Chemotactic Network Designing

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Contents

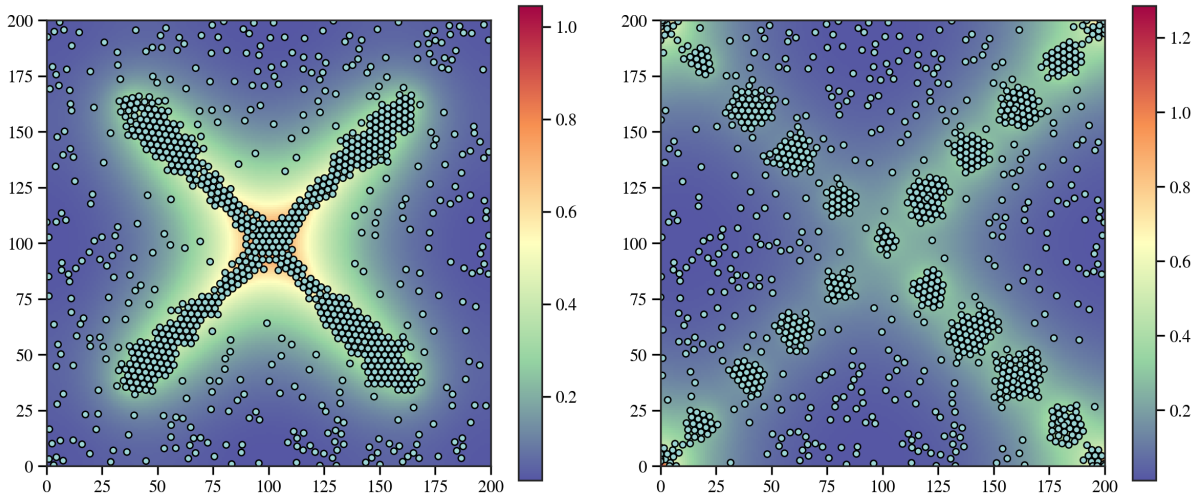
1 Models

1.1 Thinking Process

$$\dot{\mathbf{r}}_i(t) = \alpha_c \nabla c - \nabla_{\mathbf{r}_i} V + \sqrt{2D_p}\eta_i \tag{1a}$$

$$\dot{c}(\mathbf{r}, t) = D_c \nabla^2 c - k_c c + \beta_c \sum\nolimits_j \delta\left(\mathbf{r} - \mathbf{r}_j^*\right) \tag{1b}$$

for $i = 1, 2, \cdots, N$ and $j = 1, 2, \cdots, M$. Here, $\mathbf{r}_i, \mathbf{r}_j^*$ is the position of the i-th particle, j-th target node, respectively, c is the concentration of the signal chemical released by the node, $\alpha_c < 0$ is the chemotactic sensitivity, V is the potential field of short-range repulsion, D_c is the diffusion coefficient, k_c is the decay rate of the chemical, and β_c is the production rate of the target nodes.



(a) Horizontal distance of nodes $d_n = 7$ (b) $d_n = 10$

Figure 1: The simulation of the above model with $\alpha_c = -5$, $D_c = 2$, $k_c = 0.001$ and $\beta_c = 0.3$. When the horizontal distance of nodes d_n is small, the nodes are connected by the particles. While, when d_n is large, the particles are not connected.

Introduce adaptive dynamics to the particles by incorporating a new variable, s_i, representing the internal state of the i-th particle, which governs its sensitivity to the chemical signal. Additionally, assume that the particles can modulate the local chemical concentration but do not actively produce the chemical. The updated model is as follows:

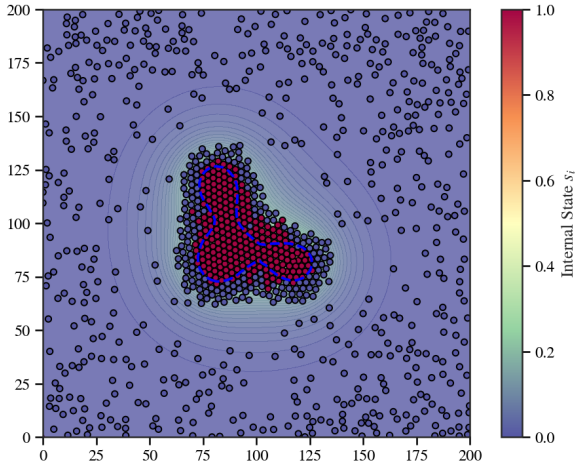
$$\dot{\mathbf{r}}_i(t) = \alpha_c \nabla c - \nabla_{\mathbf{r}_i} V + \sqrt{2D_p}\eta_i \,, \tag{2a}$$

$$\dot{s}_i(t) = s_i(1 - s_i)\left(c_{\mathbf{r}_i} - c_s\right) \,, \tag{2b}$$

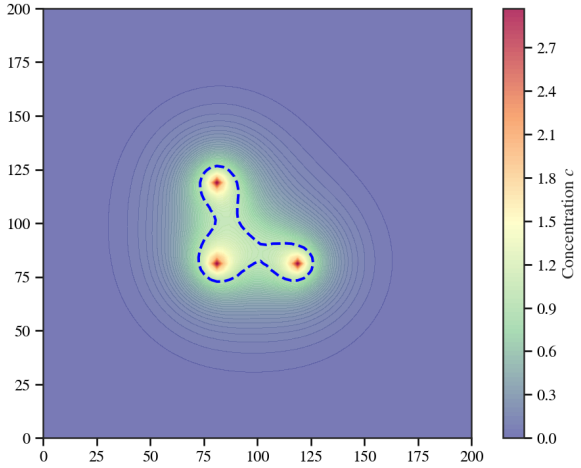
$$\dot{c}(\mathbf{r}, t) = \left[D_c \nabla^2 c - k_c(1 - s_{\mathbf{r}})c + \beta_c \sum\nolimits_j \delta\left(\mathbf{r} - \mathbf{r}_j^*\right)\right]\left(c_u - c\right) \,, \tag{2c}$$

where $c_{\mathbf{r}_i} = c(\mathbf{r}_i(t), t)$ is the chemical concentration at the position of the i-th particle, c_s is the threshold concentration for the particle's internal state, and $s_{\mathbf{r}} = \sum_i s_i \delta(\mathbf{r} - \mathbf{r}_i)$ is the local internal state of particles at position $\mathbf{r}$. Note that the term $s_i \in [0, 1]$ and short-range repulsion interation $\nabla_{\mathbf{r}_i} V$ will resilt in at most one particle at position $\mathbf{r}$, which means $s_{\mathbf{r}} \leqslant 1$ and the term $k_c(1 - s_{\mathbf{r}})c$ in the chemical equation ensures that the chemical concentration is always non-negative. In another word, the particles can only modulate the decay rate of the chemical, while the chemical is still produced by the target nodes. c_u is the upper bound of the chemical concentration to prevent unlimited growth of c in case of linear instability.

Following figure shows the simulation of the above model of three nodes.



(a) Final snapshot of the particles and chemical fields



(b) Final snapshot of only chimical fields

Figure 2: The simulation of the model (2) with $\alpha_c = -1$, $D_c = 3$, $k_c = 0.005$ and $\beta_c = 0.1$. The blue dashed line is critical contour of the network region. **(a)**, the particles are attracted to the target nodes and form a connected network like letter "L". The color of the particles represents their internal state s_i, with red indicating a high state and blue indicating a low state. **(b)**, the chemical field is shown with the critical contour (blue dashed line) of the network region. It can be noticed that the critical contour lines obtained from c are not a regular L-shape, but rather a short Y-shape, which is obviously more economic than the L-shape.

1.2 Final Definitions

2 Continuum model

3 Behaviors