Chemotactic Network Designing

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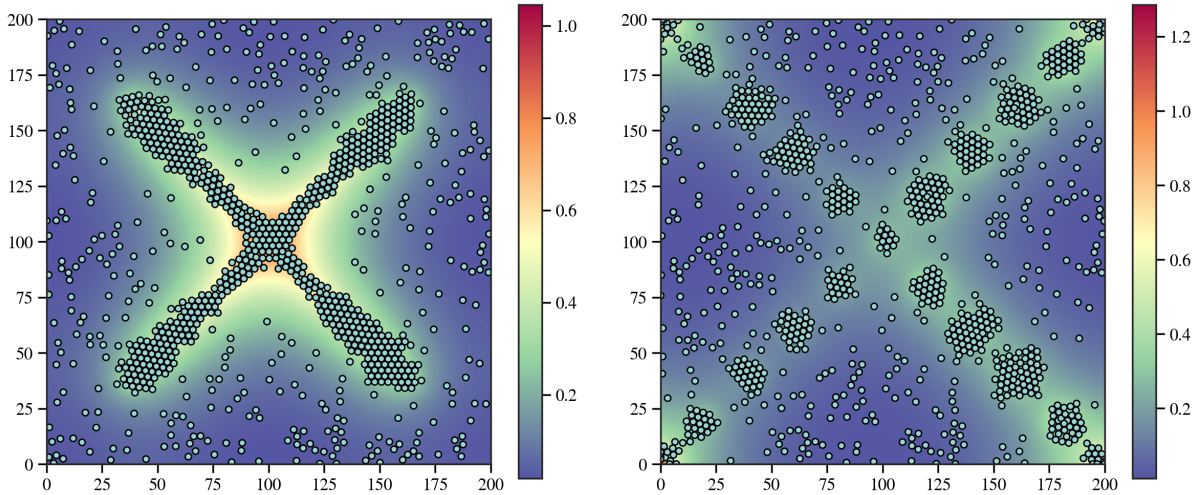
1 Models

1.1 Thinking Process

$$\dot{\mathbf{r}}_i(t) = \alpha_c \nabla c - \nabla_{\mathbf{r}_i} V + \sqrt{2D_p}\boldsymbol{\eta}_i \tag{1a}$$

$$\dot{c}(\mathbf{r}, t) = D_c \nabla^2 c - k_c c + \beta_c \sum\nolimits_j \delta\left(\mathbf{r} - \mathbf{r}_j^*\right) \tag{1b}$$

for $i = 1, 2, \cdots, N$ and $j = 1, 2, \cdots, M$. Here, $\mathbf{r}_i, \mathbf{r}_j^*$ is the position of the i-th particle, j-th target node, respectively, c is the concentration of the signal chemical released by the node, $\alpha_c < 0$ is the chemotactic sensitivity, V is the potential field of short-range repulsion, D_p is the intensity of thermal noise, s_i is the internal state of the i-th particle, D_c is the diffusion coefficient, k_c is the decay rate of the chemical, and β_c is the production rate of the target nodes.



(a) Horizontal distance of nodes $d_n = 7$ (b) $d_n = 10$

Figure 1: The simulation of the above model with $\alpha_c = -5$, $D_c = 2$, $k_c = 0.001$ and $\beta_c = 0.3$. When the horizontal distance of nodes d_n is small, the nodes are connected by the particles. While, when d_n is large, the particles are not connected.

The above model have initially realized the network formation, but it does not solve the problem when the distance between nodes is large. To address this, we introduce a convection term $\mathbf{v} \cdot \nabla c$ in the chemical equation, which represents the influence of the particles' active transport on the chemical field. The updated model is as follows:

$$\dot{\mathbf{r}}_i(t) = \alpha_c \nabla c - \nabla_{\mathbf{r}_i} V + \sqrt{2D_p}\boldsymbol{\eta}_i\,, \tag{2a}$$

$$\dot{c}(\mathbf{r}, t) = D_c \nabla^2 c - \mathbf{v} \cdot \nabla c - k_c c + \beta_c \sum\nolimits_j \delta\left(\mathbf{r} - \mathbf{r}_j^*\right)\,, \tag{2b}$$

where $\mathbf{v} = v_{\boldsymbol{c}} \sum_{i=1}^{N} \delta(\mathbf{r} - \mathbf{r}_i)(\cos\phi_{\mathbf{r}}, \sin\phi_{\mathbf{r}})^\top$ is the convection velocity of the chemical field, $v_{\boldsymbol{c}}$ is the convection speed, and

$$\phi_{\mathbf{r}} = \tan^{-1}\left(\frac{y_{\mathrm{cm}} - y_i}{x_{\mathrm{cm}} - x_i}\right) \tag{3}$$

is the angle toward the particle's center of mass at position $\mathbf{r}$, where $(x_{\mathrm{cm}}, y_{\mathrm{cm}})^\top = N^{-1} \sum_i \mathbf{r}_i$.

$$\dot{\mathbf{r}}_i(t) = \alpha_c \nabla c - \nabla_{\mathbf{r}_i} V + \sqrt{2D_p}\boldsymbol{\eta}_i\,, \tag{4a}$$

$$\dot{c}(\mathbf{r}, t) = D_c \nabla^2 c - k_c c + \sum\nolimits_j s_j \delta\left(\mathbf{r} - \mathbf{r}_j^*\right)\,, \tag{4b}$$

$$\dot{s}_j(t) = (1 + s_j)(1 - s_j)\left(s_{\mathrm{th}} - \sum\nolimits_i \delta\left(\mathbf{r}_i - \mathbf{r}_j^*\right)\right) \tag{4c}$$

2 Behaviors

3 Continuum model