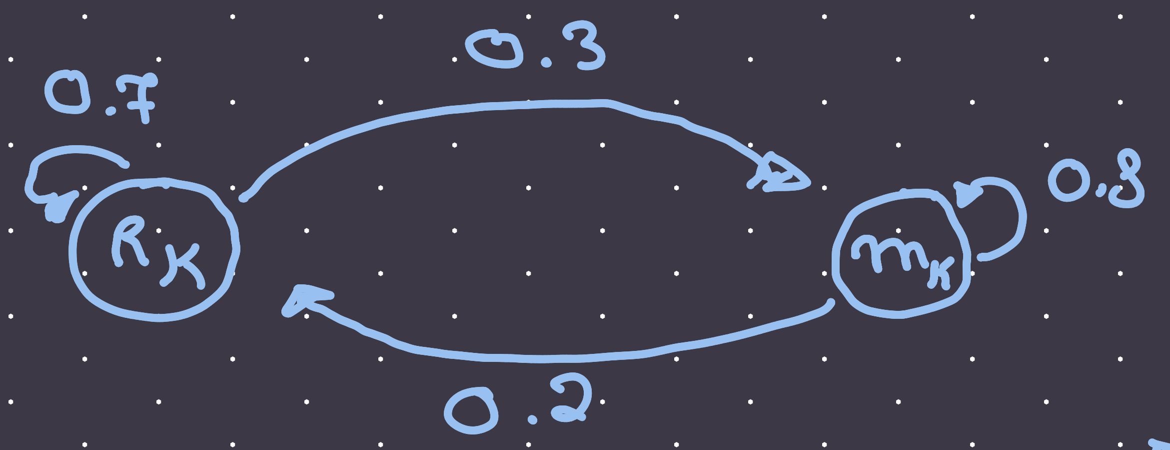
1)



$$R_{K+1} = 0.7\,R_K + 0.2\,m_K$$
$$m_{K+1} = 0.8\,m_K + 0.3\,R_K$$

$$\begin{bmatrix} 0.7 & 0.2 \\ 0.8 & 0.3 \end{bmatrix} \cdot \begin{bmatrix} R_K \\ m_K \end{bmatrix} = \begin{bmatrix} R_{K+1} \\ m_{K+1} \end{bmatrix}$$

2×2 $\quad$ 2×1

* Código em anexo, com as respostas

3)

a)

$$N_{K+1} = 0.5\,M_K + 0.2\,V_K$$
$$m_{K+1} = 0.8\,N_K$$
$$V_{K+1} = 0.5\,M_K$$

$$\begin{bmatrix} 0 & 0.5 & 0.2 \\ 0.8 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix} \cdot \begin{bmatrix} n_K \\ m_K \\ v_K \end{bmatrix} = \begin{bmatrix} n_{K+1} \\ m_{K+1} \\ v_{K+1} \end{bmatrix}$$

8) $\det(A - \lambda I) = (2-\lambda)^2 - 1$, $\lambda_1 = 1$ e $\lambda_2 = 3$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A^{1000000} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = z, \quad A^{1000000} \cdot \left[\frac{1}{2}\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) \right] = K$$

$$\frac{1}{2} \left[A^{1000000} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} + A^{1000000} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right]$$

$$\frac{1}{2} \cdot \left[\begin{bmatrix} 1 \\ -1 \end{bmatrix} + 3^{1000000} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right] = \begin{bmatrix} \dfrac{3^{1000000}+1}{2} \\ \dfrac{3^{1000000}-1}{2} \end{bmatrix} \rightarrow \boxed{\frac{z_1}{z_2} = \frac{3^{1000000}+1}{3^{1000000}-1}}$$

2) $\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} \cdot \begin{bmatrix} F_t \\ F_{t+1} \end{bmatrix} = \begin{bmatrix} F_{t+1} \\ F_{t+2} \end{bmatrix}$

$$F_t \cdot \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} + F_{t+1} \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} F_{t+1} \\ F_t + F_{t+1} \end{bmatrix}$$

$x_1 = 0,\ x_2 = 1,\ x_3 = 1,\ x_4 = 1$

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = A$$

b) $-\lambda(1-\lambda) - 1$

$\lambda^2 - \lambda - 1 = 0,\ \lambda_1 = \frac{1+\sqrt{5}}{2}$ e $\lambda_2 = \frac{1-\sqrt{5}}{2}$

$$\begin{bmatrix} -\left(\frac{1+\sqrt{5}}{2}\right) & 1 \\ 1 & \frac{1+\sqrt{5}}{2} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};\ V_{\lambda_1} = \begin{bmatrix} \frac{\sqrt{5}-1}{2} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -\left(\frac{1-\sqrt{5}}{2}\right) & 1 \\ 1 & \frac{1-\sqrt{5}}{2} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ V_{\lambda_2} = \begin{bmatrix} -\frac{1-\sqrt{5}}{2} \\ 1 \end{bmatrix}$$

a) Os autovalores e autovetores associados a essa transformação é o número de ouro, ou proporção áurea.