Phase Frustration-Induced Crystallize in Chiral Active Matter

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1 The Model

Particles have a spatial position $\mathbf{r}_i = (x_i, y_i)$ and an internal phase θ_i which evolve according to equations:

$$\dot{\mathbf{r}}_i = v\mathbf{p}\left(\theta_i\right) \ , \tag{1a}$$

$$\dot{\theta}_i = \omega_i + \frac{K}{|A_i|}\sum_{j\in A_i}\left[\sin\left(\theta_j - \theta_i + \alpha\right) - \sin\alpha\right] \ , \tag{1b}$$

for $i = 1, 2, \ldots, N$. Here in Eq. (1a), $\mathbf{p}\left(\theta\right) = (\cos\theta, \sin\theta)$, which means each particle rotates with a constant speed v in the direction of its instantaneous phase $\theta_i(t)$. The particles are treated as point-like with no direct spatial interactions, consistent with classical models of chiral self-propelled particles [1–3, 5, 6]. As per Eq. (1b), the mean runs over neighbors within a coupling radius d_0 around particle i:

$$A_i\left(t\right) = \left\{j \mid \left|\mathbf{r}_i\left(t\right) - \mathbf{r}_j\left(t\right)\right| \leqslant d_0\right\} \ , \tag{2}$$

K is the coupling strength, and ω_i is the natural frequency of the i-th particle. This means that a particle will rotate with the angular velocity $|\omega_i|$ in the absence of mutual coupling ($K = 0$), and the sign of ω_i represents the direction of rotation, namely, the tribute of the chirality of the i-th particle. A positive (negative) chirality (ω) describes the counterclockwise (clockwise) rotations of the particle in space. We will study particles including two types of chiralities with both positive and negative natural frequencies uniformly distributed in two symmetric regimes as

$$g_U^D\left(\omega;\mu,\sigma\right) = \frac{1}{2}\left[g_U\left(\omega;-\mu,\sigma\right) + g_U\left(\omega;\mu,\sigma\right)\right]. \tag{3}$$

where $g_U\left(\omega;\mu,\sigma\right)$ is the uniform distribution:

$$g_U\left(\omega;\mu,\sigma\right) = \begin{cases} \frac{1}{\sigma}, & \omega_{\min} \leqslant \omega \leqslant \omega_{\max} \\ 0, & \text{otherwise} \end{cases} . \tag{4}$$

Here $\sigma = \omega_{\max} - \omega_{\min}$ is the natural-frequency span, and $\mu = (\omega_{\max} + \omega_{\min})/2$ is the average.

Additionally, α is the phase frustration between two neighboring particles. When $\alpha_0 = 0$, the dynamics reduces to the normal chiral model [4]. The counter term $-\sin\alpha$ is introduced to ensure that frustration only interferes with the phase coupling without changing the sign of effective frequency.

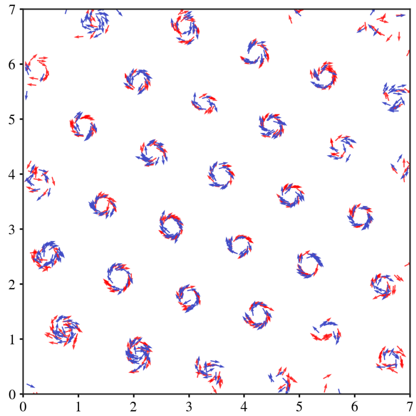

Figure 1: Snapshot of the system at $t = 80$ with $N = 1000$, $K = 19.2$, $\alpha = 0.6\pi$, and $\omega_{\min} = 0.1$, $\omega_{\max} = 1$.

2 Phase frustration-induced crystallization

2.1 Key properties

1. What does the lattice structure look like?
2. What is each cell composed of?
3. What is the unit cell structure, and what is the spatial arrangement of the unit cells?
4. What is the internal dynamics within a cell? Besides triangular, what other spatial structures exist?
5. What determines the length (periodicity)? (Interaction distance?) In which regions of frustration does it appear? (And what are the corresponding coupling conditions and natural frequency distributions?)

2.2 Mechanism

What is the mechanism behind the formation of this phenomenon?

3 Continuum model (Theoretical Analysis)

References

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