Two Coupled Swarmalators with Chirality

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1 Reference

1.1 Farrell F D C, Marchetti M C, Marenduzzo D, et al. Pattern formation in self-propelled particles with density-dependent motility[J]. Physical review letters, 2012, 108(24): 248101.

Microscopic dynamics:

$$\begin{aligned} \dot{\mathbf{r}}_i &= v\mathbf{e}_{\theta_i} \ , \\ \dot{\theta}_i &= \gamma \sum_{j=1}^{N} F\left(\theta_j - \theta_i, \mathbf{r}_j - \mathbf{r}_i\right) + \sqrt{2\epsilon}\tilde{\eta}_i\left(t\right) \ . \end{aligned} \tag{1}$$

The microscopic density of particles at position $\mathbf{r}$ with angle θ is given by

$$f\left(\mathbf{r}, \theta\right) = \sum_{i=1}^{N} \delta\left(\mathbf{r} - \mathbf{r}_i\right) \delta\left(\theta - \theta_i\right) \ . \tag{2}$$

Using Itô's formula, a stochastic dynamical equation for the density Eq. (2) can be derived:

$$\begin{aligned} &\partial_t f\left(\mathbf{r}, \theta\right) + \mathbf{e}_\theta \cdot \nabla\left[v f\right] \\ &= \epsilon \tfrac{\partial^2 f}{\partial \theta^2} - \tfrac{\partial}{\partial \theta}\sqrt{2\epsilon f}\eta - \gamma \tfrac{\partial}{\partial \theta} \textstyle\int \mathrm{d}\theta' \mathrm{d}\mathbf{r}' f\left(\mathbf{r}', \theta'\right) \times f\left(\mathbf{r}, \theta\right) F\left(\theta' - \theta, \mathbf{r} - \mathbf{r}'\right) \ . \end{aligned} \tag{3}$$

Drop the noise term, and Fourier transform Eq. (3) to get equations of motion for

$$f_k \equiv \int f\left(\mathbf{r}, \theta\right) e^{ik\theta} \mathrm{d}\theta \ . \tag{4}$$

2 Our Work

We replace the finite range alignment interaction by a pseudopotential (δ-interaction) in the model:

$$\begin{aligned} \dot{\mathbf{r}}_i &= v\mathbf{p}_i \\ \dot{\theta}_i &= \omega_i + g \sum_{j \neq i} \delta\left(\mathbf{r}_j - \mathbf{r}_i\right) \sin\left(\theta_j - \theta_i\right) \end{aligned} \tag{5}$$

where $\mathbf{p}_i = (\cos\theta_i, \sin\theta_i)$ and $g = \lambda/\pi d_0^2$. The combined probability density of finding a particle at position $\mathbf{r}$ with angle θ of the ith swarmalator is given by

$$f_i\left(\mathbf{r}, \theta\right) = \delta\left(\mathbf{r} - \mathbf{r}_i\right)\delta\left(\theta - \theta_i\right)\ . \tag{6}$$

For the disorder state, which is a uniform incoherent state, the frequency ω_i is spatiotemporal independent. Therefore, we assume frequency $\omega(\mathbf{x}, t)$ to represent spatiotemporal white noise following the distribution given by the definition of the mono-chiral and chiral swarmalators.

With above assumptions, we obtain

$$f\left(\mathbf{r}, \theta\right) = \sum_{i=1}^{N} \delta\left(\mathbf{r} - \mathbf{r}_i\right)\delta\left(\theta - \theta_i\right)\ . \tag{7}$$

Using Itô's Lemma, a stochastic dynamical equation for the density Eq. (7) can be derived:

$$\dot{f}(\mathbf{r}, \theta, t) = -v\mathbf{p}\cdot\nabla f - \omega\partial_\theta f - g\partial_\theta \int \mathrm{d}\theta' f\left(\mathbf{r}, \theta'\right)\sin\left(\theta' - \theta\right) f(\mathbf{r}, \theta) + \partial_\theta^2 f \tag{8}$$

where $\omega = \omega\left(\mathbf{r}, t\right)$ is the spatiotemporal white noise.