Comparasion NN and Perceptron 1.0

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Environment: R and Jupyter notebook

Neural Network vs. Perceptron algorithms

Initial process

In [1]:

```
# import library
library(reshape2)
library(ggplot2)
```

In [2]:

```
####################### GENERAL AUXILIARY FUNCTIONS #######################
### credit: https://stat.ethz.ch/pipermail/r-help/2004-June/053343.html
list <- structure(NA,class="result")
"[<-.result" <- function(x,...,value) {
  args <- as.list(match.call())
  args <- args[-c(1:2,length(args))]
  length(value) <- length(args)
  for(i in seq(along=args)) {
    a <- args[[i]]
    if(!missing(a)) eval.parent(substitute(a <- v,list(a=a,v=value[[i]])))
  }
  x
}

# reading the data
read.data <- function(file.name, scaling=FALSE) {
  data <- read.csv(file=file.name,head=TRUE,sep=",")
  data <- data[complete.cases(data),] # removes rows with NA values
  D <- ncol(data)
  x = data[,-D]
  y = data[,D]
  if (isTRUE(scaling)) {
    x = scale(x)
    y = scale(y)
  }
  return (list('x' = x, 'y' = y))
}

error.rate <- function(Y1, T1){
  if (length(Y1)!=length(T1)){
    stop('error.rate: size of true lables and predicted labels mismatch')
  }
  return (sum(T1!=Y1)/length(T1))
}
```

I. Load Task2B_train.csv and Task2B_test.csv sets, plot the training data with classes are marked with different colors, and attach the plot to your PDF report.

In [3]:

```
# Read the datasets
set.seed(1234)           # set random seed

list[X1,T1] <- read.data('Dataset/Task2B_train.csv') # read training data
T1[T1==0] <- -1          # convert 0 labels to -1
list[X2,T2] <- read.data('Dataset/Task2B_test.csv') # read test data
T2[T2==0] <- -1          # convert 0 labels to -1
```

In [4]:

```
ggplot(data=as.data.frame(X1),
       aes(x = x1, y = x2, color = factor(T1), label = ifelse(T1==1, '+', '
    geom_text(alpha=0.7) +
    ggtitle ('Training data') +
    theme_minimal()
```

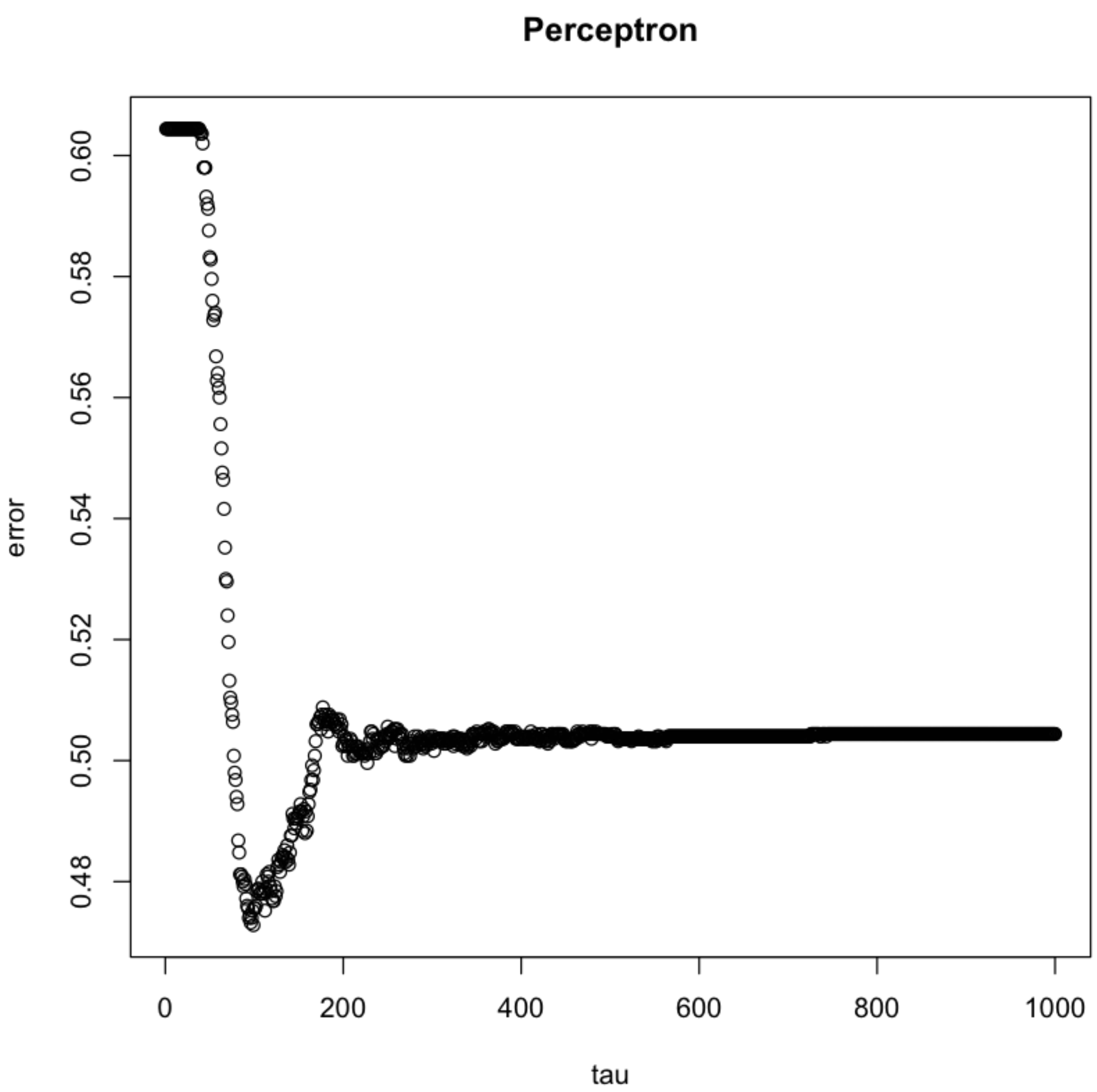
II. Train two perceptron models on the loaded training data by setting the learning rates η to .01 and .09 respectively, using a code from Activity 3.1. Calculate the test errors of two models and find the best η and its corresponding model, then plot the test data while the points are colored with their estimated class labels using the best model that you have selected; attach the plot to your PDF report.

In [5]:

```
###################### PERCEPTRON ######################
## prediction
perceptron.predict <- function(Phi, W){
  return(ifelse(Phi%*%W>=0, +1, -1))
}
## is it a misclassification? if yes, update the weight vector
is.a.miss <- function(Phi, W, T1){
  return((W%*%Phi)*T1<0)
}
## Perceptron Build function
perceptron.build <- function(X1, T1, eta=0.01, epsilon=0.001, tau.max=100,
  if (length(unique(T1))!=2){
    stop("Perceptron: The input data is not a binary classification probler
  }
  if (all(sort(unique(T1)) != c(-1, 1))) {
    stop("Perceptron: The labels are not (-1, +1).")
  }

  N1 <- nrow(X1)
  Phi <- as.matrix(cbind(1, X1)) # add a column of 1 as phi_0

  W <- matrix(NA,nrow=tau.max, ncol=ncol(Phi)) # Empty Weight vector
  W[1,] <- 2*runif(ncol(Phi))-1 # Random initial values for weight vector
  error.rec <- matrix(NA,nrow=tau.max, ncol=1) # Placeholder for errors
  error.rec[1] <- error.rate(perceptron.predict(Phi, W[1,]), T1) # record e
  tau <- 1 # iteration counter
  terminate <- FALSE # termination status

  while(!terminate){
    # resuffling train data and associated labels:
    indx <- sample(1:N1, replace = FALSE)
    Phi <- Phi[indx,]
    T1 <- T1[indx]

    for (i in 1:N1){
      if (tau >= tau.max) {break}
      # look for missclassified samples
      if (is.a.miss(Phi[i,], W[tau,], T1[i])){
        tau <- tau +1                              # update tau counter
        W[tau,] <- W[tau-1,] + eta * Phi[i,] * T1[i]  # update the weights
        error.rec[tau] <- error.rate(perceptron.predict(Phi, W[tau,]), T1)#
        eta = eta * 0.99                              # decrease eta
      }
    }

    # recalculate termination conditions
    terminate <- tau >= tau.max | (abs(error.rec[tau] - error.rec[tau-1]) <

  }
  if (plotting){
    plot(error.rec[complete.cases(error.rec),], xlab = 'tau', ylab = 'error
  }
  W <- W[complete.cases(W),]  # cut the empty part of the matrix (when the
  return(W[nrow(W),])         # return the last wight vector
}
```

learning rates η to 0.01

In [6]:
```
# Build a perceptron and plot its train error curve
W.01 <- perceptron.build(X1, T1, eta = 0.01, tau.max = 1000, plotting = TRU
```

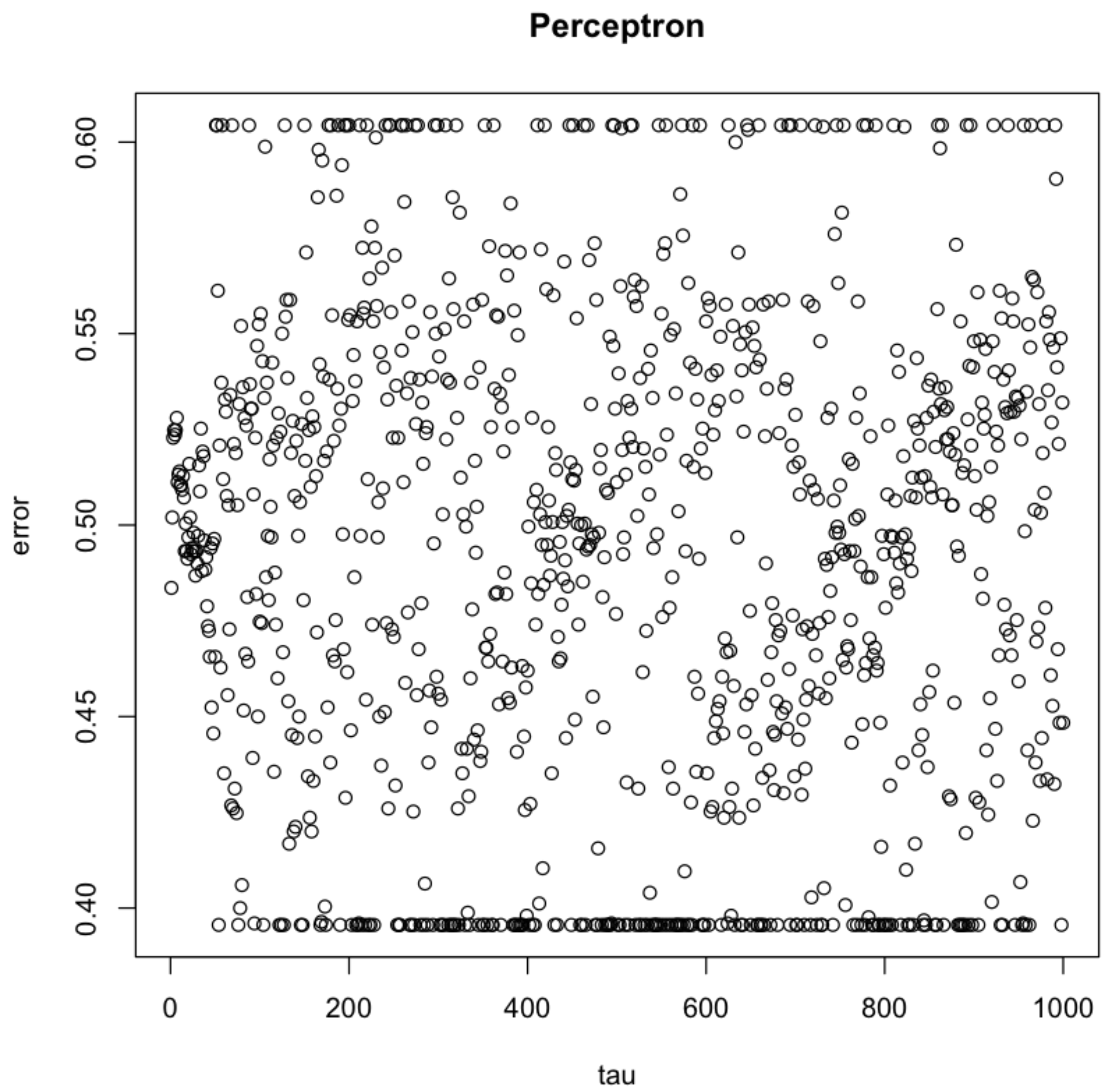


In [7]:
```
# Evaluate Perceptron
Phi.01 <- as.matrix(cbind(1, X2))
test.labels.01 <- perceptron.predict(Phi.01, W.01)
error.rate.01 = error.rate(test.labels.01, T2)
paste("Error rate = " ,error.rate.01*100 ,"%")
```

'Error rate = 51.68 %'

learning rates η to 0.09

In [8]:
```
# Build a perceptron and plot its train error curve
W.09 <- perceptron.build(X1, T1, eta = 0.09, tau.max = 1000, plotting = TRU
```



In [9]:
```
# Evaluate Perceptron
Phi.09 <- as.matrix(cbind(1, X2))
test.labels.09 <- perceptron.predict(Phi.09, W.09)
error.rate.09 = error.rate(test.labels.09,T2)
paste("Error rate = " ,error.rate.09*100 ,"%")
```

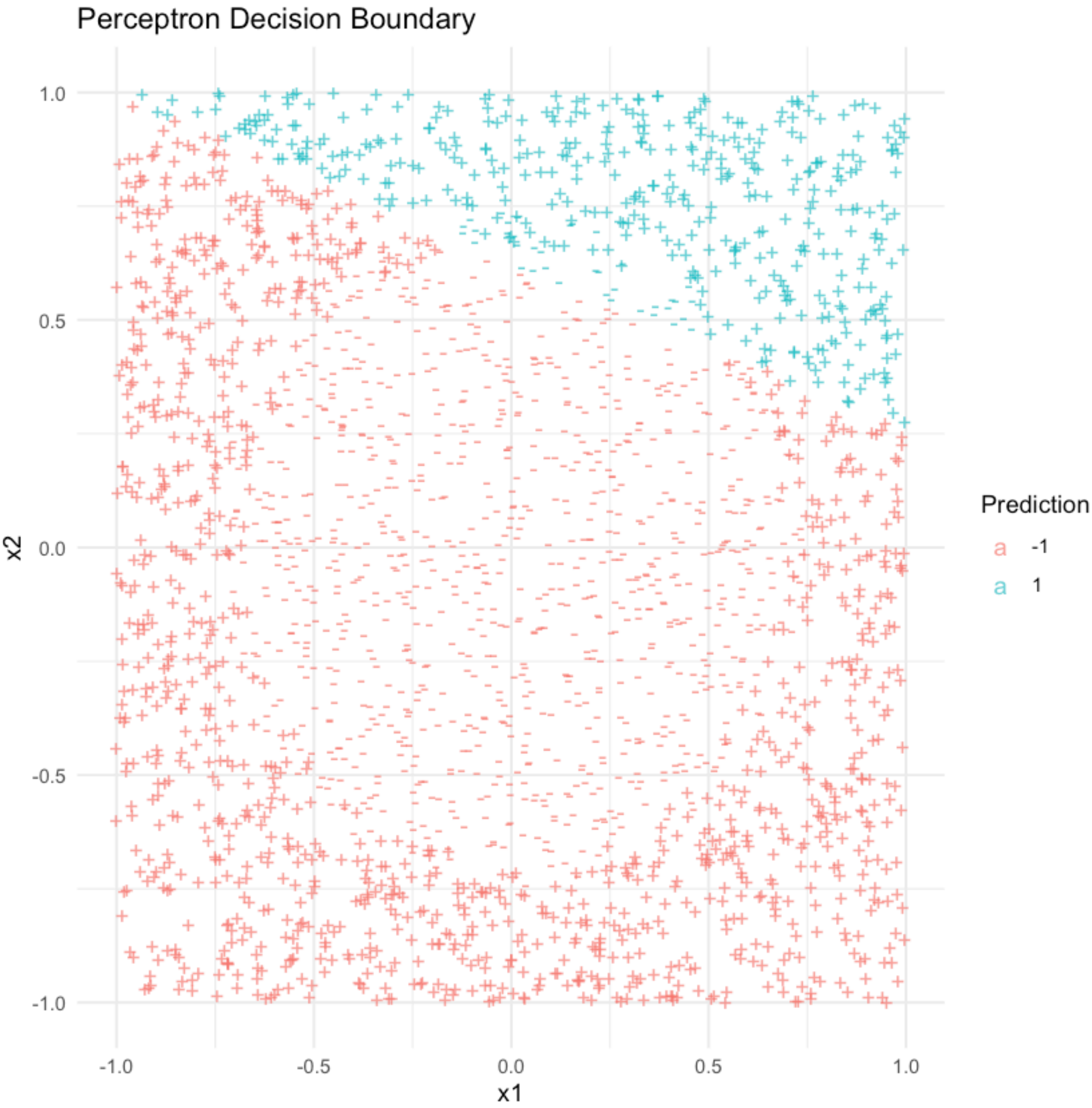
'Error rate = 46.24 %'

Find the best learning rate and plot.

In [10]:

```
# Plot Perceptron Decision Boundary
## Hint: Plot the testing data with different symbols for each class (real
## Then color each point based on its predicted label.

Phi <- 0
test.labels <- 0

# find the best learning rate
if (error.rate.01 < error.rate.09){
    print(paste('The best η is 0.01 with ', error.rate.01*100, '% error rat
    Phi <- Phi.01
    test.labels <- test.labels.01

} else {
    print(paste('The best η is 0.09 with ', error.rate.09*100, '% error rat
    Phi <- Phi.09
    test.labels <- test.labels.09
}

# plot a result
ggplot(data=as.data.frame(Phi),
       aes(x=x1, y=x2, label=ifelse(T2==1, '+', '-'),
           color=factor(test.labels))) +
geom_text(alpha=0.7) +
scale_color_discrete(guide = guide_legend(title = 'Prediction'))+
ggtitle('Perceptron Decision Boundary')+
theme_minimal()
```

```
[1] "The best η is 0.09 with 46.24% error rate."
```



III. For each combination of K (i.e, number of units in the hidden layer) in {5, 10, 15, ..., 100} and μ (learning rate) in {0.01, 0.09}, run the 3-layer Neural Network given to you in Activity 5.1 and record testing error for each of them (40 models will be developed, based on all possible combinations). Plot the error for μ 0.01 and 0.09 vs K (one line for μ 0.01 and another line for μ 0.09 in a plot). Based on this plot, find the best combination of K and μ and the corresponding model, then plot the test data while the points are colored with their estimated class labels using the best model that you have selected.

In [11]:

```
######################### NEURAL NET #########################
## the activation function (tanh here)
h <- function(z) {
  return ((exp(z)-exp(-z))/(exp(z)+exp(-z)))
}

## the derivitive of the activation function (tanh here)
h.d <- function(z) {
```

```
return (1-(h(z))^2)
}

## Class Probabilities
class.prob <- function(X, W1, W2, b1, b2){
  a2 <- h(sweep(W1 %*% X, 1, b1,'+' ))
  a3 <- h(sweep(W2 %*% a2, 1, b2,'+' ))
  return (a3)
}
## prediction
nn.predict <- function(X, W1, W2, b1, b2, threshold=0){
  return (ifelse(class.prob(X, W1, W2, b1, b2)>=threshold, 1, -1))
}
## feedforward step
feedforward <- function(Xi, Ti, W1, b1, W2, b2){
  ### 1st (input) layer
  a1 <- Xi
  y <- Ti
  ### 2nd (hidden) layer
  z2 <- W1 %*% a1 + b1
  a2 <- h(z2)
  ### 3rd (output) layer
  z3 <- W2 %*% a2 + b2
  a3 <- h(z3)
  return(list(a1, a2, a3, y, z2, z3))
}
## backpropagation step
backpropagation <- function(Ti, W2, z2, z3, a3){
  ### 3rd (output) layer
  d3 <- -(Ti-a3) * h.d(z3)
  ### 2nd (hidden) layer
  d2 <-  t(W2)%*%d3  * h.d (z2)
  return(list(d2,d3))
}
## NN build function
nn.build <- function(K, X1, T1, plotting=FALSE, epoch.max=50, eta = 0.1, la
  # initialization
  if (plotting) {error.rec <- matrix(NA,nrow=epoch.max, ncol=1)}
  D <- nrow(X1)
  if (D!=2) {stop('nn.predict: This simple version only accepts two dimensi
  N <- ncol(X1)

  W1 <- matrix(rnorm(D*K, sd=0.5), nrow=K, ncol=D)
  b1 <- matrix(rnorm(1*K), nrow=K, ncol=1)
  W2 <- matrix(rnorm(K*1, sd=0.5), nrow=1, ncol=K)
  b2 <- matrix(rnorm(1*1), nrow=1, ncol=1)

  for (epoch in 1:epoch.max){
    ## delta vectors/matrices initialization
    W1.d <- W1 *0
    b1.d <- b1 *0
    W2.d <- W2 *0
    b2.d <- b2 *0

    for (i in 1:N){
      ## Feedforward:
      list[a1, a2, a3, y, z2, z3] <- feedforward(X1[,i], T1[i], W1, b1, W2,
      ## Backpropagation:
      list[d2, d3] <- backpropagation(T1[i], W2, z2, z3, a3)
      ## calculate the delta values
      ### 1st layer
```

```
        W1.d <- W1.d + d2 %*% t(a1)
        b1.d <- b1.d + d2
        ### 2nd layer
        W2.d <- W2.d + d3 %*% t(a2)
        b2.d <- b2.d + d3
      }
      ## update weight vectors and matrices
      W1 <- W1 - eta * (W1.d/N + lambda*W1)
      b1 <- b1 - eta * (b1.d/N)
      W2 <- W2 - eta * (W2.d/N + lambda*W2)
      b2 <- b2 - eta * (b2.d/N)
      ## record the errors
      if (plotting){error.rec[epoch]<- error.rate(nn.predict(X1, W1, W2, b1,
    }
    if (plotting)
        plot(error.rec, xlab = 'epoch', ylab = 'error', main = 'Neural Net')
    return(list(W1, W2, b1, b2))
}
```

Initial varibles for NN

In [12]:

```
# Build a number of Neural Networks with different number of units in the l
X1.t <- t(as.matrix(X1))
X2.t <- t(as.matrix(X2))

epoch.max <- 1000
lambda <- 0.01
```

learning rates η to 0.01

In [13]:

```
nn.error.rate.01 = c()
i <- 1
for (k in seq(5, 100, 5)) {
    list[W1, W2, b1, b2]<- nn.build(k, X1.t, T1, plotting=FALSE, epoch.max=
    # Evaluate the model
    Y.01 <- nn.predict(X2.t, W1, W2, b1, b2)

    # Record the test errors for plotting purposes
    nn.error.rate.01[i] <- error.rate(Y.01, T2)
    i <- i + 1
}
```

In [14]:

```
nn.error.rate.01
```

1. 0.3908
2. 0.3908
3. 0.6092
4. 0.4904
5. 0.242
6. 0.3552
7. 0.1688
8. 0.1976
9. 0.2244
10. 0.3588
11. 0.178
12. 0.1476
13. 0.114
14. 0.396
15. 0.1168
16. 0.1116
17. 0.3808
18. 0.4368
19. 0.4772
20. 0.0864

learning rates η to 0.09

In [15]:

```
nn.error.rate.09 = c()
i <- 1
for (k in seq(5, 100, 5)) {
    list[W1, W2, b1, b2]<- nn.build(k, X1.t, T1, plotting=FALSE, epoch.max=
    # Evaluate the model
    Y.09 <- nn.predict(X2.t, W1, W2, b1, b2)

    # Record the test errors for plotting purposes
    nn.error.rate.09[i] <- error.rate(Y.09, T2)
    i <- i + 1
}
```

In [16]:

```
nn.error.rate.09
```

1. 0.2652
2. 0.3908
3. 0.0528
4. 0.074
5. 0.0452
6. 0.048
7. 0.0488
8. 0.0732
9. 0.0436
10. 0.0464
11. 0.0472
12. 0.0444
13. 0.0708
14. 0.2584
15. 0.126
16. 0.2464
17. 0.1768
18. 0.1236
19. 0.1272
20. 0.1888

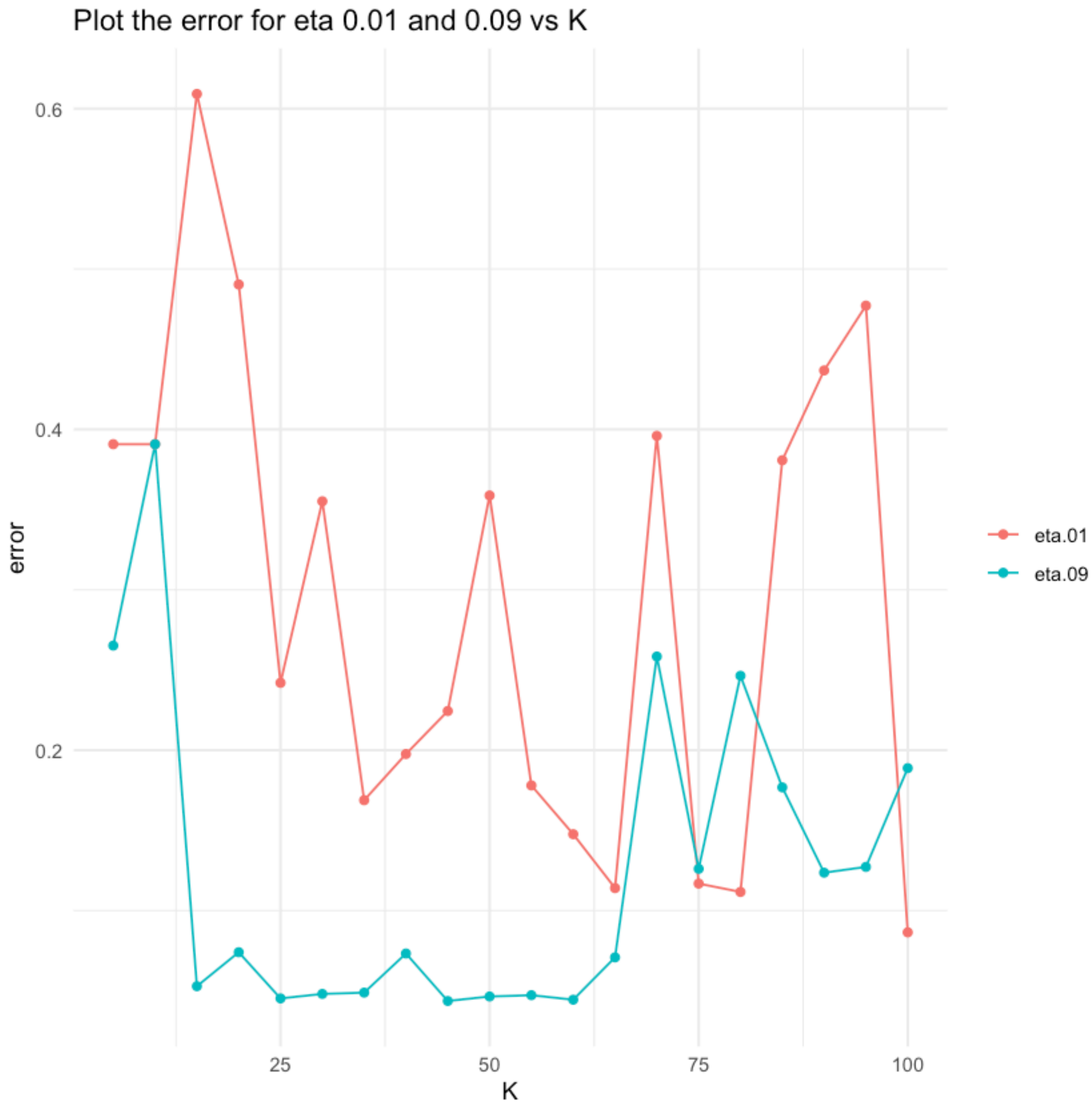
Plot the error for eta 0.01 and 0.09 vs K

In [17]:

```
# merge 2 error data
error = data.frame(t(rbind(K=seq(5, 100, 5), eta.01=nn.error.rate.01, eta.(
error.m = melt(error, id='K')
names(error.m) <- c('K', 'learning.rate', 'error')
```

In [18]:

```
# plot graph
ggplot(data=error.m, aes(x=K, y=error, color=learning.rate)) +
    geom_line() +
    geom_point()  +
    scale_color_discrete(guide = guide_legend(title = NULL)) +
    ggtitle("Plot the error for eta 0.01 and 0.09 vs K") +
    theme_minimal()
```



Find the best combination of K and μ

In [19]:
```
best.k <- error.m[which.min(error.m[,'error']),'K']
best.eta <- error.m[which.min(error.m[,'error']),'learning.rate']
best.eta <- as.double(substr(best.eta, 4, 6))

paste('The best K is ', best.k, 'and the best μ is ', best.eta)
```

'The best K is 45 and the best μ is 0.09'

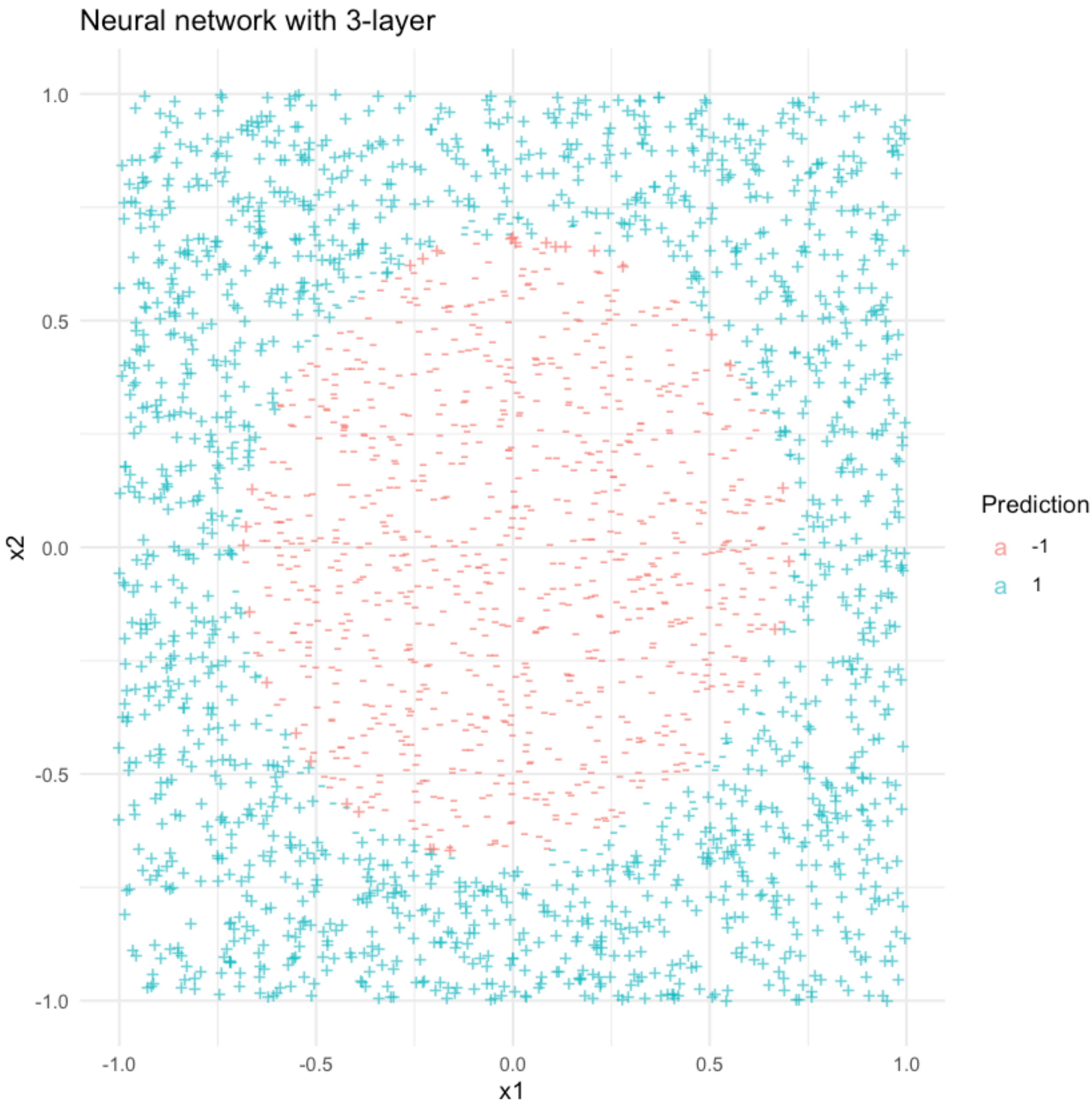
Plot the test data while the points are colored with their estimated class labels

In [20]:
```
list[W1, W2, b1, b2]<- nn.build(best.k, X1.t, T1, plotting=FALSE, epoch.max
Y <- nn.predict(X2.t, W1, W2, b1, b2)
nn.error.rate.op <- error.rate(Y, T2)
paste("Error rate = " ,nn.error.rate.op*100 ,"%")
```

'Error rate = 4.68 %'

In [21]:

```
# plot
ggplot(data=X2,
       aes(x=x1, y=x2, label=ifelse(T2==1, '+', '-'),
       color=factor(Y))) +
    geom_text(alpha=0.7) +
    scale_color_discrete(guide = guide_legend(title = 'Prediction'))+
    ggtitle('Neural network with 3-layer') +
    theme_minimal()
```



Conclusion

From the plots above, we can see that the perceptron gives a bad result with about 46% error rate since it can classify data linearly, whereas, the neural network is able to represent any kind of data due to each neuron works separately and combines an output at the end. Hence, the data in circular shape is not an issue for neural network. It can separate data correctly close to 95%, which is considerably more efficient comparing to the perceptron.