Time and Space Complexity

Lecture- 19

Raghav Garg

Ques : Given an array of size n+1 consisting of integers from 1 to n. One of the elements is duplicate in the array. Find that duplicate element.

int arr[n+1] Ex: arr =

6	1	2	4	3	2	7	5

M-I: Brute Force

Method-1 (Brute Force)

arr =

6	3	2	4	1	7	1	5

no. of ops = 7 + 6 + 5 + 4 + 2 = 24 operations

```
bool flag = false;
for(int i = 0; i< arr.size()-1 ; i++){
    for(int j = i+1; j < arr.size() ; j++){
        if(arr[i] == arr[j]){
            cout << arr[i];
            flag = true;
            break;
        }
    }
    if(flag == true) break;
}
```

Observations

1) Time Consuming → $O(n^2)$
2) Space Efficient → $O(1)$

Another method for previous problem

arr =

0	1	2	3	4	5	6	7
6	3	2	4	1	7	1	5

int check [8] =

0	1	2	3	4	5	6	7
0	1	1	1	1	0	1	1

Steps →
- 0 nai to 1 karo
- 1 nai to wahi duplicate
- Only 7 operations

Good Method ∆ it saves time

Problem : we are using extra space

Observations
1) Time Efficient : O(n)
2) Space Consuming → O(n)

* Time Complexity → Hardware se koi lena dena nahi hai

Method - 1

↓

13th Gen i9

↓

faster

Method - 2

↓

5th gen i3

↓

slower

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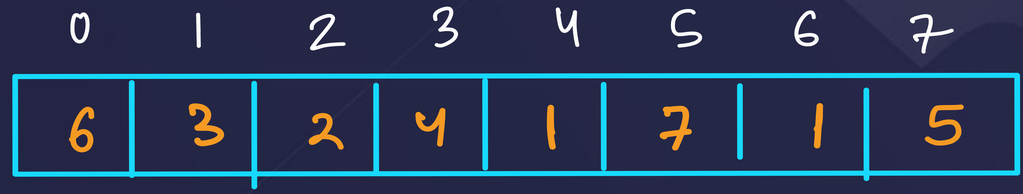
Solving the previous problem using maths

```
int sum = 0;
int n = arr.size() - 1;
for (int i = 0; i <= n; i++){
    sum += arr[i];
}
int s = n*(n+1)/2;
cout << sum - s;
```

arr =



No. of ops = 7 + 1

sum ⇒ 29
s = 28

29 - 28 = 1

Observations :

1) Time Efficient : O(n)

2) Space Efficient : O(1)

NOTE : Time Complexity can only be calculated on same programming device

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Notations for different types of Time Complexity → Jyada dhyaan nahi denge → Omega, Theta ❌

Big Oh Notation :

↓

upper bound

$O(n)$

$O(n^2)$

$O(\log n)$

$O(n^3)$

$O(2^n)$

$O(1)$

where → n is usually size of array / data structure

Ques : Calculate the time complexity for iterating in a loop.

```
for(int i = 0; i < n; i++) {
    cout << "PhysicsWallah\n";
}
```

5 operations ?

'n' times loop chalega

Ans : O(n)

n = 5

PW
PW
PW
PW
PW

What if this time we increment the pointer by 2?

$n = 10$

```
for(int i = 0; i < n; i+=2) {
    cout << "PhysicsWallah\n";
}
```

$i = 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10$

$\Rightarrow \frac{n}{2}$ iterations / rounds / ops

T.C. $O\left(\frac{n}{2}\right) \approx O(n)$

$O(Kn) \approx O(n)$
(K is constant)

PW
PW
PW
PW
PW

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```
for( int i = 1 ;  i <= n-7 ; i++ ){
|    cout << "PW";
3
```

n-7 iterations

⇒ T.C. $O(n-7) \approx O(n)$

$$O(n \pm K) \approx O(n)$$

1) $O(5n^3 + 3) = O(5n^3) = O(n^3)$

2) $O(6n^2 - 8) = O(n^2)$

3) $O(6n^2 + n) = O(n^2)$

4) $O(11n^{13/2} + 7n^4 - 2n^3 + 6n) = O(n^{13/2})$

$$O(k_1 n^m \pm k_2 n^{m-1} \pm k_3 n^{m-2}) \approx O(n^m)$$

Ques : Calculate the time complexity for traversing 2 arrays of size n and m.

```
int a[n], b[m];
for(int i = 0; i < n; i++) {
    a[i]++;
}
for(int i = 0; i < m; i++) {
    b[i]++;
}
```

→ 'n' times → O(n)

'm' times → O(m)

T.C. = O(n+m)

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Q11 Calculate the T.C. of this given code

```
for (int i = 1 ; i <= n ; i++){
|    for(int j = 1; j <= n ; j++){
|    |    cout << "PW";
|    3
3
```

No. of iterations ⇒ $n \times n = n^2$

T.C. = $O(n^2)$

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SKILLS

Q11 Calculate the T.C. of this given code

```
for (int i = 1 ; i <= n ; i++){
|    for(int j = 1; j <= i ; j++){
|    |    cout << "PW";
|    3
3
```

$i = 1 \rightarrow j = 1$ to $i \rightarrow 1$ to 1 : 1

$i = 2 \rightarrow j = 1, 2 \rightarrow$: 2

$i = 3 \rightarrow j = 1, 2, 3 \rightarrow$: 3

$i = 4 \rightarrow j = 1, 2, 3, 4$: 4

⋮

$i = n \rightarrow j = 1, 2, 3 .. n$: n

No. of iterations $\Rightarrow 1 + 2 + 3 \cdots + n$

$= \frac{n(n+1)}{2} = \frac{n^2+n}{2}$

T.C. $= O\left(\frac{n^2}{2} + \frac{n}{2}\right) \approx O\left(\frac{n^2}{2}\right) \approx O(n^2)$

C.W. : Calculate the T.C. of this given code

```
for (int i = 1 ; i <= n ; i++){
    for(int j = i; j <= n ; j++){
        cout << "PW";
    }
}
```

What if this time we traverse them in a nested manner?

```
for(int i = 0; i < n; i++) {
    for(int j = 0; j < m; j++) {
        cout<<"okay";
    }
}
```

→ 'm' times

i = 0 → j = 0, 1, 2 .. m-1 : m

i = 1 → j = 0, 1, 2 .. m-1 : m

⋮

i = n-1 → j = 0, 1, 2 .. m-1 : m

No. of iterations : n * m

T.C. : O(n*m)

***Ques : Calculate the time complexity for the below code snippet.**

$K \neq 1$

$K = 2, 3, 4, 5, 6 \ldots$

```
int c = 0;
for(int i = 1; i ≤ n; i*=k) {
    c++;
}
```

Ex: $n = 100$, $K = 2$ → no. of obs → 7

$i = 1, 2, 4, 8, 16, 32, 64, 128$

Khatam

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Ques : Calculate the time complexity for the below code snippet.

no. of iterations → no. of times 'i' is changing

```
int c = 0;
for(int i = 1; i ≤ n; i*=k) {
    c++;
}
```

$i = 1, k, k^2, k^3, k^4 \ldots\ldots k^x$

→ this loop will end when $k^x > n$

$= k^x = n$

$= \log_k n = x$

T.C. = $O(x)$

T.C. = $O(\log_k n)$

$= O(\log n)$

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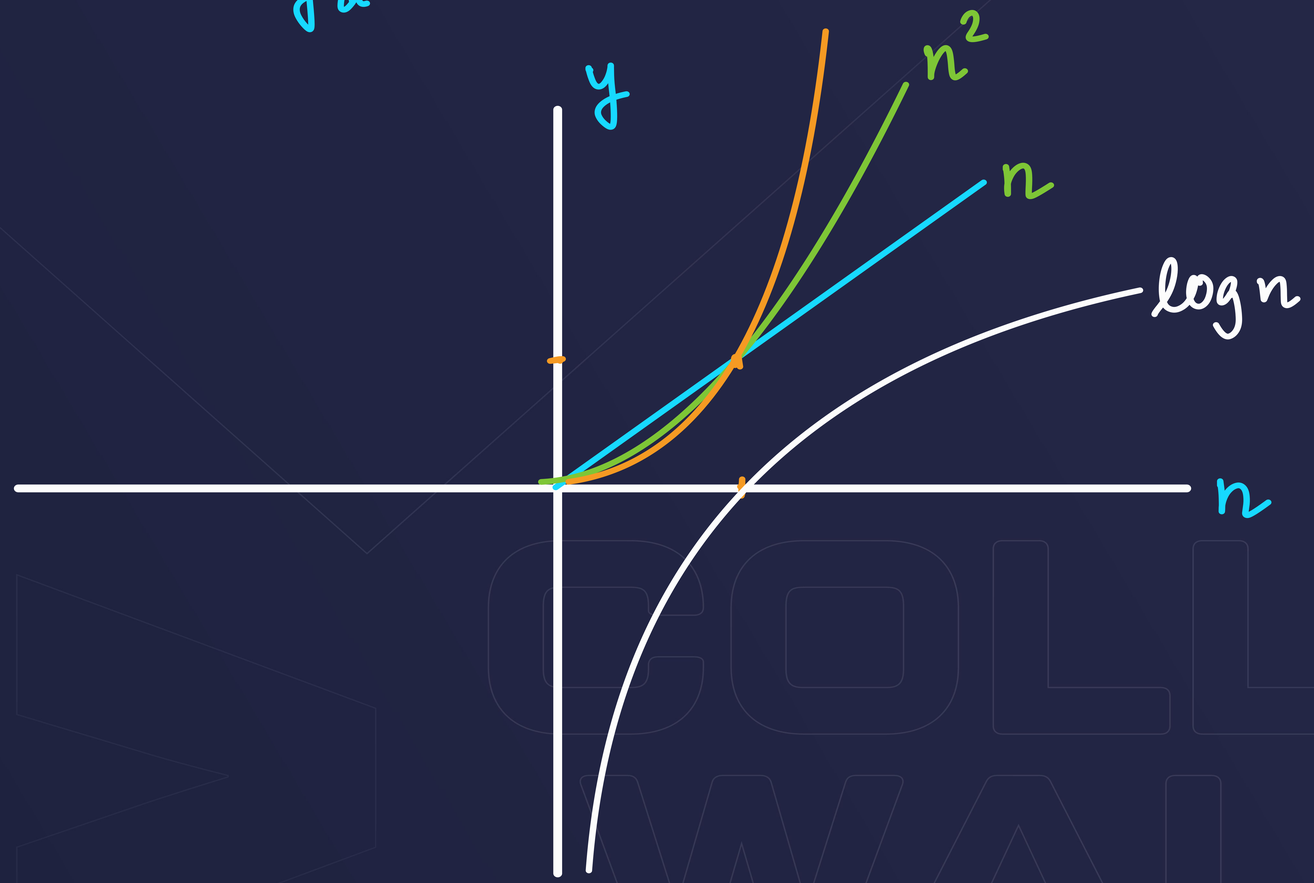
Logarithms : $a^b = c$

$\Rightarrow \log_a c = b$

$n > 1$

$n^3 > n^2 > n > \log n$



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Ques : Calculate the time complexity for the below code snippet.

```
int c = 0;
for(int i = 0; i < n; i++) {
    for(int j = i+1; j < m; j++) {
        c++;
    }
}
```

$i = 0 \rightarrow j = 1, 2, 3 \ldots m-1 : m-1$

$i = 1 \rightarrow j = 2, 3, 4 \ldots m-1 : m-2$

$i = 2 \rightarrow j = 3, 4, 5 \ldots m-1 : m-3$

$\vdots$

$i = n-1 \rightarrow j = n+1, n+2 \ldots m-1 : m-(n+1)$

$\therefore m-n-1$

total no: $(m-1) + (m-2) + (m-3) \ldots + [m-(n+1)] \rightarrow (n+1)$ terms

$$S = \frac{N}{2}[a_1 + a_l] = \frac{n}{2}[m-1+m-n-1]$$

$$S = O\left[\frac{n}{2}[2m-n-2]\right] \Rightarrow O(n*m - n^2) \approx O(m*n)$$

$i = 0 \rightarrow j = 1, 2, 3 \ldots m-1 : m-1$

$i = 1 \rightarrow j = 2, 3, 4 \ldots m-1 : m-2$

$m-3$

$\vdots$

$i = n-1 \rightarrow j = n, n+1, n+2 \ldots m-1 : m-n$

total iterations : $(m-1) + (m-2) + (m-3) \ldots (m-n)$

$= \underbrace{(m-1) + (m-2) + \ldots (m-n)}_{n \text{ terms}} + \underbrace{(m-n-1) + \ldots 2 + 1}_{m-n-1}$

$S = \frac{(m-1)(m-1+1)}{2} - \frac{(m-n-1)(m-n-1+1)}{2}$

$-[(m-n-1) + \ldots 2 + 1]$

PW SKILLS

Space Complexity : Study of all extra space used in terms of given 'n', 'm'

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Ques : Calculate the space complexity for the below code snippet.

```
int a[n];
for(int i = 0; i < n; i++) {
   a[i]++;
}
```

No. of ops : n

T.C. : O(n)

S.C. : O(n)

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What will be the space complexity if we just traverse without creating any array?

```
int c = 0;
for(int i = 0; i < n; i++) {
   c++;
}
```

No extra space

T.C. → O(n)

S.C. → O(1)

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Ques : Calculate the space complexity for the below nested loop code snippet.

```
vector<int> a;
vector<int> b;
for(int i = 0; i < n; i++) {
    for(int j = 0; j < m; j++) {
        a[i]++, b[j]++;
    }
}
```

a. push_back(10);
b. push_back(5);

m*n elemen

n times

m times

T.C. : O(n*m)

S.C. : O(n*m)

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```
vector < int > a(n) ;
vector <int > b(m) ;

for (int i = 0;  i< n; i++){
|    for (int j = 0; j<m; j++){
|    |    a[i] = i ;
|    |    b[j] = j ;
|    }
}
```

T.C. = $O(m*n)$

S.C. = $O(n+m)$

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Space Complexity of creating a 2d matrix

```
int arr[m][n];
```

rows cols

cells : $m * n$

S.C. = $O(m * n)$

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What will be the space complexity if we create 3 arrays of the same size?

```
int a[n], b[n], c[n];
for(int i = 0; i < n; i++) {
   c++;
}
```

T.C. → O(n)

S.C. → O(n+n+n)

⇒ O(3n)

≈ O(n)

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Ques : Calculate the time and space complexity for the below nested loop code snippet.

```
int a[n][n/2];
for(int i = 1; i < n; i*=2) {
    for(int j = 0; j < n/2; j++) {
        a[i][j]++;
    }
}
```

Space used : $n * \frac{n}{2} = \frac{n^2}{2}$

S.C. → $O(\frac{n^2}{2}) \approx O(n^2)$

$\log_2 n$

$\sim \frac{n}{2}$ operations

T.C. → $\frac{n}{2} * \log_2 n$

$O(n)$

T.C = $O(n \cdot \log n)$

$i = 1, 2^1, 2^2, 2^3 \cdots 2^x \rightarrow x$ steps

$2^x = n$

$\log_2 n = x$

A.P., G.P., Basic Math → Log

↓

Easy ♡

Doubts

↓

Form

↓

Telegram

Thank you!

↓

Maza aa gaya