Radii of Emergent Patterns in Swarmalator Systems

Udo Schilcher ; Christoph W. Rauter ; Christian Bettstetter All Authors

Abstract

Document Sections

1. Introduction
2. System Model
3. Derivation of Radii
4. Numerical Evaluation
5. Conclusions and Outlook

Abstract:
For a system of swarmalators converging to different types of circular patterns, we provide expressions for the outer and inner radii of these patterns and examine their dependence on the model parameters. Derivations are made for three static patterns with an infinite number of entities and a generalized swarmalator model with parameterized attraction and repulsion kernels. Simulations of finite systems show good agreement with the asymptotic expressions.

Published in: 2023 IEEE International Conference on Autonomic Computing and Self-Organizing Systems (ACSOS)

Date of Conference: 25-29 September 2023 **DOI:** 10.1109/ACSOS58161.2023.00034

Model equations:

$$\dot{x}_i = \frac{1}{N}\sum_{j\neq i}^{N} e_{ij} d_{ij}^{\alpha}\left(1 + J\cos\theta_{ij}\right) - e_{ij} d_{ij}^{\beta}$$

$$\dot{\theta}_i = \frac{K}{N}\sum_{j\neq i}^{N} d_{ij}^{\gamma}\sin\theta_{ij}$$

where $d_{ij} = \|x_j - x_i\|$, $\theta_{ij} = \theta_i - \theta_j$, $e_{ij} = \frac{1}{d_{ij}}(x_j - x_i)$.

Choosing $\alpha = 0$ and $\beta = \gamma = -1$ gives the original model.

In the converged state of the static patterns, the entities no longer move, so we have $\dot{x}_i = 0$, which yields:

$$\underbrace{r_{\text{out}}^{\alpha} \sum_{j \neq i}^{N} e_{ij} g_{ij}^{\alpha} \left(1 + J \cos \theta_{ij}\right)}_{\text{Attraction}A} = \underbrace{r_{\text{out}}^{\beta} \sum_{j \neq i}^{N} e_{ij} g_{ij}^{\alpha}}_{\text{Repulsion}R}$$

where $r_{\text{out}} = \max_{1 \leq i \leq N} \|x_i - \bar{x}\|$, $g_{ij} = \frac{d_{ij}}{r_{\text{out}}}$.