Exercícios de quadros

1)

$$\min_x \|v - x \cdot \begin{bmatrix}1\\1\\1\end{bmatrix}\|^2$$

$$\|v - x \cdot \begin{bmatrix}1\\1\\1\end{bmatrix}\|^2 = (v - x \cdot \begin{bmatrix}1\\1\\1\end{bmatrix})^t (v - x \cdot \begin{bmatrix}1\\1\\1\end{bmatrix}) =$$

$$K = v^T v - 2x v^T \cdot \begin{bmatrix}1\\1\\1\end{bmatrix} + x^2 \cdot [1\,1\,1] \cdot \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\frac{dK}{dx} = 0 \;) \; -2 v^T \cdot \begin{bmatrix}1\\1\\1\end{bmatrix} + 2x \cdot [1\,1\,1] \cdot \begin{bmatrix}1\\1\\1\end{bmatrix} = 0$$

$$-2 v^T \cdot \begin{bmatrix}1\\1\\1\end{bmatrix} + 6x = 0$$

$$x = \frac{1}{3} \cdot v^T \cdot \begin{bmatrix}1\\1\\1\end{bmatrix}$$

2) $\min_{\lambda \in \mathbb{R}} \|a - \lambda b\|^2$,

$$\frac{d}{d\lambda}((a - \lambda b)^t \cdot (a - \lambda b)) = 0$$

$$\frac{d}{d\lambda}(a^T a - a^T \lambda b - \lambda b^T a + \lambda^2 b^T b) = 0$$

$$-a^T b - b^T a + 2\lambda b^T b = 0$$

$$\lambda = \frac{1}{2}(a^T b + b^T a)(b^T b)^{-1}$$

3) $\min \|Ax-b\|^2 + \lambda \|x\|^2$

$$\frac{d}{dx}\left((Ax-b)^T(Ax-b) + \lambda x^T x\right)$$

$$\frac{d}{dx}\left(x^T A^T - b^T\right)(Ax-b) + \lambda x^T x) = 0$$

$$(A^T A + (A^T A)^T)\, x - A^T b - A^T b + 2\lambda x = 0$$

$$2A^T A x + 2\lambda x = 2A^T b$$

$$x(A^T A \cdot 2 + 2\lambda I) = 2A^T b$$

$$x = (A^T A + \lambda I)^{-1} \cdot A^T b$$

4) Com SVD, temos que:

$$A^T A = V \cdot \Sigma^T \cdot U^T \cdot U \cdot \Sigma \cdot V^T$$

$$A^T A = V \cdot \Sigma^2 \cdot V^T$$

$$cond(A^T A) = \left(\sigma_{1/\sigma_m}\right)^2, \quad cond(A^T A) = 10^6$$

6) 10^6

7) A_5 A_6 A_7 A_8

A_1 A_2 A_3

A_9 A_{10}

A_4

5) $\begin{bmatrix} 0 & 0 & 8 & -3 & 9 \\ 0 & 1 & 4 & 0 & 3 \end{bmatrix} \approx \begin{bmatrix} -1 & 6.5 \\ 1/3 & 3.5 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$