

# Chemotactic Network Designing

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# 1 Models

## 1.1 Thinking Process

$$\dot{\mathbf{r}}_i(t) = \alpha_c \nabla c - \nabla_{\mathbf{r}_i} V + \sqrt{2D_p} \boldsymbol{\eta}_i \quad (1a)$$

$$\dot{c}(\mathbf{r}, t) = D_c \nabla^2 c - k_c c + \beta_c \sum_j \delta(\mathbf{r} - \mathbf{r}_j^*) \quad (1b)$$

for  $i = 1, 2, \dots, N$  and  $j = 1, 2, \dots, M$ . Here,  $\mathbf{r}_i, \mathbf{r}_j^*$  is the position of the  $i$ -th particle,  $j$ -th target node, respectively,  $c$  is the concentration of the signal chemical released by the node,  $\alpha_c < 0$  is the chemotactic sensitivity,  $V$  is the potential field of short-range repulsion,  $D_p$  is the intensity of thermal noise,  $s_i$  is the internal state of the  $i$ -th particle,  $D_c$  is the diffusion coefficient,  $k_c$  is the decay rate of the chemical, and  $\beta_c$  is the production rate of the target nodes.

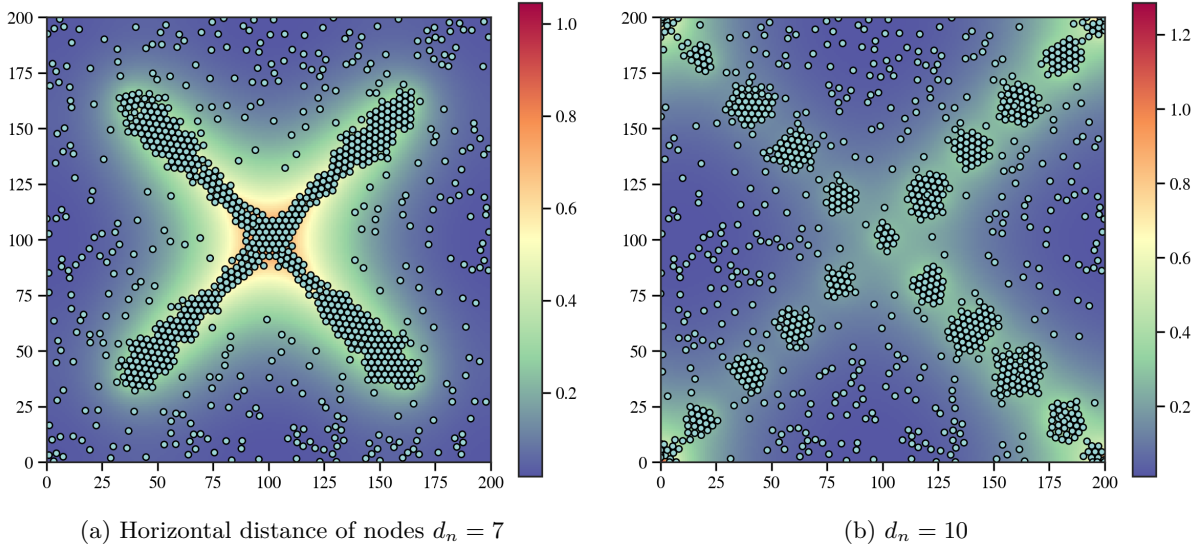


Figure 1: The simulation of the above model with  $\alpha_c = -5$ ,  $D_c = 2$ ,  $k_c = 0.001$  and  $\beta_c = 0.3$ . When the horizontal distance of nodes  $d_n$  is small, the nodes are connected by the particles. While, when  $d_n$  is large, the particles are not connected.

The above model have initially realized the network formation, but it does not solve the problem when the distance between nodes is large. To address this, we introduce a convection term  $\mathbf{v} \cdot \nabla c$  in the chemical equation, which represents the influence of the particles' active transport on the chemical field. The updated model is as follows:

$$\dot{\mathbf{r}}_i(t) = \alpha_c \nabla c - \nabla_{\mathbf{r}_i} V + \sqrt{2D_p} \boldsymbol{\eta}_i, \quad (2a)$$

$$\dot{c}(\mathbf{r}, t) = D_c \nabla^2 c - \mathbf{v} \cdot \nabla c - k_c c + \beta_c \sum_j \delta(\mathbf{r} - \mathbf{r}_j^*), \quad (2b)$$

where  $\mathbf{v} = v_c \sum_{i=1}^N \delta(\mathbf{r} - \mathbf{r}_i) (\cos \phi_{\mathbf{r}}, \sin \phi_{\mathbf{r}})^\top$  is the convection velocity of the chemical field,  $v_c$  is the convection speed, and

$$\phi_{\mathbf{r}} = \tan^{-1} \left( \frac{y_{\text{cm}} - y_i}{x_{\text{cm}} - x_i} \right) \quad (3)$$

is the angle toward the particle's center of mass at position  $\mathbf{r}$ , where  $(x_{\text{cm}}, y_{\text{cm}})^\top = N^{-1} \sum_i \mathbf{r}_i$ .

## 1.2 Final Definitions

We consider a 2D box with  $N$  chemotactic particles and  $M$  food source nodes. The particles are subject to a chemotactic force, a short-range repulsion force, and a noise term. The chemical concentration field

evolves via diffusion, decay, and production from the target nodes, while each particle's internal state regulates the local chemical concentration. The dynamics are governed by the following equations:

$$\dot{\mathbf{r}}_i(t) = \alpha_c \nabla c - \nabla_{\mathbf{r}_i} V + \sqrt{2D_p} \boldsymbol{\eta}_i, \quad (4a)$$

$$\dot{s}_i(t) = s_i(1 - s_i)(c_{\mathbf{r}_i} - c_s), \quad (4b)$$

$$\dot{c}(\mathbf{r}, t) = D_c \nabla^2 c - k_c(1 - s_{\mathbf{r}})c + \beta_c \sum_j \delta(\mathbf{r} - \mathbf{r}_j^*), \quad (4c)$$

for  $i = 1, 2, \dots, N$  and  $j = 1, 2, \dots, M$ . Here,  $\mathbf{r}_i$  and  $\mathbf{r}_j^*$  denote the positions of the  $i$ -th particle and  $j$ -th node, respectively,  $c$  is the concentration of chemical released by nodes,  $\alpha_c < 0$  is the chemotactic sensitivity,  $V$  is the potential field of short-range repulsion,  $D_p$  is the intensity of thermal noise,  $s_i$  is the internal state of the  $i$ -th particle,  $c_{\mathbf{r}_i} = c(\mathbf{r}_i(t), t)$  is the local chemical concentration at the position of the  $i$ -th particle,  $D_c$  and  $k_c$  are the diffusion coefficient and decay rate of the chemical,  $s_{\mathbf{r}} = \sum_i s_i \delta(\mathbf{r} - \mathbf{r}_i)$  aggregates the internal states of particles at  $\mathbf{r}$  (zero if no particle exists there),  $\beta_c$  is the saturation concentration preventing unbounded growth. The term  $s_i \in [0, 1]$  and the repulsive interaction  $\nabla_{\mathbf{r}_i} V$  ensure at most one particle occupies any position  $\mathbf{r}$ . Thus,  $s_{\mathbf{r}} \leq 1$ , guaranteeing the decay term  $-k_c(1 - s_{\mathbf{r}})c$  remains non-positive. In other words, particles modulate the chemical decay rate but do not contribute to its production, which is solely driven by the nodes.

## 2 Behaviors

## 3 Continuum model