Two Coupled Oscillators with Chirality

April 24, 2024

1 The Model

1.1 Raw model

$$
\begin{cases}
\dot{x}_1 = v\cos\theta_1 \\
\dot{y}_1 = v\sin\theta_1 \\
\dot{\theta}_1 = \omega_1 + \lambda A\sin\left(\theta_2 - \theta_1\right) \\
\dot{x}_2 = v\cos\theta_2 \\
\dot{y}_2 = v\sin\theta_2 \\
\dot{\theta}_2 = \omega_2 + \lambda A\sin\left(\theta_1 - \theta_2\right)
\end{cases}
$$

where

$$
A = \begin{cases}
1, & |\mathbf{r}_1 - \mathbf{r}_2| \leqslant d_0 \\
0, & |\mathbf{r}_1 - \mathbf{r}_2| > d_0
\end{cases}
$$

1.2 Model under polar coordinates

Let

$$
x_i = r_i\cos\varphi_i\ ,
$$

$$
y_i = r_i\sin\varphi_i\ ,
$$

then we have

$$
\begin{aligned}
\dot{r}_i &= \frac{1}{r_i}\left(x_i\dot{x}_i + y_i\dot{y}_i\right) \\
&= v\cos\varphi_i\cos\theta_i + v\sin\varphi_i\sin\theta_i \\
&= v\cos\left(\varphi_i - \theta_i\right)\ , \\
\dot{\varphi} &= \frac{1}{r_i^2}\left(x\dot{y} - y\dot{x}\right) \\
&= \frac{v}{r_i}\left(\sin\varphi_i\cos\theta_i - \cos\varphi_i\sin\theta_i\right) \\
&= \frac{v}{r_i}\sin\left(\varphi_i - \theta_i\right)\ .
\end{aligned}
$$

Introduce $\alpha_i = \varphi_i - \theta_i$, $\Delta\theta = \theta_2 - \theta_1$, $\Delta\varphi = \varphi_2 - \varphi_1$, $\Delta\omega = \omega_2 - \omega_1$, then we have

$$\begin{cases} \dot{r}_1 = v\cos\alpha_1 \\ \dot{r}_2 = v\cos\alpha_2 \\ \dot{\alpha}_1 = \frac{v}{r_1}\sin\alpha_1 - \omega_1 - \lambda A\sin\Delta\theta \\ \dot{\alpha}_2 = \frac{v}{r_2}\sin\alpha_2 - \omega_2 + \lambda A\sin\Delta\theta \\ \Delta\dot{\varphi} = \frac{v}{r_2}\sin\alpha_2 - \frac{v}{r_1}\sin\alpha_1 \\ \Delta\dot{\theta} = \Delta\omega - 2\lambda A\sin\Delta\theta \end{cases}, \tag{1}$$

where

$$\begin{aligned} d &= \sqrt{\left(r_1\cos\varphi_1 - r_2\cos\varphi_2\right)^2 + \left(r_1\sin\varphi_1 - r_2\sin\varphi_2\right)^2} \\ &= \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\Delta\varphi} \end{aligned}.$$

1.3 Single direction driving

Assuming that $\dot{\theta}_2 = \omega_2$, the model becomes

$$\begin{cases} \dot{r}_1 = v\cos\alpha_1 \\ \dot{\alpha}_1 = \frac{v}{r_1}\sin\alpha - \omega_i - \lambda\sin\Delta\theta \\ \Delta\dot{\theta} = \Delta\omega - \lambda\sin\Delta\theta \end{cases},$$