Ising model

Speaker: Yichen Lu

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Ising model

In an approximation called mean-field theory, the equation governing the equilibrium value of m(average spin or magnetization) is

$$h = T \tanh^{-1} m - Jnm \tag{1}$$

where J and n are constants; $J > 0$ is the ferromagnetic coupling strength and n is the number of neighbors of each spin; $T > 0$ is the temperature.

- a) Analyze the solutions m^* of equation 1, using a graphical approach.

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- For the special case $h = 0$, find the critical temperature T_c at which a phase transition occurs.

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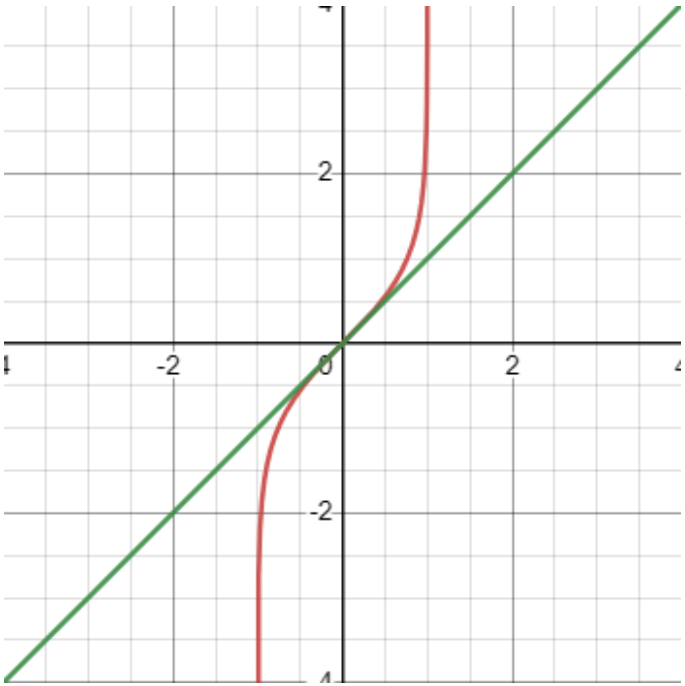
Solution to a)

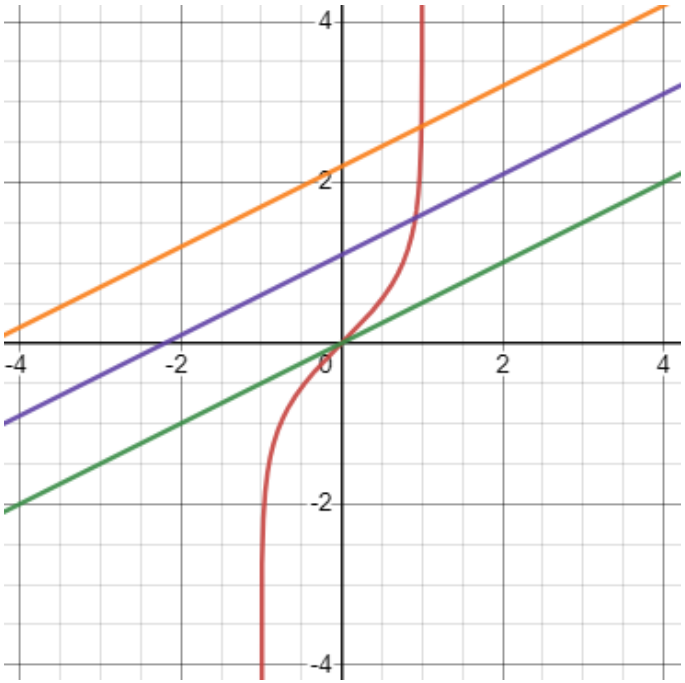
$$h = T \tanh^{-1} m - Jnm$$
$$Jnm + h = T \tanh^{-1} m$$

We plot the graphs of $y = km + h, (k = Jn)$ and $y = T \tanh^{-1} m$ on the same axes, and look for intersections



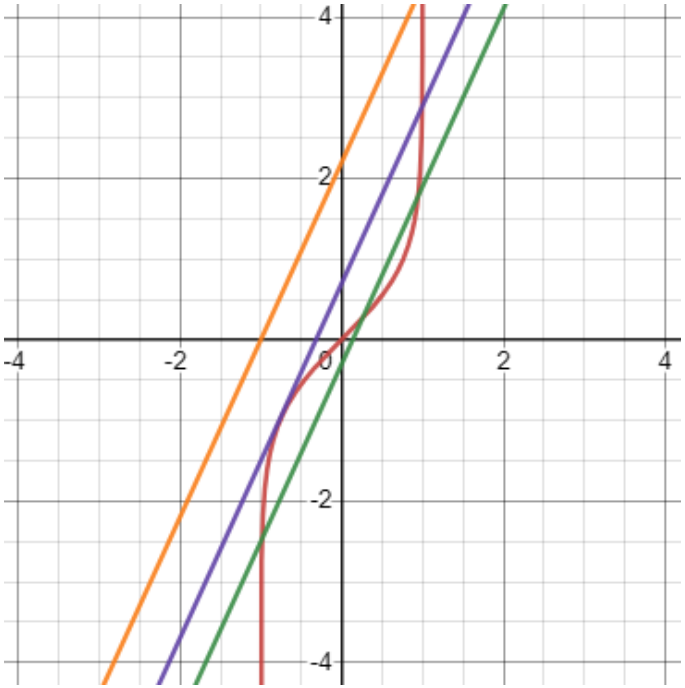
Solution to a)

Due to $\left(\tanh^{-1} m\right)' = 1/(1-m^2)$, we know that $T \tanh^{-1} m$ develops a slope of T at the origin. So when $k = Jn \in (0, T\,]$, $h \in \mathbb{R}$, the two curves intersect at only one point.



Solution to a)

When $k = Jn > T$, there are one, two, or three solutions depending on the values depending on h.



Solution to b)

For the special case $h = 0$, find the critical temperature T_c at which a phase transition occurs.

- If $h = 0$ and the phase transition occurs when the slope of the line $k = Jn = T$, so the critical temperature is $T_c = Jn$.

Thank you for listening!