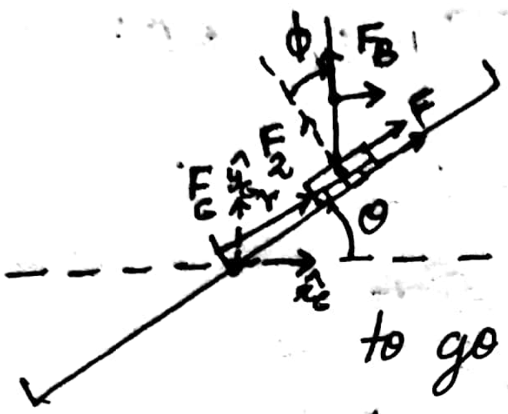
~~Inverted~~ Pendulum on platform :



Try to find the rotation matrix from F_B to F_G

i.e. $\underline{v}^G = R\,\underline{v}^B$

to go from $F_B \longrightarrow F_2$: $R_3(\phi)$

to go from $F_2 \longrightarrow F_G$: $R_3(-\theta)$

$\therefore$ ~~to~~ $R = R_3(-\theta)\,R_3(\phi)$ where, $R_3(\theta) = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

calculating velocities in the ground frame, F_G :

$\cancel{\underline{v}^G_{cart} = \underline{v}^B_c}$ Cart :-

~~assume the cart is a point mass and~~ so

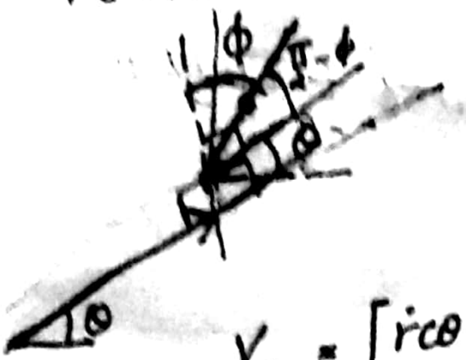
~~There is n~~

$x_{cm} = r\cos\theta \quad y_{cm} = r\sin\theta$

$$\Rightarrow \underline{v}_{cm} = \begin{bmatrix} \dot{r}\cos\theta - r\sin\theta\,\dot{\theta} \\ \dot{r}\sin\theta + r\cos\theta\,\dot{\theta} \end{bmatrix}$$

$\therefore KE_{cart} = \frac{1}{2}M(\dot{r}^2 + r^2\dot{\theta}^2) + 0$ (cart is assumed to be point mass)

Pendulum:



$$x_{cm} = r\cos\theta + \frac{l}{2}\cos\left(\frac{\pi}{2}+\theta-\phi\right)$$

$$y_{cm} = r\sin\theta + \frac{l}{2}\sin\left(\frac{\pi}{2}+\theta-\phi\right)$$

$$v_{cm} = \begin{bmatrix} \dot{r}c\theta - rs\theta\,\dot{\theta} - \frac{l}{2}\cos(\theta-\phi)(\dot{\theta}-\dot{\phi}) \\ \dot{r}s\theta + rc\theta\,\dot{\theta} - \frac{l}{2}\sin(\theta-\phi)(\dot{\theta}-\dot{\phi}) \end{bmatrix}$$

$$\Omega^{B}_{cm} = R^T\dot{R} = R^T\left(\frac{\partial R_3(-\theta)}{\partial\theta}R_3(\phi)\,\dot{\theta} + R_3(-\theta)\frac{\partial R_3(\phi)}{\partial\phi}\dot{\phi}\right)$$

$$= \begin{bmatrix} 0 & \dot{\theta}-\dot{\phi} & 0 \\ \dot{\phi}-\dot{\theta} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \omega^{B}_{cm} = [0\ 0\ \dot{\phi}-\dot{\theta}]^T$$

$$I_{cm} = \begin{bmatrix} \frac{ml^2}{12}+\frac{mR^2}{4} & 0 & 0 \\ 0 & \frac{mR^2}{2} & 0 \\ 0 & 0 & \frac{ml^2}{12}+\frac{mR^2}{4} \end{bmatrix} = \mathrm{diag}(I_i)$$

$$\Rightarrow KE_p = \frac{1}{2}\left(m v_{cm}^T v_{cm} + \omega^{B\,T}_{cm} I_{cm}\,\omega^{B}_{cm}\right)$$

$$\cancel{= \frac{m}{24}\left((l^2+3r^2)(\dot{\theta}-\dot{\phi})^2 + 12(\right.}$$

$$= \frac{1}{2}\left(I_3(\dot{\theta}-\dot{\phi})^2 + m\left(\left(r\dot{\theta}s\theta + \frac{l}{2}\cos(\theta-\phi)(\dot{\theta}-\dot{\phi}) - \dot{r}\cos\theta\right)^2 + \left(r\dot{\theta}c\theta - \frac{l}{2}\sin(\theta-\phi)(\dot{\theta}-\dot{\phi}) + \dot{r}\sin\theta\right)^2\right)\right)$$

$$KE = KE_{cart} + KE_p$$

$$PE = gr\,s\theta(m+M) + \frac{mgl}{2}\cos(\theta-\phi)$$

$\therefore\ \mathcal{L} = KE - PE$ and let $\underline{q} := [r, \phi]^T$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\underline{q}}}\right) - \frac{\partial \mathcal{L}}{\partial \underline{q}} = \begin{bmatrix} F \\ 0 \end{bmatrix}$$

$$\Rightarrow \ddot{r} = \frac{-\left(8\left(-\frac{1}{4}(4I_3+ml^2)\left(-F - mr\dot{\theta}^2 - \frac{1}{2}mls\phi(\dot{\theta}-\dot{\phi})^2 - \frac{1}{2}mlc\phi\ddot{\theta}\right) - \frac{1}{8}mlc\phi\left(-2glms(\theta-\phi) + 4mls\phi\dot{r}\dot{\theta} + 4I_3\ddot{\theta} + ml^2\ddot{\theta} + 2mlr(c\phi\dot{\theta}^2 + s\phi\ddot{\theta})\right)\right)\right)}{\left(m(-8I_3 - ml^2 + ml^2c2\phi)\right)}$$

$$\ddot{\phi} = \left(4Flc\phi + 4mgl\sin(\theta-\phi) - 8mls\phi\dot{r}\dot{\theta} + ml^2s2\phi\dot{\theta}^2 - 2ml^2s2\phi\dot{\theta}\dot{\phi} + ml^2s2\phi\dot{\phi}^2 - 8I_3\ddot{\theta} - ml^2\ddot{\theta} + ml^2c2\phi\ddot{\theta} - 4mlrs\phi\ddot{\theta}\right) / \left(-8I_3 - ml^2 + ml^2c2\phi\right)$$