

# Phase Plane

Speaker: Yichen Lu


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# Diverse behaviors in non-uniform chiral and non-chiral swarmalators

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Steven Ceron <sup>1,2</sup>, Kevin O’Keeffe<sup>3</sup> & Kirstin Petersen <sup>4</sup> 

We study the emergent behaviors of a population of swarming coupled oscillators, dubbed swarmalators. Previous work considered the simplest, idealized case: identical swarmalators with global coupling. Here we expand this work by adding more realistic features: local coupling, non-identical natural frequencies, and chirality. This more realistic model generates a variety of new behaviors including lattices of vortices, beating clusters, and interacting phase waves. Similar behaviors are found across natural and artificial micro-scale collective systems, including social slime mold, spermatozoa vortex arrays, and Quincke rollers. Our results indicate a wide range of future use cases, both to aid characterization and understanding of natural swarms, and to design complex interactions in collective systems from soft and active matter to micro-robotics.

In the original model:

$$|\mathbf{v}_i| = v_0 = 0$$

In this paper:

$$\begin{aligned}\mathbf{v}_i &= c_i \mathbf{n}_i \\ c_i &= \omega_i R_i \\ \mathbf{n}_i &= \begin{bmatrix} \cos\left(\theta_i + \frac{\pi}{2}\right) \\ \sin\left(\theta_i + \frac{\pi}{2}\right) \end{bmatrix}\end{aligned}$$

New phase offset terms,  $Q_{\dot{x}}$ ,  $Q_{\dot{\theta}}$ , which enable ‘frequency coupling’ .

$$Q_{\dot{x}} = \frac{\pi}{2} \left| \frac{\omega_j}{|\omega_j|} - \frac{\omega_i}{|\omega_i|} \right|$$
$$Q_{\dot{\theta}} = \frac{\pi}{4} \left| \frac{\omega_j}{|\omega_j|} - \frac{\omega_i}{|\omega_i|} \right|$$

My understanding:

$$Q_{\dot{x}} = \frac{\pi}{2} |\operatorname{sgn} \omega_j - \operatorname{sgn} \omega_i| = \begin{cases} \pi, & \operatorname{sgn} \omega_j \neq \operatorname{sgn} \omega_i \\ 0, & \operatorname{sgn} \omega_j = \operatorname{sgn} \omega_i \end{cases}$$
$$J \cos(\theta_j - \theta_i - Q_{\dot{x}}) = \begin{cases} -J \cos(\theta_j - \theta_i), & Q_{\dot{x}} = \pi \\ J \cos(\theta_j - \theta_i), & Q_{\dot{x}} = 0 \end{cases}$$

$$\dot{\mathbf{x}}_i = \mathbf{v}_i + \frac{1}{N} \sum_{j \neq i}^N \left[ \frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|} (A + J \cos(\theta_j - \theta_i - Q_x)) - B \frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|^2} \right]$$

$$J \cos(\theta_j - \theta_i - Q_x) = \begin{cases} -J \cos(\theta_j - \theta_i), & \text{sgn } \omega_j \neq \text{sgn } \omega_i \\ J \cos(\theta_j - \theta_i), & \text{sgn } \omega_j = \text{sgn } \omega_i \end{cases}$$

How can I ensure that  $J \cos(\theta_j - \theta_i) > 0$ , which means  $\theta_j - \theta_i > \pi$  when  $\text{sgn } \omega_j \neq \text{sgn } \omega_i$ ?