Merge Sort - 1

Lecture-31

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Already discussed sorting algorithms

Revisiting their time complexity!

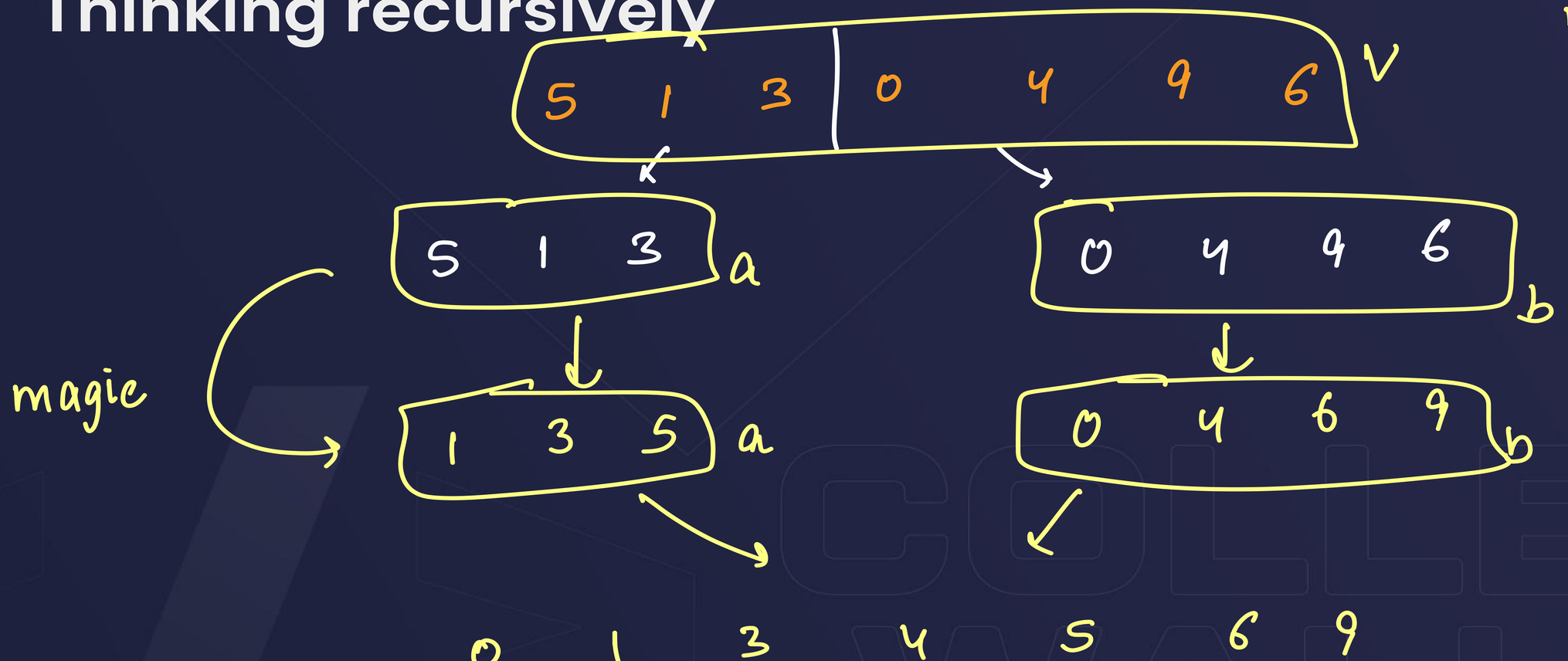
Bubble Sort	$O(n^2)$
Selection Sort	$O(n^2)$
Insertion Sort	$O(n^2)$
Merge Sort	$O(n \cdot \log n)$
Quick Sort	$O(n \cdot \log n)$

What if we had two sorted arrays?

Building the intuition

Introducing divide and conquer

Thinking recursively



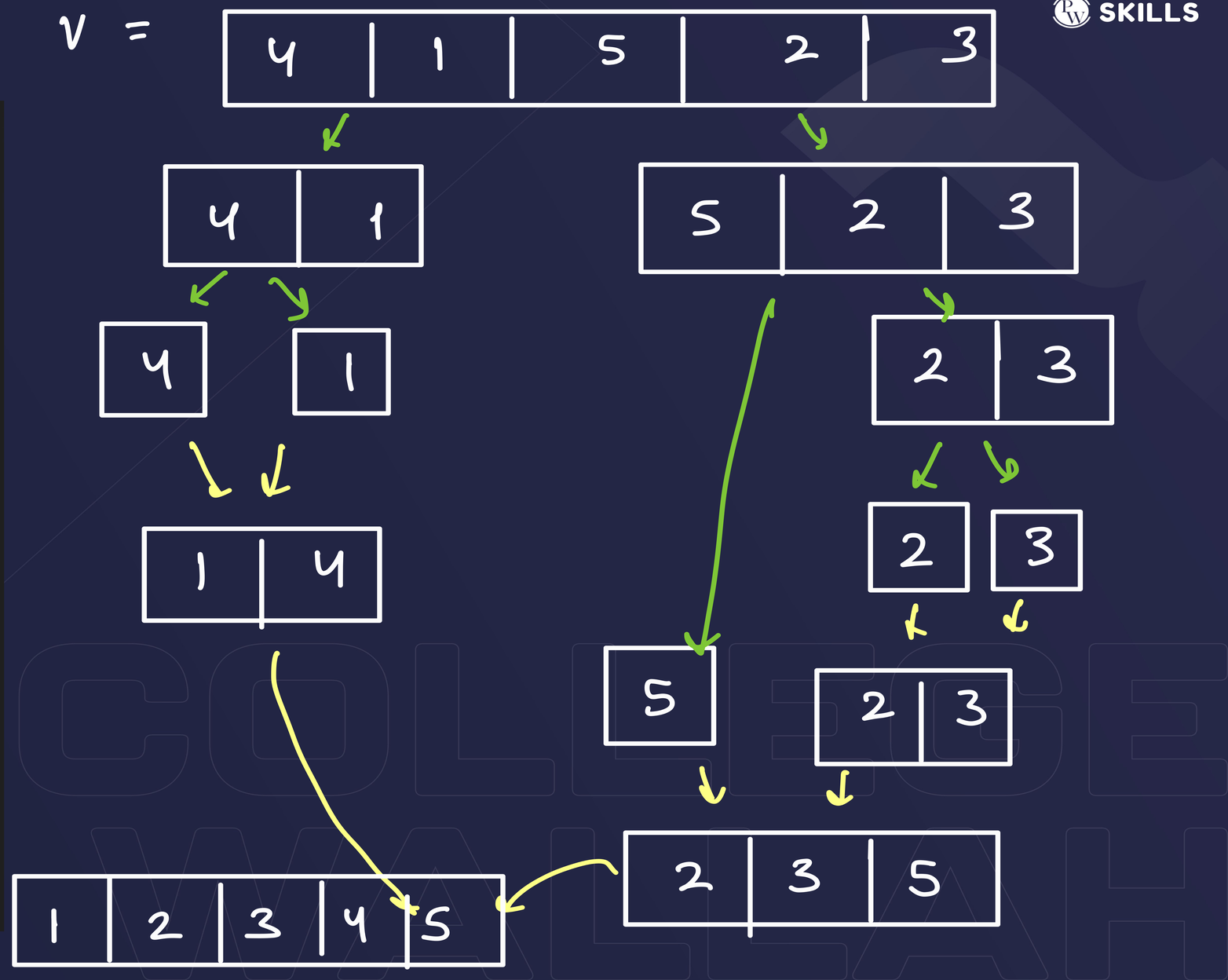
Merge sort algorithm

then apply magic

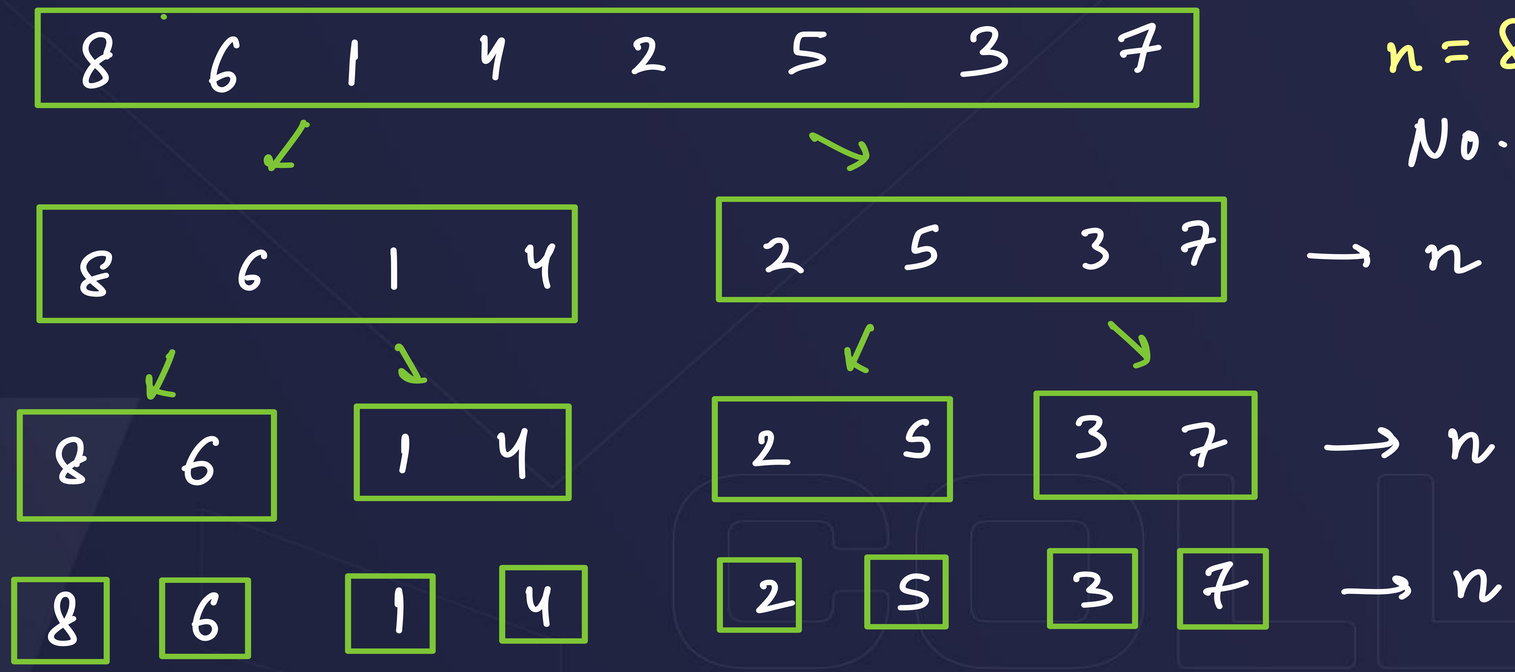
Dividing the array into two equal parts and then merging them.

Code for Merge Sort

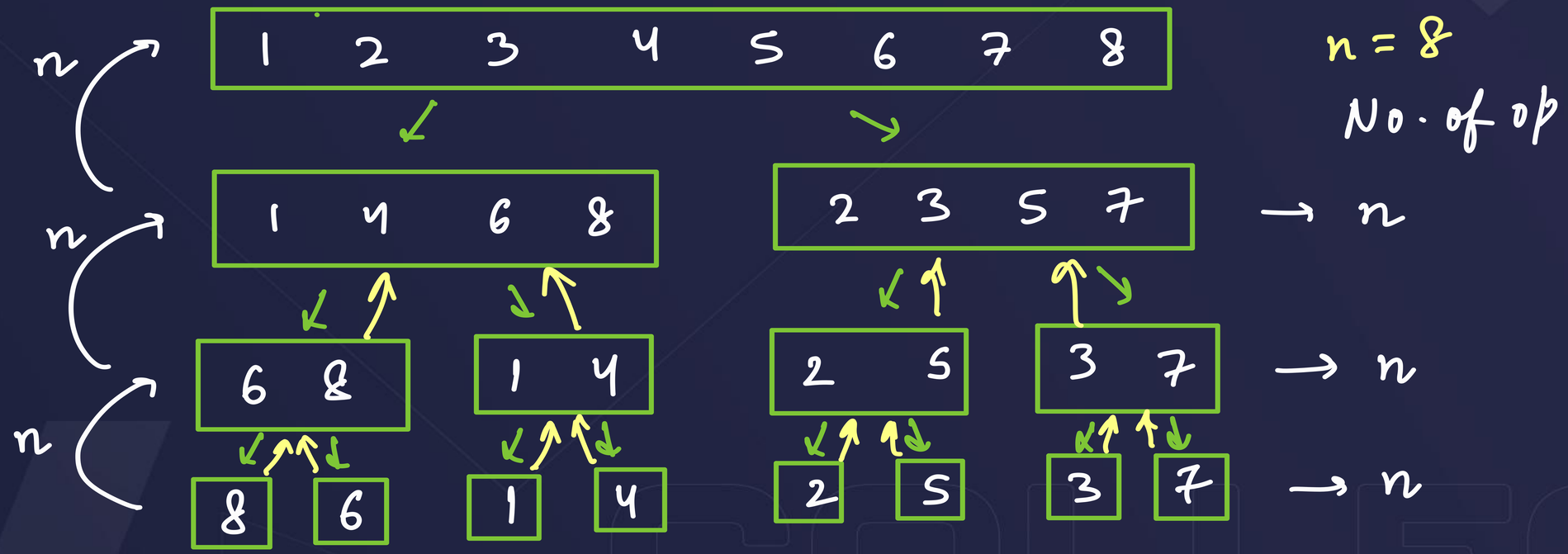
```
void mergeSort(vector<int>& v){
    int n = v.size();
    if(n==1) return;
    int n1 = n/2, n2 = n - n/2;
    vector<int> a(n1), b(n2);
    // copy pasting
    for(int i=0;i<n1;i++)
        a[i] = v[i];
    for(int i=0;i<n2;i++)
        b[i] = v[i+n1];
    // magic aka recursion
    mergeSort(a);
    mergeSort(b);
    // merge
    merge(a,b,v);
}
```



Does it improve the time complexity?



Does it improve the time complexity?



Total no. of operations = $Kn + Kn = X.(2n)$

T.C. = $O(Xn) \rightarrow O(n \cdot \log n)$

Does it improve the time complexity?

$n, \frac{n}{2}, \frac{n}{4}, \frac{n}{8} \ldots \ldots$

$\rightarrow \quad n, \underbrace{\frac{n}{2^1}, \frac{n}{2^2}, \frac{n}{2^3} \ldots \frac{n}{2^x}}_{x \text{ terms}}$

$\Rightarrow \quad \frac{n}{2^x} = 1 \Rightarrow n = 2^x$

$\Rightarrow 2^x = n$

$\Rightarrow \boxed{x = \log_2 n}$

$n = 128$

$\downarrow$

$B.S. = \frac{n(n-1)}{2} = 64 \cdot 127$

$= 8128$ ops

M.S $\rightarrow 2n \log_2 n$

$\Rightarrow 2 \times 128 \times \log_2 128$

$= 256 \times 7$

$= 1792$ operations

Does it improve the time complexity?

Yess!!

$n = 1024$

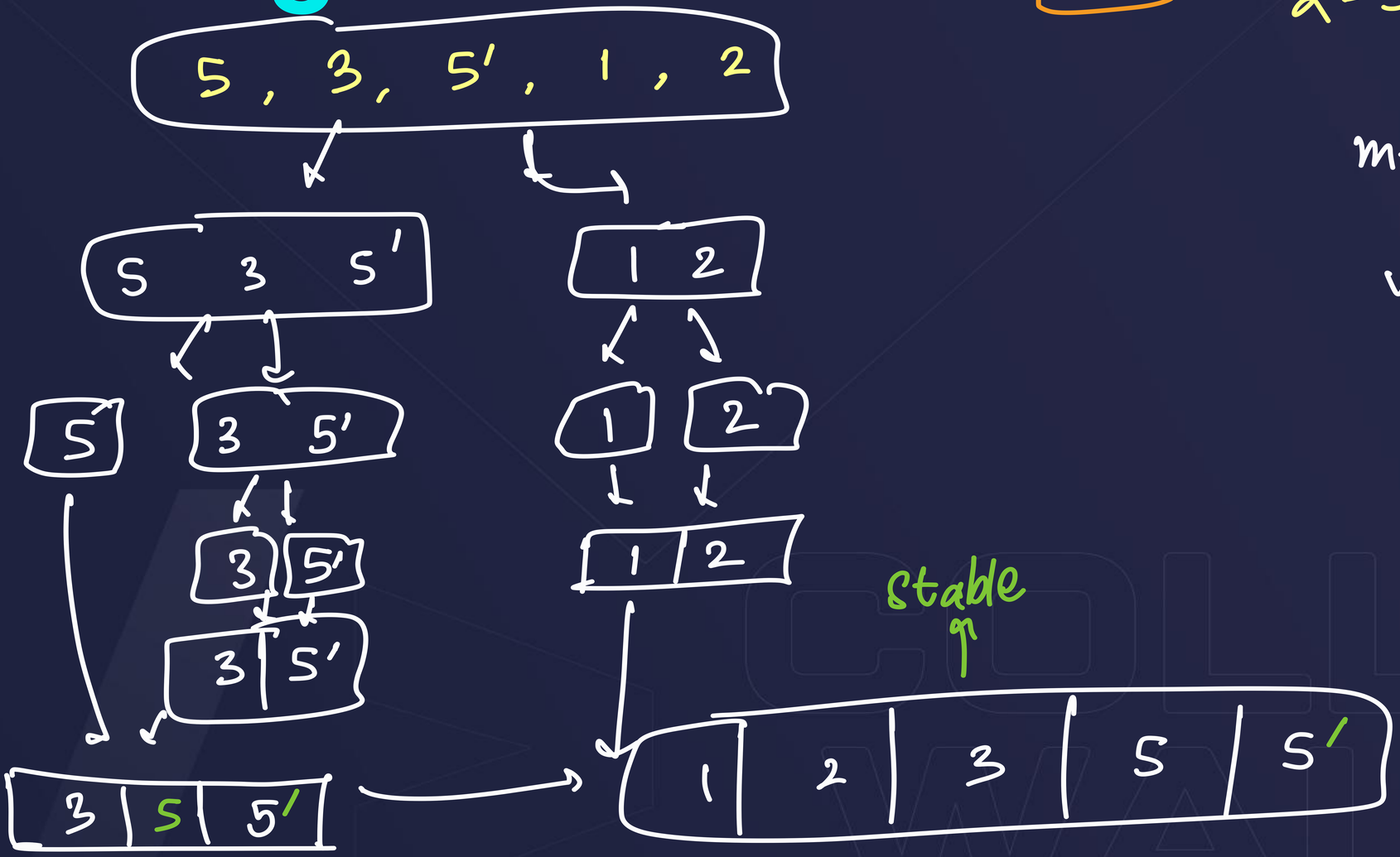
$B.S. = \frac{n(n-1)}{2} = \frac{1024}{2} \cdot 1023 = 5,23,776$ ops

$M.S. = 2 \times n \times \log n = 2 \times 1024 \times \log_2 1024$

$= 2048 \times 10$

$= 20480$ ops

Is merge sort stable? Yes



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Applications of Merge Sort

1) It is used in sorting linked list

2) It is used in count inversion problem

3) External sorting

Drawbacks of Merge Sort

- Time Complexity → $O(n \log n)$
- Space Complexity → $O(n \log n)$ → improved
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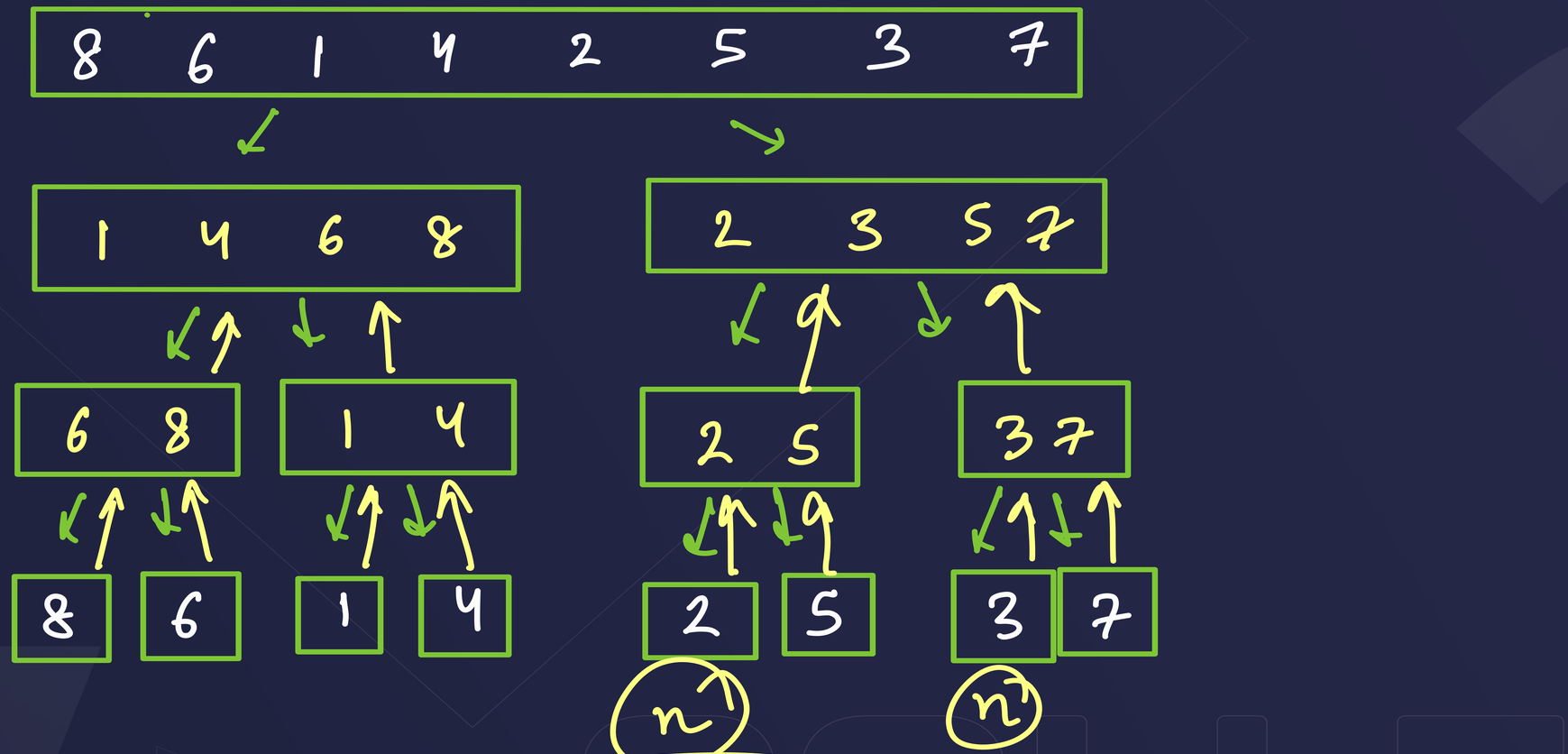
  - $O(n)$

Drawbacks of Merge Sort

Extra Space Used: $n + \frac{n}{2} + \frac{n}{4} \rightarrow < 2n \rightarrow O(n)$



Extra Space Used : $n + \frac{n}{2} + \frac{n}{4} + \frac{n}{4} + \frac{n}{2} + \frac{n}{4} + \frac{n}{4} \rightarrow 3n$

Practice

→ Homework

'Implement' merge sort algorithm to sort an array of elements in decreasing order.

Count Inversions

Two elements of an array a, a[i] and a[j] form an inversion if a[i] > a[j] and i < j. Given an array of integers. Find the Inversion Count in the array.

	0	1	2	3	4
a →	5	1	8	2	3

(5, 1)
(5, 2)
(5, 3)
(8, 2)
(8, 3)

Total no. of inversions = 5

a[0] > a[1]

0 < 1

M-I: Brute Force :

total no. of ops = $\frac{n(n+1)}{2}$

	0	1	2	3	4
a →	5	1	8	2	3

```
for(int i=0; i<n-1; i++){
    for(int j=i+1 ; j<n; j++){
        if(a[i] > a[j]) count++;
    }
}
```

T.C. = $O(n^2)$

S.C. = $O(1)$

What if...?

We have an array made up of two subarrays, both sorted.

What can be said about the inversions including a certain element?

{ 3 5 6 7 }
  i

{ 2, 2, 3, 4 }
      j

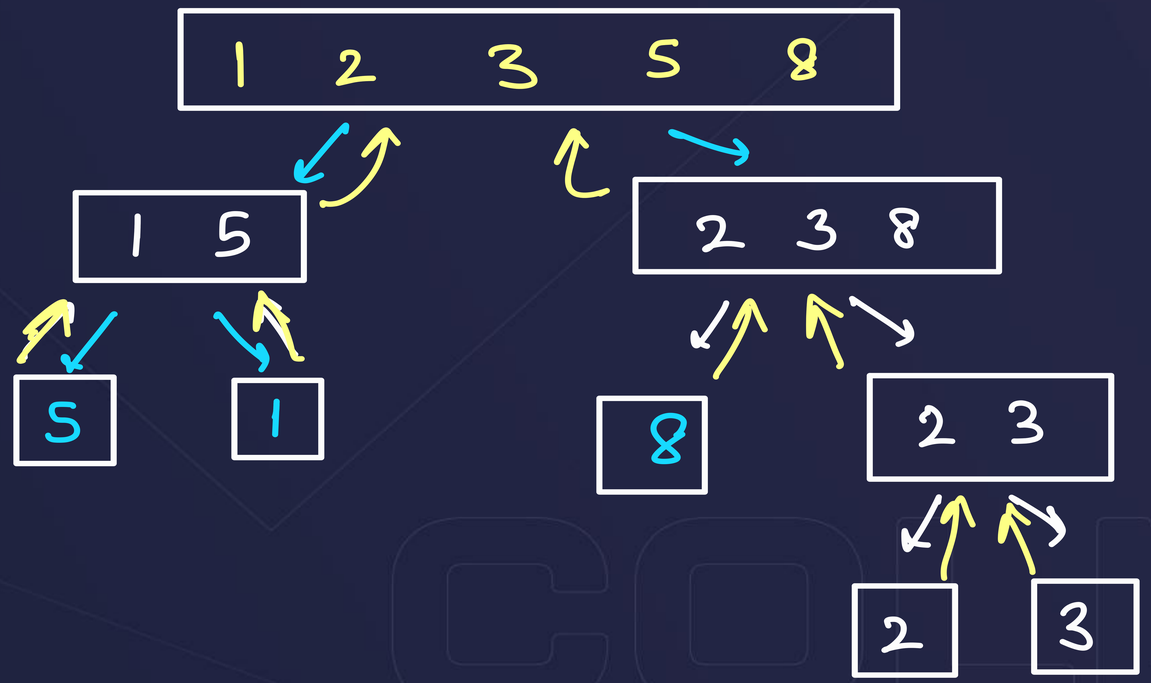
Count = 0 4 8 11 14 inversions

Recalling Merge Sort with this respect

The efficient solution

Algorithm

count = 0 1 2 3 4 5



Coding and dry run

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Time complexity

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Merge Sort

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T.C. = O(n log n)

S.C. = O(n)

Next Lecture

Next sorting algorithm: Quick Sort & Quick Select

THANK YOU