Exercícios do quadro

1)

$$\min_x \| v - x \cdot \begin{bmatrix}1\\1\\1\end{bmatrix} \|^2$$

$$\| v - x \cdot \begin{bmatrix}1\\1\\1\end{bmatrix} \|^2 = \left(v - x \cdot \begin{bmatrix}1\\1\\1\end{bmatrix}\right)^t \left(v - x \cdot \begin{bmatrix}1\\1\\1\end{bmatrix}\right) =$$

$$K = v^T v - 2x v^T \cdot \begin{bmatrix}1\\1\\1\end{bmatrix} + x^2 \cdot [1\,1\,1] \cdot \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\frac{dK}{dx} = 0, \quad -2 v^T \cdot \begin{bmatrix}1\\1\\1\end{bmatrix} + 2x \cdot [1\,1\,1] \cdot \begin{bmatrix}1\\1\\1\end{bmatrix} = 0$$

$$-2 v^T \begin{bmatrix}1\\1\\1\end{bmatrix} + 6x = 0$$

$$x = \frac{1}{3} \cdot v^T \cdot \begin{bmatrix}1\\1\\1\end{bmatrix}$$

2)

$$\min_{\lambda \in \mathbb{R}} \| a - \lambda b \|^2,$$

$$\frac{d}{d\lambda}((a - \lambda b)^t \cdot (a - \lambda b)) = 0$$

$$\frac{d}{d\lambda}(a^T a - a^T \lambda b - \lambda b^T a + \lambda^2 b^T b) = 0$$

$$-a^T b - b^T a + 2\lambda b^T b = 0$$

$$\lambda = \frac{1}{2}(a^T b + b^T a)(b^T b)^{-1}$$

3) $\min \|Ax - b\|^2 + \lambda \|x\|^2$

$$\frac{d}{dx}\left((Ax-b)^T \cdot (Ax-b) + \lambda x^T \cdot x \right)$$

$$\frac{d}{dx}\left(x^T A^T - b^T \right)(Ax - b) + \lambda x^T x) = 0$$

$$(A^T A + (A^T A)^T)\, x - A^T b - A^T b + 2\lambda x = 0$$

$$2A^T A x + 2\lambda x = 2A^T b$$

$$x\,(A^T A \cdot 2 + 2\lambda I) = 2A^T b$$

$$x = (A^T A + \lambda I)^{-1} \cdot A^T b$$

4) Com SVD, temos que:

$$A^T A = V \cdot \Sigma^T \cdot U^T \cdot U \cdot \Sigma \cdot V^T$$

$$A^T A = V \cdot \Sigma^2 \cdot V^T$$

$$\text{cond}(A^T A) = \left(\sigma_1 / \sigma_n\right)^2, \quad \text{cond}(A^T \cdot A) = 10^6$$

6) 10^6

7) A_5 A_6 A_7 A_8

A_1 A_2

A_3

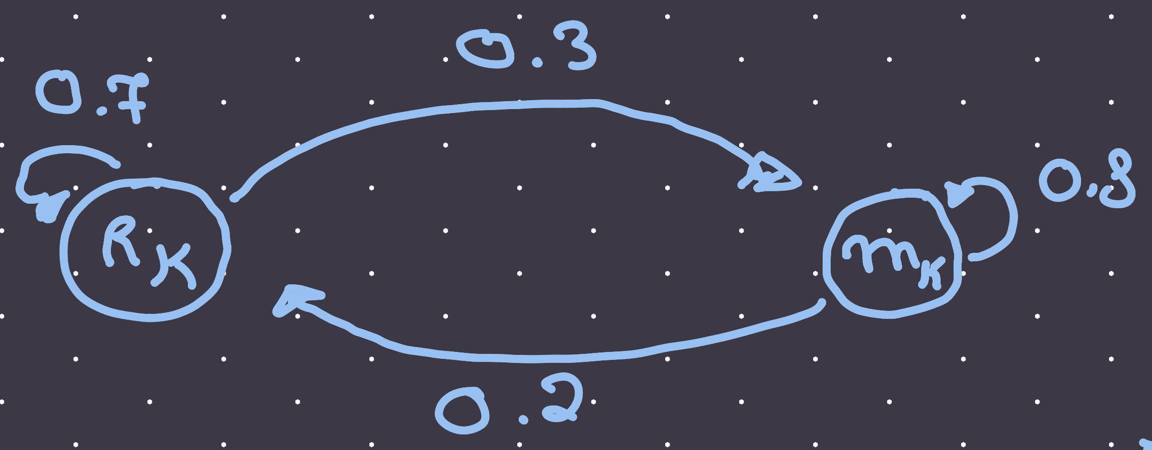
A_9 A_{10}

A_4

5) $$\begin{bmatrix} 0 & 0 & 8 & -3 & 9 \\ 0 & 1 & 4 & 0 & 3 \end{bmatrix} \approx \begin{bmatrix} -1 & 6.5 \\ 1/3 & 3.5 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Lista 3

1)



$R_{K+1} = 0.7 R_K + 0.2 m_K$

$m_{K+1} = 0.8 m_K + 0.3 R_K$

$$\underset{2\times 2}{\begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix}} \cdot \underset{2\times 1}{\begin{bmatrix} R_K \\ m_K \end{bmatrix}} = \begin{bmatrix} R_{K+1} \\ m_{K+1} \end{bmatrix}$$

* Código em anexo, com as respostas

3)

a)

$N_{K+1} = 0.5 M_K + 0.2 V_K$

$m_{K+1} = 0.8 N_K$

$V_{K+1} = 0.5 M_K$

$$\begin{bmatrix} 0 & 0.5 & 0.2 \\ 0.8 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix} \cdot \begin{bmatrix} n_K \\ m_K \\ v_K \end{bmatrix} = \begin{bmatrix} n_{K+1} \\ m_{K+1} \\ v_{K+1} \end{bmatrix}$$

8) $\det(A - \lambda I) = (2-\lambda)^2 - 1$, $\lambda_1 = 1$ e $\lambda_2 = 3$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A^{1000000} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = z, \quad A^{1000000} \cdot \left[\frac{1}{2} \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) \right] = K$$

$$\frac{1}{2} \left[A^{1000000} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} + A^{1000000} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right]$$

$$\frac{1}{2} \cdot \left[\begin{bmatrix} 1 \\ -1 \end{bmatrix} + 3^{1000000} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right] = \begin{bmatrix} \dfrac{3^{1000000} + 1}{2} \\ \dfrac{3^{1000000} - 1}{2} \end{bmatrix} \rightarrow \boxed{\frac{z_1}{z_2} = \frac{3^{1000000} + 1}{3^{1000000} - 1}}$$

2) $\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} \cdot \begin{bmatrix} F_t \\ F_{t+1} \end{bmatrix} = \begin{bmatrix} F_{t+1} \\ F_{t+2} \end{bmatrix}$

$$F_t \cdot \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} + F_{t+1} \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} F_{t+1} \\ F_t + F_{t+1} \end{bmatrix}$$

$x_1 = 0,\ x_2 = 1,\ x_3 = 1,\ x_4 = 1$

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = A$$

b) $-\lambda(1-\lambda) - 1$

$\lambda^2 - \lambda - 1 = 0,\ \lambda_1 = \frac{1+\sqrt{5}}{2}$ e $\lambda_2 = \frac{1-\sqrt{5}}{2}$

$$\begin{bmatrix} -\left(\frac{1+\sqrt{5}}{2}\right) & 1 \\ 1 & \frac{1+\sqrt{5}}{2} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};\ V_{\lambda_1} = \begin{bmatrix} \frac{\sqrt{5}-1}{2} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -\left(\frac{1-\sqrt{5}}{2}\right) & 1 \\ 1 & \frac{1-\sqrt{5}}{2} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ V_{\lambda_2} = \begin{bmatrix} -\frac{1-\sqrt{5}}{2} \\ 1 \end{bmatrix}$$

a) Os autovalores e autovetores associados a essa transformação é o número de ouro, ou proporção áurea.