

# Chemotactic Chiral Active Matter

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# 1 Models

## 1.1 Definitions

### 1.1.1 General Model

$$\dot{\mathbf{r}}_i(t) = v\mathbf{p}(\theta_i) + \sum_{j \in A_i^{1,2}} \mathbf{I}(\Delta\mathbf{r}_{ij}), \quad (1a)$$

$$\dot{\theta}_i(t) = \omega_i + G(\mathbf{r}, \theta, c) + \sum_{j \neq i} H(\Delta\theta_{ij}, \Delta\mathbf{r}_{ij}), \quad (1b)$$

$$\dot{c}(\mathbf{r}, t) = D\nabla^2 c + F(c) \sum_{j=1} \delta(\mathbf{r} - \mathbf{r}_i), \quad (1c)$$

for  $i = 1, 2, \dots, N$ . Here,  $\mathbf{r}_i$  is the position of the  $i$ -th particle,  $\theta_i$  is the orientation of the  $i$ -th particle,  $v$  is the self-propulsion velocity,  $\mathbf{p}(\theta_i) = (\cos \theta_i, \sin \theta_i)$  is the unit vector pointing in the direction of the  $i$ -th particle,  $\omega_i$  is the natural frequency of the  $i$ -th particle,  $G(\mathbf{r}, \theta, c)$  is the coupling function between particles and chemical fields,  $H(\Delta\theta_{ij}, \Delta\mathbf{r}_{ij})$  is the coupling function between partials,  $c(\mathbf{r}, t)$  is the chemical concentration,  $D$  is the diffusion coefficient,  $F(c)$  is the production rate of the chemical field,  $A_i^{1,2} = \{j \mid r_c \geq |\mathbf{r}_j - \mathbf{r}_i|\}$ ,  $\Delta\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i$ ,  $\Delta\theta_{ij} = \theta_j - \theta_i$ ,  $\mathbf{I}(\Delta\mathbf{r}_{ij}) = \frac{\Delta\mathbf{r}_{ij}}{|\Delta\mathbf{r}_{ij}|^2}$ . The natural frequencies  $\omega_i$  are distributed with following two cases:

- Single-chiral particles:** The natural frequencies  $\omega_i$  are distributed in  $U(\omega_{\min}, \omega_{\max})$  for all particles and  $\omega_{\min}\omega_{\max} > 0$ . The chemotactic dynamics of Single-chiral particles has been studied by Liebchen et al. [1]:

$$\dot{\rho} = -v_0 \nabla \cdot (\rho \mathbf{p}) + D_\rho \nabla^2 \rho + K \nabla^2 \rho^3 \quad (2a)$$

$$\dot{\phi} = \omega + \beta \hat{\mathbf{p}} \times \nabla c; \mathbf{p} = (\cos \phi, \sin \phi)^\top \quad (2b)$$

$$\dot{c} = k_0 \rho - k_d c + D_c \nabla^2 c + \varepsilon (c_0 - c)^3 \quad (2c)$$

where  $\rho$  is the density of particles,  $\phi$  is the angle of the particle,  $c$  is the concentration of the chemical field,  $k_0$  is the production rate of the chemical field,  $k_d$  is the degradation rate of the chemical field,  $D_c$  is the diffusion coefficient of the chemical field,  $\varepsilon$  is a constant, and  $c_0$  is a constant.

- Double-chiral particles:** The frequencies are distributed in two symmetric uniform distributions, representing two types of chirality. Exactly half of the particles have natural frequencies  $\omega_i \sim U(\omega_{\min}, \omega_{\max})$  and the other half have natural frequencies  $\omega_i \sim U(-\omega_{\max}, -\omega_{\min})$ .

### 1.1.2 Polar alignment Interaction

- Additive coupling:

$$\dot{\theta}_i = \omega_i + K \sum_{j=1}^N f(r_{ij}) \sin(\theta_j - \theta_i + \alpha), \quad (3)$$

- Mean-field coupling by oscillator number:

$$\dot{\theta}_i = \omega_i + \frac{K}{N} \sum_{j=1}^N f(r_{ij}) \sin(\theta_j - \theta_i + \alpha), \quad (4)$$

Here,  $f(r_{ij})$  is a function of  $r = |\mathbf{r}_i - \mathbf{r}_j|$ , and  $K$  is the coupling strength. The function  $f(r)$  can be defined as

$$1. f_H(r) = H(d_0 - r), r_0 > 0;$$

$$2. f_E(r) = e^{-\frac{r}{d_0}}, r_0 > 0.$$

### 1.1.3 Chemotactic Interaction

#### General Chemotactic Model For Two Species

Type 1:

$$\dot{\mathbf{r}}_i^{1,2} = v \mathbf{p}(\theta_i^{1,2}) - \sum_{j \in A_i^{1,2}} \mathbf{I}_{ij}^{1,2}, \quad (5a)$$

$$\dot{\theta}_i^{1,2} = \omega + |\nabla c_{1,2}| \sin(\varphi_{c_{1,2}} - \theta_i^{1,2}) + F(\theta, \mathbf{r}), \quad (5b)$$

$$\dot{c}_1 = D_1 \nabla^2 c_1 + F_1(c_1, c_2) \sum_{j=1}^N \delta(\mathbf{r} - \mathbf{r}_j^1), \quad (5c)$$

$$\dot{c}_2 = D_2 \nabla^2 c_2 + F_2(c_1, c_2) \sum_{j=1}^N \delta(\mathbf{r} - \mathbf{r}_j^2), \quad (5d)$$

where  $\mathbf{I}_{ij}^{1,2} = \frac{\mathbf{r}_j - \mathbf{r}_i^{1,2}}{|\mathbf{r}_j - \mathbf{r}_i^{1,2}|^2}$ ,  $\varphi_{c_{1,2}} = \arctan\left(\frac{\partial_y c_{1,2}}{\partial_x c_{1,2}}\right)$  and  $A_i^{1,2} = \{j \mid r_c \geq |\mathbf{r}_j - \mathbf{r}_i^{1,2}|\}$ .

Type 2 (Now Considering):

$$\dot{\mathbf{r}}_i^{1,2} = v \mathbf{p}(\theta_i^{1,2}) + \alpha_{1,2} \nabla c_{1,2} - \sum_{j \in A_i^{1,2}} \mathbf{I}_{ij}^{1,2}, \quad (6a)$$

$$\dot{\theta}_i^{1,2} = \omega + F(\theta, \mathbf{r}), \quad (6b)$$

$$\dot{c}_1 = D_1 \nabla^2 c_1 + F_1(c_1, c_2) \sum_{j=1}^N \delta(\mathbf{r} - \mathbf{r}_j^1), \quad (6c)$$

$$\dot{c}_2 = D_2 \nabla^2 c_2 + F_2(c_1, c_2) \sum_{j=1}^N \delta(\mathbf{r} - \mathbf{r}_j^2), \quad (6d)$$

#### Chemotactic Model with Lotka-Volterra Functions

Let  $F_1(c_1, c_2) = k_1 c_1 (1 - c_2)$  and  $F_2(c_1, c_2) = k_2 c_2 (c_1 - 1)$ , where  $k_1, k_2$  are constants.

$$\dot{\mathbf{r}}_i^{1,2} = v \mathbf{p}(\theta_i^{1,2}) - \sum_{j \in A_i^{1,2}} \mathbf{I}_{ij}^{1,2}, \quad (7a)$$

$$\dot{\theta}_i^{1,2} = \omega + |\nabla c_{1,2}| \sin(\varphi_{c_{1,2}} - \theta_i^{1,2}) + F(\theta, \mathbf{r}), \quad (7b)$$

$$\dot{c}_1 = D_1 \nabla^2 c_1 + k_1 c_1 (1 - c_2) \sum_{j=1}^N \delta(\mathbf{r} - \mathbf{r}_j^1), \quad (7c)$$

$$\dot{c}_2 = D_2 \nabla^2 c_2 + k_2 c_2 (c_1 - 1) \sum_{j=1}^N \delta(\mathbf{r} - \mathbf{r}_j^2), \quad (7d)$$

## 2 Continuum model

We start with the model as given by Eqs. (7) in the main text but replace the finite range alignment interaction by a pseudopotential ( $\delta$ -interaction), which is justified if the interaction is short ranged enough, such that the shape of the associated interaction potential is irrelevant to the many particle dynamics:

In the thermodynamic limit  $N \rightarrow \infty$ , the Eqs. (7a) and (7b) give rise to the following continuum model:

$$\frac{\partial}{\partial t} f^{1,2}(\mathbf{r}, \theta, t) = -\frac{\partial}{\partial \theta} \left( f^{1,2} v_\theta^{1,2} \right) - \nabla \cdot (f^{1,2} \mathbf{v}_\mathbf{r}^{1,2}), \quad (8)$$

where  $f^{1,2}(\mathbf{r}, \theta, t)$  is the probability density of particles of species 1 or 2 at position  $\mathbf{r}$  and orientation  $\theta$  at time  $t$ , and  $\mathbf{v}_\theta^{1,2}$  and  $v_\theta^{1,2}$  are the velocity fields in the position and orientation space, respectively. The velocity fields are given by

$$v_\theta^{1,2}(\mathbf{r}, \theta, t) = \omega + |\nabla c_{1,2}| \sin(\varphi_{c_{1,2}} - \theta) + F(\theta, \mathbf{r}), \quad (9a)$$

$$\mathbf{v}_\theta^{1,2}(\mathbf{r}, \theta, t) = v\mathbf{p}(\theta) - \int d\theta' d\mathbf{r}' f^{1,2}(\mathbf{r}', \theta', t) \mathbf{I}(\mathbf{r}' - \mathbf{r}), \quad (9b)$$

where  $\mathbf{I}(\mathbf{r}) = |\mathbf{r}|^{-2}\mathbf{r}$ . By substituting Eqs. (9) into Eq. (8), we obtain

$$\begin{aligned} \frac{\partial}{\partial t} f^{1,2}(\mathbf{r}, \theta, t) &= -v\mathbf{p}(\theta) \cdot \nabla f^{1,2} - \nabla \cdot \mathbf{F}_{\text{coll}}^{1,2}(\mathbf{r}, \theta, t) \\ &\quad - \omega \partial_\theta f^{1,2} - |\nabla c_{1,2}| \partial_\theta [f^{1,2} \sin(\varphi_{c_{1,2}} - \theta)], \end{aligned} \quad (10)$$

where

$$\mathbf{F}_{\text{coll}}^{1,2}(\mathbf{r}, \theta, t) = \int d\theta' d\mathbf{r}' f^{1,2}(\mathbf{r}', \theta', t) f^{1,2}(\mathbf{r}, \theta, t) \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r}' - \mathbf{r}|^2}, \quad (11)$$

and

$$\begin{aligned} \nabla \cdot \mathbf{F}_{\text{coll}}^{1,2}(\mathbf{r}, \theta, t) &= \nabla \cdot \int d\theta' d\mathbf{r}' f^{1,2}(\mathbf{r}', \theta', t) f^{1,2}(\mathbf{r}, \theta, t) \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r}' - \mathbf{r}|^2} \\ &= \int d\theta' d\mathbf{r}' f^{1,2}(\mathbf{r}', \theta', t) \nabla \cdot \left[ f^{1,2}(\mathbf{r}, \theta, t) \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r}' - \mathbf{r}|^2} \right] \\ &= \int d\theta' d\mathbf{r}' f^{1,2}(\mathbf{r}', \theta', t) \left[ -2\pi\delta(\mathbf{r}' - \mathbf{r}) f^{1,2}(\mathbf{r}, \theta, t) + \nabla f^{1,2}(\mathbf{r}, \theta, t) \cdot \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r}' - \mathbf{r}|^2} \right] \\ &= -2\pi f^{1,2}(\mathbf{r}, \theta, t) \int_0^{2\pi} d\theta' f^{1,2}(\mathbf{r}, \theta', t) + \nabla f^{1,2}(\mathbf{r}, \theta, t) \cdot \int d\theta' d\mathbf{r}' f^{1,2}(\mathbf{r}', \theta', t) \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r}' - \mathbf{r}|^2} \end{aligned} \quad (12)$$

The equation accounts for self-propulsion, and short-range repulsion between particles, rotational diffusion and chemotactic alignment. From here, we expand  $f^{1,2}(\mathbf{r}, \theta, t)$  in a Fourier series

$$f^{1,2}(\mathbf{r}, \theta, t) = \sum_{k=-\infty}^{\infty} f_k^{1,2}(\mathbf{r}, t) e^{ik\theta}, \quad (13)$$

with the projection

$$f_k^{1,2}(\mathbf{r}, t) = \frac{1}{2\pi} \int d\theta f^{1,2}(\mathbf{r}, \theta, t) e^{-ik\theta}, \quad (14)$$

and define the coefficients as the particle density  $\rho(\mathbf{r}, t)$  and the density-weighted polar order  $\mathbf{p}(\mathbf{r}, t)$  by relating them to the harmonics via the Fourier expansion:

$$\rho^{1,2}(\mathbf{r}, t) \equiv \int_0^{2\pi} d\theta f^{1,2}(\mathbf{r}, \theta, t) = 2\pi f_0^{1,2}(\mathbf{r}, t) \quad (15a)$$

$$\begin{aligned} \mathbf{p}^{1,2}(\mathbf{r}, t) \equiv \int_0^{2\pi} d\theta \mathbf{p}(\theta) f^{1,2}(\mathbf{r}, \theta, t) &= \int_0^{2\pi} d\theta \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} f^{1,2}(\mathbf{r}, \theta, t) = \int_0^{2\pi} d\theta \frac{1}{2} \begin{bmatrix} e^{i\theta} + e^{-i\theta} \\ i(e^{-i\theta} - e^{i\theta}) \end{bmatrix} f^{1,2}(\mathbf{r}, \theta, t) \\ &= \pi \begin{bmatrix} \frac{1}{2\pi} \int_0^{2\pi} d\theta (e^{i\theta} + e^{-i\theta}) f^{1,2}(\mathbf{r}, \theta, t) \\ \frac{1}{2\pi} \int_0^{2\pi} d\theta i(e^{-i\theta} - e^{i\theta}) f^{1,2}(\mathbf{r}, \theta, t) \end{bmatrix} = \pi \begin{bmatrix} f_1^{1,2}(\mathbf{r}, t) + f_{-1}^{1,2}(\mathbf{r}, t) \\ i(f_1^{1,2}(\mathbf{r}, t) - f_{-1}^{1,2}(\mathbf{r}, t)) \end{bmatrix} \end{aligned} \quad (15b)$$

In the following, the different contributions to the continuum model, Eq. (8), are analyzed separately. First, in order to derive expressions for the self-propulsion,  $-v\mathbf{p}(\theta) \cdot \nabla f^{1,2}$ , we apply the projection

operator, Eq. (14), onto the corresponding term in Eq. (8) and obtain

$$\begin{aligned}
\frac{\partial}{\partial t} f_{k,\text{prop}}^{1,2} &= \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{-ik\theta} [-v \mathbf{p}(\theta) \cdot \nabla f^{1,2}] \\
&= -\frac{v}{4\pi} \int_0^{2\pi} d\theta e^{-ik\theta} \left[ \partial_x \sum_{m=-\infty}^{\infty} f_m^{1,2} (e^{i(m+1)\theta} + e^{i(m-1)\theta}) + i\partial_y \sum_{k=-\infty}^{\infty} f_m^{1,2} (e^{i(m-1)\theta} - e^{i(m+1)\theta}) \right] \\
&= -\frac{v}{4\pi} \int_0^{2\pi} d\theta \left[ \partial_x \sum_{m=-\infty}^{\infty} f_m^{1,2} (e^{i(m-k+1)\theta} + e^{i(m-k-1)\theta}) + i\partial_y \sum_{k=-\infty}^{\infty} f_m^{1,2} (e^{i(m-k-1)\theta} - e^{i(m-k+1)\theta}) \right] \\
&= -\frac{v}{2} \left\{ \partial_x \sum_{m=-\infty}^{\infty} f_m^{1,2} [\delta(m-k+1) + \delta(m-k-1)] + i\partial_y \sum_{k=-\infty}^{\infty} f_m^{1,2} [\delta(m-k-1) - \delta(m-k+1)] \right\}. \tag{16}
\end{aligned}$$

With the definitions, Eqs. (15a) and (15b), we obtain for the field variables

$$\begin{aligned}
\frac{\partial}{\partial t} \rho_{\text{prop}}^{1,2} &= 2\pi \frac{\partial}{\partial t} f_{0,\text{prop}}^{1,2} \\
&= -v\pi \left[ \partial_x \sum_{m=-\infty}^{\infty} f_m^{1,2} [\delta(m+1) + \delta(m-1)] + i\partial_y \sum_{k=-\infty}^{\infty} f_m^{1,2} [\delta(m-1) - \delta(m+1)] \right] \\
&= -v\pi \left[ \partial_x (f_{-1}^{1,2} + f_1^{1,2}) + i\partial_y (f_1^{1,2} - f_{-1}^{1,2}) \right] \\
&= -v\nabla \cdot \mathbf{p}_{\text{prop}}^{1,2}, \tag{17}
\end{aligned}$$

and because of

$$\begin{aligned}
\partial_t f_1^{1,2} &= -\frac{v}{2} \left\{ \partial_x \sum_{m=-\infty}^{\infty} f_m^{1,2} [\delta(m) + \delta(m-2)] + i\partial_y \sum_{k=-\infty}^{\infty} f_m^{1,2} [\delta(m-2) - \delta(m)] \right\} \\
&= -\frac{v}{2} \left[ \partial_x (f_0^{1,2} + f_2^{1,2}) + i\partial_y (f_2^{1,2} - f_0^{1,2}) \right] \tag{18a}
\end{aligned}$$

$$\begin{aligned}
\partial_t f_{-1}^{1,2} &= -\frac{v}{2} \left\{ \partial_x \sum_{m=-\infty}^{\infty} f_m^{1,2} [\delta(m-2) + \delta(m)] + i\partial_y \sum_{k=-\infty}^{\infty} f_m^{1,2} [\delta(m) - \delta(m-2)] \right\} \\
&= -\frac{v}{2} \left[ \partial_x (f_0^{1,2} + f_2^{1,2}) + i\partial_y (f_0^{1,2} - f_2^{1,2}) \right], \tag{18b}
\end{aligned}$$

we have

$$\frac{\partial}{\partial t} \mathbf{p}_{\text{prop}}^{1,2} = \pi \begin{bmatrix} \partial_t (f_1^{1,2} + f_{-1}^{1,2}) \\ i\partial_t (f_1^{1,2} - f_{-1}^{1,2}) \end{bmatrix} = \pi \begin{bmatrix} -v\partial_x (f_0^{1,2} + f_2^{1,2}) \\ v\partial_y (f_2^{1,2} - f_0^{1,2}) \end{bmatrix} \approx -\frac{v}{2} \begin{bmatrix} \partial_x f_0^{1,2} \\ \partial_y f_0^{1,2} \end{bmatrix} = -\frac{v}{2} \nabla \rho_{\text{prop}}^{1,2}. \tag{19}$$

Next, we turn to the short-range repulsion,  $-\nabla \cdot \mathbf{F}_{\text{coll}}^{1,2}(\mathbf{r}, \theta, t)$ . We apply the projection operator, Eq. (14), onto the corresponding term in Eq. (8) and obtain

$$\begin{aligned}
\frac{\partial}{\partial t} f_{k,\text{coll}}^{1,2} &= \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{-ik\theta} [\nabla \cdot \mathbf{F}_{\text{coll}}^{1,2}(\mathbf{r}, \theta, t)] \\
&= -4\pi^2 f_{0,\text{coll}}^{1,2}(\mathbf{r}, t) f_{k,\text{coll}}^{1,2}(\mathbf{r}, t) + 2\pi \nabla f_{k,\text{coll}}^{1,2}(\mathbf{r}, t) \cdot \int d\mathbf{r}' f_{0,\text{coll}}^{1,2}(\mathbf{r}', t) \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r}' - \mathbf{r}|^2}, \tag{20}
\end{aligned}$$

With the definitions, Eqs. (15a) and (15b), we obtain for the field variables

$$\begin{aligned}
\frac{\partial}{\partial t} \rho_{\text{coll}}^{1,2} &= 2\pi \frac{\partial}{\partial t} f_{0,\text{coll}}^{1,2} \\
&= -2\pi \left[ f_{0,\text{coll}}^{1,2}(\mathbf{r}, t) \right]^2 + \nabla f_{0,\text{coll}}^{1,2}(\mathbf{r}, t) \cdot \int d\mathbf{r}' f_{0,\text{coll}}^{1,2}(\mathbf{r}', t) \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r}' - \mathbf{r}|^2}, \tag{21}
\end{aligned}$$

and

$$\frac{\partial}{\partial t} \mathbf{p}_{\text{coll}}^{1,2} = -2\pi \rho_{\text{coll}}^{1,2} \mathbf{p}_{\text{coll}}^{1,2} + \dots \tag{22}$$

Next, we turn to the rotational diffusion,  $-\omega\partial_\theta f^{1,2}$ . We apply the projection operator, Eq. (14), onto the corresponding term in Eq. (8) and obtain

$$\begin{aligned}
\frac{\partial}{\partial t} f_{k,\text{rota}}^{1,2}(\mathbf{r}, t) &= -\frac{\omega}{2\pi} \int_0^{2\pi} d\theta e^{-ik\theta} \partial_\theta f^{1,2}(\mathbf{r}, \theta, t) \\
&= -\frac{\omega}{2\pi} \int_0^{2\pi} d\theta e^{-ik\theta} \sum_{m=-\infty}^{\infty} f_{m,\text{rota}}^{1,2}(\mathbf{r}, t) \partial_\theta e^{im\theta} \\
&= -\frac{\omega}{2\pi} \int_0^{2\pi} d\theta \sum_{m=-\infty}^{\infty} i m f_{m,\text{rota}}^{1,2}(\mathbf{r}, t) e^{i(m-k)\theta} \\
&= -\frac{\omega}{2\pi} \sum_{m=-\infty}^{\infty} i m f_{m,\text{rota}}^{1,2}(\mathbf{r}, t) \int_0^{2\pi} d\theta e^{i(m-k)\theta} \\
&= -\omega \sum_{m=-\infty}^{\infty} i m f_{m,\text{rota}}^{1,2}(\mathbf{r}, t) \delta(m-k) \\
&= -ik\omega f_{k,\text{rota}}^{1,2}(\mathbf{r}, t)
\end{aligned} \tag{23}$$

With the definitions, Eqs. (15a) and (15b), we obtain for the field variables

$$\frac{\partial}{\partial t} \rho_{\text{rota}}^{1,2} = 0, \tag{24}$$

and

$$\frac{\partial}{\partial t} \mathbf{p}_{\text{rota}}^{1,2} = \pi \left[ \begin{array}{c} \partial_t f_{1,\text{rota}}^{1,2} + \partial_t f_{-1,\text{rota}}^{1,2} \\ i \left( \partial_t f_{1,\text{rota}}^{1,2} - \partial_t f_{-1,\text{rota}}^{1,2} \right) \end{array} \right] = \omega \pi \left[ \begin{array}{c} i \left( f_{-1,\text{rota}}^{1,2} - f_{1,\text{rota}}^{1,2} \right) \\ f_{-1,\text{rota}}^{1,2} + f_{1,\text{rota}}^{1,2} \end{array} \right] = \omega \mathbf{p}_{\text{rota},\perp}^{1,2}, \tag{25}$$

where  $\mathbf{p}_{\text{rota},\perp}^{1,2} = (-p_{\text{rota},y}^{1,2}, p_{\text{rota},x}^{1,2})$ .

Next, we turn to the chemotactic alignment,  $-\|\nabla c_{1,2}\| \partial_\theta [f^{1,2} \sin(\varphi_{c_{1,2}} - \theta)]$ . We apply the projection operator, Eq. (14), onto the corresponding term in Eq. (8) and obtain

$$\frac{\partial}{\partial t} \rho_{\text{chem}}^{1,2} = 0, \tag{26}$$

and

$$\frac{\partial}{\partial t} \mathbf{p}_{\text{chem}}^{1,2} = -\rho_{\text{chem}}^{1,2} \|\nabla c_{1,2}\| \nabla c_{1,2}. \tag{27}$$

To summarize, the continuum model, Eq. (8), can be written as

$$\dot{\rho}^{1,2}(\mathbf{r}, t) = -v \nabla \cdot \mathbf{p}^{1,2} + D \nabla^2 \rho^{1,2}, \tag{28a}$$

$$\dot{\mathbf{p}}^{1,2}(\mathbf{r}, t) = -\frac{v}{2} \nabla \rho^{1,2} + D \nabla^2 \mathbf{p}^{1,2} + \omega \mathbf{p}_\perp^{1,2} - \rho^{1,2} \|\nabla c_{1,2}\| \nabla c_{1,2}, \tag{28b}$$

$$\dot{c}_1(\mathbf{r}, t) = D_1 \nabla^2 c_1 + c_1 (k_1 - k_2 c_2) \sum_{j=1}^N \delta(\mathbf{r} - \mathbf{r}_j^1), \tag{28c}$$

$$\dot{c}_2(\mathbf{r}, t) = D_2 \nabla^2 c_2 + c_2 (k_3 c_1 - k_4) \sum_{j=1}^N \delta(\mathbf{r} - \mathbf{r}_j^1), \tag{28d}$$

where  $\mathbf{p}_\perp^{1,2} = (-p_y^{1,2}, p_x^{1,2})$ , and  $D$  describes the effective diffusion caused by particle collisions.

### 3 Behaviors

#### 3.1 Snapshots of Particle Distributions

The time varying version of the snapshots can be found at the tail end of the document.

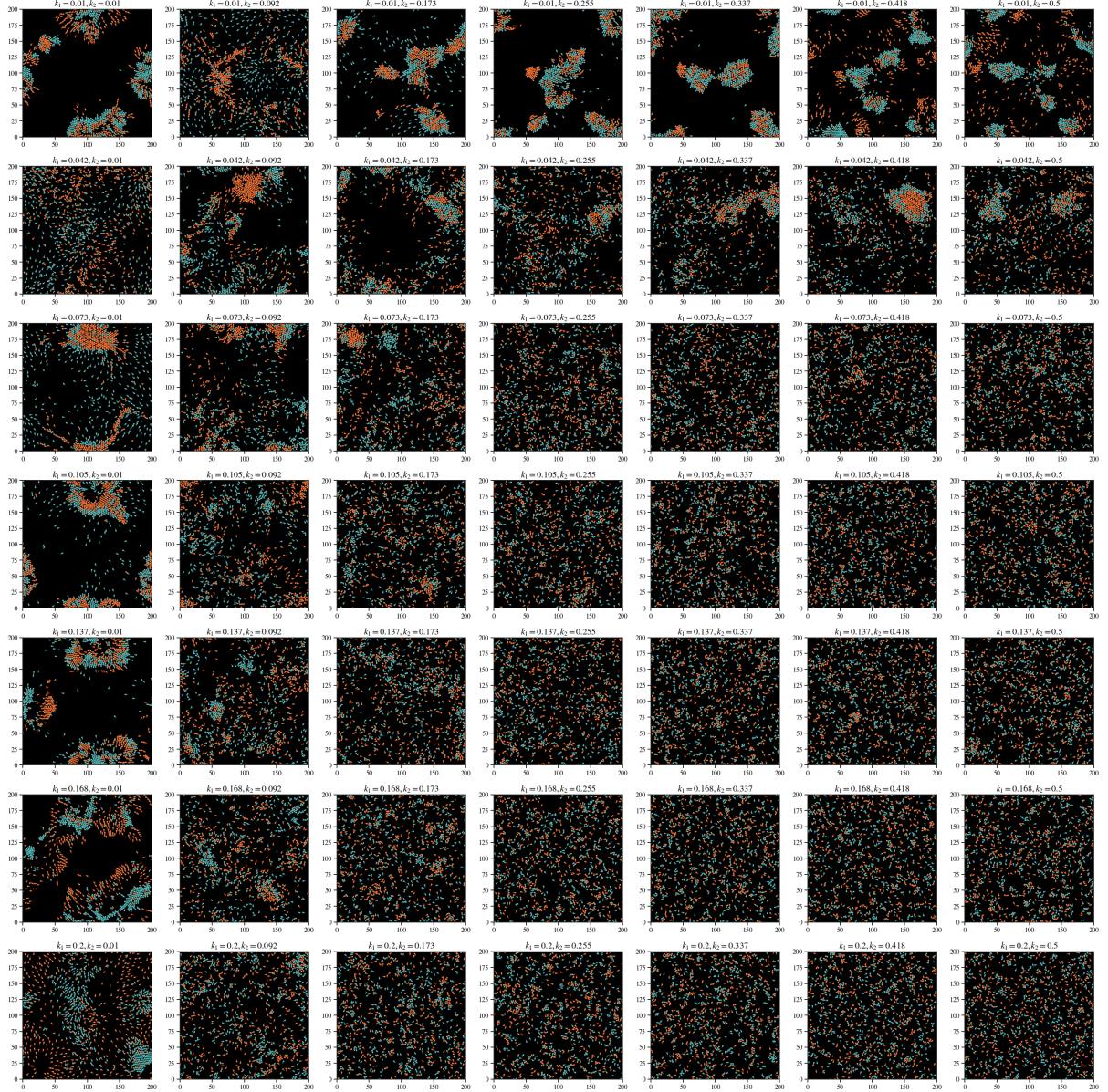


Figure 1: Particle distribution of the chemotactic model with Lotka-Volterra functions in Eqs. (7). The parameters used are:  $\omega = 0$ ,  $D_1 = D_2 = 1$ ,  $v = 0.01$ .

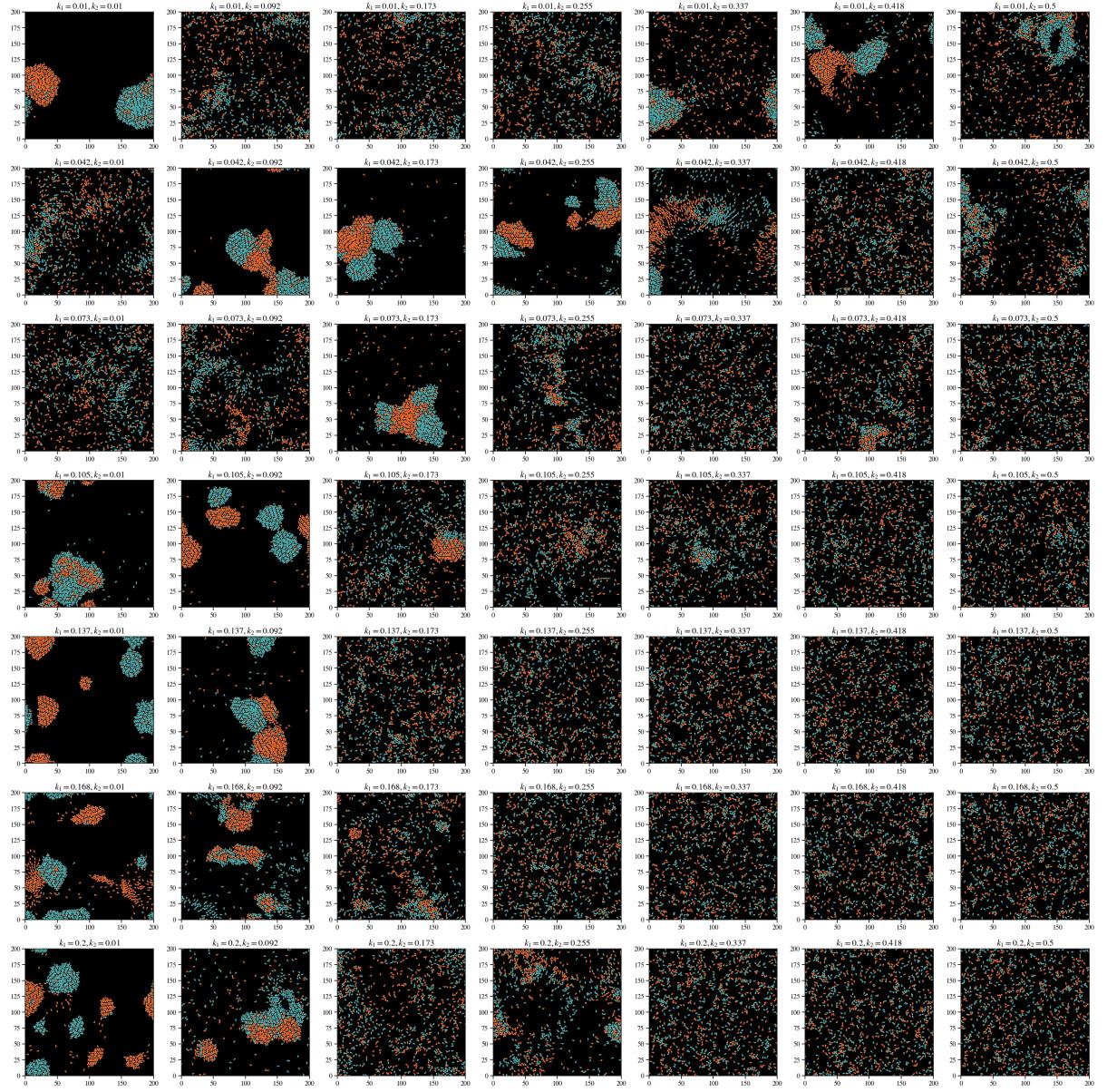


Figure 2: Particle distribution of the chemotactic model with Lotka-Volterra functions in Eqs. (7). The parameters used are:  $\omega = 0$ ,  $D_1 = D_2 = 1$ ,  $v = 0.1$ .

### 3.2 Snapshots of Chemical Fields

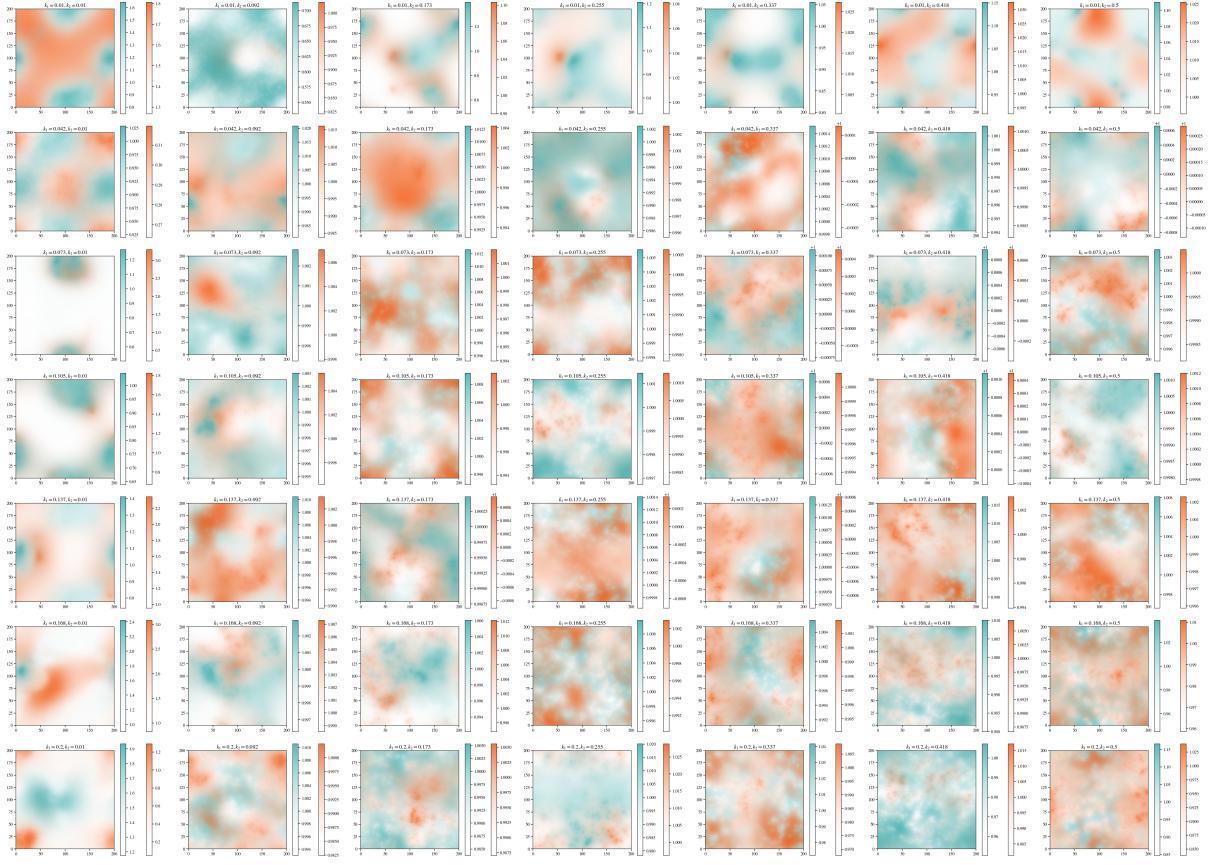


Figure 3: Chemical fields of the chemotactic model with Lotka-Volterra functions in Eqs. (7). The parameters used are:  $\omega = 0$ ,  $D_1 = D_2 = 1$ ,  $v = 0.01$ .

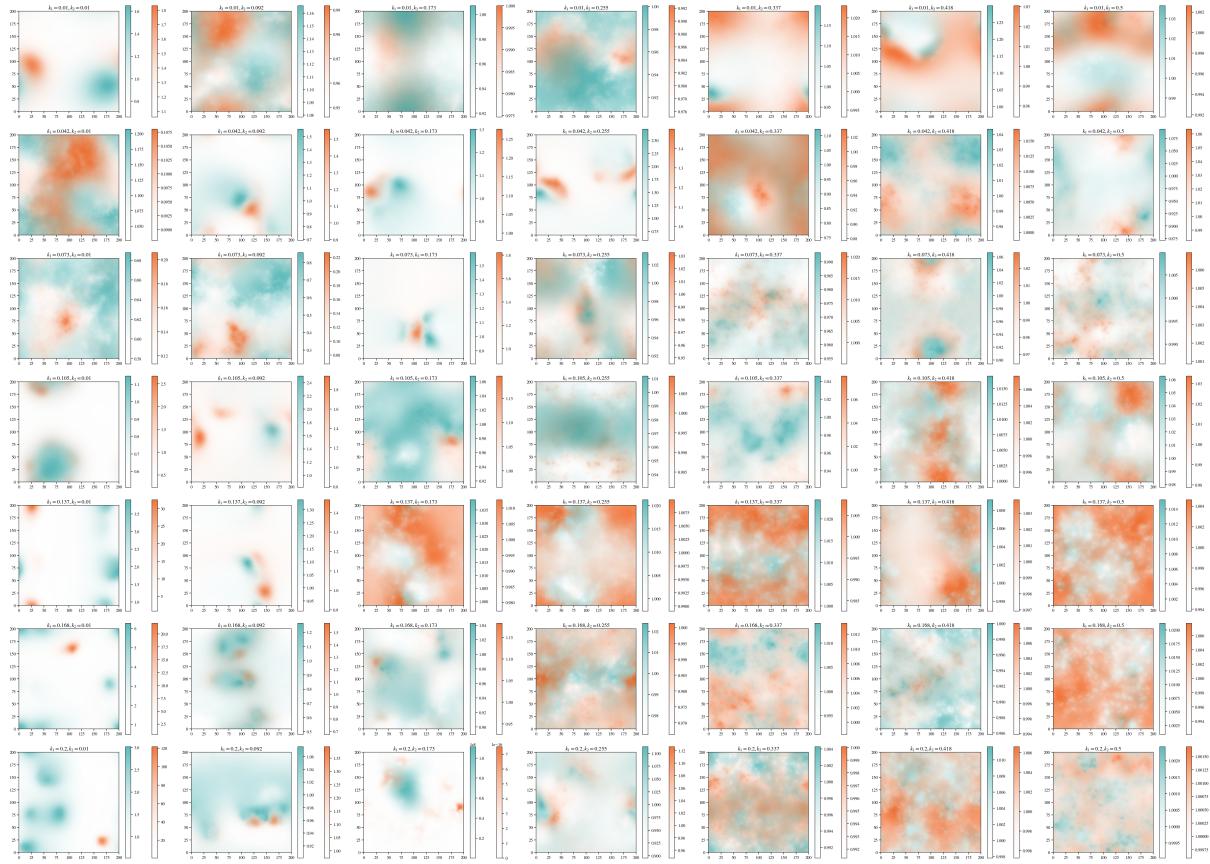
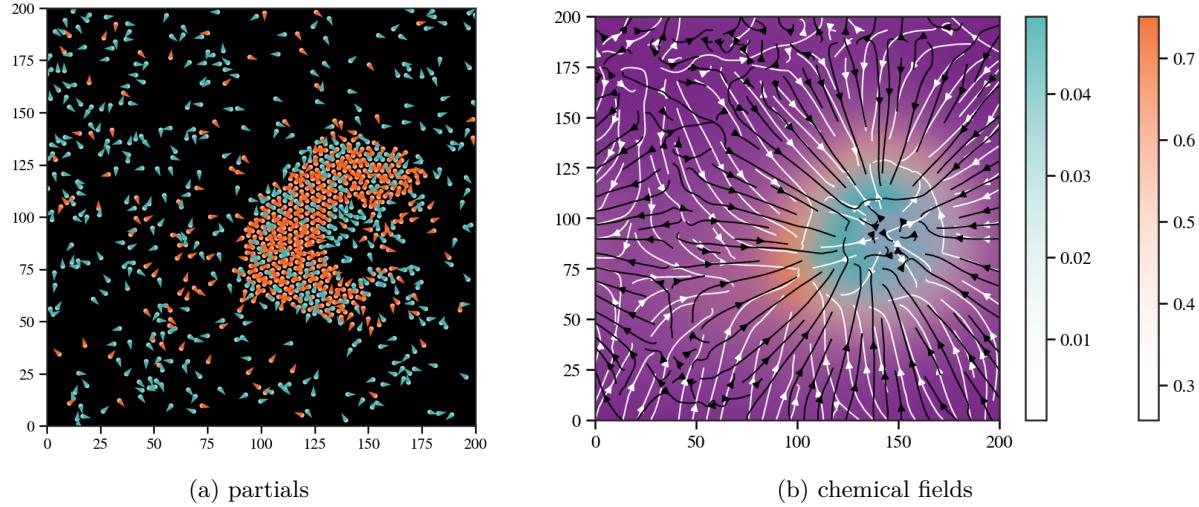


Figure 4: Chemical fields of the chemotactic model with Lotka-Volterra functions in Eqs. (7). The parameters used are:  $\omega = 0$ ,  $D_1 = D_2 = 1$ ,  $v = 0.1$ .



## References

- [1] Benno Liebchen, Michael E. Cates, and Davide Marenduzzo. Pattern formation in chemically interacting active rotors with self-propulsion. *Soft Matter*, 12:7259–7264, 2016.