

Unit 1: Relations and Functions

Example 1.1

If $A = \{1, 3, 5\}$ and $B = \{2, 3\}$, then:

- (i) Find $A \times B$ and $B \times A$.
- (ii) Is $A \times B = B \times A$? If not, why?
- (iii) Show that $n(A \times B) = n(B \times A) = n(A) \times n(B)$.

Example 1.2

If $A \times B = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$, then find A and B .

Example 1.3

Let $A = \{x \in \mathbb{N} \mid 1 < x < 4\}$, $B = \{x \in \mathbb{W} \mid 0 \leq x < 2\}$ and $C = \{x \in \mathbb{N} \mid x < 3\}$. Then verify that:

- (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
- (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Example 1.4

Let $A = \{3, 4, 7, 8\}$ and $B = \{1, 7, 10\}$. Which of the following sets are relations from A to B ?

- (i) $R_1 = \{(3, 7), (4, 7), (7, 10), (8, 1)\}$
- (ii) $R_2 = \{(3, 1), (4, 12)\}$
- (iii) $R_3 = \{(3, 7), (4, 10), (7, 7), (7, 8), (8, 11), (8, 7), (8, 10)\}$

Example 1.5

The arrow diagram shows a relationship between the sets P and Q . Write the relation in:

- (i) Set builder form
- (ii) Roster form
- (iii) What is the domain and range of R ?

Example 1.6

Let $X = \{1, 2, 3, 4\}$ and $Y = \{2, 4, 6, 8, 10\}$ and $R = \{(1, 2), (2, 4), (3, 6), (4, 8)\}$. Show that R is a function and find its domain, co-domain, and range.

Example 1.7

A relation $f: X \rightarrow Y$ is defined by $f(x) = x^2 - 2$ where, $X = \{-2, -1, 0, 3\}$ and $Y = \mathbb{R}$.

- (i) List the elements of f .
- (ii) Is f a function?

Example 1.8

If $X = \{-5, 1, 3, 4\}$ and $Y = \{a, b, c\}$, then which of the following relations are functions from X to Y ?

- (i) $R_1 = \{(-5, a), (1, a), (3, b)\}$
- (ii) $R_2 = \{(-5, b), (1, b), (3, a), (4, c)\}$
- (iii) $R_3 = \{(-5, a), (1, a), (3, b), (4, c), (1, b)\}$

Example 1.9

Given $f(x) = 2x - x^2$, find:

- (i) $f(1)$
- (ii) $f(x + 1)$
- (iii) $f(x) + f(1)$

Example 1.10

Using the vertical line test, determine which of the given curves represent a function.

Example 1.11

Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 5, 8, 11, 14\}$ be two sets. Let $f: A \rightarrow B$ be a function given by $f(x) = 3x - 1$. Represent this function:

- (i) by arrow diagram
- (ii) in a table form
- (iii) as a set of ordered pairs
- (iv) in a graphical form

Example 1.12

Using the horizontal line test, determine which of the following functions are one-one.

Example 1.13

Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B . Show that f is one-one but not an onto function.

Example 1.14

If $A = \{-2, -1, 0, 1, 2\}$ and $f: A \rightarrow B$ is an onto function defined by $f(x) = x^2 + x + 1$, then find B .

Example 1.15

Let f be a function $f: \mathbb{N} \rightarrow \mathbb{N}$ be defined by $f(x) = 3x + 2$, $x \in \mathbb{N}$.

- (i) Find the images of 1, 2, 3.
- (ii) Find the pre-images of 29, 53.
- (iii) Identify the type of function.

Example 1.16

Forensic scientists can determine the height (in cm) of a person based on the length of the thigh bone. They usually do so using the function $h(b) = 2.47b + 54.10$, where b is the length of the thigh bone.

- (i) Verify the function h is one-one or not.
- (ii) Also find the height of a person if the length of his thigh bone is 50 cm.
- (iii) Find the length of the thigh bone if the height of a person is 147.96 cm.

Example 1.17

Let f be a function from \mathbb{R} to \mathbb{R} defined by $f(x) = 3x - 5$. Find the values of a and b given that $(a, 4)$ and $(1, b)$ belong to f .

Example 1.18

If the function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = \{ 2x+7, x < -2; x^2-2, -2 \leq x < 3; 3x-2, x \geq 3 \}$, then find the values of:

- (i) $f(4)$
- (ii) $f(-2)$
- (iii) $f(4) + 2f(1)$
- (iv) $[f(1) - 3f(4)] / f(-3)$

Example 1.19

Find $f \circ g$ and $g \circ f$ when $f(x) = 2x + 1$ and $g(x) = x^2 - 2$.

Example 1.20

Represent the function $f(x) = \sqrt{2x^2 - 5x + 3}$ as a composition of two functions.

Example 1.21

If $f(x) = 3x - 2$, $g(x) = 2x + k$ and if $f \circ g = g \circ f$, then find the value of k .

Example 1.22

Find k if $f \circ f(k) = 5$, where $f(k) = 2k - 1$.

Example 1.23

If $f(x) = 2x + 3$, $g(x) = 1 - 2x$ and $h(x) = 3x$. Prove that $(f \circ g) \circ h = f \circ (g \circ h)$.

Example 1.24

Find x if $gff(x) = fgg(x)$, given $f(x) = 3x + 1$ and $g(x) = x + 3$.

Unit 2: Numbers and Sequences

Example 2.1

We have 34 cakes. Each box can hold 5 cakes only. How many boxes do we need to pack and how many cakes are unpacked?

Example 2.2

Find the quotient and remainder when a is divided by b in the following cases:

(i) $a = -12$, $b = 5$

(ii) $a = 17$, $b = -3$

(iii) $a = -19$, $b = -4$

Example 2.3

Show that the square of an odd integer is of the form $4q + 1$, for some integer q .

Example 2.4

If the Highest Common Factor of 210 and 55 is expressible in the form $55x - 325$, find x .

Example 2.5

Find the greatest number that will divide 445 and 572 leaving remainders 4 and 5 respectively.

Example 2.6

Find the HCF of 396, 504, 636.

Example 2.7

In the given factorisation, find the numbers m and n.

Example 2.8

Can the number 6^n , n being a natural number, end with the digit 5? Give a reason for your answer.

Example 2.9

Is $7 \times 5 \times 3 \times 2 + 3$ a composite number? Justify your answer.

Example 2.10

'a' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.

Example 2.11

Find the remainders when 70004 and 778 are divided by 7.

Example 2.12

Determine the value of d such that $15 \equiv 3 \pmod{d}$.

Example 2.13

Find the least positive value of x such that:

(i) $67 + x \equiv 1 \pmod{4}$

(ii) $98 \equiv (x + 4) \pmod{5}$

Example 2.14

Solve $8x \equiv 1 \pmod{11}$.

Example 2.15

Compute x, such that $10^4 \equiv x \pmod{19}$.

Example 2.16

Find the number of integer solutions of $3x \equiv 1 \pmod{15}$.

Example 2.17

A man starts his journey from Chennai to Delhi by train. He starts at 22.30 hours on Wednesday. If it takes 32 hours of travelling time and assuming that the train is not late, when will he reach Delhi?

Example 2.18

Kala and Vani are friends. Kala says, “Today is my birthday” and she asks Vani, “When will you celebrate your birthday?” Vani replies, “Today is Monday and I celebrated my birthday 75 days ago”. Find the day when Vani celebrated her birthday.

Example 2.19

Find the next three terms of the sequences:

- (i) $1/2, 1/6, 1/10, 1/14, \dots$
- (ii) $5, 2, -1, -4, \dots$
- (iii) $1, 0.1, 0.01, \dots$

Example 2.20

Find the general term for the following sequences:

- (i) $3, 6, 9, \dots$
- (ii) $1/2, 2/3, 3/4, \dots$
- (iii) $5, -25, 125, \dots$

Example 2.21

The general term of a sequence is defined as $a_n = \{ n(n+3); n \in \mathbb{N} \text{ is odd}; n^2+1; n \in \mathbb{N} \text{ is even} \}$. Find the eleventh and eighteenth terms.

Example 2.22

Find the first five terms of the following sequence: $a_1 = 1, a_2 = 1, a_n = (a_{n-1} / a_{n-2}) + 3; n \geq 3, n \in \mathbb{N}$.

Unit 3: Algebra**Example 3.1**

The father's age is six times his son's age. Six years hence the age of the father will be four times his son's age. Find the present ages (in years) of the son and father.

Example 3.2

Solve $2x - 3y = 6$, $x + y = 1$.

Example 3.3

Solve the following system of linear equations in three variables: $3x - 2y + z = 2$, $2x + 3y - z = 5$, $x + y + z = 6$.

Example 3.4

In an interschool athletic meet, with a total of 24 individual prizes, securing a total of 56 points, a first place secures 5 points, a second place secures 3 points, and a third place secures 1 point. Having as many third-place finishers as first and second place finishers, find how many athletes finished in each place.

Example 3.5

Solve $x + 2y - z = 5$; $x - y + z = -2$; $-5x - 4y + z = -11$.

Example 3.6

Solve $3x + y - 3z = 1$; $-2x - y + 2z = 1$; $-x - y + z = 2$.

Example 3.7

Solve $(x/2) - 1 = (y/6) + 1 = (z/7) + 2$; $(y/3) + (z/2) = 13$.

Example 3.8

Solve : $(1/2x) + (1/4y) - (1/3z) = 1/4$; $(1/x) = (1/3y)$; $(1/x) - (1/5y) + (4/z) = 2 \frac{2}{15}$.

Example 3.9

The sum of thrice the first number, the second number, and twice the third number is 5. If thrice the second number is subtracted from the sum of the first number and thrice the third, we get 2. If the third number is subtracted from the sum of twice the first and thrice the second, we get 1. Find the numbers.

Example 3.10

Find the GCD of the polynomials $x^3 + x^2 - x + 2$ and $2x^3 - 5x^2 + 5x - 3$.

Example 3.11

Find the GCD of $6x^3 - 30x^2 + 60x - 48$ and $3x^3 - 12x^2 + 21x - 18$.

Example 3.12

Find the LCM of the following:

(i) $8x^4y^2, 48x^2y^4$

(ii) $5x - 10, 5x^2 - 20$

(iii) $x^4 - 1, x^2 - 2x + 1$

(iv) $x^3 - 27, (x - 3)^2, x^2 - 9$

Example 3.13

Reduce the rational expressions to its lowest form:

(i) $(x-3) / (x^2-9)$

(ii) $(x^2-16) / (x^2+8x+16)$

Example 3.14

Find the excluded values of the following expressions (if any):

(i) $(x+10) / 8x$

(ii) $(7p+2) / (8p^2+13p+5)$

(iii) $x / (x^2+1)$

Example 3.15

(i) Multiply $(x^3/9y^2)$ by $(27y/x^5)$

(ii) Multiply $(x^4b^2 / (x-1))$ by $((x^2-1)/a^4b^3)$

Example 3.16

Find:

(i) $(14x^4/y) \div (7x/3y^4)$

(ii) $(x^2-16)/(x+4) \div (x-4)/(x+4)$

(iii) $(16x^2-2x-3)/(3x^2-2x-1) \div (8x^2+11x+3)/(3x^2-11x-4)$

Example 3.17

Find $(x^2+20x+36)/(x^2-3x-28) - (x^2+12x+4)/(x^2-3x-28)$.

Example 3.18

Simplify $1/(x^2-5x+6) + 1/(x^2-3x+2) - 1/(x^2-8x+15)$.

Example 3.19

Find the square root of the following expressions:

(i) $256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}$

(ii) $(144a^8b^{12}c^{16}) / (81f^{12}g^4h^{14})$

Example 3.20

Find the square root of the following expressions:

(i) $16x^2 + 9y^2 - 24xy + 24x - 18y + 9$

(ii) $(6x^2+x-1)(3x^2+2x-1)(2x^2+3x+1)$

(iii) $[\sqrt{(15)x^2 + (\sqrt{3} + \sqrt{10})x + \sqrt{2}}][\sqrt{5x^2 + (2\sqrt{5} + 1)x + 2}][\sqrt{3x^2 + (\sqrt{2} + 2\sqrt{3})x + 2\sqrt{2}}]$

Example 3.21

Find the square root of $64x^4 - 16x^3 + 17x^2 - 2x + 1$.

Example 3.22

If $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square, find the values of a and b.

Example 3.23

Find the zeroes of the quadratic expression $x^2 + 8x + 12$.

Example 3.24

Write down the quadratic equation in general form for which the sum and product of the roots are given below:

(i) 9, 14

(ii) $-7/2$, $5/2$

(iii) $-3/5$, $-1/2$

Example 3.25

Find the sum and product of the roots for each of the following quadratic equations:

(i) $x^2 + 8x - 65 = 0$

(ii) $2x^2 + 5x + 7 = 0$

(iii) $kx^2 - k^2x - 2k^3 = 0$

Example 3.26

Solve $2x^2 - 2\sqrt{6}x + 3 = 0$.

Example 3.27

Solve $2m^2 + 19m + 30 = 0$.

Example 3.28

Solve $x^4 - 13x^2 + 42 = 0$.

Example 3.29

Solve $(x / (x-1)) + ((x-1) / x) = 2 \frac{1}{2}$.

Example 3.30

Solve $x^2 - 3x - 2 = 0$ by completing the square method.

Example 3.31

Solve $2x^2 - x - 1 = 0$ by completing the square method.

Example 3.32

Solve $x^2 + 2x - 2 = 0$ by formula method.

Example 3.33

Solve $2x^2 - 3x - 3 = 0$ by formula method.

Example 3.34

Solve $3p^2 + 2\sqrt{5}p - 5 = 0$ by formula method.

Example 3.35

Solve $pqx^2 - (p+q)^2x + (p+q)^2 = 0$.

Example 3.36

The product of Kumaran's age (in years) two years ago and his age four years from now is one more than twice his present age. What is his present age?

Example 3.37

A ladder 17 feet long is leaning against a wall. If the ladder, vertical wall, and the floor form a right triangle, find the height of the wall where the top of the ladder meets if the distance between the bottom of the wall to the bottom of the ladder is 7 feet less than the height of the wall.

Example 3.38

A flock of swans contained x^2 members. As the clouds gathered, $10x$ went to a lake and one-eighth of the members flew away to a garden. The remaining three pairs played about in the water. How many swans were there in total?

Example 3.39

A passenger train takes 1 hour more than an express train to travel a distance of 240 km from Chennai to Virudhachalam. The speed of the express train is more than that of the passenger train by 20 km per hour. Find the average speed of both the trains.

Example 3.40

Determine the nature of roots for the following quadratic equations:

(i) $x^2 - x - 20 = 0$

(ii) $9x^2 - 24x + 16 = 0$

(iii) $2x^2 - 2x + 9 = 0$

Example 3.41

(i) Find the values of 'k' for which the quadratic equation $kx^2 - (8k+4)x + 81 = 0$ has real and equal roots.

(ii) Find the values of 'k' such that the quadratic equation $(k+9)x^2 + (k+1)x + 1 = 0$ has no real roots.

Example 3.42

Prove that the equation $x^2(p^2+q^2) + 2x(pr+qs) + r^2+s^2 = 0$ has no real roots. If $ps = qr$, then show that the roots are real and equal.

Example 3.43

If the difference between the roots of the equation $x^2 - 13x + k = 0$ is 17, find k .

Example 3.44

If α and β are the roots of $x^2 + 7x + 10 = 0$, find the values of:

- (i) $(\alpha - \beta)$
- (ii) $\alpha^2 + \beta^2$
- (iii) $\alpha^3 - \beta^3$
- (iv) $\alpha^4 + \beta^4$
- (v) $(\alpha/\beta) + (\beta/\alpha)$
- (vi) $(\alpha^2/\beta) + (\beta^2/\alpha)$

Example 3.45

If α, β are the roots of the equation $3x^2 + 7x - 2 = 0$, find the values of:

- (i) $(\alpha/\beta) + (\beta/\alpha)$
- (ii) $(\alpha^2/\beta) + (\beta^2/\alpha)$

Example 3.46

If α, β are the roots of the equation $2x^2 - x - 1 = 0$, then form the equation whose roots are:

- (i) $1/\alpha, 1/\beta$
- (ii) $\alpha^2\beta, \beta^2\alpha$
- (iii) $2\alpha+\beta, 2\beta+\alpha$

Example 3.47

Varshika drew 6 circles with different sizes. Draw a graph for the relationship between the diameter and circumference of each circle as shown in the table and use it to find the circumference of a circle when its diameter is 6 cm.

Diameter (x) cm	1	2	3	4	5	
:---	:---	:---	:---	:---	:---	
Circumference (y) cm	3.1	6.2	9.3	12.4	15.5	

Example 3.48

A bus is travelling at a uniform speed of 50 km/hr. Draw the distance-time graph and hence find:

- (i) the constant of variation
- (ii) how far will it travel in 90 minutes?
- (iii) the time required to cover a distance of 300 km from the graph.

Example 3.49

A company initially started with 40 workers to complete the work by 150 days. Later, it decided to increase the number of workers as shown below.

| Number of workers (x) | 40 | 50 | 60 | 75 |

| :--- | :--- | :--- | :--- | :--- |

| Number of days (y) | 150 | 120 | 100 | 80 |

- (i) Graph the above data and identify the type of variation.
- (ii) From the graph, find the number of days required to complete the work if the company decides to opt for 120 workers.
- (iii) If the work has to be completed by 200 days, how many workers are required?

Example 3.50

Nishanth is the winner in a Marathon race of 12 km. He ran at a uniform speed of 12 km/hr and reached the destination in 1 hour. He was followed by others with their respective speeds of 6 km/hr, 4 km/hr, 3 km/hr and 2 km/hr, covering the distance in 2 hrs, 3 hrs, 4 hrs and 6 hours respectively. Draw the speed-time graph and use it to find the time taken by Kaushik with his speed of 2.4 km/hr.

Example 3.51

Discuss the nature of solutions of the following quadratic equations by graphing:

(i) $x^2 + x - 12 = 0$

(ii) $x^2 - 8x + 16 = 0$

(iii) $x^2 + 2x + 5 = 0$

Example 3.52

Draw the graph of $y = 2x^2$ and hence solve $2x^2 - x - 6 = 0$.

Example 3.53

Draw the graph of $y = x^2 + 4x + 3$ and hence find the roots of $x^2 + x + 1 = 0$.

Example 3.54

Draw the graph of $y = x^2 + x - 2$ and hence solve $x^2 + x - 2 = 0$.

Example 3.55

Draw the graph of $y = x^2 - 4x + 3$ and use it to solve $x^2 - 6x + 9 = 0$.

Example 3.56

Consider the following information regarding the number of men and women workers in three factories I, II, and III. Represent the information as a matrix and state what the entry in the second row and first column represents.

Factory	Men	Women
I	23	18
II	47	36
III	15	16

Example 3.57

If a matrix has 16 elements, what are the possible orders it can have?

Example 3.58

Construct a 3×3 matrix whose elements are $a_{ij} = i^2 j^2$.

Example 3.59

Find the value of a, b, c, d from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$$

Example 3.60

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{bmatrix}$, find $A+B$.

Example 3.61

Given two matrices A and B representing average marks for three groups in two different exams. Find the total marks of both examinations for all three groups by finding $A+B$.

Example 3.62

If $A = \begin{bmatrix} 1 & 3 & -2 \\ 5 & -4 & 6 \\ -3 & 2 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 8 \\ 3 & 4 \\ 9 & 6 \end{bmatrix}$, find $A+B$.

Example 3.63

If $A = \begin{bmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{bmatrix}$, find $2A+B$.

Example 3.64

If $A = \begin{bmatrix} 5 & 4 & -2 \\ 1/2 & 3/4 & \sqrt{2} \\ 1 & 9 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -7 & 4 & -3 \\ 1/4 & 7/2 & 3 \\ 5 & -6 & 9 \end{bmatrix}$, find $4A - 3B$.

Example 3.65

Find the value of a, b, c, d from the matrix equation:

$$\begin{bmatrix} d & 8 \\ 3b & a \end{bmatrix} + \begin{bmatrix} 3a & -2 \\ -4 & b-5 \end{bmatrix} = \begin{bmatrix} 2 & 2a+1 \\ b-5 & 4c \end{bmatrix}$$

Example 3.66

If A, B , and C are given 3×3 matrices, compute:

(i) $3A + 2B - C$

(ii) $(1/2)A - (3/2)B$

Example 3.67

If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{bmatrix}$, find AB .

Example 3.68

If $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$, find AB and BA . Verify if $AB = BA$.

Example 3.69

If $A = \begin{bmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{bmatrix}$, show that A and B satisfy the commutative property with respect to matrix multiplication.

Example 3.70

Solve: $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

Example 3.71

If $A = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, show that $(AB)C = A(BC)$.

Example 3.72

If $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -4 & 2 \end{bmatrix}$, and $C = \begin{bmatrix} -7 & 6 \\ 3 & 2 \end{bmatrix}$, verify that $A(B+C) = AB + AC$.

Example 3.73

If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{bmatrix}$, show that $(AB)^T = B^T A^T$.

Unit 4: Geometry

Example 4.1

Show that $\Delta PST \sim \Delta PQR$ in both cases shown in the figures.

Example 4.2

Is $\Delta ABC \sim \Delta PQR$?

Example 4.3

Observe the figure and find $\angle P$.

Example 4.4

A boy of height 90cm is walking away from the base of a lamp post at a speed of 1.2 m/sec. If the lamppost is 3.6m above the ground, find the length of his shadow cast after 4 seconds.

Example 4.5

In the given figure, $\angle A = \angle CED$. Prove that $\Delta CAB \sim \Delta CED$. Also, find the value of x .

Example 4.6

In the figure, QA and PB are perpendiculars to AB. If AO = 10 cm, BO = 6 cm and PB = 9 cm, find AQ.

Example 4.7

The perimeters of two similar triangles ABC and PQR are respectively 36 cm and 24 cm. If PQ = 10 cm, find AB.

Example 4.8

If ΔABC is similar to ΔDEF such that BC = 3 cm, EF = 4 cm and the area of $\Delta ABC = 54 \text{ cm}^2$, find the area of ΔDEF .

Example 4.9

Two poles of height 'a' metres and 'b' metres are 'p' metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $(ab)/(a+b)$ metres.

Example 4.10

Construct a triangle similar to a given triangle PQR with its sides equal to $3/5$ of the corresponding sides of the triangle PQR (scale factor $3/5 < 1$).

Example 4.11

Construct a triangle similar to a given triangle PQR with its sides equal to $7/4$ of the corresponding sides of the triangle PQR (scale factor $7/4 > 1$).

Example 4.12

In $\triangle ABC$, if $DE \parallel BC$, $AD = x$, $DB = x-2$, $AE = x+2$ and $EC = x-1$, then find the lengths of the sides AB and AC.

Example 4.13

D and E are respectively the points on the sides AB and AC of a $\triangle ABC$ such that $AB = 5.6$ cm, $AD = 1.4$ cm, $AC = 7.2$ cm and $AE = 1.8$ cm. Show that $DE \parallel BC$.

Example 4.14

In the figure, $DE \parallel AC$ and $DC \parallel AP$. Prove that $BE/EC = BC/CP$.

Example 4.15

In the figure, AD is the bisector of $\angle A$. If $BD = 4$ cm, $DC = 3$ cm and $AB = 6$ cm, find AC.

Example 4.16

In the figure, AD is the bisector of $\angle BAC$. If $AB = 10$ cm, $AC = 14$ cm and $BC = 6$ cm, find BD and DC.

Example 4.17

Construct a $\triangle PQR$ in which $PQ = 8$ cm, $\angle R = 60^\circ$ and the median RG from R to PQ is 5.8 cm. Find the length of the altitude from R to PQ.

Example 4.18

Construct a triangle ΔPQR such that $QR = 5$ cm, $\angle P = 30^\circ$ and the altitude from P to QR is of length 4.2 cm.

Example 4.19

Draw a triangle ABC of base $BC = 8$ cm, $\angle A = 60^\circ$ and the bisector of $\angle A$ meets BC at D such that $BD = 6$ cm.

Example 4.20

An insect 8 m away initially from the foot of a lamp post which is 6 m tall, crawls towards it moving through a distance. If its distance from the top of the lamp post is equal to the distance it has moved, how far is the insect away from the foot of the lamp post?

Example 4.21

P and Q are the mid-points of the sides CA and CB respectively of a ΔABC , right angled at C . Prove that $4(AQ^2 + BP^2) = 5AB^2$.

Example 4.22

What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall? Round off your answer to the next tenth place.

Example 4.23

An Aeroplane after taking off from an airport flies due north at a speed of 1000 km/hr. At the same time, another aeroplane takes off from the same airport and flies due west at a speed of 1200 km/hr. How far apart will the two planes be after $1\frac{1}{2}$ hours?

Example 4.24

Find the length of the tangent drawn from a point whose distance from the centre of a circle is 5 cm and the radius of the circle is 3 cm.

Example 4.25

PQ is a chord of length 8 cm to a circle of radius 5 cm. The tangents at P and Q intersect at a point T . Find the length of the tangent TP .

Example 4.26

In the figure, O is the centre of a circle. PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Find $\angle POQ$.

Example 4.27

In the figure, $\triangle ABC$ is circumscribing a circle. Find the length of BC.

Example 4.28

If the radii of two concentric circles are 4 cm and 5 cm, then find the length of the chord of one circle which is a tangent to the other circle.

Example 4.29

Draw a circle of radius 3 cm. Take a point P on this circle and draw a tangent at P.

Example 4.30

Draw a circle of radius 4 cm. At a point L on it, draw a tangent to the circle using the alternate segment theorem.

Example 4.31

Draw a circle of diameter 6 cm from a point P, which is 8 cm away from its centre. Draw the two tangents PA and PB to the circle and measure their lengths.

Example 4.32

Show that in a triangle, the medians are concurrent.

Unit 5: Coordinate Geometry

Example 5.1

Find the area of the triangle whose vertices are $(-3, 5)$, $(5, 6)$ and $(5, -2)$.

Example 5.2

Show that the points $P(-1.5, 3)$, $Q(6, -2)$, $R(-3, 4)$ are collinear.

Example 5.3

If the area of the triangle formed by the vertices $A(-1, 2)$, $B(k, -2)$ and $C(7, 4)$ (taken in order) is 22 sq. units, find the value of k .

Example 5.4

If the points $P(-1, -4)$, $Q(b, c)$ and $R(5, -1)$ are collinear and if $2b + c = 4$, then find the values of b and c .

Example 5.5

The floor of a hall is covered with identical tiles which are in the shapes of triangles. One such triangle has the vertices at $(-3, 2)$, $(-1, -1)$ and $(1, 2)$. If the floor of the hall is completely covered by 110 tiles, find the area of the floor.

Example 5.6

Find the area of the quadrilateral formed by the points $(8, 6)$, $(5, 11)$, $(-5, 12)$ and $(-4, 3)$.

Example 5.7

The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost ₹1300 per square foot. What will be the total cost for making the parking lot?

Example 5.8

- (i) What is the slope of a line whose inclination is 30° ?
- (ii) What is the inclination of a line whose slope is $\sqrt{3}$?

Example 5.9

Find the slope of a line joining the given points:

- (i) $(-6, 1)$ and $(-3, 2)$
- (ii) $(1/3, 1/2)$ and $(2/7, 3/7)$
- (iii) $(14, 10)$ and $(14, -6)$

Example 5.10

The line r passes through the points $(-2, 2)$ and $(5, 8)$ and the line s passes through the points $(-8, 7)$ and $(-2, 0)$. Is the line r perpendicular to s ?

Example 5.11

The line p passes through the points $(3, -2)$, $(12, 4)$ and the line q passes through the points $(6, -2)$ and $(12, 2)$. Is p parallel to q ?

Example 5.12

Show that the points $(-2, 5)$, $(6, -1)$ and $(2, 2)$ are collinear.