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Spatial groups and cyclic oscillations induced by positive correlation between moving direction and phase of mobile oscillators

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ABSTRACT

Lots of studies on synchronization focus on networks with static topologies, but ignore the synchronization in time-varying networks. We study the synchronization of the system in which agents carrying phase oscillators move in two-dimensional lattice with periodic boundary conditions and only interact with their neighbors. In particular, we assume that the direction of movement of an agent is positively correlated to the phase of the oscillator. By numerical simulations, we find cyclic oscillations in the phase transition to synchronization. Remarkably, the oscillators cluster spatially toward the oscillators with higher natural frequency to form groups. And the groups will collapse from the center, as coupling strength exceeds the critical value. In addition, we investigate the effect of moving speed on synchronization and find that the cyclic oscillation during synchronization will disappear with the increment of moving speed.

1. Introduction

Synchronization is one of the most important collective phenomena that interacting units adjust their rhythm over time to achieve a coherent behavior [1–3], which plays a critical role in many contexts, such as social [4,5], biological [6–9], electrical systems [10] and swarmalators [11–14]. In recent years, with the developments of network science, heterogeneous interactions in real systems can be characterized picturesquely in terms of complex networks. Therefore, taking advantage of network science, the understanding of the interplay between the topology of interacting units and the synchronization has been greatly improved in the last decade [15–19].

In the context of complex networks, considerable previous studies on the investigation of synchronization in the dynamic of complex networks mostly have assumed that the topology of the network is fixed, which refers to the interactions between oscillators being static for all course of the time [20–22]. But notably, this sort of assumption may not resemble for many practical situations. In effect, the most prominent feature of the networks in real world is that the interactions among agents are not fixed in time, conversely, they have an explicit temporal nature (their strength may be time-dependent, or may be suppressed in some moments and activated in other moments). There are so many typical examples taking place in such time-varying system, such as animal groups [23], time-dependent plasticity in neural networks, power transmission systems [24], person-to-person communication [25], wireless sensor networks [26], and so on. Networks with time-varying topology, which are called time-varying networks or temporal networks, are one of the currently most significant extensions of complex networks. The time-varying networks provide an effective method to account for the time-vary interactions which are the most realistic category of inter-node interactions. It's evident that synchronization is one of the major facets of such networks, and a review about synchronization on time-varying networks can be found in Ref. [27]. Ref. [27] has elucidated the dependence of the structural evolution of the graph on the state space associated with the individual units, time and the position of nodes in physical space.

Among the studies of synchronization in time-varying networks, a common approach is to consider the systems where mobile agents perform random walk in a certain space and only interact with the nearby ones. This kind of network has been called as moving neighborhood network [28]. The network of in-

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teractions is determined by the way in which connections are established between agents and by the characteristics of the agent motion. In this system, agents can only couple with the neighbors within their communication radius. Each mobile agent is assumed to carry an oscillator and perform synchronization with locally coupled neighbors. Thus, the rule of motion adopted by agents has an impact on the emergence and stability of the synchronization. The problem of synchronization of moving oscillators has enormous applications in various fields [8,29,30] such as epidemic modeling [31], moving robots with wireless sensors [32], the expression of segmentation clock genes [33], and any more. This framework provides a significant platform for better study of synchronization on mobile networks. Now, a fundamental question is how the mobility of agents affects the dynamic of synchronization. The synchronous behavior in moving neighborhood network has been studied [34,35]. N. Fujiwara et al. introduced two timescales in moving neighborhood networks: one related to local synchronization in clusters and the other related to the topology change, and found that a greater timescale of topology change can prevent the occurrence of synchronization [36]. S. Majhi and D. Ghosh presented a network model of interaction between moving oscillators where each oscillator is moving in three-dimensional space [37]. G. Petrungaro studied the synchronization dynamics of mobile phase oscillators in the presence of coupling delays and found that mobility can speed up synchronization when coupling delays are present [38]. In addition, S. Majhi et al. explored the interlayer and intralayer synchronization states in a multiplex dynamical network comprising of layers having mobile Rössler oscillators performing two-dimensional lattice random walk [6]. Since locally coupled oscillators can move around and exchange their neighbors, allowing oscillators to interact with previously distant neighbors, the mobility of coupled oscillators may promote synchronization by extending the effective range of coupling. In the above studies, the movement of oscillators is independent of their states. The moving speed, moving direction, and communication radius of the oscillators will not change with the dynamic of synchronization. A different situation arises when the movement of oscillators is affected by their states.

In the current work, we propose a simple but effective scheme to synchronize an ensemble of moving dynamical agents. We combine the agent dynamics and oscillator dynamics in a complex network to explore the impact of the interweaving of two processes on synchronization. In contrast to the free motion of the oscillator in previous studies, however, the direction of motion of oscillator cannot be completely random in real world and must be influenced by its own state and surroundings. Therefore, we make the direction of movement of the oscillator correlate with the phase. For the sake of simplicity, we assume that the magnitude of the direction of travel is equal to the magnitude of the phase. R. Sepulcher and N.E. Leonard have proposed a framework called particles with coupled oscillators dynamics (PCOD) where the headings of particles are equal to the phases of the oscillators [39]. However, most of their work focus on the studies of collective motion under different control laws and cooperate control algorithms in PCOD, and the synchronization process is less studied [40–45]. We analyze both the macroscopic and microscopic collective dynamics of locally coupled, moving phase oscillators in the proposed Kuramoto model. Remarkably, an interesting, cyclic oscillation phenomenon in the transition to coherence was revealed by numerical simulations when the moving direction of agents and phase of oscillators are positively correlated. The cyclic oscillation is similar to what Dai et al. found in the positive feedback D-dimensional Kuramoto model [46]. The effect of oscillators' natural frequency on the formation of the final groups and the impact of agents' speed of movement on the synchronization process have also been explored by numerical simulations.

2. Model

For simplicity, we consider a two-dimensional plane with length L, where N mobile agents are randomly arranged in it with periodic boundary conditions. We model the mobility of the agents according to the general scheme proposed in Ref. [21]. Each mobile agent is associated with a phase oscillator and moves with the speed v. Their position $(x_i(t_k), y_i(t_k))$ at discrete time steps t_k is updated according to

$$\begin{aligned} x_i\,(t_k+\Delta t) &= x_i\,(t_k) + v cos\theta_i(t_k)\Delta t \mod L,\\ y_i\,(t_k+\Delta t) &= y_i\,(t_k) + v sin\theta_i(t_k)\Delta t \mod L, \end{aligned} \tag{1}$$

where $x_i\,(t_k)$ and $y_i\,(t_k)$ denote the horizontal ordinate and vertical ordinate of node i at time t, respectively. $\theta_i(t) \in [-\pi, \pi)$ represents the angle of the ith agent's motion at time t, $i \in [1, N]$, the step size $\Delta t = t_{k+1} - t_k$. In our work, the state of oscillators can influence the movement of mobile agents. We consider that the motion direction of the agent is affected by the phase of the oscillator such that the angle is positively correlated with the phase variable of oscillator, namely $\theta_i(t) = \varphi_i(t)$. The phase variable $\varphi_i \in [-\pi, \pi)$ of oscillator i evolves in terms of the following equation:

$$\dot{\varphi}_i(t) = \omega_i + \lambda \sum_{j=1}^{N} A_{ij} \sin(\varphi_j(t) - \varphi_i(t)). \tag{2}$$

Here, $\dot{\varphi}_i$ is the instantaneous angular velocity of the ith oscillator, ω_i is the natural frequency of ith oscillator and λ represents the coupling strength of the whole system. The adjacency matrix A defines the connections of the network, in which $A_{ij} = 1$ represents the presence of edge between the ith node and jth node, otherwise, $A_{ij} = 0$. The synchronization of locally coupled oscillators depends on the coupling topology, that is, how the oscillators are connected with each other. Interactions can take place whenever agents are sufficiently close. We calculate the Euclidean distance $D_{ij}(t) = \sqrt{[(x_i(t) - x_j(t)]^2 + [(y_i(t) - y_j(t)]^2}$ between two agents. For a particular instant, phase oscillators can only interact with their local neighbors within an interaction range r, which can also be called communication radius. In other words, $A_{ij} = 1$, if $D_{ij}(t) \le r$, and $A_{ij} = 0$, if $D_{ij}(t) > r$. Obviously, the mobility of the agents arises the variation in interaction.

In order to quantify transition to synchronization of N oscillators in the network, we define the global order parameter R of system, and R is given by:

$$R(t)e^{i\psi(t)} = \frac{1}{N} \sum_{j=1}^{N} e^{i\varphi_i(t)}, \tag{3}$$

here i is the imaginary unit and $\psi(t)$ represents the average phase of the system. Here, $0 \le R(t) \le 1$. $R(t) = 0$ corresponds to incoherent state where all the oscillators have distinct phases. Conversely, $R(t) = 1$ denotes a fully synchronized state.

3. Simulation results

In this letter, the dynamics of oscillators are explored through the numerical simulations. Starting from the random initial phases and positions, we let the system evolves according to Eq. (1) and Eq. (2) and integrate Eq. (2) by using a standard RK4 algorithm with step length $\delta h = 0.01$. Unless otherwise stated, the simulation is performed on a 10×10 square lattice ($L = 10$) with periodic boundary conditions and our results refer to networks consisting

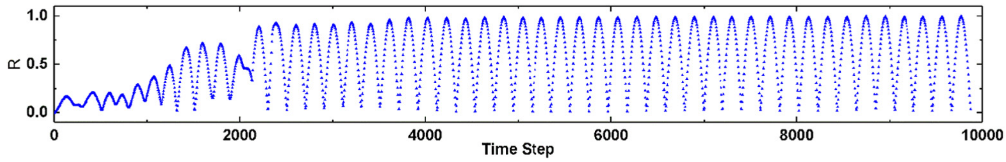

Fig. 1. Time evolution of global order parameter of system at coupling strength $\lambda = 0.1$, speed of movement $v = 0.03$, communication radius $r = 1$.

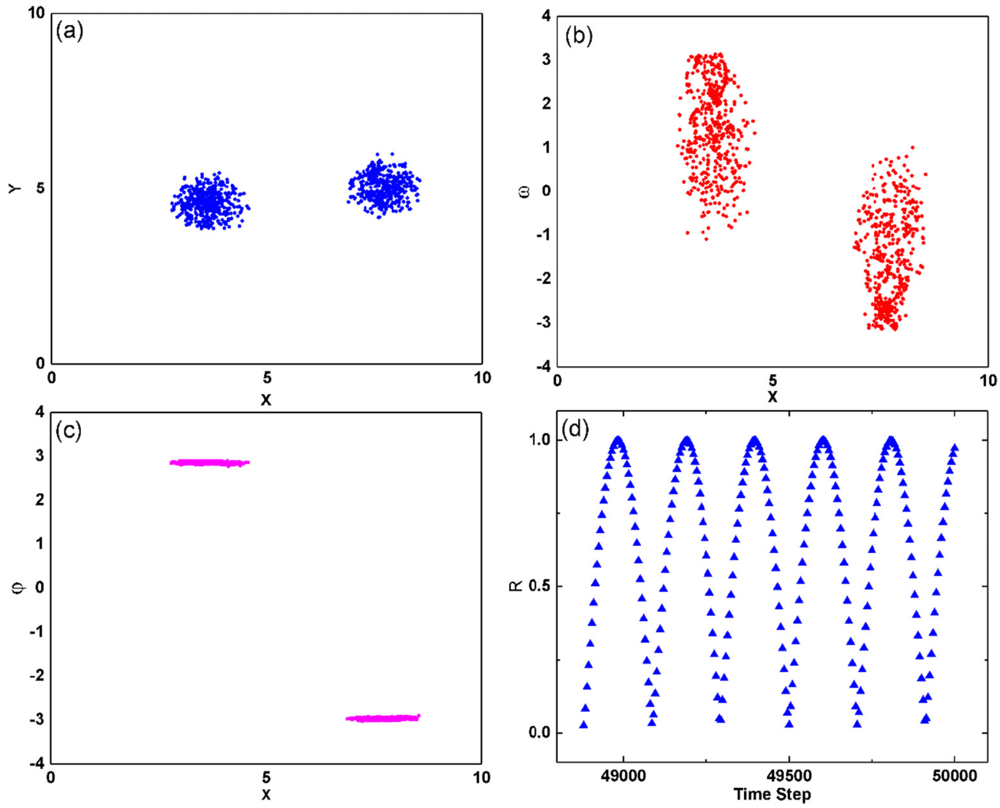

Fig. 2. (a) The spatial distribution of the agents, (b) natural frequency verses horizontal ordinate for each oscillator, (c) phase verses horizontal ordinate for each oscillator, (d) time evolution of global order parameter when the system reaches to stable state. Parameters are consistent with those in Fig. 1. The microscopic details of the cyclic oscillations can be best viewed in supplementary movie [51]. (For interpretation of the colors in the figures, the reader is referred to the web version of this article.)

of $N = 1000$ oscillators, with the distribution of oscillators' natural frequency ω_i independent uniformly drawn from interval $[-\pi, \pi]$.

For the numerical simulations, we begin our study by exploring the time evolution of global order parameter of system. In Fig. 1, we fix the coupling strength $\lambda = 0.1$ and display the global order parameter R as a function of time step for a network with $N = 1000$, communication radius $r = 1$, speed of movement $v = 0.03$. As shown in Fig. 1, the value of R does not increase gradually to reach stability as expected, but oscillates constantly in the path to synchronization. The synchronization degree (respected by the value of R) of the system spirals from zero to a certain value firstly. As time goes on, the magnitude of R begins to evolve into cyclic oscillation and fluctuates at the range from 0 and 1.

To intuitively understand how the cyclic oscillations occur, we plot the horizontal ordinate, vertical ordinate in two-dimensional lattice, phase and natural frequency of each oscillator in Fig. 2 when the speed of movement $v = 0.03$, and communication radius $r = 1$. Blue dots in Fig. 2(a) show the spatial distribution of oscillators. We can see that all oscillators are aggregated into two groups at the coupling strength $\lambda = 0.1$. And as depicted in Fig. 2(c), oscillators in each group have the same phase, thus two groups both have reached synchronization. Furthermore, one can see from Fig. 2(b) that the natural frequencies of most oscillators in the left group are positive, on the contrary, the natural frequencies of most oscillators in the right group are negative. Because of the existence of communication radius r, if the distance between the two groups is greater than r, oscillators in the two communities cannot interact and can only synchronized with oscillators in the local community. Therefore, the two communities can never be combined into one and achieve stable state of complete synchronization. In addition, since the phases of the oscillators in the two communities are the same, the second term on the right-hand side of Eq. (2) is 0, so the update of phase only depends on its own natural frequency. The average frequency of one community is positive and the other is negative, leading to the cyclic oscillation of global order parameter of the whole system which plotted in Fig. 2(d).

To gain further insights, we now move to numerically investigate phase transition to synchronization. As for the stipulations followed in our simulations, the state of the network is monitored by gradually increasing the coupling strength λ from 0 to $\lambda + n\delta\lambda$ (where already one has $R \sim 1$) with step $\delta\lambda = 5 \times 10^{-6}$. Fig. 3 shows the global order parameter R as a function of coupling strength λ when the speed of motion $v = 0.03$, communication radius $r = 1$. As plotted in Fig. 3, one can clearly see that, different from the phase transition in previous studies, an oscillating state emerges between the incoherent ($R \backsim 0$) and coherent ($R \backsim 1$) states in our work. We use dashed lines to divide the phase tran-

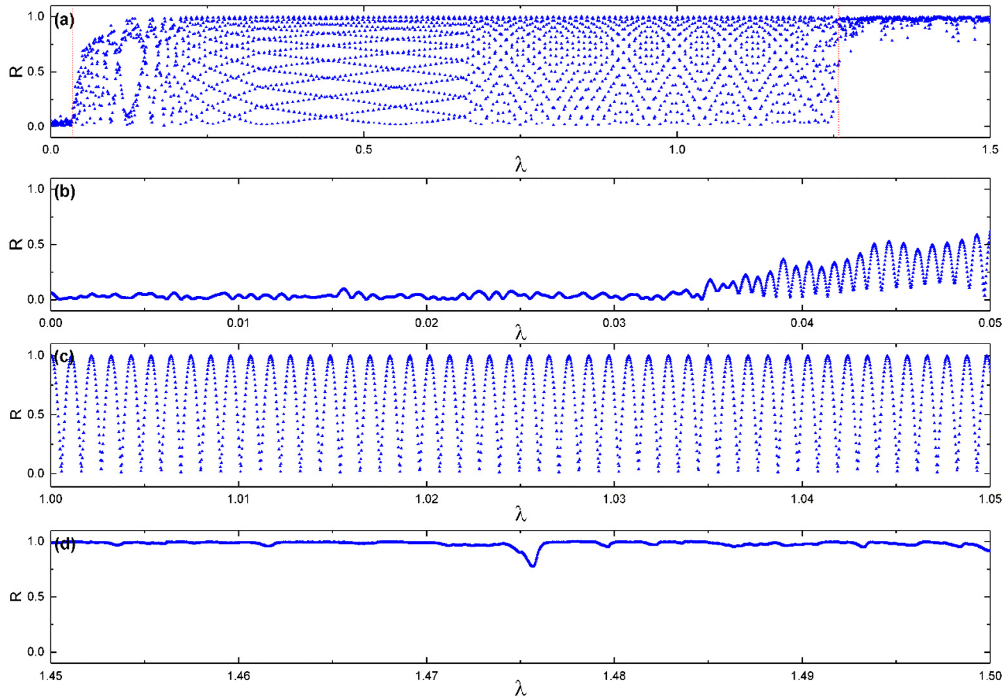

Fig. 3. (a) The transition to synchronization. The red dashed lines are straight lines that mark the critical points at different stages of the synchronization process. Plot of Fig. 3(b) depicts that R gradually increases at the beginning and begins to fluctuate. Plot of Fig. 3(c) depicts that the value of R starts to fluctuate wildly from 0 to 1, and the oscillations occur periodically. Plot of Fig. 3(d) shows that the system reaches synchronization and the cyclic oscillation disappears. Other parameters are consistent with those in Fig. 1.

sition process into three stages. As shown in Fig. 3(a), there are two critical points during the phase transition, λ_{c1} and λ_{c2}. In the case of $\lambda < \lambda_{c1}$, the system of oscillators goes to an incoherent state, $R \approx 0$ (as shown in Fig. 3(b)). As the coupling strength just exceeds λ_{c1}, $\lambda > \lambda_{c1}$, R is time-dependent and displays a phenomenon of cyclic oscillations. Notice that, the value of R in this stage ranges from 0 to 1 (as shown in Fig. 3(c)). Furthermore, as coupling strength exceeds λ_{c2}, $\lambda > \lambda_{c2}$, the cyclic oscillations disappear and global order parameter R converges to constant values close to 1 (as shown in Fig. 3(d)). All oscillators ultimately acquire the same phase.

We now move to investigate the microscopic details behind the observed oscillations in R with the increment of coupling strength λ. In Fig. 4, five typical phases are illustrated, corresponding to the values of λ equal to 0.01, 0.05, 0.5, 0.6, and 1.5, respectively. The panels in left column plot the spatial distribution of the agents and the color represents the natural frequency of the oscillators (carried by agents). The middle and right column panels plot the natural frequency and phase of oscillators in two-dimensional lattice, respectively. In Fig. 4(a1)-4(a3), $\lambda = 0.01$, all oscillators are randomly distributed on a 2D plane and $R \sim 0$ as shown in Fig. 3(b). At $\lambda = 0.05$ [Fig. 4(b1)-4(b3)], we first notice that oscillators have a tendency to cluster and two small groups begin to emerge. Noticing that, oscillators with large absolute values of the natural frequencies take the lead in gathering to groups. As shown in Fig. 4(b3), the oscillators in the groups have been synchronized. In previous studies, low-frequency oscillators were "tamed" by the mean field first, while high-frequency oscillators required a larger mean field to be pulled into the synchronous group [47,48]. However, because of the positive correlation between direction and phase in our work, the oscillators with large natural frequency can move in a larger range and interact with a large number of oscillators. As a result, oscillators with high natural frequency tend to synchronize with each other first, and thus move in the same direction, forming a group and constantly attracts their neighbors. At $\lambda = 0.5$ [Fig. 4(c1)-4(c3)], according to the positive and negative of the value of natural frequency, almost all drifting oscillators are divided into two groups. The oscillators with the higher absolute natural frequency locate in the center of the group, and the oscillators with the lower absolute natural frequency are closely surrounded by them. At this coupling strength, the oscillators in different groups have been synchronized respectively, but due to the opposition in sign of natural frequency, the oscillators in different groups rotate in the opposite direction. Hence, the global order parameter R of the system will exhibit cyclic oscillations as shown in Fig. 3(b). When λ gradually increases to 0.6, as shown in the Fig. 4(d1)-(d3), a hollowing area appears in the center of both groups. Two local groups collapse first from oscillators in the center. At $\lambda = 1.5$ [Fig. 4(e1)-4(e3)], the two local groups collapse completely and the system reaches to synchronization state. As shown in Fig. 4(e3), although spatially scattered, the phases of all the oscillators are identical.

As mentioned above in Fig. 2, the oscillators are spatially divided into two groups based on the positive or negative natural frequencies, and the number of oscillators in both groups is the same. Previously, we generally took a uniform distribution of natural frequencies between $[-\pi, \pi]$. Thus, a question arises: will there be a difference in the number of oscillators of a group if the natural frequencies are asymmetrically distributed? In Fig. 5, we no longer take a uniform distribution and make the p proportion oscillators randomly selected between $[-\pi, 0]$ and the other 1 - p oscillators selected between $[0, \pi]$. As depicted in Fig. 5, the number of oscillators in the group is indeed related to the positive or negative of the natural frequency. With the proportion of oscillators with natural frequencies between $[-\pi, 0]$ changing from 0 to 0.4, the distribution asymmetry of oscillators' natural frequency gradually decreases. So, if all oscillators' natural frequencies are positive ($p = 0$), there will be only one group exists.

Lastly, we shift our focus to the effect of the constant absolute velocity on the synchronization process. We plot the transitions to synchronization with different speeds of movement, while other

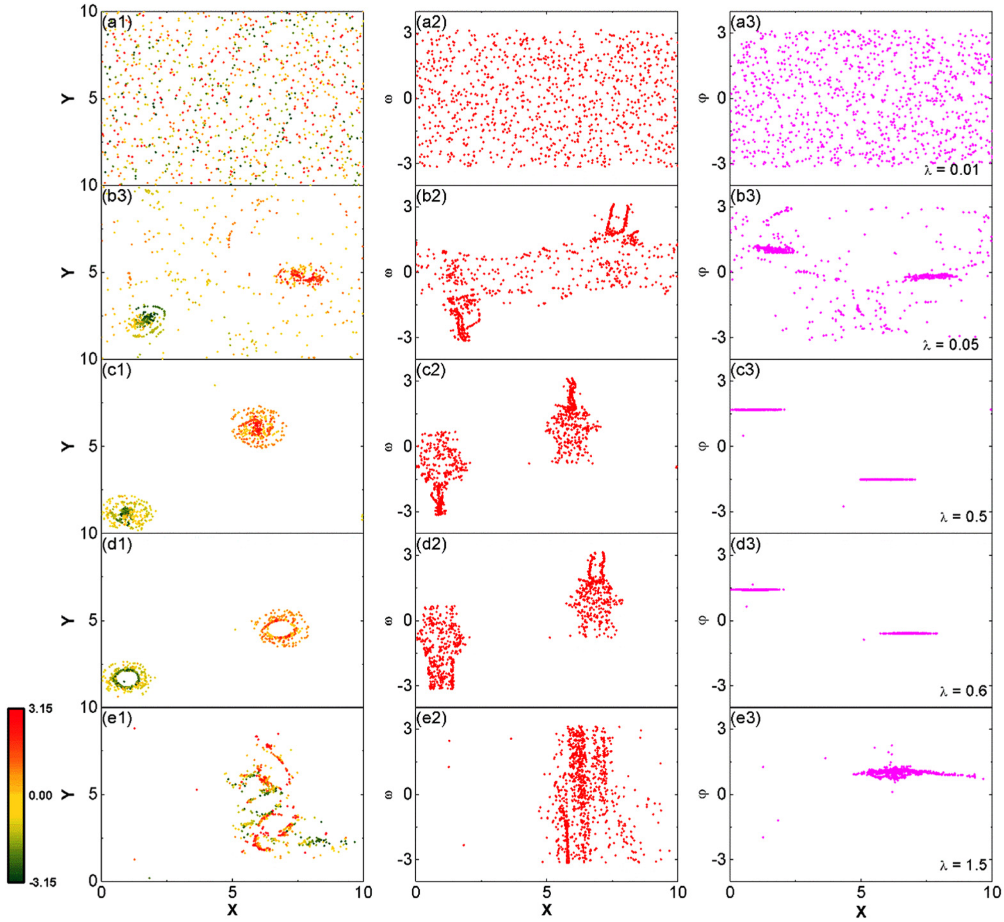

Fig. 4. The spatial distribution (left column panels), natural frequency (middle column panels), and phase (right column panels) of oscillators in 2-dimensional lattice for coupling strength $\lambda = 0.01, 0.05, 0.5, 0.6$ and 1.5, respectively. The color in left column panels represents the natural frequency of oscillators. The network consists of $N = 1000$ mobile oscillators with $\omega_i \in [-\pi, \pi]$, $v = 0.03$, $r = 1$ and $L = 10$.

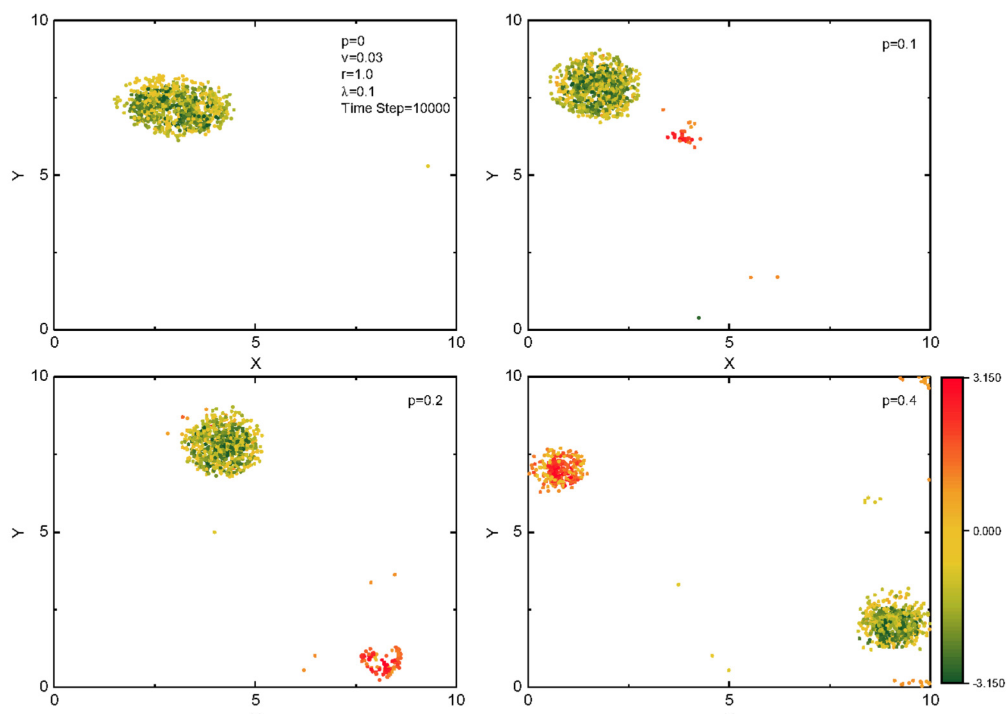

Fig. 5. The spatial distribution of oscillators in 2-dimensional lattice for different natural frequency distribution. (a) $p = 0$, (b) $p = 0.1$, (c) $p = 0.2$, (d) $p = 0.4$ when $v = 0.03$, $r = 1$, $\lambda = 0.1$. Color represents natural frequency ω_i of oscillator ranges from $[-\pi, \pi]$. p represents the proportion of oscillators with natural frequencies between $[-\pi, 0]$.

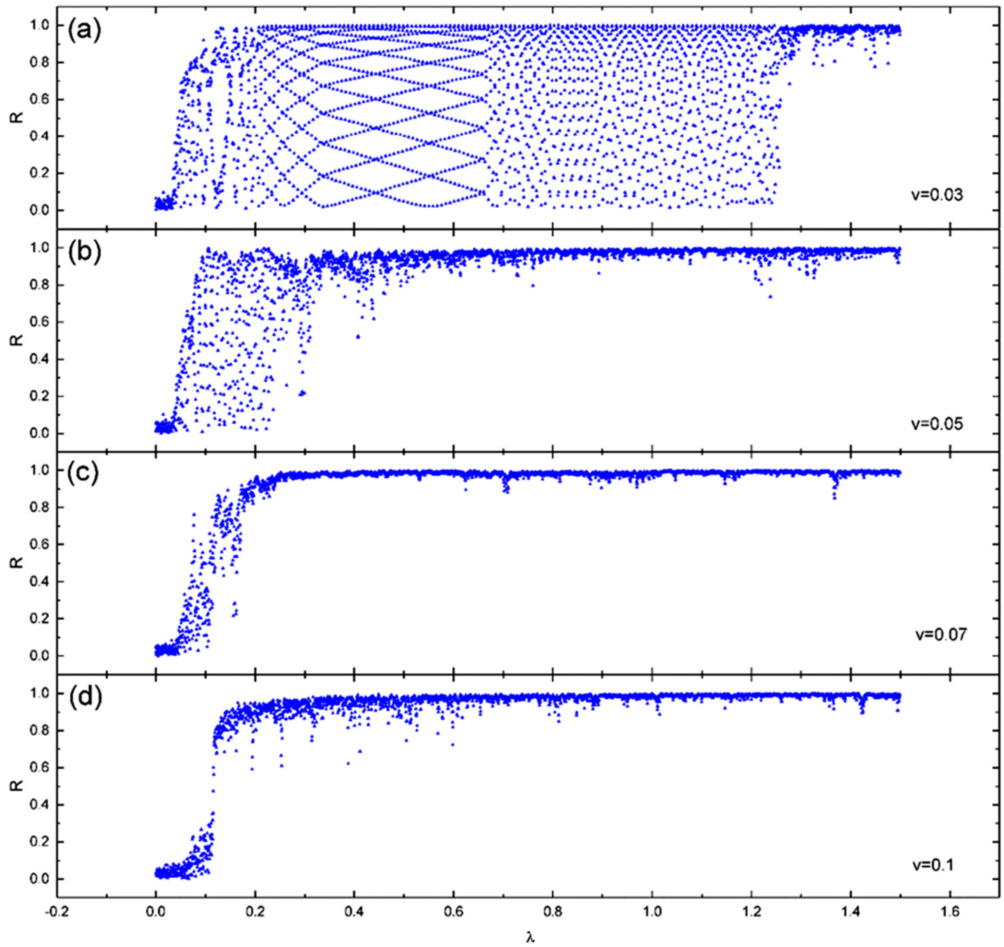

Fig. 6. Order parameter as a function of coupling strength for different speeds of movement. Speeds of oscillators in each panel are (a) $v = 0.03$, (b) $v = 0.05$, (c) $v = 0.07$, and (d) $v = 0.1$.

parameters the same as those in Fig. 3(a). In Fig. 6, we report the global order parameter R vs coupling strength λ for $v = 0.03$, 0.05, 0.07, and 0.1. Here, the coupling strength λ is gradually increased from 0 to 1.5 with step $\delta\lambda = 5 \times 10^{-6}$. Notice that increasing the velocity of the oscillators results in a smaller area of oscillation. At $v = 0.1$ [Fig. 6(d)], the cyclic oscillation of R disappears completely and there is a sudden jump in R from 0.3 to 0.7. In addition, the coupling strength, which required for the system to reach synchronization for the first time, does not smell to be affected by the speed of movement, if oscillations are out of consideration. On the basis of this founding, we deduce that the agents, which move faster, can own more neighbors and interact with more agents. Therefore, local groups cannot emerge.

4. Conclusion

In summary, we study the synchronization of mobile oscillators in two-dimensional lattice. In particular, we propose a Kuramoto-like model in which the direction of motion of the agent is positively correlated with the phase of the oscillator. Compared with the previous studies, a rich and novel scenario is discovered in the transition. We show that the cyclic oscillation of global order parameter will emerge when the coupling strength exceeds the threshold value. Increasing coupling strength again, the oscillation disappears and the system reaches to the complete synchronization state. Therein, the natural frequency plays a significant role as the oscillators spontaneously form groups around the oscillators with large absolute frequencies. Our method makes the oscillators move in the same direction after synchronization to achieve spatial aggregation. In addition, it was shown that the asymmetry of the natural frequency distribution of the oscillators will affect the formation of the final groups. Lastly, our simulation results also reveal the effects of mobility on the macroscopic collective dynamics of locally coupled, moving phase oscillators. The cyclic oscillations will disappear with increasing of the speed of movement. Synchronization on mobile networks has always been an interesting issue. For example, synchronization of ad hoc networks [49], the formation of consensus in a crowd [50]. Real-world systems are often complex and intertwined with multiple processes. In our work, the rich nonlinear dynamics, which emerge in the relatively simple case that direction of movement of agents equals to phase of oscillators, highlights the complexity that may arise via simplicial interactions in systems.

CRediT authorship contribution statement

Xiang Ling: Writing – review & editing, Visualization, Methodology, Conceptualization, Writing – original draft. **Qing-Yang Liu:** Writing – review & editing, Writing – original draft, Conceptualization, Methodology, Software. **Bo Hua:** Writing – original draft, Writing – review & editing. **Kong-Jin Zhu:** Writing – review & editing, Funding acquisition. **Ning Guo:** Writing – review & editing, Funding acquisition. **Ling-Lin Li:** Writing – review & editing, Funding acquisition. **Jia-Jia Chen:** Writing – review & editing, Funding acquisition. **Chao-Yun Wu:** Writing – review & editing, Funding acquisition. **Qing-Yi Hao:** Writing – review & editing, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.physleta.2022.128428.

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[51] See Supplemental Material to understand how the cyclic oscillations occur intuitively.