Hybrid1: A Local Search Algorithm That Switches Between Two Heuristics

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1 Local Search Algorithms $adaptG^2WSAT_P$ and VW

The local search algorithm $adaptG^2WSAT_P$ [3, 4] flips the promising decreasing variable with the largest computed promising score if there are promising decreasing variables. It selects a variable to flip from a randomly chosen unsatisfied clause using heuristic $Novelty{+}{+}_P$ [3, 4] otherwise. This heuristic $Novelty{+}{+}_P$ is described as follows.

$Novelty{+}{+}_P(p, dp)$: With probability dp (diversification probability), flip a variable in c whose flip can falsify the least recently satisfied clause. With probability $1 - dp$, do as $Novelty$, but flip $second$ if $best$ is more recently flipped than $second$ and if $pscore(second) \geq pscore(best)$.

The local search algorithm VW [6] introduces variable weighting. This algorithm initializes the weight of a variable x, $variable\_weight[x]$, to 0 and updates and smoothes $variable\_weight[x]$ each time x is flipped, using the following equation:

$$variable\_weight[x] = (1 - s)(variable\_weight[x] + 1) + s \times t \tag{1}$$

where s is a parameter and $0 \leq s \leq 1$, and t denotes the time when x is flipped. This algorithm uses a unique variable selection rule. We call this rule the *low variable weight favoring rule*. If a randomly selected unsatisfied clause c contains freebie variables,[3] VW randomly flips one of them. Otherwise, with probability p, it flips a variable chosen randomly from c, and with probability $1 - p$, it flips a variable in c according to the low variable weight favoring rule.

Based on $adaptG^2WSAT_P$ and VW, we develop a new local search algorithm called $Hybrid1$, which can switch between $adaptG^2WSAT_P$ and VW, according to the evenness or non-evenness of the distribution of variable weights. Although several local search algorithms can switch between heuristics [1, 5, 2–4], none of the alternating or switching heuristics uses any weighting.

1.1 Algorithm $Hybrid1$

Assume that variable weights are updated using Equation 1. Assume that γ is an integer and that $\gamma > 1$. If the maximum variable weight is at least γ times as high as the average variable weight, the distribution of variable weights is considered *uneven*. Otherwise, the distribution of variable weights is considered *even*.

$Hybrid1$ is described in Fig. 1. This algorithm switches between two heuristics. When the distribution of variable weights is uneven, $Hybrid1$ chooses a variable to flip according to heuristic VW. Otherwise, it selects a variable to flip according to heuristic $adaptG^2WSAT_P$.

To distinguish these heuristics from the original algorithms $adaptG^2WSAT_P$ introduced in [3, 4] and VW introduced in [6], we call these heuristics *heuristic* $adaptG^2WSAT_P$ and *heuristic*

[3] A freebie variable is a variable with a break of 0.

Algorithm: $Hybrid1$(SAT-formula $\mathcal{F}$)

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$A$← randomly generated truth assignment;
for each variable $x$ do initialize $flip\_time[x]$ and $variable\_weight[x]$ to 0;
initialize $p$, $wp$, $max\_weight$, and $ave\_weight$ to 0;
store promising decreasing variables in stack DecVar;
for flip=1 to Maxsteps do
    if $A$ satisfies $\mathcal{F}$ then return $A$;
    if $max\_weight \geq \gamma \times ave\_weight$
    then $y$←heuristic $VW(p)$;
    else $y$←heuristic $adaptG^2WSAT_P(p, wp)$
    $A$←$A$ with $y$ flipped; adapt $p$ and $dp$;
    update $flip\_time[y]$, $variable\_weight[y]$, $max\_weight$, $ave\_weight$, and DecVar;
return Solution not found;
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Fig. 1. Algorithm $Hybrid1$

VW. In $Hybrid1$, heuristic $adaptG^2WSAT_P$ is improved in two ways, based on the preliminary $adaptG^2WSAT_P$ described in [3, 4]. The first improvement is that, when promising decreasing variables exist, heuristic $adaptG^2WSAT_P$ no longer computes the promising scores for the δ promising decreasing variables with higher scores, where δ is a parameter, but chooses the least recently flipped promising decreasing variable among all promising decreasing variables to flip. As a result, heuristic $adaptG^2WSAT_P$ no longer needs parameter δ. The second improvement is that, when there is no promising decreasing variable, heuristic $adaptG^2WSAT_P$ uses $Novelty+_P$ instead of $Novelty{+}{+}_P$ [3, 4], to select a variable to flip from a randomly chosen unsatisfied clause c. The difference between $Novelty+_P$ and $Novelty{+}{+}_P$ is that, with wp (random walk probability), $Novelty+_P$ randomly chooses a variable to flip from c, but with dp (diversification probability), $Novelty{+}{+}_P$ chooses a variable in c, whose flip will falsify the least recently satisfied clause. In $Hybrid1$, heuristic VW is the same as the algorithm VW described in [6].

2 Contest Implementation

For the SAT 2007 competition, parameter γ in $Hybrid1$ is set to 15.

References

1. E. A. Hirsch and A. Kojevnikov. UnitWalk: A New SAT Solver That Uses Local Search Guided by Unit Clause Elimination. *Ann. Math. Artif. Intell.*, 43(1):91–111, 2005.
2. C. M. Li and W. Q. Huang. Diversification and Determinism in Local Search for Satisfiability. In *Proceedings of SAT-2005*, pages 158–172. Springer, LNCS 3569, 2005.
3. C. M. Li, W. Wei, and H. Zhang. Combining Adaptive Noise and Look-Ahead in Local Search for SAT. In *Proceedings of LSCS-2006*, pages 2–16, 2006.
4. C. M. Li, W. Wei, and H. Zhang. Combining Adaptive Noise and Look-Ahead in Local Search for SAT. In F. Benhamou, N. Jussien, and B. O'Sullivan, editors, *Trends in Constraint Programming*, chapter 2. Hermes Science, 2007 (to appear).
5. X. Y. Li, M. F. M. Stallmann, and F. Brglez. A Local Search SAT Solver Using an Effective Switching Strategy and an Efficient Unit Propagation. In *Proceedings of SAT-2003*, pages 53–68. Springer, LNCS 2919, 2003.
6. S. Prestwich. Random Walk with Continuously Smoothed Variable Weights. In *Proceedings of SAT-2005*, pages 203–215. Springer, LNCS 3569, 2005.