Transcritical Bifurcation

Introducer: Yichen Lu

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Contents

Bifurcation

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- Fixed points can be created or destroyed,

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- Fixed points can be created or destroyed,
- or their stability can change (stable $\leftrightarrow$ unstable).

Transcritical Bifurcation

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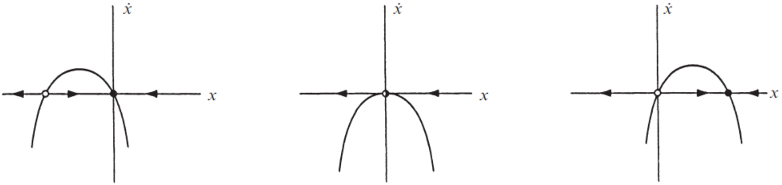
- 

(a) $r<0$ (b) $r=0$ (c) $r>0$

Contents

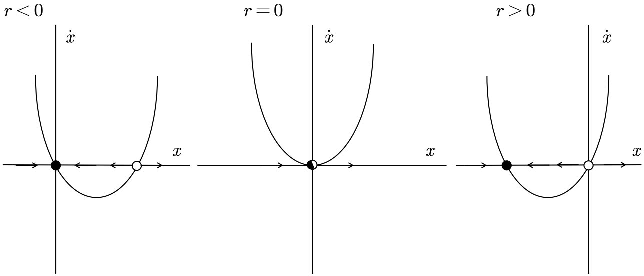
Exercises 3.2.1

Sketch all the qualitatively different vector fields that occur as r is varied. Show that a transcritical bifurcation occurs at a critical value of r, to be determined. Finally, sketch the bifurcation diagram of fixed points x^* vs. r.
Consider the following system of ODEs:

$$\dot{x} = rx + x^2$$

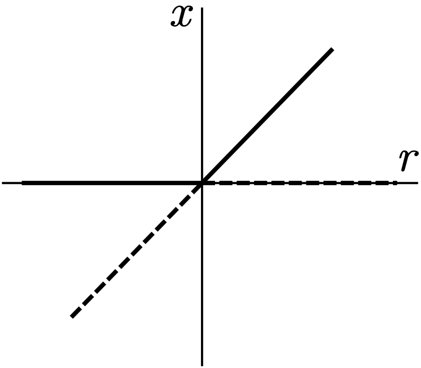
Note that $x = 0$ is a fixed point for all value of r

All the qualitatively different vector fields that occur as r is varied:



Transcritical bifurcation occurs at r = 0.

Bifurcation diagram of fixed points x^* vs. r :



Exercises 3.2.5

$$A + X \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} 2X \quad X + B \xrightarrow{k_2} C.$$

According to the law of mass action of chemical kinetics, the rate of an elementary reaction is proportional to the product of the concentrations of the reactants. Then the equation for the kinetics of x is

$$\dot{x} = k_1 ax - k_{-1} x^2 - k_2 bx$$
$$\dot{x} = (k_1 a - k_2 b)x - k_{-1} x^2$$
$$\dot{x} = c_1 x - c_2 x^2$$

,where $c_1 = k_1 a - k_2 b$ and $c_2 = k_{-1}$.
For $c_1 < 0$, $x^* = 0$ become stable, so $k_1 a < k_2 b$

Thank you for listening!