# 1 Models

## 1.1 Definitions

### 1.1.1 Self-propelled dynamics

$$\dot{x}_i = v\cos\theta_i \ , \tag{1a}$$
$$\dot{y}_i = v\sin\theta_i \ , \tag{1b}$$

### 1.1.2 Phase coupling dynamics

- Additive coupling:

$$\dot{\theta}_i = \omega_i + K\sum_{j=1}^{N} f\left(r_{ij}\right)\sin\left(\theta_j - \theta_i + \alpha\right) \ , \tag{2}$$

- Mean-field coupling by oscillator number:

$$\dot{\theta}_i = \omega_i + \frac{K}{N}\sum_{j=1}^{N} f\left(r_{ij}\right)\sin\left(\theta_j - \theta_i + \alpha\right) \ , \tag{3}$$

  which is similar to the swarmalator model.

Here, $f\left(r_{ij}\right)$ is a function of $r = \left|\mathbf{r}_i - \mathbf{r}_j\right|$, and $K$ is the coupling strength. The function $f\left(r\right)$ can be defined as

1. $f_H\left(r\right) = H\left(d_0 - r\right),\ r_0 > 0$;
2. $f_E\left(r\right) = e^{-\frac{r}{d_0}},\ r_0 > 0$.

The natural frequencies $\omega_i$ are distributed with following two cases:

1. **Single-chiral swarmalators:** The natural frequencies $\omega_i$ are distributed in $U\left(\omega_{\min}, \omega_{\max}\right)$ for all swarmalators and $\omega_{\min}\omega_{\max} > 0$.
2. **Double-chiral swarmalators:** The frequencies are distributed in two symmetric uniform distributions, representing two types of chirality. Exactly half of the swarmalators have natural frequencies $\omega_i \sim U\left(\omega_{\min}, \omega_{\max}\right)$ and the other half have natural frequencies $\omega_i \sim U\left(-\omega_{\max}, -\omega_{\min}\right)$.

### 1.1.3 Chemotactic Dynamics

Consider two chemical fields $u\left(\mathbf{r}, t\right), v\left(\mathbf{r}, t\right)$ that are produced by the ensemble of two symmetrically chiral swarmalators. Swarmalators interact with the chemical field and move towards/against the regions with higher concentration, which can be described by the following equation:

$$\dot{\mathbf{r}}_i = v\mathbf{p}_i \tag{4a}$$
$$\dot{\theta}_i = \omega_i + \beta_i^u \mathbf{p}_i \times \nabla u + \beta_i^v \mathbf{p}_i \times \nabla v + \frac{K}{N}\sum_{j=1}^{N} f\left(\left|\mathbf{r}_j - \mathbf{r}_i\right|\right)\sin\left(\theta_j - \theta_i\right) \ , \tag{4b}$$

where $\beta_i^{u,v}$ denote the 'chemotactic' coupling strength and $\mathbf{p}_i = \left(\cos\theta_i, \sin\theta_i\right)$ is the unit vector pointing in the direction of the $i$-th swarmalator. Here, we used the notation $\mathbf{a} \times \mathbf{b} = a_1 b_2 - a_2 b_1$.

These two fields evolve as

$$\dot{u} = k_0 \sum_{j\in S_+} \delta\left(\mathbf{r} - \mathbf{r}_j\right) - k_d u + D_u \nabla^2 u \ , \tag{5a}$$
$$\dot{v} = k_0 \sum_{j\in S_-} \delta\left(\mathbf{r} - \mathbf{r}_j\right) - k_d v + D_v \nabla^2 v \ , \tag{5b}$$

where $S_+$ and $S_-$ are the sets of two chiral swarmalators, $k_0$ is the production rate, $k_d$ is the decay rate, $D_{u,v}$ are the diffusion coefficients.