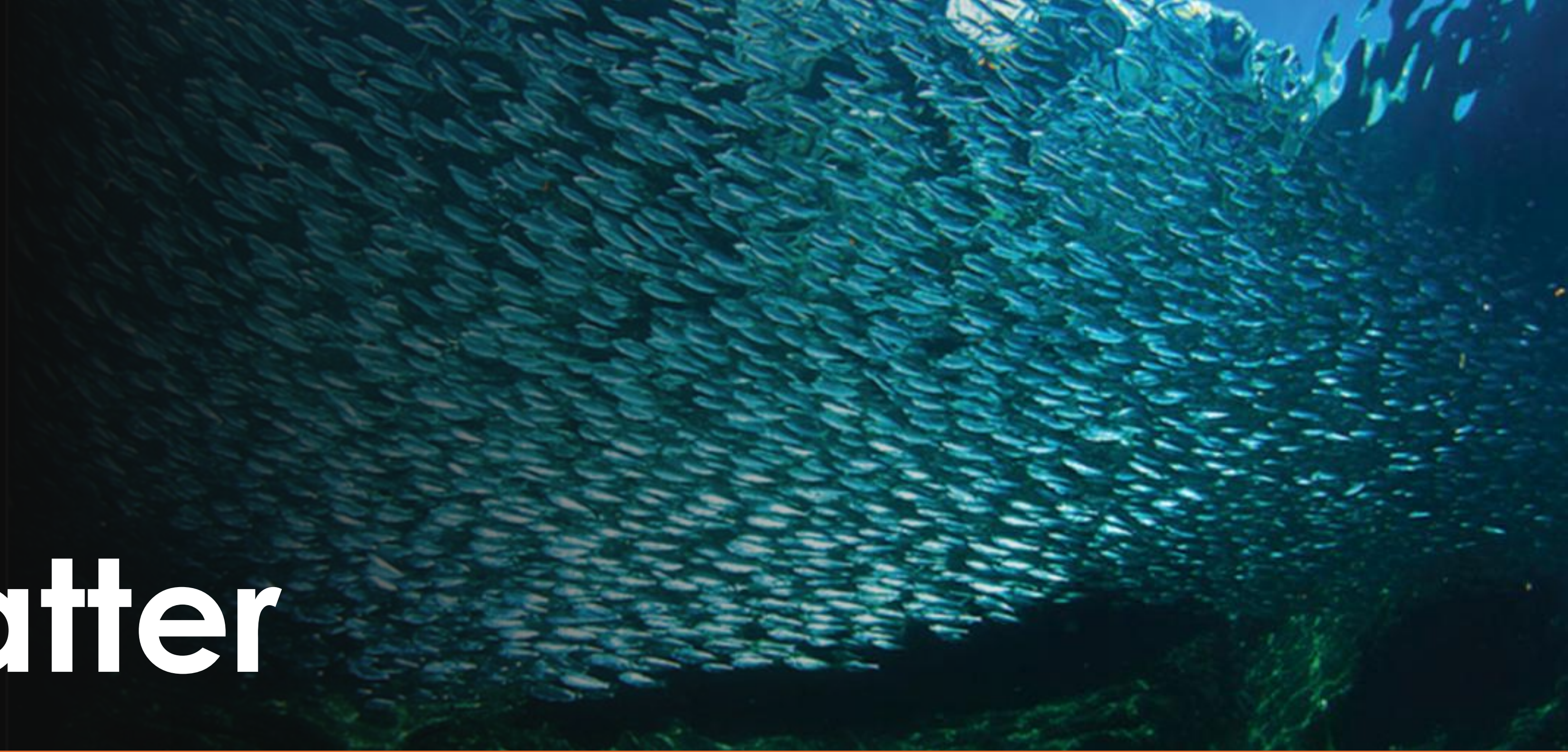
Active Matter



Flocking, Self-propelled, Chiral & Chemotactic

Quantification of Long-Range Correlation

Scale-free correlations in starling flocks

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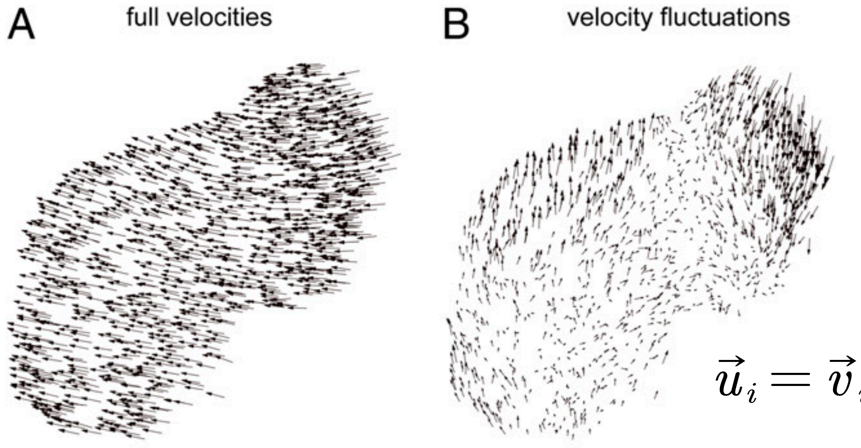
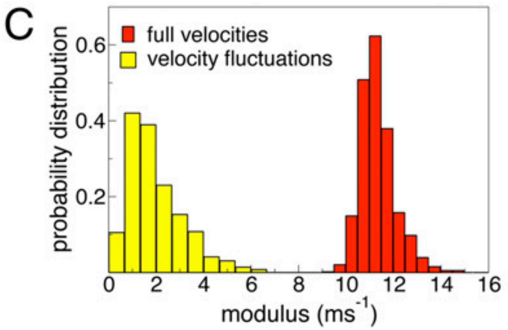
Contributed by Giorgio Parisi, May 11, 2010 (sent for review December 6, 2009)

$$\Phi = \left\| \frac{1}{N} \sum_{i=1}^{N} \frac{\vec{v}_i}{\| v_i \|} \right\|,$$

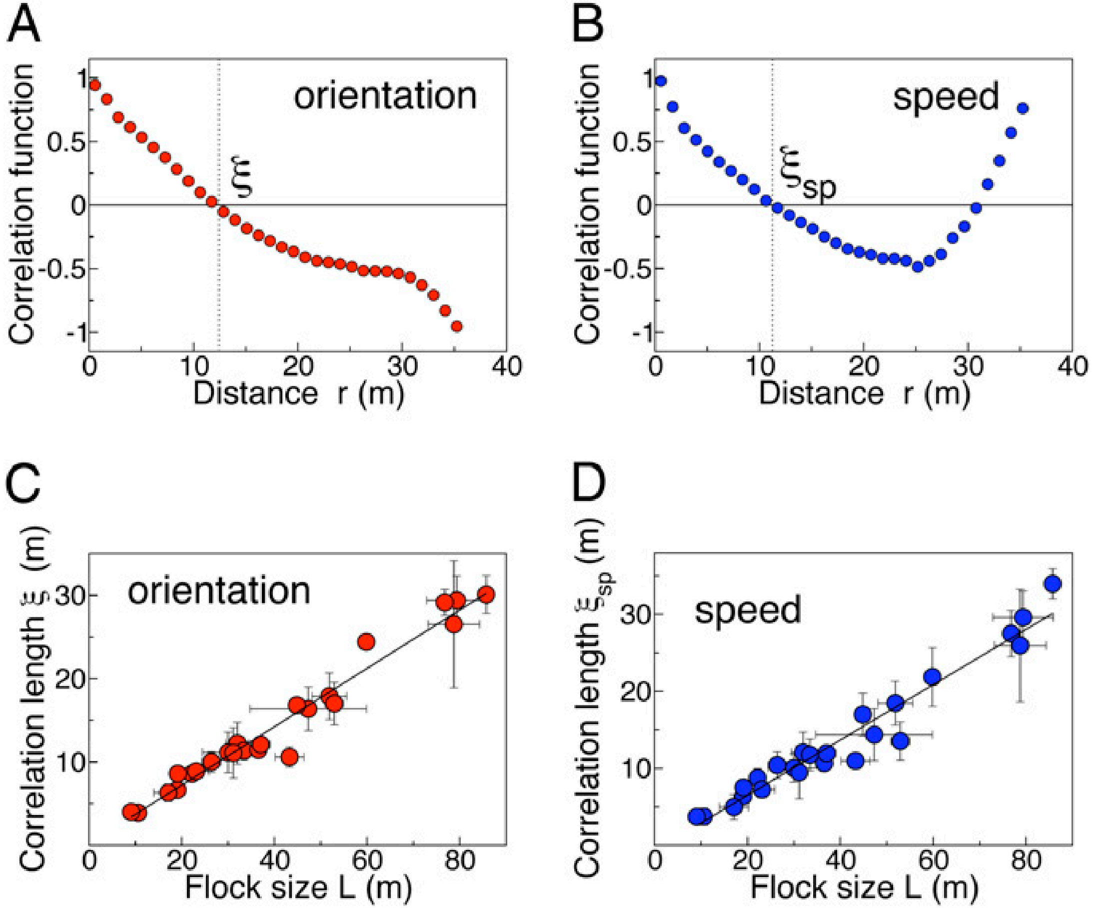




$$\vec{u}_i = \vec{v}_i - \frac{1}{N} \sum_{k=1}^{N} \vec{v}_k,$$

$$\sum_{i=1}^{N} \bar{u}_i = 0.$$

$$C(r) = \frac{1}{c_0} \frac{\sum_{ij} \bar{u}_i \cdot \bar{u}_j \delta(r - r_{ij})}{\sum_{ij} \delta(r - r_{ij})}$$



A. Cavagna, A. Cimarelli, I. Giardina, G. Parisi, R. Santagati, F. Stefanini, and M. Viale, Scale-free correlations in starling flocks, Proc. Natl. Acad. Sci. U.S.A. 107, 11865 (2010).

Also seen in insect swarms

Selected for a Viewpoint in *Physics*
PRL **113,** 238102 (2014) PHYSICAL REVIEW LETTERS week ending 5 DECEMBER 2014

Finite-Size Scaling as a Way to Probe Near-Criticality in Natural Swarms

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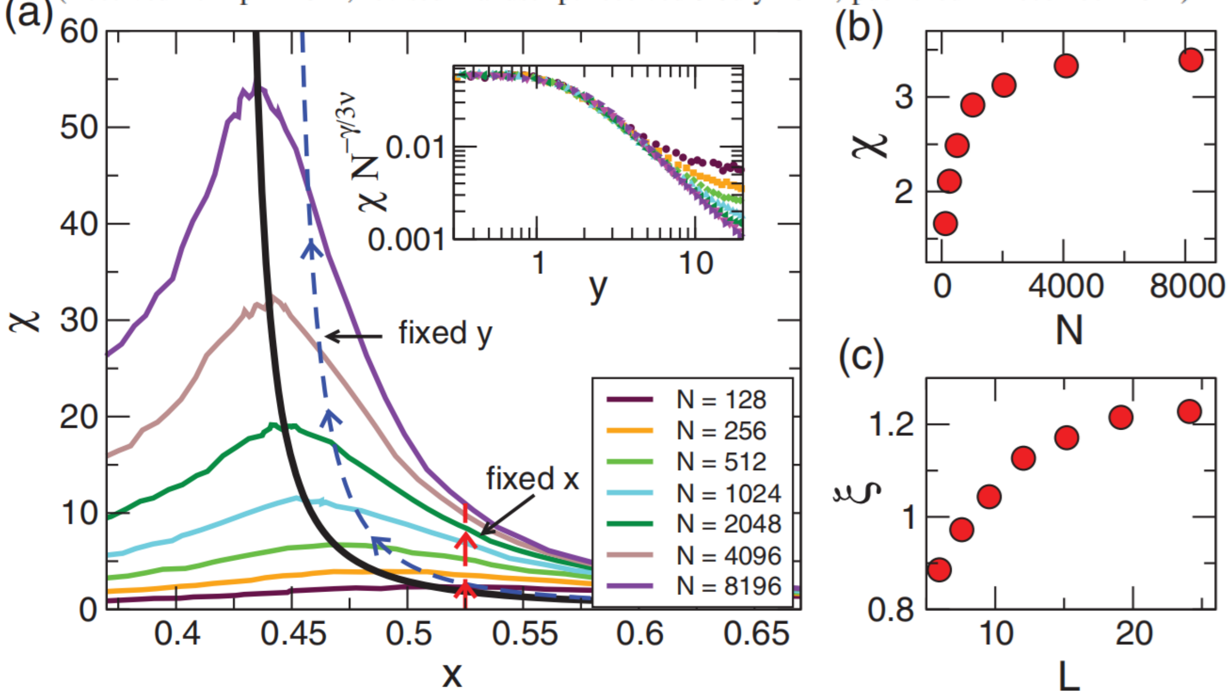
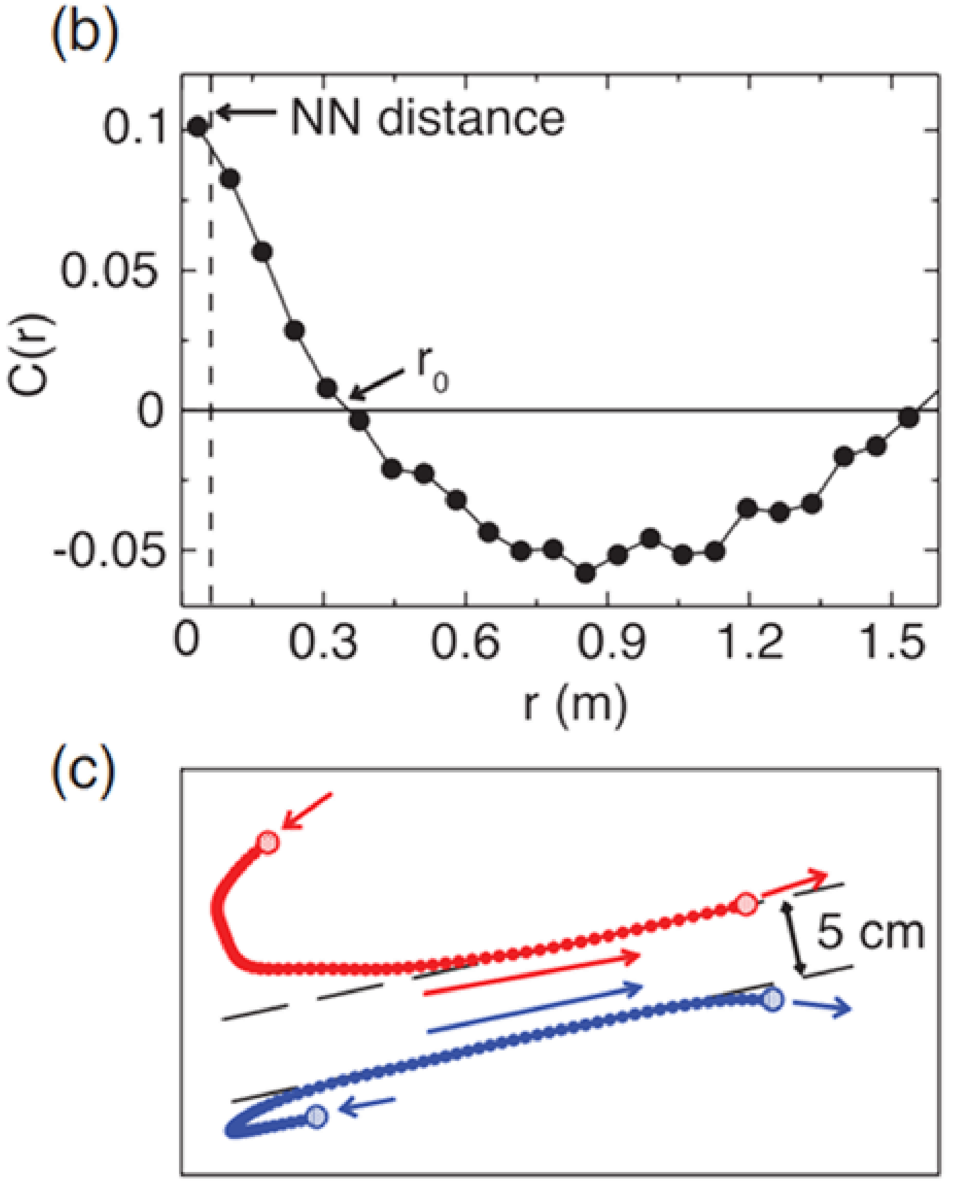




Correlation in a 2D Space

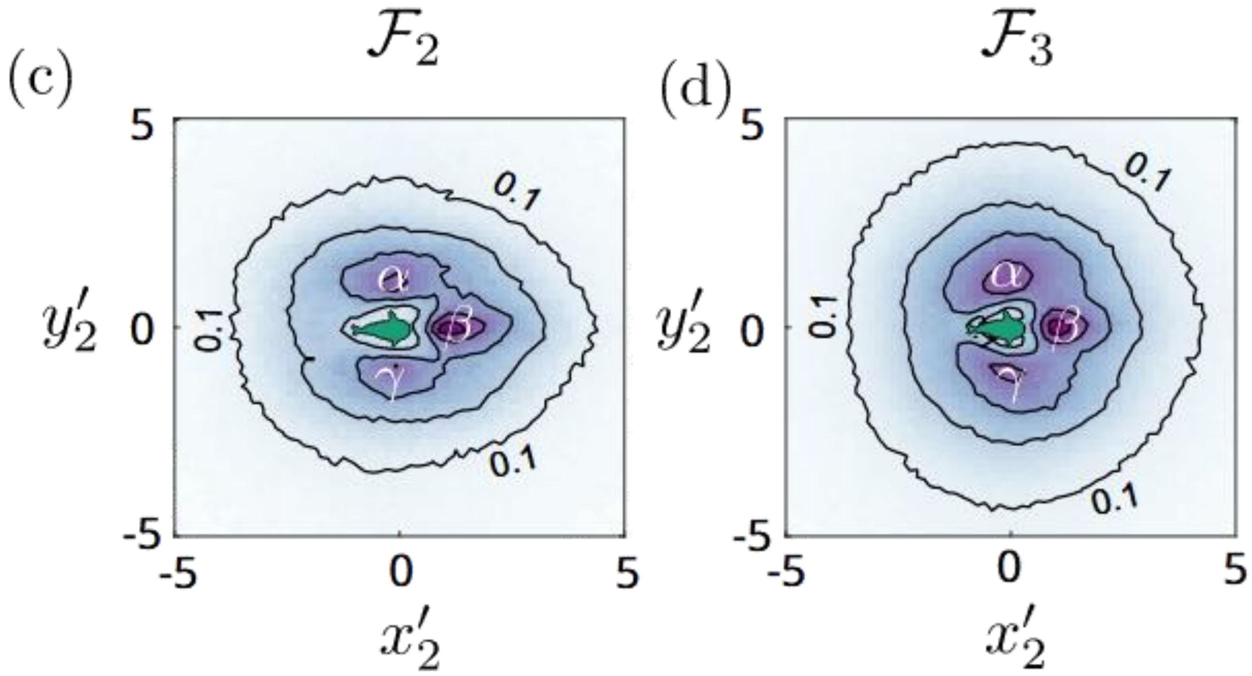
nature communications

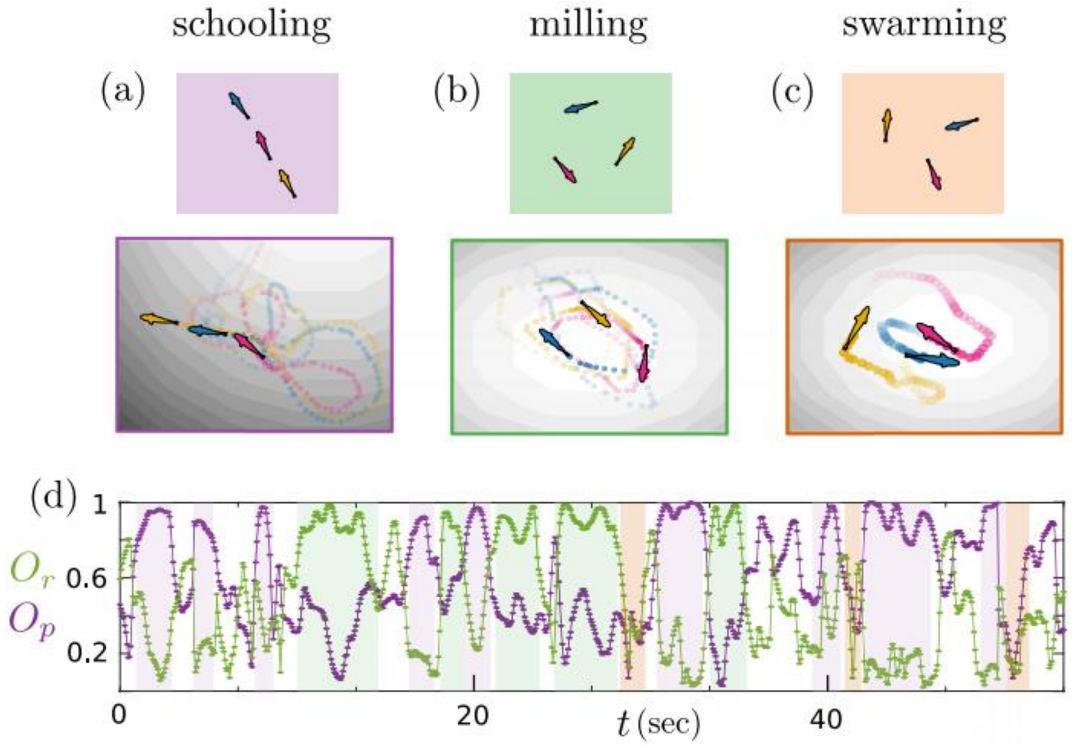
Article https://doi.org/10.1038/s41467-024-46426-1

Dynamical order and many-body correlations in zebrafish show that three is a crowd

$$O_p = \frac{1}{N}\left|\sum_{i=1}^{N}\frac{\mathbf{v}_i}{|\mathbf{v}_i|}\right|,$$

$$O_r = \frac{1}{N}\left|\sum_{i=1}^{N}\frac{\mathbf{v}_i}{|\mathbf{v}_i|}\times\left(\frac{\mathbf{r}_i-\mathbf{r}_{\mathrm{cm}}}{|\mathbf{r}_i-\mathbf{r}_{\mathrm{cm}}|}\right)\right|$$





A. Zampetaki, Y. Yang, H. Löwen, and C. P. Royall, Dynamical order and many-body correlations in zebrafish show that three is a crowd, Nat Commun 15, 2591 (2024).

Quantification of transient behavior

$$\dot{\mathbf{r}}_i = v\,\mathbf{p}(\theta_i)\ ,$$

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j\in A_i} \sin(\theta_j - \theta_i)\ ,$$

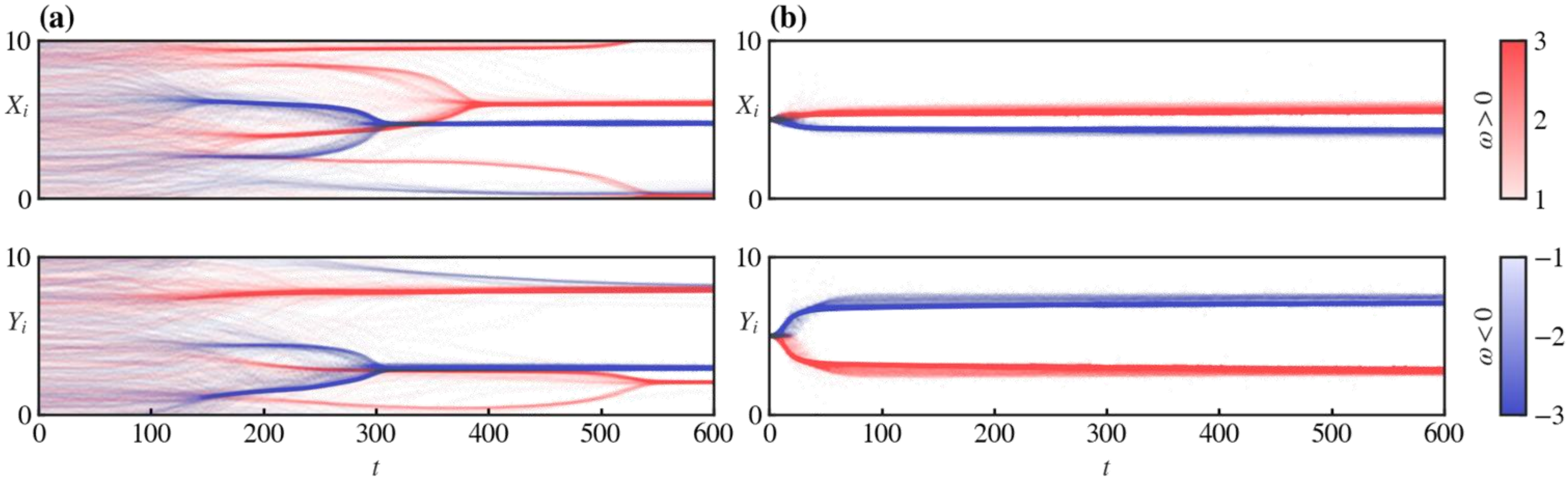
$$\mathbf{p}(\theta) = (\cos\theta, \sin\theta)^{\top}$$

$$A_i(t) = \{ j \,|\, |\mathbf{r}_i(t) - \mathbf{r}_j(t)| \leqslant d_0 \}\ ,$$

Instantaneous rotation centers of CAPs :

$$X_i(t) = x_i(t) - \frac{v}{\dot{\theta}_i(t)} \sin\theta_i(t)\ ,$$

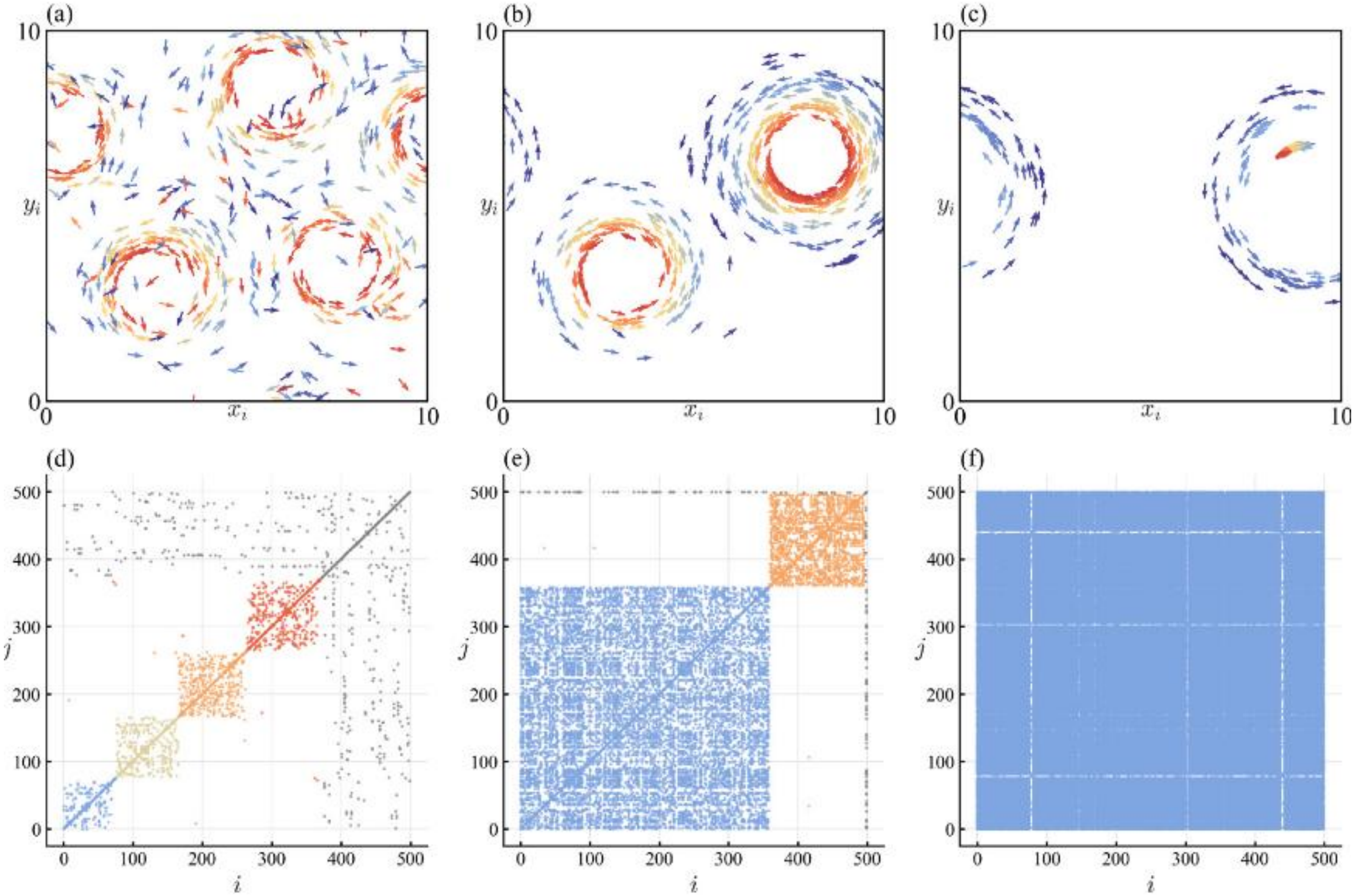
$$Y_i(t) = y_i(t) + \frac{v}{\dot{\theta}_i(t)} \cos\theta_i(t)\ ,$$



Quantification of transient behavior

The adjacency matrix A is then defined as:

$$A_{ij} = \begin{cases} 1\,, & |\mathbf{r}_i - \mathbf{r}_j| \leqslant d_0 \\ 0\,, & \text{otherwise} \end{cases}$$



Quantification of Phase Separation

The distance between mean centers of two chiralities:

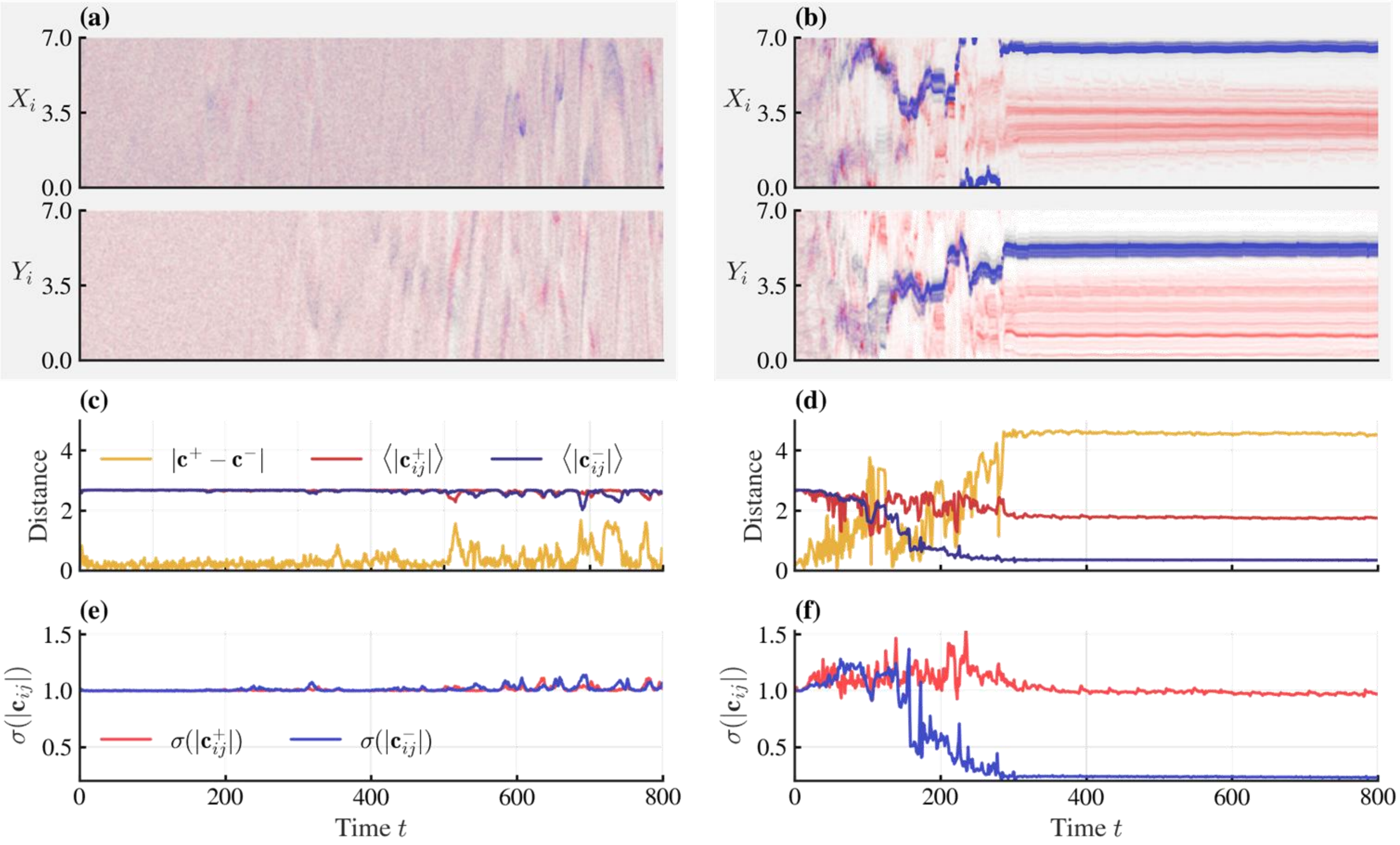
$$|\mathbf{c}^{+}-\mathbf{c}^{-}|=\left|\frac{1}{N_c}\sum_{i=1}^{N_c}\mathbf{c}_i^{+}-\frac{1}{N_c}\sum_{i=1}^{N_c}\mathbf{c}_i^{-}\right|,$$

The mean distance between the centers of the same chirality:

$$\langle|\mathbf{c}|_{ij}^{\pm}\rangle=\frac{2}{N_c^2}\sum_{i=1}^{N_c}\sum_{j=1}^{i}|\mathbf{c}_i^{\pm}-\mathbf{c}_i^{\pm}|,$$

The standard deviation of the distances between centers of the same chirality:

$$\sigma(|\mathbf{c}|_{ij}^{\pm})=\sqrt{\frac{2}{N_c^2}\sum_{i=1}^{N_c}\sum_{j=1}^{i}\left(|\mathbf{c}_i^{\pm}-\mathbf{c}_i^{\pm}|-\langle|\mathbf{c}|_{ij}^{\pm}\rangle\right)^2},$$



Quantification of Phase Separation

Some order parameters can be introduced to measure the chiral demixing:

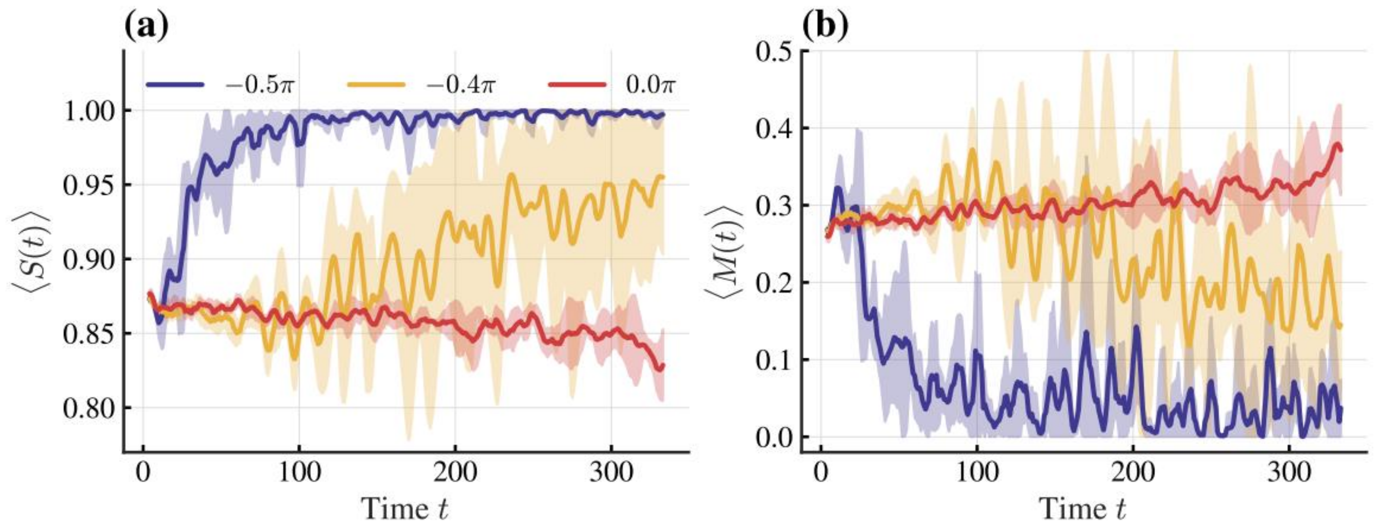
$$S(t) = \frac{1}{N}\sum_{i=1}^{N}\frac{\sum_{j\in A_i} H(\omega_i\omega_j)}{|A_i(t)|} ,$$

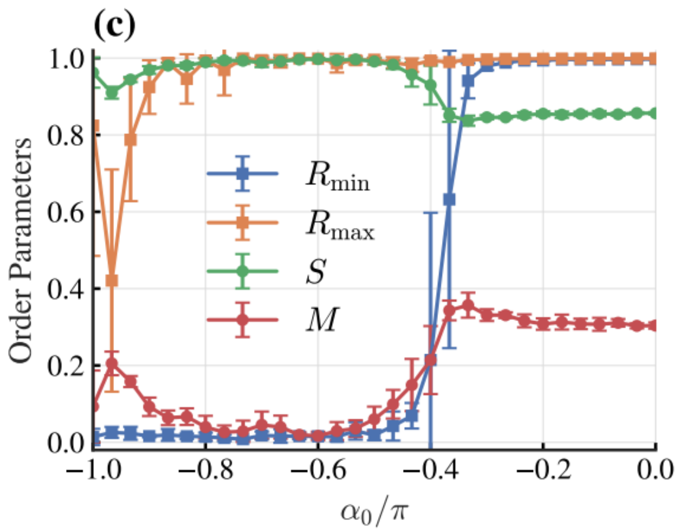
$$M(t) = \frac{1}{N}\sum_{i=1}^{N} H\left[\sum_{j\in A_i} H(-\omega_i\omega_j)\right] .$$

$$Z(t) = R(t)\mathrm{e}^{\mathrm{i}\psi(t)} = \frac{1}{N}\sum_{j=1}^{N}\mathrm{e}^{\mathrm{i}\theta_j(t)} ,$$

$$R_{\max/\min} = \max_{t\in W}/\min\{R(t)\} ,$$

where $W = [Y - h, T + h]$ is the time rolling window and $H(x)$ is the Heaviside step function.





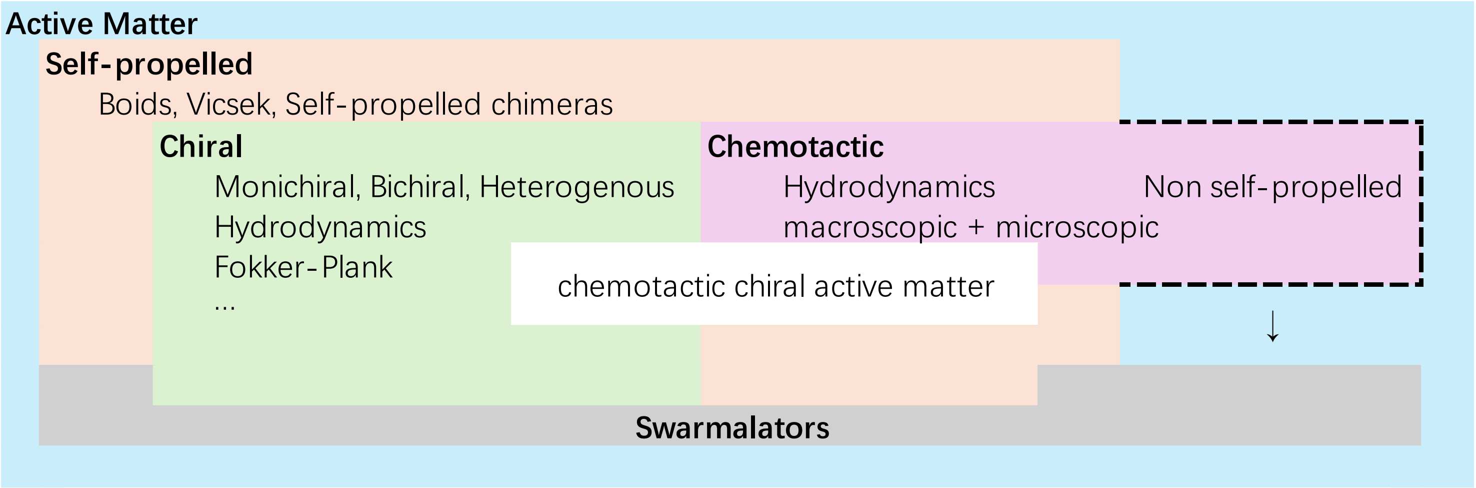
Retracing

1. Modeling
   Biods, Vicsek, CAPs, Chemotactic, Swarmalators……

2. Continuum/Hydrodynamics Model
   Continuity Equation, Fourier Expansion, Polar Density……

3. Linear Stability Analysis of PDE
   For all conditions, wave numbers ≤ 0

4. Data-driven Analysis of Swarming Behaviors



Keller–Segel instability

Smoluchowski Equation

$$\dot{\rho} = -\nabla\cdot(\beta_D \rho \nabla c) + D\nabla^2 \rho$$

$$\dot{c} = D_c \nabla^2 c + k_0 \rho - k_d c$$

Linear stability analysis $\Downarrow$ $(\rho, c) = (\rho_0, k_0\rho_0/k_d)$

$$\rho = \rho_0 + \delta\rho,\ c = c_0 + \delta c \ \ (|\delta\rho| \ll \rho_0,\ |\delta c| \ll c_0)$$

$$\begin{aligned}\dot{\delta\rho} &= -\beta_D \nabla\cdot(\rho\nabla\delta c) + D\nabla^2\delta\rho \\ &= -\beta_D\nabla\cdot[(\rho_0+\delta\rho)\nabla(c_0+\delta c)] + D\nabla^2(\rho_0+\delta\rho) \\ &= D\nabla^2\delta\rho - \beta_D\rho_0\nabla^2\delta c - \beta_D\nabla\cdot\underbrace{(\delta\rho\nabla\delta c)}_{\approx 0} \\ &= D\nabla^2\delta\rho - \beta_D\rho_0\nabla^2\delta c\end{aligned}$$

$$\begin{aligned}\dot{\delta c} &= D_c\nabla^2\delta c + k_0\delta\rho - k_d\delta c + \underbrace{k_0\delta_0 - k_d c_0}_{=0} \\ &= D_c\nabla^2\delta c + k_0\delta\rho - k_d\delta c\end{aligned}$$

Take the form of a plane wave:

$$\delta\rho = \delta\rho_0 \mathrm{e}^{\sigma t + i\mathbf{k}\cdot\mathbf{r}},\ \ \delta c = \delta c_0 \mathrm{e}^{\sigma t + i\mathbf{k}\cdot\mathbf{r}}$$

Symbol	Means
σ	growth rate
$\mathbf{k}$	wave vector (representing the oscillation mode in space)
$k = \lvert\mathbf{k}\rvert$	wave numbers (LSA usually focuses on the σ corresponding to different wave numbers)

For $\delta\rho = \delta\rho_0 \mathrm{e}^{\sigma t + i\mathbf{k}\cdot\mathbf{r}}$

$$\text{left} = \dot{\delta\rho} = \sigma\delta\rho_0 \mathrm{e}^{\sigma t + i\mathbf{k}\cdot\mathbf{r}}$$

$$\text{right} = D(-k^2)\delta\rho_0 \mathrm{e}^{\sigma t + i\mathbf{k}\cdot\mathbf{r}} - \beta_D\rho_0(-k^2)\delta c_0 \mathrm{e}^{\sigma t + i\mathbf{k}\cdot\mathbf{r}}$$

$$\sigma\delta\rho_0 = -Dk^2\delta\rho_0 + \beta_D\rho_0 k^2 \delta c_0$$

$$\begin{cases}\sigma\delta\rho_0 = -Dk^2\delta\rho_0 + \beta_D\rho_0 k^2\delta c_0 \\ \sigma\delta c_0 = -D_c k^2\delta c_0 + k_0\delta\rho_0 - k_d\delta c_0\end{cases}$$