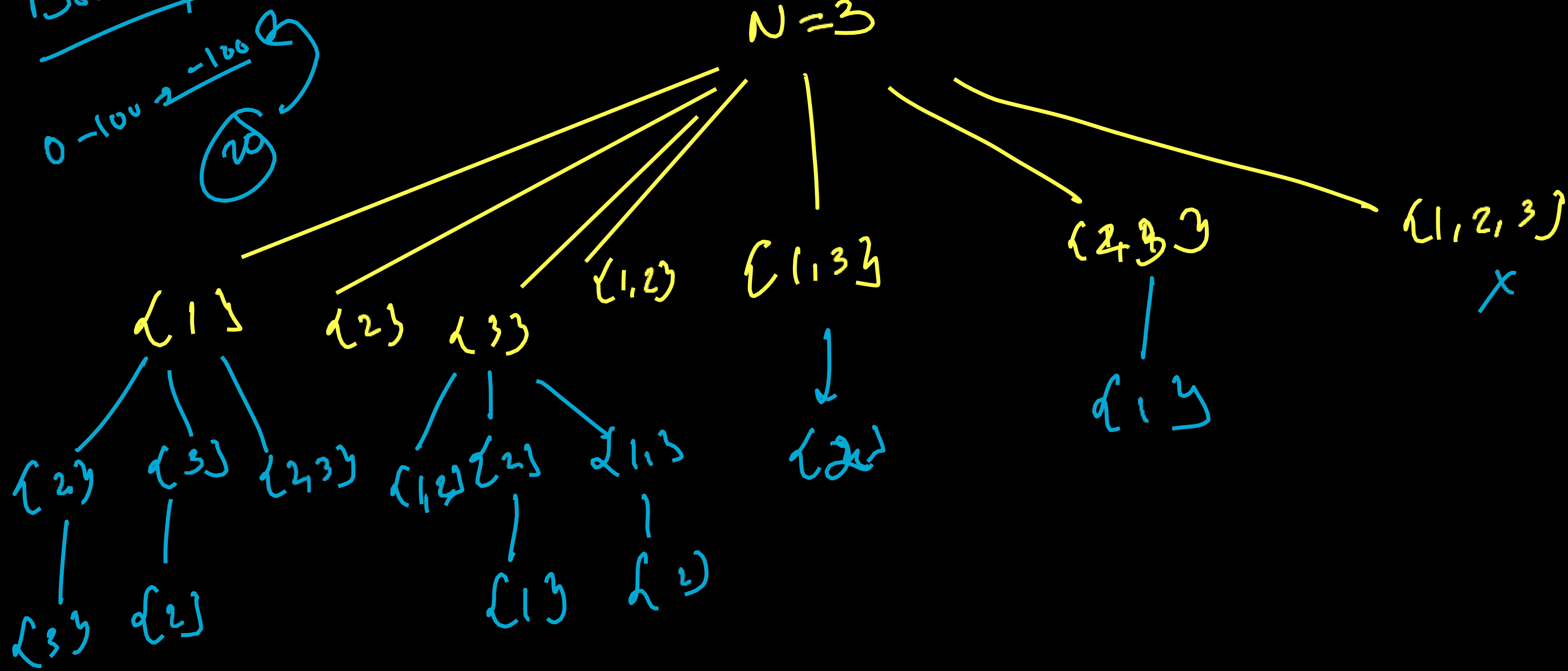
$\{2,3\}\ \{1\}$

$\{1,2,3\}$

Brute fa

0 - 100 2 - 100

20



1 0 0 1 h

R ~ ~ R

1 0 1 0 1 →

$a_{s3} + a_{s1} + a_{31}$

$$f(S) = \max\Big(f(S - G) + \underline{\text{sum}(G)}\Big)$$

$S \rightarrow$ bit

2

$S - G \rightarrow$ bitmask

max possible score achievable by grouping rabbits in set $\underline{S}$.

$G \rightarrow$ all possible groups

$\{r_1, r_2, r_3\} \rightarrow \boxed{a_{12} + a_{23} + a_{13}}$

$\rightarrow$ bitmask

$\underline{1111} \rightarrow$

$f(2^n - 1)$

JOIN THE DARKSIDE

$2^3 - 1 \rightarrow (111)_2 \rightarrow (7)_{10}$

$R_1 R_2 R_3$

R_1 R_2 R_3 R_2R_3 R_1R_3 R_1R_2 $R_1R_2R_3$

10110 → 10010 → 10100 → 00110 —

2

$g = mask$; $g != 0$; $g = (g-1)$ & mask

101010
101001

111
110

110

101
111

101

g = 111 → 111

110 → 110

b = b-1

11010 → 11001

1111

1101

0010

16 + 8 + 2

16 + 8 + 1

20 → 25

11111 → 11110 → 11101 → 11100

11011 → 11010 → 11001

15 14 13 12

1111 → 1110 → 1101 → 1100 → 1011

→ 1101 S

↓

S = 1101

→ 1100 & 1101

1100 → 1011 & 1101

1001

$R_4\ R_2 = R_3\ R_4$

1 1 1 1

R_3

R_4

$\wedge \quad R_1\ R_2$

1 1 0 1

0 0 1 0

R_3

Q2

10110

10101

10100

THANK YOU