Chemotactic Network Designing

Yichen Lu

May 24, 2025

Contents

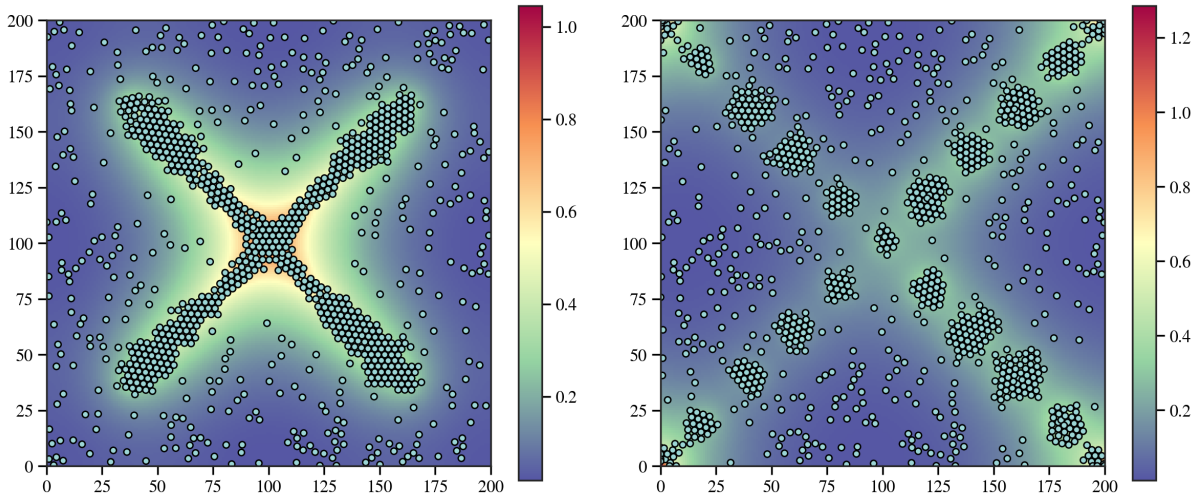
1 Models

1.1 Thinking Process

$$\dot{\mathbf{r}}_i(t) = \alpha_c \nabla c - \nabla_{\mathbf{r}_i} V + \sqrt{2D_p}\eta_i \tag{1a}$$

$$\dot{c}(\mathbf{r}, t) = D_c \nabla^2 c - k_c c + \beta_c \sum\nolimits_j \delta\left(\mathbf{r} - \mathbf{r}_j^*\right) \tag{1b}$$

for $i = 1, 2, \cdots, N$ and $j = 1, 2, \cdots, M$. Here, $\mathbf{r}_i, \mathbf{r}_j^*$ is the position of the i-th particle, j-th target node, respectively, c is the concentration of the chemical, $\alpha_c < 0$ is the chemotactic sensitivity, V is the potential field of short-range repulsion, D_c is the diffusion coefficient, k_c is the decay rate of the chemical, and β_c is the production rate of the target nodes.



(a) Horizontal distance of nodes $d_n = 7$ (b) $d_n = 10$

Figure 1: The simulation of the above model with $\alpha_c = -5$, $D_p = 0$, $D_c = 2$, $k_c = 0.001$ and $\beta_c = 0.3$. When the horizontal distance of nodes d_n is small, the nodes are connected by the particles. While, when d_n is large, the particles are not connected.

Add new chemical u to the model, which is produced by the particles and decays with time. The chemical u will also affect the movement of the particles.

$$\dot{\mathbf{r}}_i(t) = \alpha_u \nabla u + \alpha_c \nabla c - \nabla_{\mathbf{r}_i} V + \sqrt{2D_p}\eta_i \tag{2a}$$

$$\dot{u}(\mathbf{r}, t) = D_u \nabla^2 c - k_u u + \beta_u \sum\nolimits_i \delta\left(\mathbf{r} - \mathbf{r}_i\right) \tag{2b}$$

$$\dot{c}(\mathbf{r}, t) = D_c \nabla^2 c - k_c c + \beta_c \sum\nolimits_j \delta\left(\mathbf{r} - \mathbf{r}_j^*\right) \tag{2c}$$

New ideas:

- **Reverse density production**: too high density of particles will lead to the decrease of the production rate of the chemical.
- **Nonlinear coupling and decay**: the chemical u will decay when c is too high (almost useless).

$$\dot{u}(\mathbf{r}, t) = D_u \nabla^2 c - (k_u + \gamma c) u + \beta_u \frac{c}{K_1 + c} \cdot \frac{K_2}{K_2 + c} \sum\nolimits_i \delta\left(\mathbf{r} - \mathbf{r}_i\right) \tag{3}$$

1.2 Definitions

$$\dot{\mathbf{r}}_i(t) = \alpha_u \nabla u + \alpha_v \nabla v - \nabla_{\mathbf{r}_i} V + \sqrt{2D_p}\eta_i \tag{4a}$$

$$\dot{u}(\mathbf{r},t) = D_u \nabla^2 u - uv^2 + k_f(1-u) + k_u \sum\nolimits_i \delta(\mathbf{r}-\mathbf{r}_i) + \beta_u \sum\nolimits_i \delta(\mathbf{r}-\mathbf{r}_i^*) \tag{4b}$$

$$\dot{v}(\mathbf{r},t) = D_v \nabla^2 v + uv^2 - k_d v + k_v \sum\nolimits_i \delta(\mathbf{r}-\mathbf{r}_i) + \beta_v \sum\nolimits_i \delta(\mathbf{r}-\mathbf{r}_i^*) \tag{4c}$$

for $i = 1, 2, \cdots, N$. Here, $\mathbf{r}_i$ is the position of the i-th particle, $D_{u,v}$ is the diffusion coefficient, $k_{f,d}$ is the production rate, $k_{u,v}$ is the degradation rate, α_u and α are the chemotactic sensitivity, V is the potential field, $\beta_{u,v}$ is the chemotactic sensitivity to the target, and η_i is a Gaussian white noise with zero mean and unit variance.

$$\dot{\mathbf{r}}_i(t) = \alpha_u \nabla u + \alpha_v \nabla v + \alpha_c \nabla c - \nabla_{\mathbf{r}_i} V + \sqrt{2D_p}\eta_i \tag{5a}$$

$$\dot{u}(\mathbf{r},t) = D_u \nabla^2 u - uv^2 + k_f(1-u) + k_u \sum\nolimits_i \delta(\mathbf{r}-\mathbf{r}_i) \tag{5b}$$

$$\dot{v}(\mathbf{r},t) = D_v \nabla^2 v + uv^2 - k_d v + k_v \sum\nolimits_i \delta(\mathbf{r}-\mathbf{r}_i) \tag{5c}$$

$$\dot{c}(\mathbf{r},t) = D_c \nabla^2 c - k_c c + \beta_c \sum\nolimits_i \delta(\mathbf{r}-\mathbf{r}_i^*) \tag{5d}$$

2 Continuum model

3 Behaviors