Congratulations! You passed!

Grade received 100% To pass 80% or higher

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1. Let $f(\theta)$ be a nonlinear function of θ mapping an n-dimensional space (the dimension of θ) to an m-dimensional space (the dimension of f). We want to find a θ_d, which may not be unique, that satisfies $x_d = f(\theta_d)$, i.e., $x_d - f(\theta_d) = 0$. If our initial guess at a solution is θ^0, then a first-order Taylor expansion approximation of $f(\theta)$ at θ^0 tells us

1 / 1 point

$x_d \approx f(\theta^0) + J(\theta^0)(\theta_d - \theta^0)$

where $J(\theta^0)$ is the matrix of partial derivatives $\partial f/\partial\theta$ evaluated at θ^0. Which of the following is a good next guess θ^1?

- (●) $\theta^1 = \theta^0 + J^\dagger(\theta^0)(x_d - f(\theta^0))$
- ( ) $\theta^1 = \theta^0 - J^\dagger(\theta^0)(x_d - f(\theta^0))$
- ( ) $\theta^1 = J^{-1}(\theta^0)(x_d - f(\theta^0))$

Correct

2. We want to solve the linear equation $Ax = b$ where A is a 3x2 matrix, x is a 2-vector, and b is a 3-vector. For a randomly chosen A matrix and vector b, how many solutions x can we expect?

1 / 1 point

- (●) None.
- ( ) One.
- ( ) More than one.

Correct
This equation implies three constraints on the two unknowns of x, so in general there are no solutions.

3. We want to solve the linear equation $Ax = b$, where

1 / 1 point

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

and $b = [7 \;\; 8]^{\mathrm{T}}$. Since x is a 3-vector and b is a 2-vector, we can expect a one-dimensional set of solutions in the 3-dimensional space of possible x values. The following are all solutions of the linear equation. Which is the solution given by $x = A^\dagger b$? (You should be able to tell by inspection, without using software.)

- ( ) $(-1.06, -3.89, 5.28)$
- (●) $(-3.06, 0.11, 3.28)$
- ( ) $(-5.06, 4.11, 1.28)$

Correct
The solution given by the pseudoinverse $A^\dagger$ minimizes the 2-norm (the square root of the sum of the squares of the vector elements) among all possible solutions, and it is apparent upon inspection that this solution has the shortest length among the 3 solutions given. (For example, comparing to the solution $(-1.06, -3.89, 5.28)$, we see that $0.11^2 < (-1.06)^2$, $(-3.06)^2 < (-3.89)^2$, and $3.28^2 < 5.28^2$.) The space of all solutions is given by this solution (or any solution) plus any value in the null space of the matrix A, where the null space of a matrix is the space of vectors v such that $Av = 0$. In other words, adding such a v to your solution x satisfies $Ax = A(x + v)$ (see any reference on linear algebra or null spaces). In this example, a basis for the null space is given by the vector $(1, -2, 1)$.

4. If we would like to find an x satisfying $Ax = b$, but A is "tall" (meaning it has more rows than columns, i.e., the dimension of b is larger than the dimension of x), then in general we would see there is no exact solution. In this case, we might want to find the x^* that comes closest to satisfying the equation, in the sense that x^* minimizes $\|Ax^* - b\|$ (the 2-norm, or the square root of the sum of the squares of the vector). This solution is given by $x^* = A^\dagger b$. Which of the two answers below satisfies this condition if

1 / 1 point

$$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 4 \end{bmatrix}?$$

- (●) $x^* = 2.2$
- ( ) $x^* = 1$

Correct
Calculating Ax^*, we get $[2.2 \;\; 4.4]^{\mathrm{T}}$. This clearly has a smaller error from the desired b than the other option, which gives $[1 \;\; 2]^{\mathrm{T}}$.