Uniform Oscillator

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- The solution is $\theta(t) = \omega t + \theta_0$.
- which corresponds to uniform motion around the circle at an angular frequency ω. This solution is **periodic**, in the sense that $\theta(t)$ changes by 2π, and therefore returns to the same point on the circle, after a time $T = 2\pi/\omega$. We call T the period of the oscillation.

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Exercises 4.2.1

- (Church bells) The bells of two different churches are ringing. One bell rings every 3 seconds, and the other rings every 4 seconds. Assume that the bells have just rung at the same time. How long will it be until the next time they ring together? Answer the question in two ways: using common sense, and using the method of Example 4.2.1.

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- Common sense: the first time they ring together will be after 12
  (the least common multiple of 3 and 4) seconds.

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- Use the method of Example 4.2.1:

$$T_{\text{lap}} = \frac{2\pi}{\omega_1 - \omega_2} = \left(\frac{1}{T_1} - \frac{1}{T_2}\right)^{-1} = \left(\frac{1}{3} - \frac{1}{4}\right)^{-1} = \left(\frac{1}{12}\right)^{-1} = 12$$

Thank you for listening!