



# Dynamic Programming 1 (Part 2)



[1, 2, 3, 1]  
↑    ↑

[2, 2, 9, 3, 1]  
↑    ↑    ↑

[2, 1, 1, 9]  
↑    ↑

( 1 , 2 , 3 )

X X X

[13]

✓ X X

[13]

X ✓ X

[23]

X X ✓

[3]

✓ ✓ X

[1,2]

✓ X ✓

[1,3]

X ✓ ✓

[2,3]

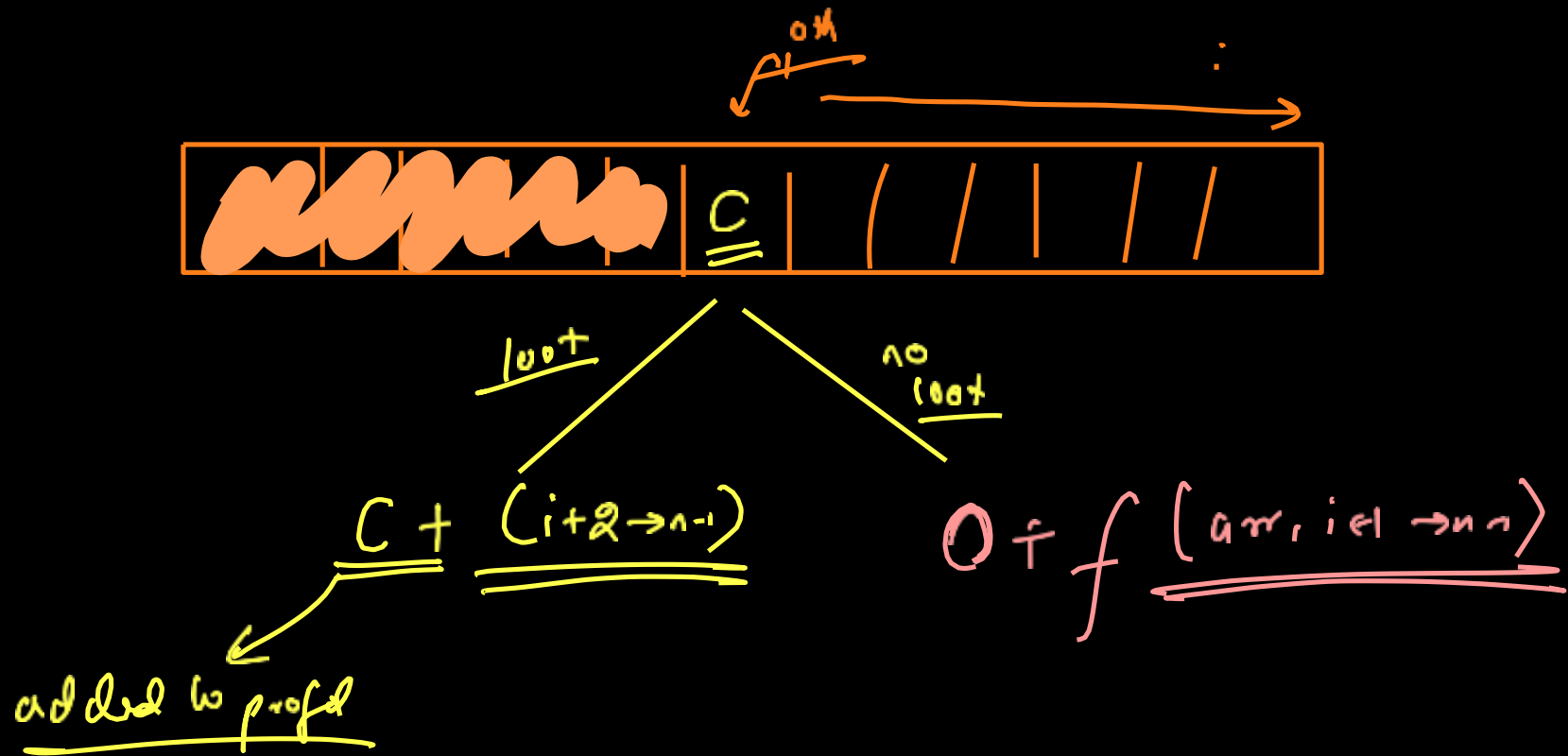
✓ ✓ ✓

[1,2,3]

→ we have a lot of ways to decide -

[1, 2, 3]  
(2<sup>3</sup>)

# Brute force → try all possibilities of loot



$$f(arr, i)$$

this recursive func<sup>n</sup>  
returns max profit

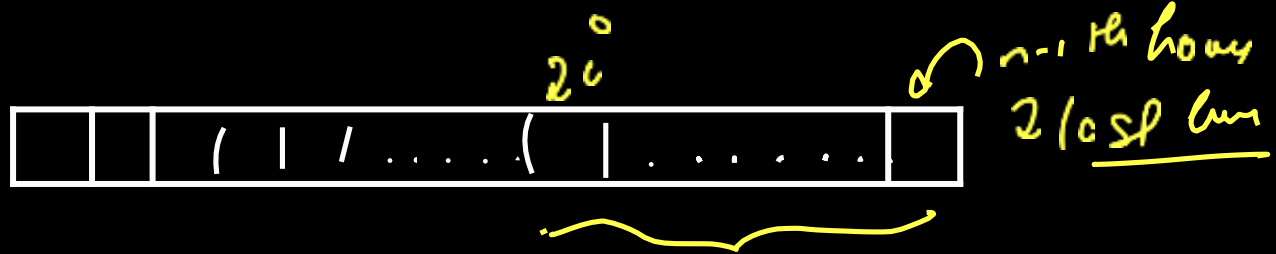
by looting houses from  
index  $i$  to  $n-1$ .

Such that no 2 adjacent  
houses are looted.

$$= \max \begin{cases} arr[i] + f(arr, i+2) \\ 0 + f(arr, i+1) \end{cases}$$

you decide to  
rob the  $i$ th  
house

you don't  
rob the  
 $i$ th house



final ans  $\rightarrow f(arr, 0) \leftarrow$  (call from main)

BC

if  $(i == n)$   
 $\rightarrow$  size arr  
return arr[i];  
if  $(i == n-1)$   
return  $\max(arr[i], arr[i+1])$ .

$dp \rightarrow \text{array}$   
 $dp[i] \leftarrow -1$   
 subproblem not  
 yet computed

$([2, 1, 1, 9], 0)$

$100+$   
 $2+$

$n/100+$   
 $0$

$([2, 1, 1, 9], 2)$

$([2, 1, 1, 9], 1)$

$1+$   
 $\checkmark$

$\times$   
 $\uparrow 9$

$\checkmark$   
 $1f \uparrow 9$

$\times 0$

$([2, 1, 1, 9], 4)$

$([2, 1, 1, 9], 3)$

$([2, 1, 1, 9], 3)$

$([2, 1, 1, 9], 2)$

B.C

$dp[i] \neq -1$

the state is already  
 computed. Return it

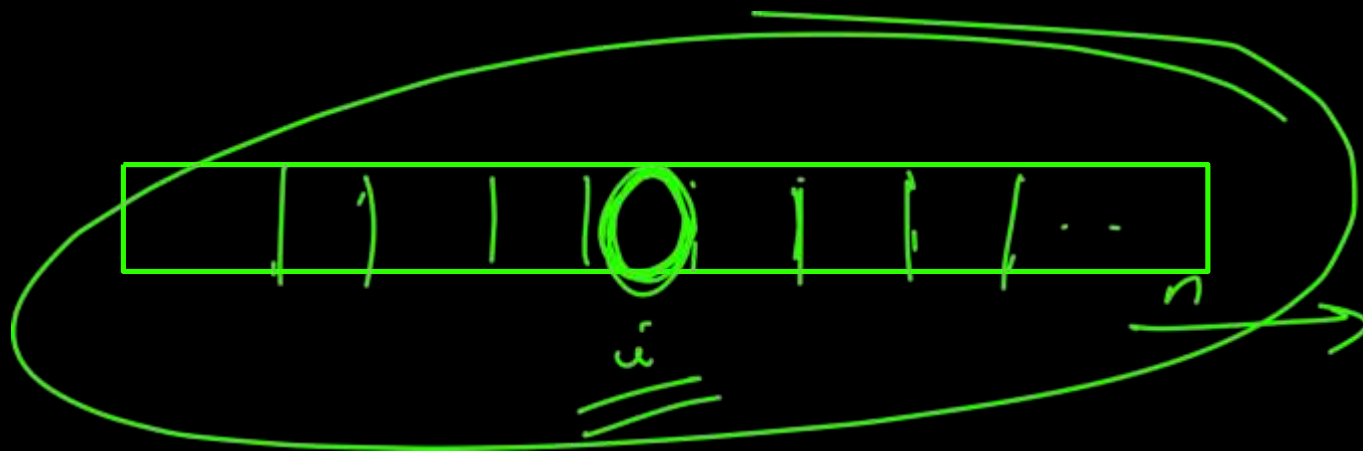
# State of Hdp  $\rightarrow$  a set of all the parameters using which we can identify a subproblem uniquely.

# How many unique subproblem will be there ??

$\Rightarrow$  no. of subproblems depend on  $i$   $[0, n-1]$

$\hookrightarrow$  Total  $n$  unique subproblems

1 variable  $\rightarrow$  1 Dimensional dp  $\rightarrow$  1d array



JOIN THE DARKSIDE



Bottom Up  
↓  
iterates

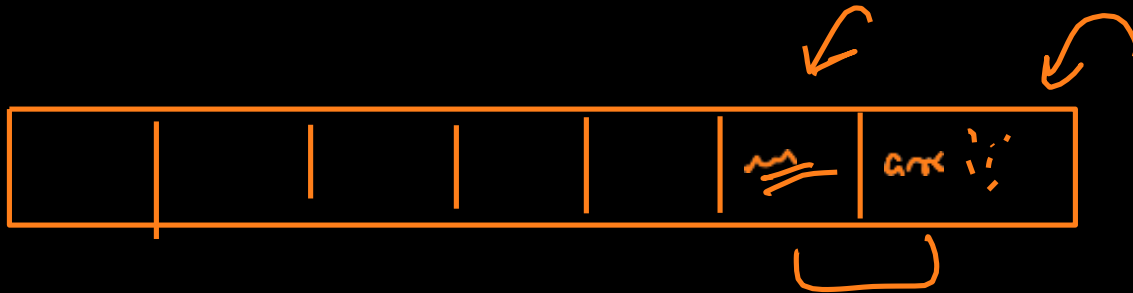


to calculate ans of any  $i^{\text{th}}$  state, we need to find  
ans of all the states  $\geq i$

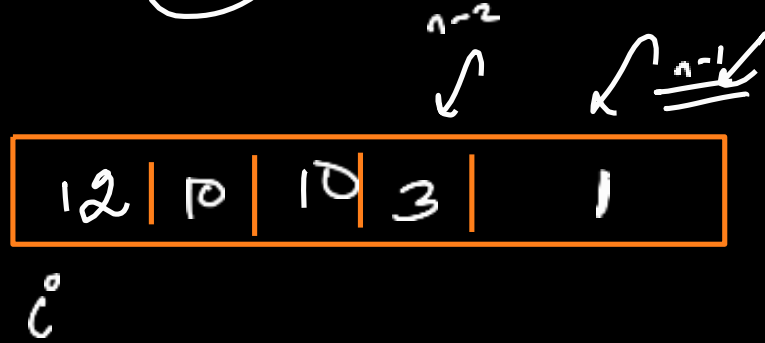
$$\underline{f}(\underline{arr}, \underline{i}) = \max \begin{cases} \underline{f}(\underline{arr}, \underline{i+1}) \\ arr[i] + \underline{f}(\underline{arr}, \underline{i+2}) \end{cases}$$

2  $\Downarrow$

$$dp(i) = \max \begin{cases} dp(i+1) \\ arr[i] + dp(i+2) \end{cases}$$

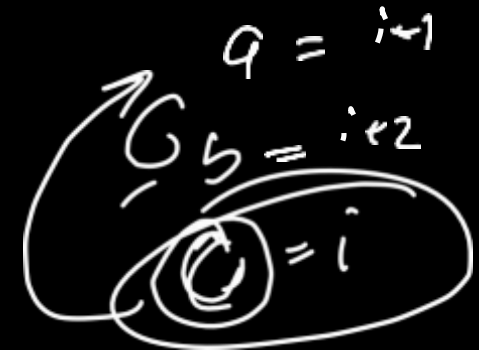


(2, 7, 9, 3, 1)



$f(arr, i)$   
profit for  $[0, i]$

$$dp(i) = \max \left( dp(i+1), dp(i+2) + arr[i] \right)$$



final ans  $\rightarrow$   $dp(0)$



$$f(arr, i) = \max \left( arr[i] + f(arr, i-2), \right. \\ \left. \underline{\underline{f(arr, i-1)}} \right)$$

if  $(i == 0)$  return  $arr[0]$   
 if  $(i == 1)$  max(-)

Q<sub>1</sub> Given a no.  $n$ , you can perform any of the following ops on it some no. of times.

①  $\rightarrow$  Reduce  $n$  to  $n-1$

②  $\rightarrow$  if  $n$  is divisible by 2 to make it  $n/2$

③  $\rightarrow$  if  $n$  is divisible by 3 make it  $n/3$

find out in how many minimum steps you can reduce  $n$  to 1.

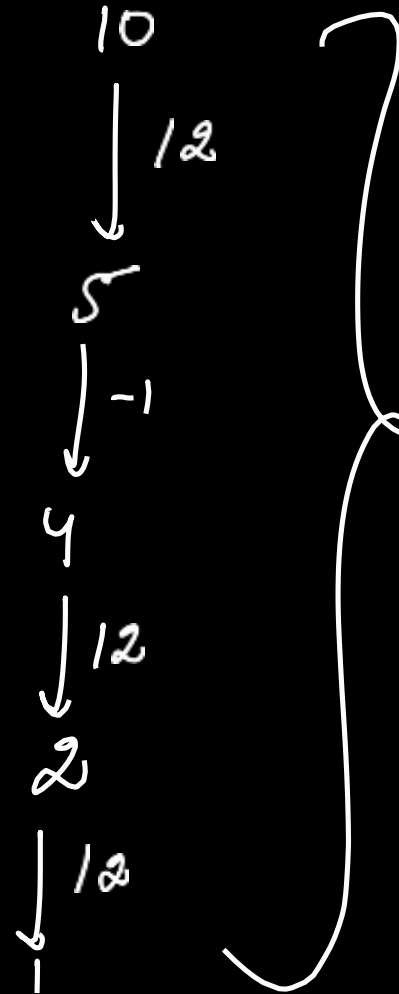
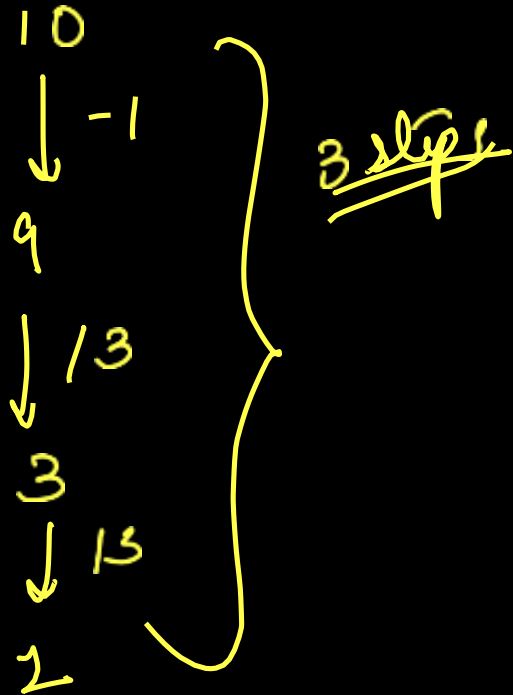
$$\underline{\underline{E_L}} \rightarrow n = 7$$

$$\begin{array}{c} 7 \\ \downarrow -1 \\ 6 \\ \downarrow /2 \\ 3 \\ \downarrow /3 \\ 1 \end{array}$$

$$\underline{\underline{ans}} \rightarrow 3$$

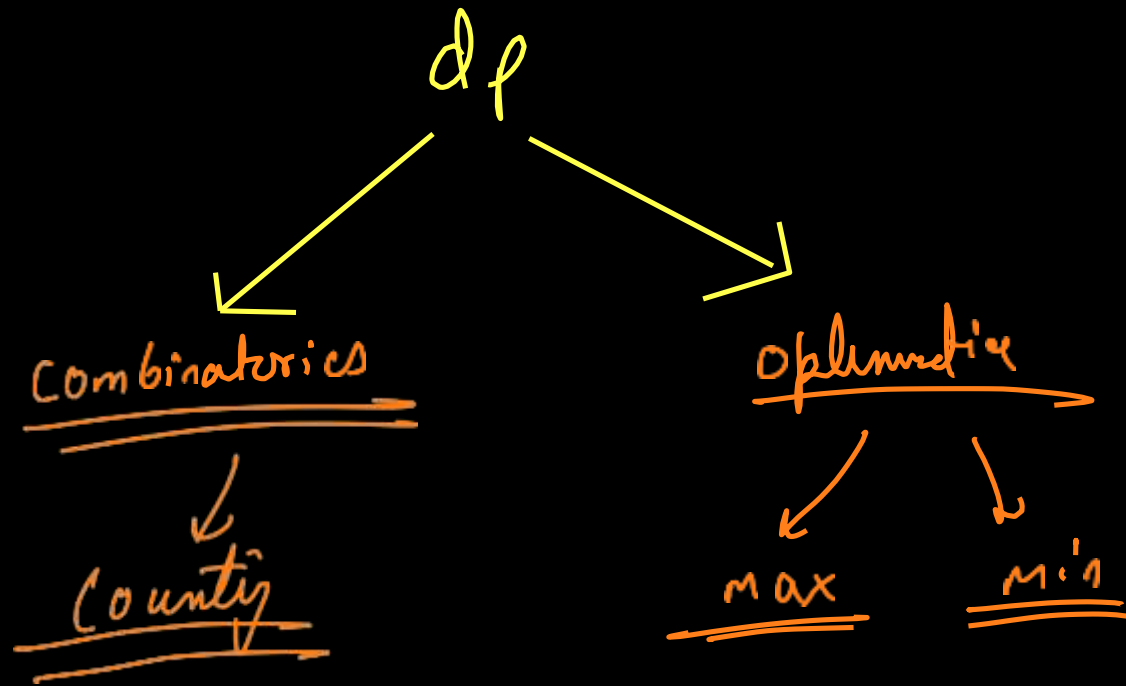
greedy will give u wrong ans.

$n = 10$



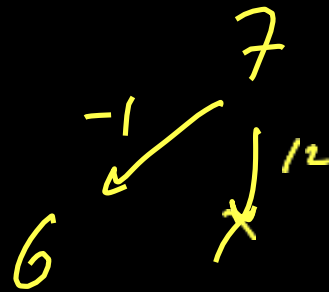
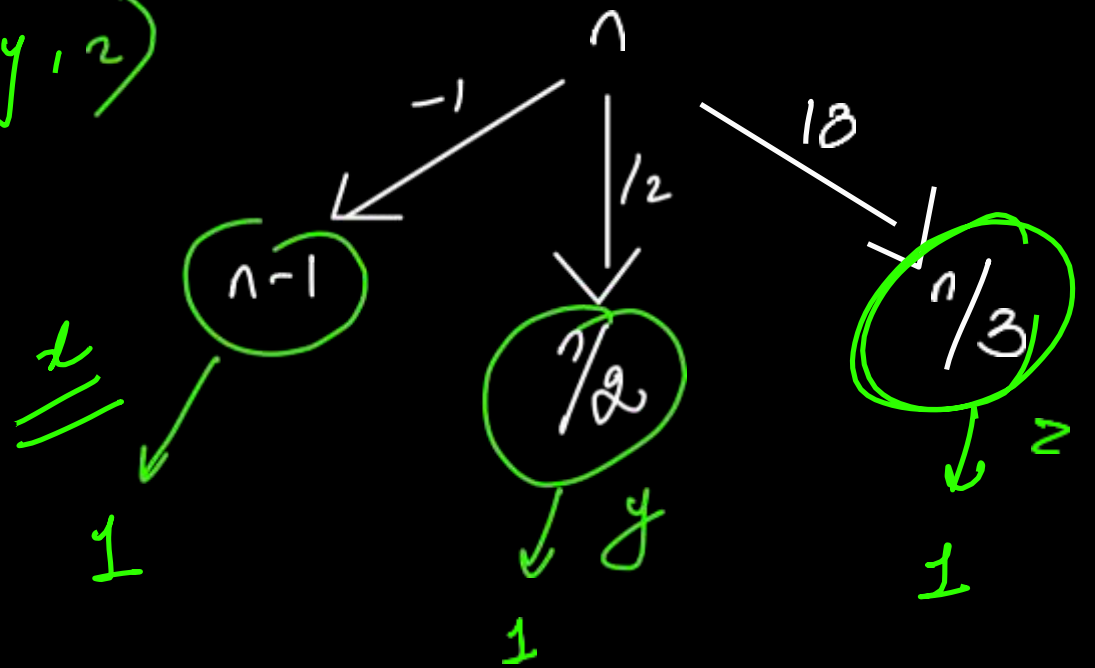
greedy + 4 steps

↳ dp → optimisation technique





$\min(x, y, z)$

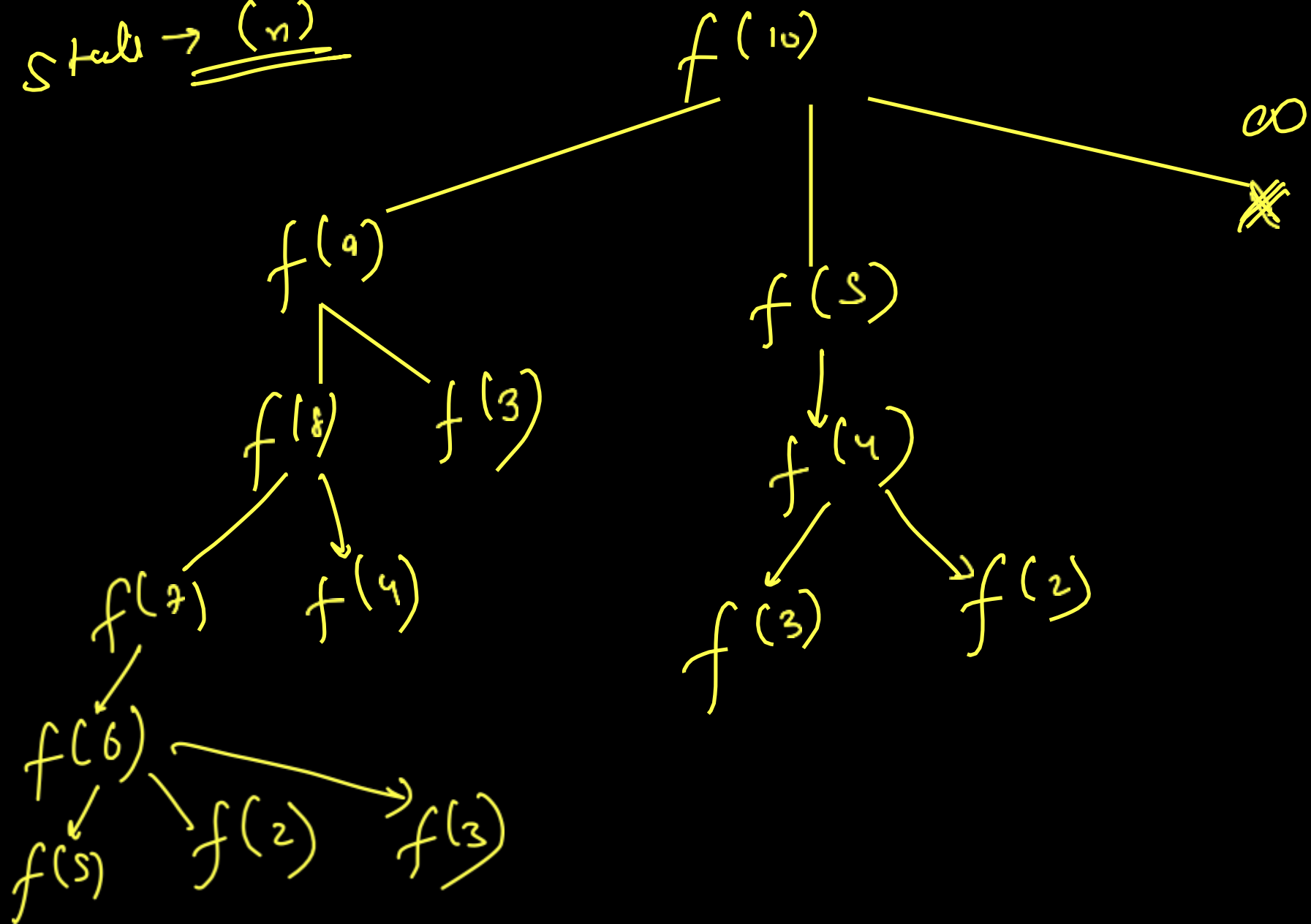


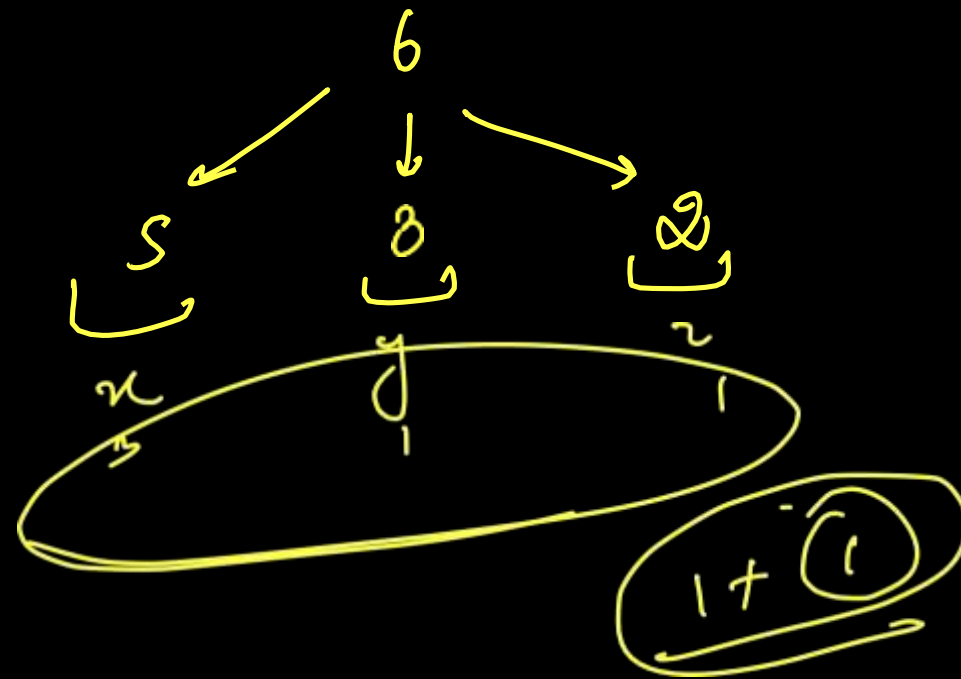
$$f(n) = 1 + \min \begin{cases} f(n-1) \\ (n \% 2 == 0) ? f(n/2) : \infty \\ (n \% 3 == 0) ? f(n/3) : \infty \end{cases}$$

$f(n)$   
 ↓  
 min steps to reduce  
 $n$  to 1.

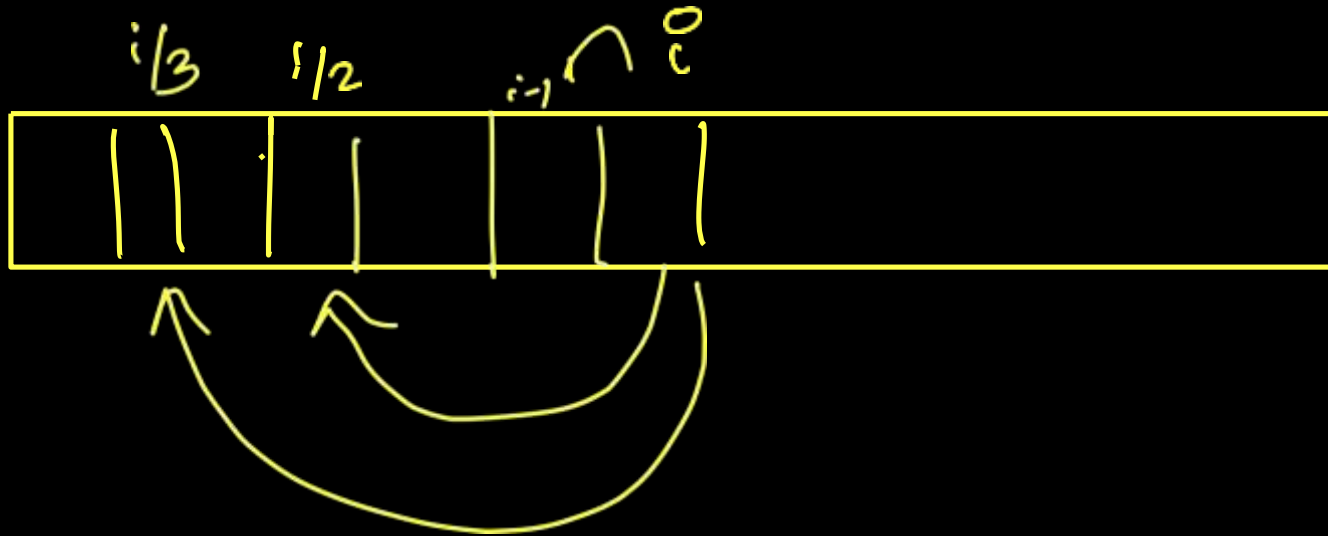
Base Case →  $n == 1 \rightarrow 0$   
 $n == 2 \rightarrow 1$   
 $n == 3 \rightarrow 1$

Starts  $\rightarrow \underline{\underline{(n)}}$





$$dp[i] = \min(dp[i-1], (i \% 2 == 0) ? dp[i/2] : \infty, \\ (i \% 3 == 0) ? dp[i/3] : \infty)$$



0	1	2	3	4	5	6	7	8	9	10
X	0	1	1	2	3	2	3	3		

10

n+1



▶ **THANK YOU** ◀