Solvable Two-dimensional Swarmalator Model for Realistic Spatial Interactions

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1 The Model

O'Keeffe et al. proposed a solvable 2D swarmalator model with periodic boundary conditions:

$$\dot{x}_i = \frac{J}{N}\sum_{j=1}^{N} \sin\left(x_j - x_i\right)\cos\left(\theta_j - \theta_i\right) \ , \tag{1a}$$

$$\dot{y}_i = \frac{J}{N}\sum_{j=1}^{N} \sin\left(y_j - y_i\right)\cos\left(\theta_j - \theta_i\right) \ , \tag{1b}$$

$$\dot{\theta}_i = \frac{K}{N}\sum_{j=1}^{N} \sin\left(\theta_j - \theta_i\right)\left(\cos\left(x_j - x_i\right) + \cos\left(y_j - y_i\right)\right) \ , \tag{1c}$$

which loses the spatial repulsion term.

Swarmalators have a spatial position $\mathbf{r}_i = (x_i, y_i)$ and an internal phase θ_i which evolve according to equations:

$$\dot{x}_i = \frac{1}{N}\sum_{j=1}^{N} \left[\sin\left(x_j - x_i\right)\left(1 + J\cos\left(\theta_j - \theta_i\right)\right) - P\sin 2\left(x_j - x_i\right)\right] , \tag{2a}$$

$$\dot{y}_i = \frac{1}{N}\sum_{j=1}^{N} \left[\sin\left(y_j - y_i\right)\left(1 + J\cos\left(\theta_j - \theta_i\right)\right) - P\sin 2\left(y_j - y_i\right)\right] , \tag{2b}$$

$$\dot{\theta}_i = \frac{K}{N}\sum_{j=1}^{N} \sin\left(\theta_j - \theta_i\right)\left(\cos\left(x_j - x_i\right) + \cos\left(y_j - y_i\right)\right) , \tag{2c}$$