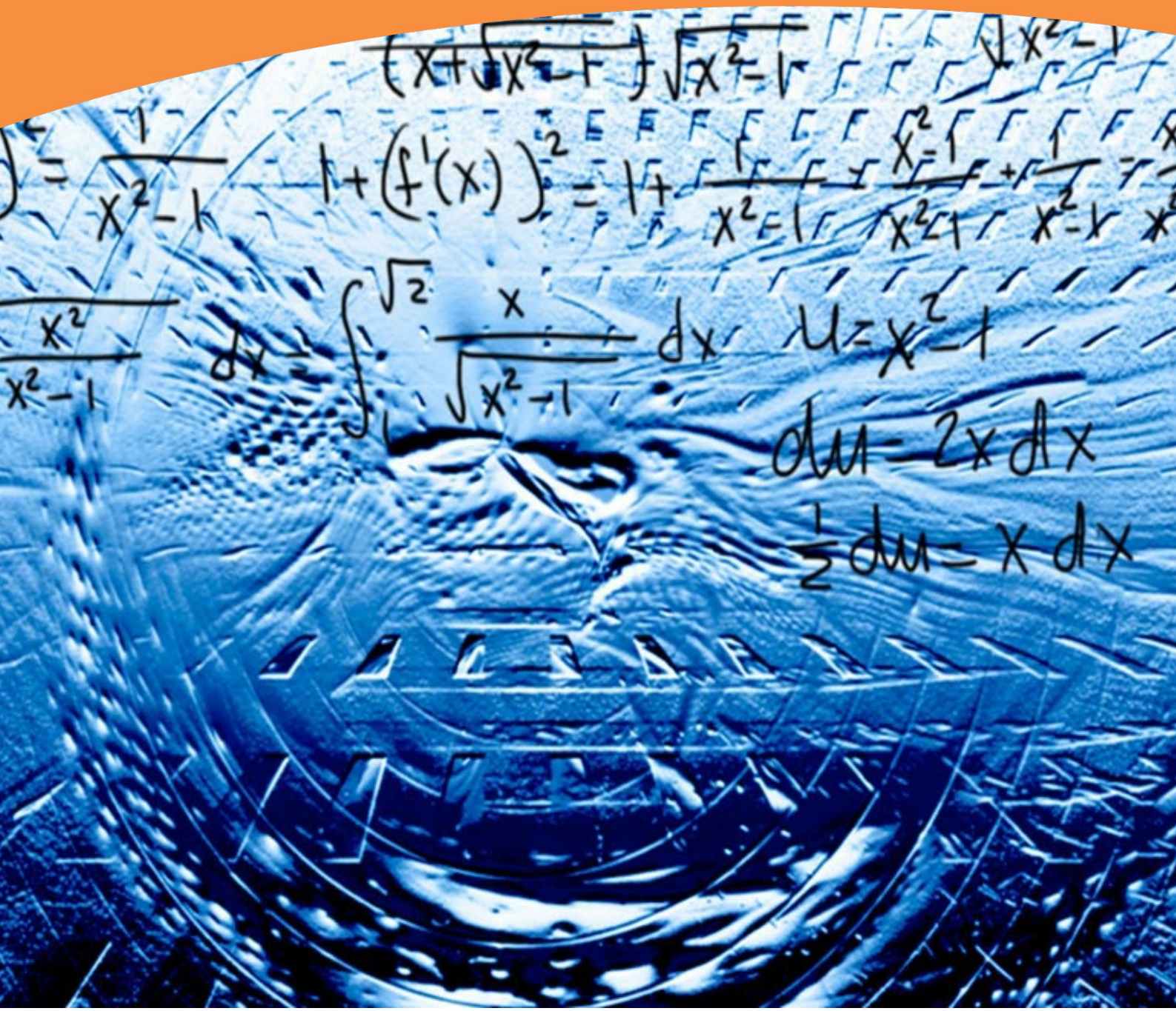
Calculus II YouTube Workbook

Albert Bronstein



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Calculus II YouTube Workbook
1st edition

ISBN 978-87-403-0470-1

Contents

Preface

This workbook contains examples for a standard Calculus II course in an American University. Many problems were assigned as homework problems at University of Illinois at Chicago. Before viewing solution videos, students should first try solving these problems on their own.

1 Area Problems

The area of the region bounded by the graphs of $y = f(x)$ and $y = g(x)$ for $a \leq x \leq b$, where $f(x)$ and $g(x)$ are continuous and $f(x) \geq g(x)$ for all $x \in [a, b]$, is

$$\int_a^b (f(x) - g(x))\, dx$$

The area of the region bounded by the graphs of $x = f(y)$ and $x = g(y)$ for $c \leq y \leq d$, where $f(y)$ and $g(y)$ are continuous and $f(y) \geq g(y)$ for all $y \in [c, d]$, is

$$\int_c^d (f(y) - g(y))\, dy$$

1. Find the area of the region enclosed by the graphs of $f(x) = x$ and $g(x) = x^2$.
   ► Link to Solution Video ◄

2. Find the area of the region bounded above by $y = \sqrt{x+2}$, bounded below by $y = \dfrac{1}{x+1}$, and bounded on the sides by $x = 0$ and $x = 2$.
   ► Link to Solution Video ◄

3. Find the area of the region enclosed by the graphs $y = -x^2 - 2x + 2$ and $y = 1 - 2x$.
   ► Link to Solution Video ◄

4. Find the area of the region enclosed by the graphs $y = 2x^2$, $y = -4x + 6$ and the x-axis.
   ► Link to Solution Video ◄

5. Find the area of the region enclosed by the graphs of $y = x - 1$ and $y^2 = 2x + 6$.
   ► Link to Solution Video ◄

6. Find the area of the region enclosed by the graphs of $y = \sin x$ and $y = \sin 2x$ for $0 \leq x \leq \pi$.
   ► Link to Solution Video ◄

7. Find the area of the region enclosed by the graphs $y = x^2 - 2$ and $y = |x|$.
   ► Link to Solution Video ◄

8. Evaluate $\displaystyle\int_{-10}^{10} \sqrt{100 - x^2}\, dx$

   ► Link to Solution Video ◄

9. Evaluate $\int_{-\sqrt{2}/2}^{\sqrt{2}/2} (\sqrt{1-x^2} - |x|)\, dx$

   ► Link to Solution Video ◄

10. Evaluate $\int_{-6}^{4} \sqrt{24 - 2x - x^2}\, dx$

    ► Link to Solution Video ◄

11. Find the area of the region enclosed by the graphs of $x + y = 4$, $y = x$, and $y + 3x = 4$

    ► Link to Solution Video ◄

12. Find the area of the region enclosed by the graph of $y^2 = x^3$ and the line x = 1

    ► Link to Solution Video ◄

13. Find the area of the region enclosed by the graphs of $y = \frac{1}{2}x$ and $y = x\sqrt{1-x^2}$

    ► Link to Solution Video ◄

14. Find the area of the region enclosed by the graphs of $y = x^2 - 5x$ and $y = -x^2 + x + 20$

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15. Find the area of the region enclosed by the graphs of $y = x^2 - 5x$ and $y = -x^2 + x + 20$ for $4 \le x \le 8$

    ► Link to Solution Video ◄

16. Evaluate $\int_{-2}^{2} (x+3)\sqrt{4-x^2}\, dx$

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17. 1Find the area of the region enclosed by the graphs of $x = -y^2 + 9$ and $x = -\frac{1}{2}y^2 - 6y - 9$

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18. Find the area of the region bounded by the graphs of $y = x^2$, the tangent line to this parabola at the point (1, 1) and the *x*-axis

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2 Volume by Cross-Section

Let S be a solid that lies between $x = a$ and $x = b$. If the cross-sectional area obtained by intersecting a solid with a plane perpendicular to the x-axis is $\boldsymbol{A}(x)$, where $\boldsymbol{A}(x)$ is a continuous function, then the volume of the solid is

$$\int_a^b \boldsymbol{A}(\boldsymbol{x})\, \boldsymbol{dx}$$

Let S be a solid that lies between $y = c$ and $y = d$. If the cross-sectional area obtained by intersecting a solid with a plane perpendicular to the y-axis is $\boldsymbol{A}(y)$, where $\boldsymbol{A}(y)$ is a continuous function, then the volume of the solid is

$$\int_c^d \boldsymbol{A}(\boldsymbol{y})\, \boldsymbol{dy}$$

1. Find the volume of the solid with a circular base of radius 1 and whose cross-sections perpendicular to the x-axis are equilateral triangles.
   ► Link to Solution Video ◄

2. Find the volume of the solid whose base is a parabolic region $\boldsymbol{\{(x, y)|x^2 \leq y \leq 1\}}$ with cross-sections perpendicular to the y-axis are isosceles right triangles with hypotenuse in the base.
   ► Link to Solution Video ◄

3. Find the volume of the solid whose base is bounded by the graphs of $4x + 5y = 20$, the x-axis, and the y-axis. Cross sections perpendicular to the x-axis are squares
   ► Link to Solution Video ◄

4. Find the volume of the solid whose base is bounded by the graphs of $4x + 5y = 20$, the x-axis, and the y-axis. Cross sections perpendicular to the x-axis are semi circles
   ► Link to Solution Video ◄

5. Find the volume of the solid whose base is bounded by the graph of $\boldsymbol{x^2 + y^2 = 2x}$. Cross sections perpendicular to the x-axis are equilateral triangles
   ► Link to Solution Video ◄

6. Find the volume of the solid whose base is bounded by the graph of $x^2 + y^2 = 2x$. Cross sections perpendicular to the y-axis are equilateral triangles
   ► Link to Solution Video ◄

7. Find the volume of a wedge cut out of a circular cylinder of radius 4 by a plane that intersects the cylinder along a diameter at an angle of 30°.
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3 Disk/Washer Method

Let S be a solid obtained by rotating a region bounded by the graphs of $y = f(x)$ and $y = g(x)$ for $a \leq x \leq b$ about the x-axis. If $f(x)$ and $g(x)$ are continuous functions and $f(x) \geq g(x)$ for all $x \in [a, b]$, then the volume of the solid is

$$\pi \int_a^b \left[(f(x))^2 - (g(x))^2 \right] dx$$

Let S be a solid obtained by rotating a region bounded by the graphs of $x = f(y)$ and $x = g(y)$ for $c \leq y \leq d$ about the y-axis. If $f(y)$ and $g(y)$ are continuous functions and $f(y) \geq g(y)$ for all $y \in [c, d]$, then the volume of the solid is

$$\pi \int_c^d \left[(f(y))^2 - (g(y))^2 \right] dy$$

1. Show that the volume of a sphere with radius R is $V = \frac{4}{3}\pi R^3$.
   ► Link to Solution Video ◄

2. Show that the volume of a cone with radius R and height h is $V = \frac{1}{3}\pi R^2 h$.
   ► Link to Solution Video ◄

3. Find the volume of the solid obtained by rotating the region under $f(x) = x + 1$ over the interval $[0, 3]$, about the x-axis
   ► Link to Solution Video ◄

4. Find the volume of the solid of revolution obtained by revolving the area bounded by the graphs of $f(x) = \cos x$ over the interval $[0, \pi/2]$, about the x-axis
   ► Link to Solution Video ◄

5. Find the volume of the solid of revolution obtained by revolving the area bounded by the graphs of $y = x^3$, $y = 8$, and the y-axis about the y-axis
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6. Find the volume of the solid of revolution obtained by revolving the area bounded by the graphs of $y = \sqrt{x + 5}$, $y = \sqrt{5 - x}$, and the x-axis about the x-axis
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7. Find the volume of the solid of revolution obtained by revolving the area bounded by the graphs of $y = x^2$ and $y = x$ about the x-axis.
   ► Link to Solution Video ◄

8. Find the volume of the solid of revolution obtained by revolving the area bounded by the graphs of $y = x^2$ and $y = x$ about the y-axis.
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9. Find the volume of the solid of revolution obtained by revolving the area bounded by the graphs of $y = x^2$ and $y = x$ about the line $y = -3$.
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10. Find the volume of the solid of revolution obtained by revolving the area bounded by the graphs of $y = x^2$ and $y = x$ about the line $x = -3$.
    ► Link to Solution Video ◄

11. Find the volume of the solid of revolution obtained by revolving the area bounded by the graphs of $y = x^2$ and $y = x$ about the line $y = 4$.
    ► Link to Solution Video ◄

12. Find the volume of the solid of revolution obtained by revolving the area bounded by the graphs of $y = x^2$ and $y = x$ about the line $x = 4$.
    ► Link to Solution Video ◄

13. Find the volume of the solid of revolution obtained by revolving the area bounded by the graphs of $\boldsymbol{y = 2x - x^2}$ and the x-axis about the line y-axis.
    ► Link to Solution Video ◄

14. Find the volume of a torus obtained by revolving the area bounded by the graph of $\boldsymbol{(x-2)^2 + (y-3)^2 = 9}$ about the line $x = 10$.
    ► Link to Solution Video ◄

15. Find the volume of the solid of revolution obtained by revolving the area bounded by the graphs of $y = x^2$ and $y = 3 - 2x$ about the x-axis
    ► Link to Solution Video ◄

16. Find the volume of the solid of revolution obtained by revolving the area bounded by the graphs of $y = x^2 + x + 1$ and $y = 4x - 1$ about the y-axis
    ► Link to Solution Video ◄

4 Method of Cylindrical Shells

Let S be a solid obtained by rotating a region bounded by the graphs of $y = f(x)$ and $y = g(x)$ for $a \leq x \leq b$ about the y-axis. If $f(x)$ and $g(x)$ are continuous functions and $f(x) \geq g(x)$ for all $x \in [a, b]$, then the volume of the solid is

$$2\pi \int_a^b x[f(x) - g(x)]\, dx$$

Let S be a solid obtained by rotating a region bounded by the graphs of $x = f(y)$ and $x = g(y)$ for $c \leq y \leq d$ about the x-axis. If $f(y)$ and $g(y)$ are continuous functions and $f(y) \geq g(y)$ for all $y \in [c, d]$, then the volume of the solid is

$$2\pi \int_c^d y[f(y) - g(y)]\, dy$$

1. Find the volume of the solid of revolution obtained by rotating the region bounded by the graphs of $y = x^2 + x + 1$ and $y = 4x - 1$ about the y-axis.
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2. Find the volume of the solid of revolution obtained by rotating the region bounded by the graphs of $y = 2x - x^2$ and $y = 0$ about the line y-axis
   ► Link to Solution Video ◄

3. Find the volume of the solid of revolution obtained by rotating the region bounded by the graphs of $y = 2x^2 - x^3$ and $y = 0$ about the y-axis.
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4. Find the volume of the solid of revolution obtained by rotating the region bounded by the graphs of $y = 2x^2 - x^3$ and $y = 0$ about the line $x = -4$
   ► Link to Solution Video ◄

5. Find the volume of the solid of revolution obtained by rotating the region bounded by the graphs of $y = 2x^2 - x^3$ and $y = 0$ about the line $x = 4$
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6. Find the volume of the solid of revolution obtained by rotating the region bounded by the graphs of $y = 4(x - 2)^2$ and $y = x^2 - 4x + 7$ about the y-axis.
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7. Find the volume of the solid of revolution obtained by rotating the region inside the circle $(x - 2)^2 + (y - 3)^2 = 9$ about the line $x = 10$.
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8. Find the volume of the solid of revolution obtained by rotating the region bounded by $y = 3x$, $y = 4 - x^2$ and the x-axis about the x-axis.
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5 Integration by Parts

If f and g are differentiable functions, then $(f(x) \cdot g(x))' = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

If we integrate both sides, $\int (f(x) \cdot g(x))' \, dx = \int [f(x) \cdot g'(x) + g(x) \cdot f'(x)] \, dx$

or

$$f(x) \cdot g(x) = \int [f(x) \cdot g'(x) + g(x) \cdot f'(x)] \, dx$$

If we let $u = f(x)$ and $v = g(x)$, then $du = f'(x) \, dx$ and $dv = g'(x) \, dx$ and the formula above becomes

$$u \cdot v = \int u \cdot dv + \int v \cdot du$$

or

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

1. Evaluate $\int \ln x \, dx$.

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2. Evaluate $\int \sqrt{x} \ln \sqrt[3]{x} \, dx$.

   ► Link to Solution Video ◄

3. Evaluate $\int x^3 (\ln x)^2 \, dx$.

   ► Link to Solution Video ◄

4. Evaluate $\int (\ln x)^2 \, dx$.

   ► Link to Solution Video ◄

5. Evaluate $\int \cos(\ln x) \, dx$.

   ► Link to Solution Video ◄

6. Evaluate $\int xe^{5x}\,dx$.

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7. Evaluate $\int x^2e^{-3x}\,dx$.

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8. Evaluate $\int x\cos(10x)\,dx$.

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9. Evaluate $\int x^2\sin 5x\,dx$.

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10. Evaluate $\int e^{3x}\sin(5x)\,dx$.

► Link to Solution Video ◄

11. Evaluate $\int \tan^{-1} x \, dx$.

► Link to Solution Video ◄

12. Evaluate $\int x \tan^{-1} x \, dx$.

► Link to Solution Video ◄

13. Evaluate $\int x^3 \tan^{-1} x \, dx$.

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14. Evaluate $\int \tan^{-1}(1/x) \, dx$.

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15. Evaluate $\int \sin^{-1} x \, dx$.

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16. Evaluate $\int (\sin^{-1} x)^2 \, dx$.

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17. Evaluate $\int e^{\sqrt{x}} \, dx$.

► Link to Solution Video ◄

18. Evaluate $\int \sin(\sqrt[3]{x}) \, dx$.

► Link to Solution Video ◄

19. Evaluate $\int \cos x \ln(\sin x) \, dx$.

► Link to Solution Video ◄

20. Evaluate $\int e^{\sin x}(x \cos x - \sec x \tan x) \, dx$.

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21. Evaluate $\int x^3 \cos(x^2)\, dx$.

► Link to Solution Video ◄

22. Evaluate $\int \sec^{-1}(\sqrt{x})\, dx$.

► Link to Solution Video ◄

23. Evaluate $\int x \tan^2 x\, dx$.

► Link to Solution Video ◄

24. Evaluate $\int \sec^3 x\, dx$.

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25. Evaluate $\int \sin^{-1}(\sqrt{x})\, dx$.

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6 Trigonometric Integrals

We will learn strategies for evaluating integrals of the form:

$$\int \sin^m(x)\cos^n(x)\,dx$$

$$\int \tan^m(x)\sec^n(x)\,dx$$

$$\int \sin(mx)\cos(nx)\,dx$$

$$\int \sin(mx)\sin(nx)\,dx$$

$$\int \cos(mx)\cos(nx)\,dx$$

1. Evaluate $\int \cos^2 x\,dx$

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2. Evaluate $\int \sin^2 x\,dx$

   ► Link to Solution Video ◄

3. Evaluate $\int \sec x\,dx$

   ► Link to Solution Video ◄

4. Evaluate $\int \csc x\,dx$

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5. Evaluate $\int \sec^3 x\,dx$

   ► Link to Solution Video ◄

6. Evaluate $\int \tan^3 x\,dx$

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7. Evaluate $\int \sin^3 x \cos^{10} x \, dx$

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8. Evaluate $\int \sin^2 x \cos^2 x \, dx$

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9. Evaluate $\int \tan^3 x \sec^4 x \, dx$

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10. Evaluate $\int \tan^2 x \sec x \, dx$

► Link to Solution Video ◄

11. Evaluate $\int \sin^4 x \, dx$

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12. Evaluate $\int \frac{1}{\sqrt{1+\cos 4x}} \, dx$

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13. Evaluate $\int \sin 3x \cos 7x \, dx$

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14. Evaluate $\int_0^{\pi/4} \sec x \tan^3 x \, dx$

► Link to Solution Video ◄

15. Evaluate $\int \cos^7(5x) \, dx$

► Link to Solution Video ◄

16. Evaluate $\int \frac{\sin x + \cos x}{\sin 2x} \, dx$

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17. Evaluate $\int \frac{1}{\cos x - 1}\, dx$

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18. Evaluate $\int \frac{\sin x}{\sin x + \cos x}\, dx$

► Link to Solution Video ◄

19. Evaluate $\int_0^{\pi/2} \frac{1}{1 + \tan x}\, dx$

► Link to Solution Video ◄

20. Evaluate $\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}\, dx$

► Link to Solution Video ◄

21. Evaluate $\int_0^{\pi/4} \ln(1 + \tan x)\, dx$

► Link to Solution Video ◄

22. Evaluate $\int \frac{1}{1 + \sin x}\, dx$

► Link to Solution Video ◄

23. Show that for any value of m, $\int_0^{\pi/2} \frac{1}{1 + \tan^m x}\, dx = \frac{\pi}{4}$

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7 Trigonometric Substitution

Use trigonometric substitution to evaluate integrals with expressions like $\sqrt{a^2 - x^2},\ \sqrt{a^2 + x^2},\ \sqrt{x^2 - a^2}$

For $\sqrt{a^2 - x^2}$ use $x = a \sin \theta$

For $\sqrt{a^2 + x^2}$ use $x = a \tan \theta$

For $\sqrt{x^2 - a^2}$ use $x = a \sec \theta$

1. Evaluate $\int \sqrt{1 - x^2}\, dx$

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2. Evaluate $\int \sqrt{1 + x^2}\, dx$

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3. Evaluate $\int \sqrt{x^2 - 1}\, dx$

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4. Evaluate $\int \sqrt{25 - x^2}\, dx$

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5. Evaluate $\int \sqrt{5 + x^2}\, dx$

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6. Evaluate $\int \frac{\sqrt{x^2 - 9}}{x^3}\, dx$

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7. Evaluate $\int \frac{x^2}{\sqrt{9x^2 + 4}}\, dx$

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8. Evaluate $\int \sqrt{x^2 - 2x + 17}\, dx$

► Link to Solution Video ◄

9. Evaluate $\int \sqrt{5 - 4x - x^2}\, dx$

► Link to Solution Video ◄

10. Evaluate $\int \frac{x}{\sqrt{3 - 2x - x^2}}\, dx$

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11. Evaluate $\int \frac{\sqrt{x^2 + 25}}{x^4}\, dx$

► Link to Solution Video ◄

12. Evaluate $\int \frac{dx}{\sqrt{10x - x^2}}$

► Link to Solution Video ◄

13. Evaluate $\displaystyle\int \frac{\sqrt{x^2+1}}{x}$

► Link to Solution Video ◄

14. Evaluate $\displaystyle\int \frac{x^3}{\sqrt{1-x^2}}\,dx$

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15. Evaluate $\displaystyle\int \sec^{-1} x\,dx$

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8 Method of Partial Fraction

We will integrate rational functions – a ratio of two polynomials – by expressing them as sums of simpler rational functions, that are much easier to integrate.

1. Evaluate $\int \frac{x+5}{x^2+x-2}\,dx$

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2. Evaluate $\int \frac{2x^2+1}{(x-1)(x-2)(x-3)}\,dx$

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3. Evaluate $\int \frac{x^2-4}{x^3-3x^2-x+3}\,dx$

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4. Evaluate $\int \frac{x}{x^4-13x^2+36}\,dx$

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5. Evaluate $\int \frac{x^4-4x^2+x+1}{x^2-4}\,dx$

   ► Link to Solution Video ◄

6. Evaluate $\int \frac{x+2}{x^2(x-4)}\,dx$

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7. Evaluate $\int \frac{4x^3-x^2-12x+28}{(x-2)^2(x+2)^2}\,dx$

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8. Evaluate $\int \frac{2x^2-x+4}{x^3+4x}\,dx$

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9. Evaluate $\int \frac{x^3 + x^2 + 2x + 1}{(x^2 + 1)(x^2 + 2)}\, dx$

► Link to Solution Video ◄

10. Evaluate $\int \frac{1}{(x + 4)(x^2 + 6x + 25)}\, dx$

► Link to Solution Video ◄

11. Evaluate $\int \frac{1 - x + 2x^2 - x^3}{x(x^2 + 1)^2}\, dx$

► Link to Solution Video ◄

12. Evaluate $\int \frac{1}{x^3 + 1}\, dx$

► Link to Solution Video ◄

13. Evaluate $\int \frac{x^2 - 1}{x^4 + x^2 + 1}\, dx$

► Link to Solution Video ◄

14. Evaluate $\int \sqrt{\tan x}\, dx$

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15. Evaluate $\int \ln(x^2 - x + 2)\, dx$

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9 Arc Length

If a curve has the equation $y = f(x)$ for $a \leq x \leq b$ and $f'(x)$ is continuous on $[a, b]$, then the length of the curve is

$$\int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

If a curve has the equation $x = g(y)$ for $c \leq y \leq d$ and $g'(y)$ is continuous on $[c, d]$, then the length of the curve is

$$\int_c^d \sqrt{1 + [g'(y)]^2}\, dy$$

1. Show that the circumference of a circle with radius r is $2\pi r$
   ► Link to Solution Video ◄

2. Find the length of the curve $f(x) = \frac{2}{3}(x^2 - 1)^{3/2}$ over the interval $1 \leq x \leq 3$.
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3. Find the length of the curve $f(x) = \frac{x^3}{6} + \frac{1}{2x}$ over the interval $\frac{1}{2} \leq x \leq 1$.
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4. Find the length of the curve $f(x) = \frac{x^2}{2} - \frac{\ln x}{4}$ over the interval $2 \leq x \leq 4$.
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5. Find the length of the curve $f(x) = \ln(\sec x)$ over the interval $0 \leq x \leq \pi/4$.
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6. Find the length of the curve $f(x) = \sin^{-1} x + \sqrt{1 - x^2}$ over the interval $0 \leq x \leq 1$.
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7. Find the length of the curve $f(x) = \frac{x^5}{6} + \frac{1}{10x^3}$ over the interval $1 \leq x \leq 2$.
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8. Find the length of the curve $f(x) = x^2 - \frac{\ln x}{8}$ over the interval $1 \leq x \leq 3$.
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9. Find the length of the curve $x^{2/3} + y^{2/3} = 4$ over the interval $1 \leq x \leq 8$.
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10. Find the length of the curve $f(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$ over the interval $1 \leq x \leq \sqrt{2}$.
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11. Find the length of the curve $f(x) = \ln x$ over the interval $1 \leq x \leq \sqrt{3}$.
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12. Find the length of the parabola $y = x^2$ from (0 , 0) to (1, 1).
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13. Find the length of the curve $y = e^x$ over the interval $0 \leq x \leq 1$
► Link to Solution Video ◄

14. Show that the length of a curve $y = f(x)$ for $a \leq x \leq b$ is $L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$
► Link to Solution Video ◄

10 Infinite Series Test for Divergence

If $\lim_{n\to\infty} a_n \neq 0$ or if $\lim_{n\to\infty} a_n$ does not exist, then the series $\sum_{n=1}^{\infty} a_n$ diverges

If $\lim_{n\to\infty} a_n = 0$ it does not mean that the series converges.

1. Determine if the series $\sum_{n=1}^{\infty} \frac{3n^2+2n+5}{10n^2+5n+12}$ converges or diverges.

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2. Determine if the series $\sum_{n=4}^{\infty} \ln\left(\frac{2n^3+9n+1}{5n^3+3n+15}\right)$ converges or diverges.

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3. Determine if the series $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^3+3n^2+2n+86}}{\sqrt{2n^2+9n+285}}$ converges or diverges.

   ► Link to Solution Video ◄

4. Determine if the series $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$ converges or diverges.

   ► Link to Solution Video ◄

5. Determine if the series $\sum_{n=1}^{\infty} \tan^{-1}(n)$ converges or diverges.

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6. Determine if the series $\sum_{n=1}^{\infty} \sin(n)$ converges or diverges.

   ► Link to Solution Video ◄

7. Determine if the series $\sum_{n=1}^{\infty} (-1)^n 2^{1/n}$ converges or diverges.

   ► Link to Solution Video ◄

8. Show that if $\lim_{x\to\infty} a_n \neq 0$ then $\sum_{n=1}^{\infty} a_n$ diverges

   ► Link to Solution Video ◄

11 Infinite Series Geometric Series

The geometric series $\sum_{n=0}^{\infty} ar^n = \sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$ if $-1 < r < 1$ and diverges otherwise.

1. Express **0.5555555…** as a rational number.
   ► Link to Solution Video ◄

2. Express **0.232323…** as a rational number.
   ► Link to Solution Video ◄

3. Express **0.34595959…** as a rational number.
   ► Link to Solution Video ◄

4. Express **1.4322222222…** as a rational number.
   ► Link to Solution Video ◄

5. Express **1.54828282…** as a rational number.
   ► Link to Solution Video ◄

6. Express **0.93333333**… as a rational number.
   ► Link to Solution Video ◄

7. Express **2.45123123123…** as a rational number.
   ► Link to Solution Video ◄

8. Find the sum of the series $12 + 4 + \frac{4}{3} + \frac{4}{9} + \frac{4}{27} + \cdots$
   ► Link to Solution Video ◄

9. Find the sum of the series $\frac{1}{3} - \frac{2}{9} + \frac{4}{27} - \frac{8}{81} + \cdots$
   ► Link to Solution Video ◄

10. Find the sum of the series $\frac{1}{3} - \frac{1}{5} + \frac{3}{25} - \frac{9}{125} + \cdots$
    ► Link to Solution Video ◄

11. Determine if the series $\sum_{n=0}^{\infty} 3\left(\frac{2}{5}\right)^n$ converges or diverges. If it converges, find its sum.
    ► Link to Solution Video ◄

12. Determine if the series $\sum_{n=0}^{\infty} 2\left(-\frac{3}{4}\right)^n$ converges or diverges. If it converges, find its sum.
    ► Link to Solution Video ◄

13. Determine if the series $\sum_{n=0}^{\infty} \frac{3^{n+1}}{5^{n-1}}$ converges or diverges. If it converges, find its sum.
    ► Link to Solution Video ◄

14. Determine if the series $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{4}{3^{n-1}}$ converges or diverges. If it converges, find its sum.
    ► Link to Solution Video ◄

15. Determine if the series $\sum_{n=1}^{\infty} \frac{4^n + 5^n}{9^{n-1}}$ converges or diverges. If it converges, find its sum.
    ► Link to Solution Video ◄

16. Determine if the series $\sum_{n=1}^{\infty} 5\frac{2^{n+1}}{3^n}$ converges or diverges. If it converges, find its sum.

► Link to Solution Video ◄

17. Determine if the series $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{2^{n-2}}{3^{n+1}}$ converges or diverges. If it converges, find its sum.

► Link to Solution Video ◄

18. Determine if the series $\sum_{n=2}^{\infty} (-1)^{n}\frac{3^{n-1}}{5^{n-3}}$ converges or diverges. If it converges, find its sum.

► Link to Solution Video ◄

19. Determine if the series $\sum_{n=3}^{\infty} \frac{1}{2^{2n+1}}$ converges or diverges. If it converges, find its sum.

► Link to Solution Video ◄

20. Determine if the series $\sum_{n=3}^{\infty} \frac{2^{3n-1}}{3^{2n-2}}$ converges or diverges. If it converges, find its sum.

► Link to Solution Video ◄

21. Determine if the series $\sum_{n=6}^{\infty} (-1)^{n}\frac{2^{n+3}}{3^{n}}$ converges or diverges. If it converges, find its sum.

► Link to Solution Video ◄

22. Determine if the series $\sum_{n=2}^{\infty} (-1)^{n}\frac{16^{n/2}}{3^{2n+1}}$ converges or diverges. If it converges, find its sum.

► Link to Solution Video ◄

23. Show that the series $\sum_{n=0}^{\infty} ar^n$ converges to $\frac{a}{1-r}$ if $-1 < r < 1$ and diverges otherwise.

► Link to Solution Video ◄

24. A ball is dropped from a height of 2 meters. The height after each bounce is 3/4 of the previous height. Find the total distance travelled.

► Link to Solution Video ◄

25. Show that the series $\sum_{n=1}^{\infty} \frac{n}{5^n}$ converges and find its sum.

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12 Infinite Series Telescoping Series

In most cases it is not possible to find a formula for the nth partial sum of a series. It is possible to find a formula for S_n for a telescoping series. Because of cancellation, S_n will have two or three terms.

1. Determine if the series $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ converges or diverges. If it converges, find its sum.

   ► Link to Solution Video ◄

2. Determine if the series $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$ converges or diverges. If it converges, find its sum.

   ► Link to Solution Video ◄

3. Determine if the series $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$ converges or diverges. If it converges, find its sum.

   ► Link to Solution Video ◄

4. Determine if the series $\sum_{n=1}^{\infty} \frac{4}{n^2+4n}$ converges or diverges. If it converges, find its sum

   ► Link to Solution Video ◄

5. Determine if the series $\sum_{n=1}^{\infty} \frac{2}{n^3+3n^2+2n}$ converges or diverges. If it converges, find its sum

   ► Link to Solution Video ◄

13 Infinite Series Integral Test

Suppose $f(x)$ is a continuous, positive, decreasing function on $[1, \infty)$ and $f(n) = a_n$.

Then if $\int_1^\infty f(x)\,dx$ is convergent, then $\int_{n=1}^\infty a_n$ is convergent and

$\int_1^\infty f(x)\,dx$ is divergent, then $\int_{n=1}^\infty a_n$ is divergent.

1. Determine if the series $\sum_{n=1}^{\infty} \frac{1}{n}$ converges or diverges.

   ► Link to Solution Video ◄

2. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges for $0 < p \leq 1$.

   ► Link to Solution Video ◄

3. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $1 \leq p$.

   ► Link to Solution Video ◄

4. Determine if the series $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$ converges or diverges.

   ► Link to Solution Video ◄

5. Determine if the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converges or diverges.

   ► Link to Solution Video ◄

6. Determine if the series $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$ converges or diverges.

   ► Link to Solution Video ◄

7. Determine if the series $\sum_{n=1}^{\infty} \frac{n}{e^n}$ converges or diverges.

   ► Link to Solution Video ◄

14 Limit Comparison Test

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

If $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ and $0 < L < \infty$, then either both series converge or both series diverge.

1. Determine if the series $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$ converges or diverges.

   ► Link to Solution Video ◄

2. Determine if the series $\sum_{n=1}^{\infty} \frac{3n^2+4n+5}{16n^4+9n^3+12n+14}$ converges or diverges.

   ► Link to Solution Video ◄

3. Determine if the series $\sum_{n=1}^{\infty} \sqrt{\frac{4n^3+9n^2+1}{n^5+16n^2+4}}$ converges or diverges.

   ► Link to Solution Video ◄

4. Determine if the series $\sum_{n=1}^{\infty} \frac{n^2+3n}{\sqrt{2n^5+4n^2+5}}$ converges or diverges.

► Link to Solution Video ◄

5. Determine if the series $\sum_{n=1}^{\infty} \frac{\sqrt{3n^3+4n+2}}{\sqrt[3]{5n^8+2n^2+10}}$ converges or diverges.

► Link to Solution Video ◄

6. Determine if the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ converges or diverges.

► Link to Solution Video ◄

7. Determine if the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^3}\right)$ converges or diverges.

► Link to Solution Video ◄

8. Determine if the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}+\sqrt{n+2}+\sqrt{5n+2}}$ converges or diverges.

► Link to Solution Video ◄

9. Determine if the series $\sum_{n=1}^{\infty} \frac{2^n+3^n+5^n}{5^n+9^n}$ converges or diverges.

► Link to Solution Video ◄

10. Determine if the series $\sum_{n=1}^{\infty} \tan\left(\frac{n+2}{n^2+3n+15}\right)$ converges or diverges.

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11. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2+5}$ converges by using three different tests

► Link to Solution Video ◄

12. Show that the series $\sum_{n=10}^{\infty} \frac{1}{n^2-4}$ converges by trying three different tests

► Link to Solution Video ◄

13. Determine if the series $\sum_{n=1}^{\infty} \left(5^{1/n}-1\right)$ and $\sum_{n=1}^{\infty} \left(5^{1/n^2}-1\right)$ converge or diverge

► Link to Solution Video ◄

14. Determine if the series $\sum_{n=1}^{\infty}\left(1-\cos\left(\frac{1}{n}\right)\right)$ converges or diverges

► Link to Solution Video ◄

15. Determine if the series $\sum_{n=3}^{\infty}\frac{n+\ln(n)+\sin(n+5)}{2n^2+\cos\left(n^2+3n\right)}$ converges or diverges

► Link to Solution Video ◄

16. Determine if the series $\sum_{n=5}^{\infty}\frac{\ln(n)}{n^2+n+1}$ converges or diverges

► Link to Solution Video ◄