SUPPORT VECTOR MACHINES

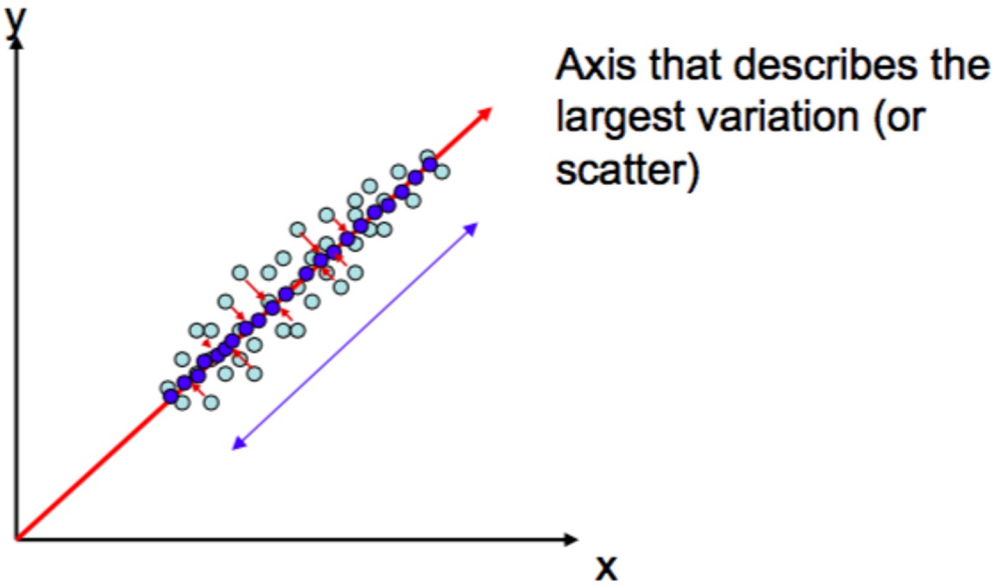
Many slides courtesy of Marios Savvides

Last time summary

- PCA
- LDA

What is PCA?

- We want to reduce the dimensionality but keep useful information
  - What is useful information? Variation
- We want to find a projection (a transformation) that describe maximum variation



Formulation

- Maximize the variance after projection ie
  - argmax $\mathrm{Var}(w^T x) = w^T \Sigma w$
- Subject to w is a unit vector
- Use Lagrangian multiplier to turn the constraint to a simple maximization
- $L(w, \lambda) = w^T \Sigma w - \lambda(w^T w - 1)$
- Take derivative with respect to w
- $\Sigma w = \lambda w$ <- eigenvector

Selecting eigenvectors

- Remember the variance of projected data is

$$\omega^T \Sigma \omega. \quad (1)$$

- And our solution yielded

$$\Sigma\omega = \lambda\omega \quad (2)$$

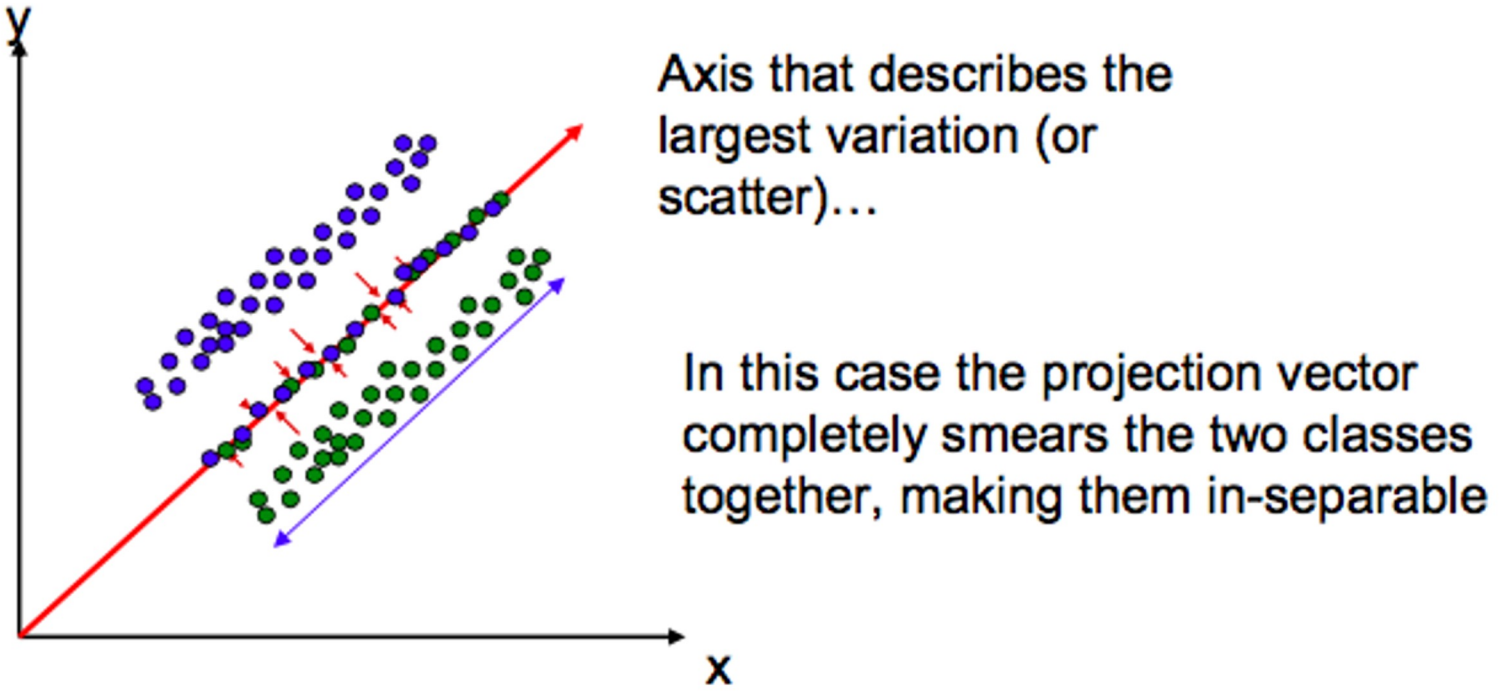
- Plug (2) in (1) and we get

$$\begin{aligned} \text{projected variance} &= \omega^T \Sigma \omega = \omega^T \lambda\omega \\ &= \lambda\, \omega^T \omega \qquad (\text{remember } \|\omega\|=1) \\ &= \lambda \end{aligned}$$

Eigenfaces

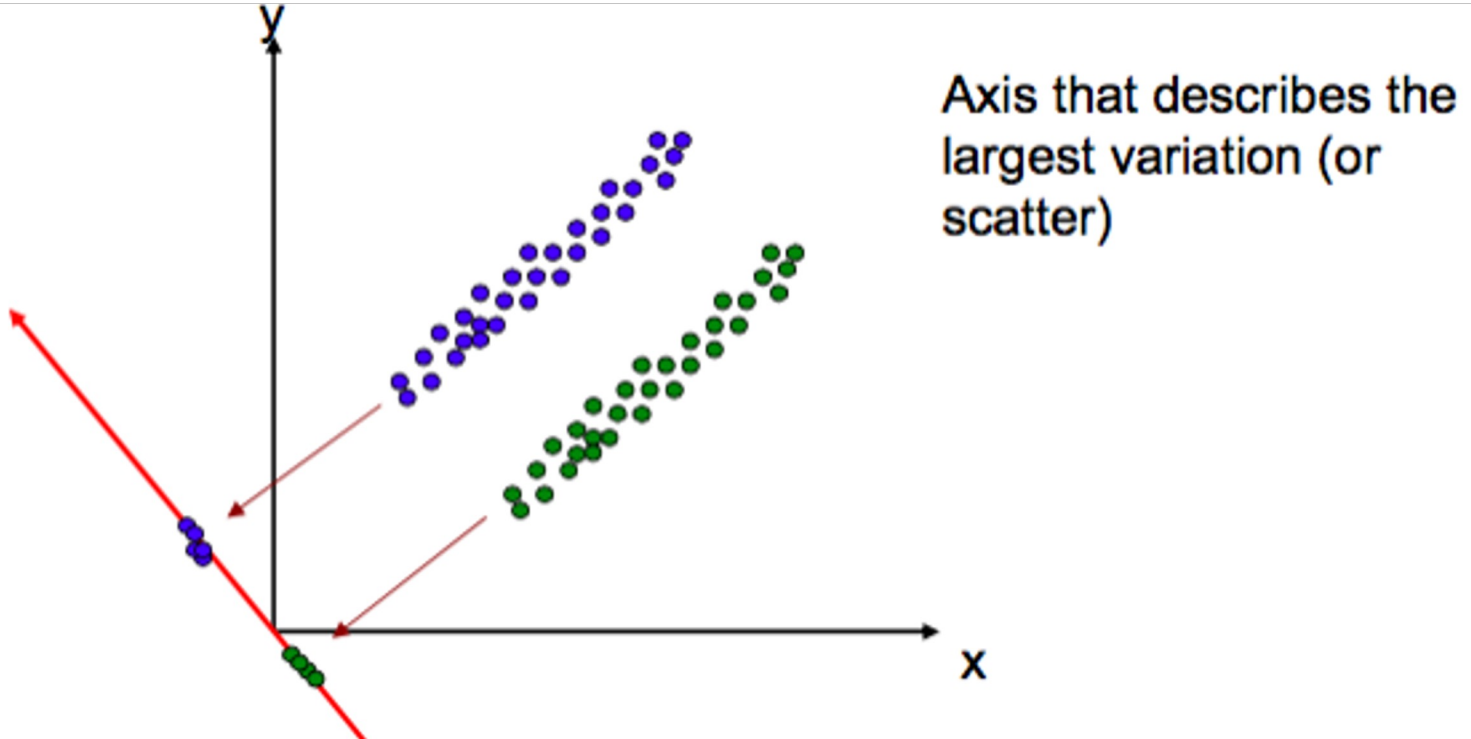
PCA for classification

- PCA does not cares about the class labels



What is LDA

- Find the projections that separate the classes.
- Assumes unimodal Gaussian model for each class
  - Maximize the distance between the means and minimize the variance of each class -> best classification performance



Fisher Linear Discriminant Criterion

- We want to maximize between class scatter
- We want to minimize within class scatter

- We have an objective function as a ratio so we can achieve both!

$$J(\mathbf{w}) = \frac{|(\tilde{\mu}_1 - \tilde{\mu}_2)|^2}{\tilde{s}_1^2 + \tilde{s}_2^2}$$

$$J(\mathbf{w}) = \frac{\mathbf{w}^T \mathbf{S}_B \mathbf{w}}{\mathbf{w}^T \mathbf{S}_W \mathbf{w}}$$

LDA solution

- If you do calculus

$$\mathbf{S}_B\mathbf{w} = \lambda\mathbf{S}_W\mathbf{w}$$

$$\mathbf{S}_W^{-1}\mathbf{S}_B\mathbf{w} = \lambda\mathbf{w}$$

If S_w is non-singular and invertible.

- Generalized eigenvalue problem. The number of solutions is min(rankS_B, rankS_W) = C-1 or N-C
- For 2 class this simplifies to
- Note this is only one projection direction

$$\mathbf{w} = \mathbf{S}_W^{-1}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)$$

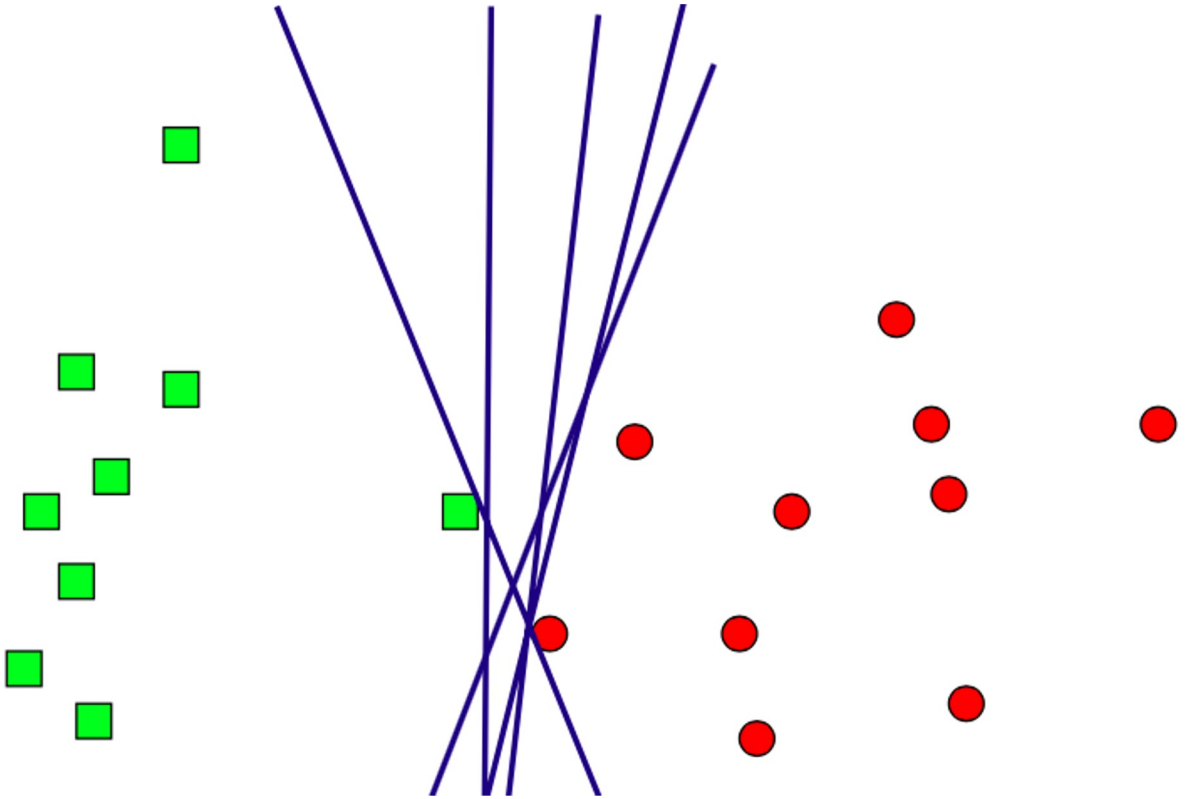
LDA+PCA

- First do PCA to reduce dimension
- Then do LDA to maximize classification ability
- How many dimensions to PCA?
  - Do PCA to keep N-C eigenvectors -> Makes S_w full rank and invertible
  - Then, do LDA and compute C-1 projections in this N-C subspace
- PCA+LDA = Fisher projection

SUPPORT VECTOR MACHINES

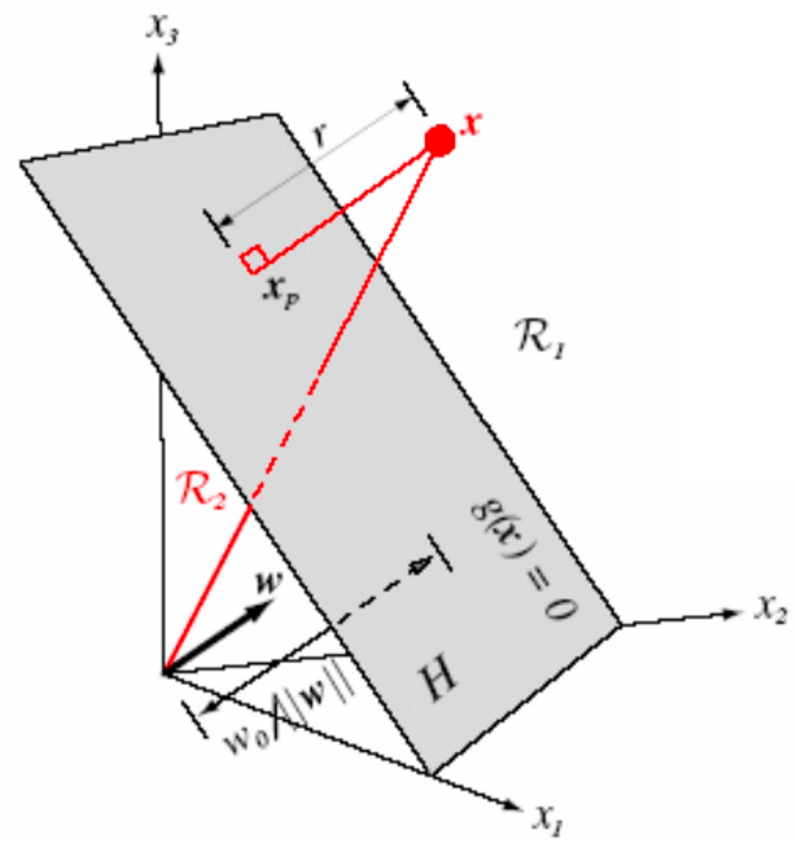
Linear classification problem

- Find a line that separates two classes
- Many solutions exist!
- Which one is the best?



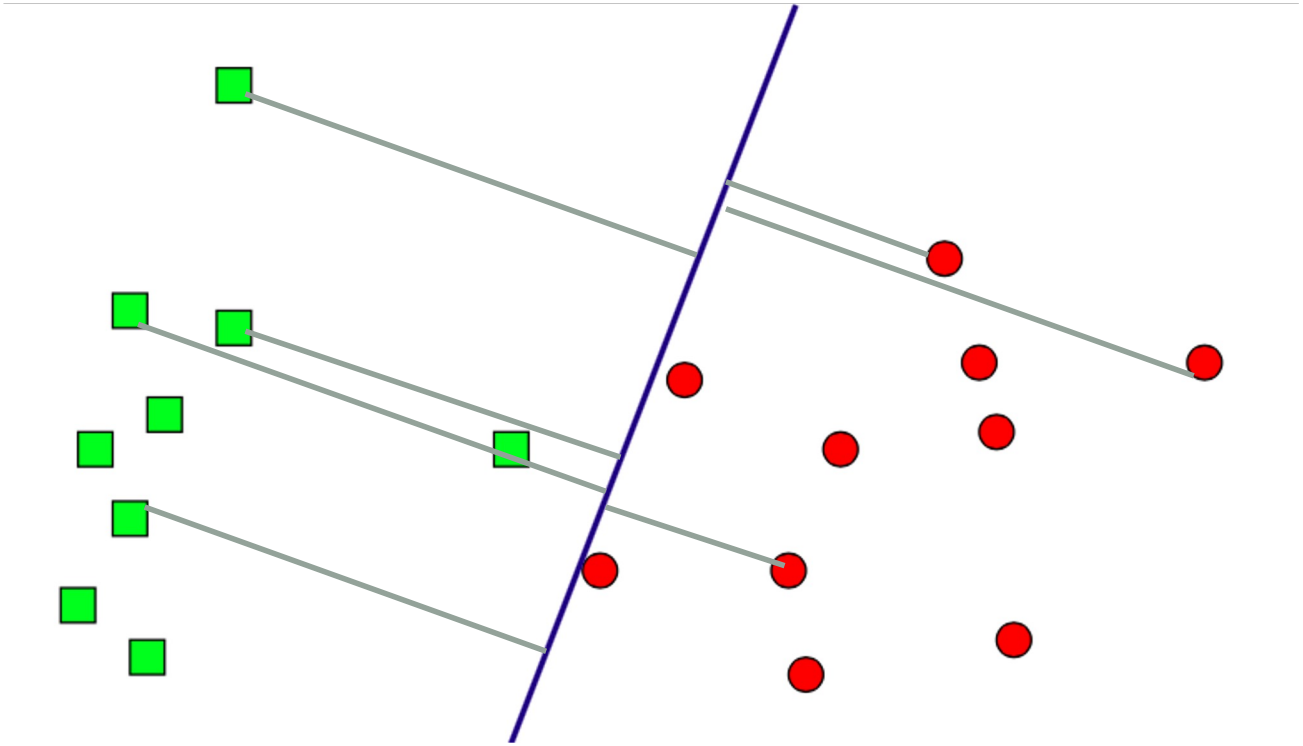
Geometric interpretation of a decision boundary

- Recall a linear classifier (without the logistic part)
  - $g(x) = \mathbf{w}^T\mathbf{x} + w_0$
- If $\mathbf{x}_1$ and $\mathbf{x}_2$ is on the decision boundary, then
  - $\mathbf{w}^T\mathbf{x}_1 + w_0 = \mathbf{w}^T\mathbf{x}_2 + w_0 = 0$
  - $w^T(\mathbf{x}_1 - \mathbf{x}_2) = 0$
    - w is normal to any vector lying in the decision boundary
    - w is normal to the decision boundary hyperplane
    - If $w_0 = 0$, the hyperplane passes through the origin
- Note we can scale w and w_0 without affecting the hyperplane

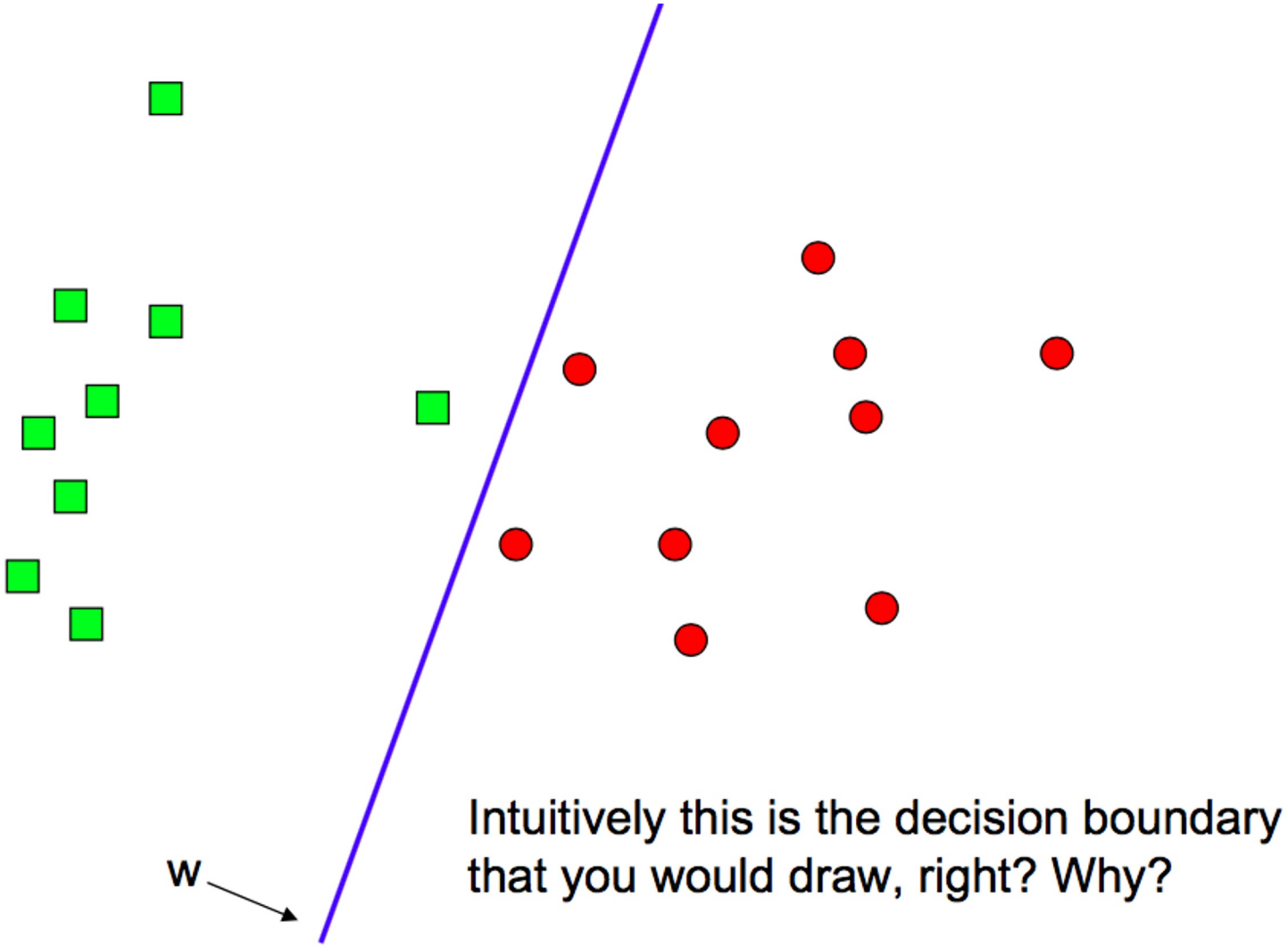


Logistic Regression

- Minimizes sum of L2 distance (square error) between all points to the line



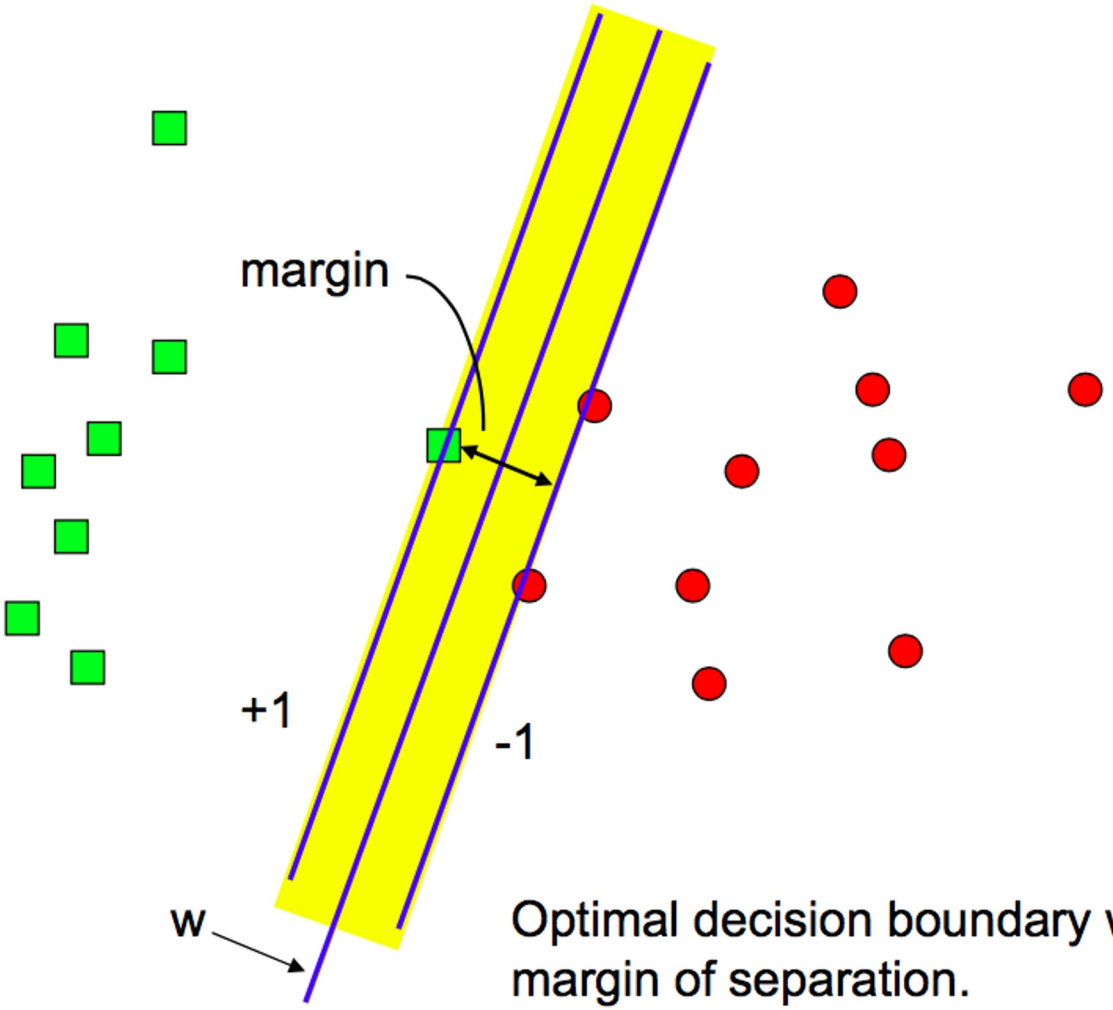
Support vectors



Support Vector Machines (SVM)

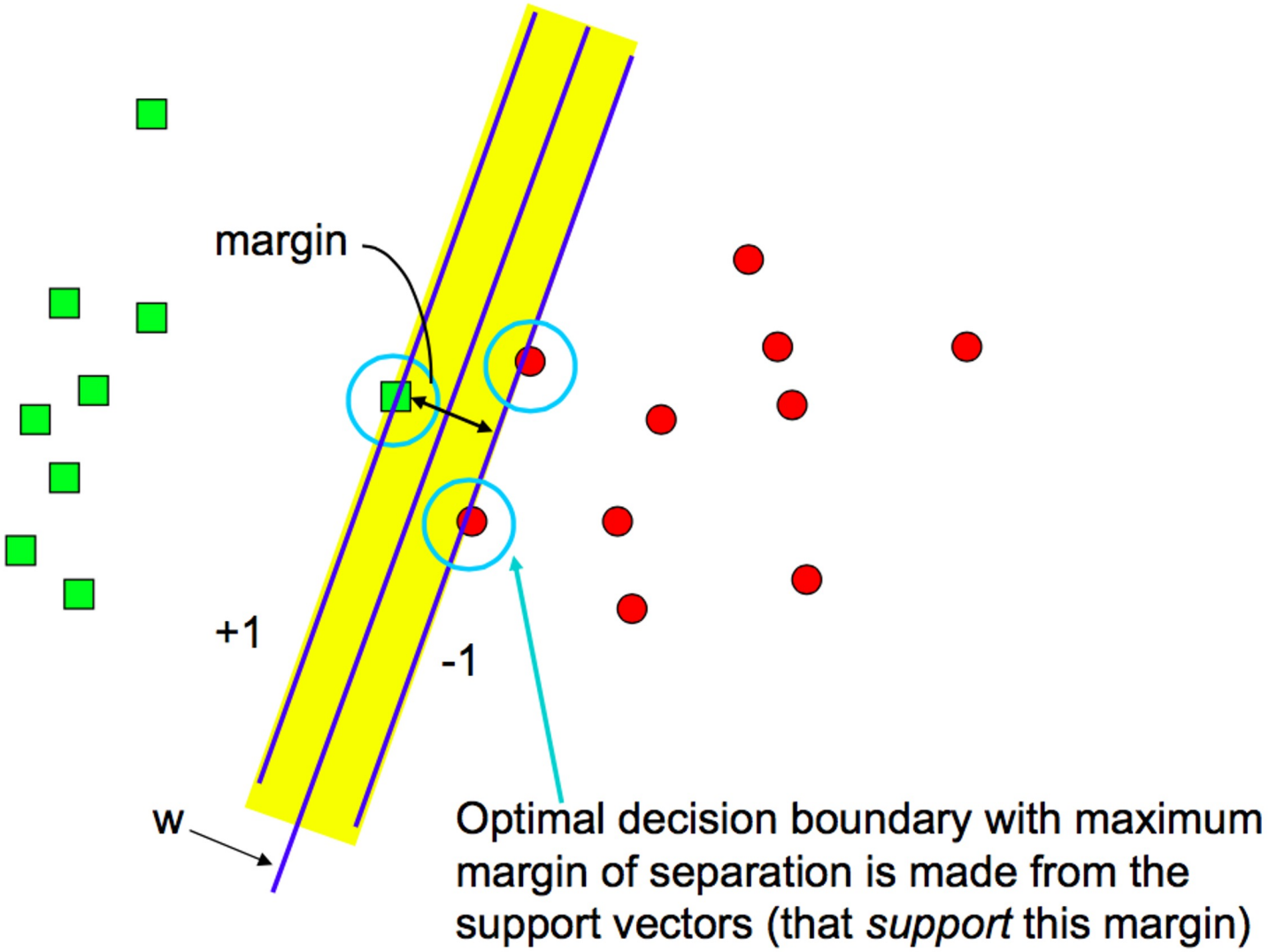
- Goal: improve generalization!
  - Care more about reducing classifier variance than reducing classifier bias
- How?
- Find the decision boundary that gives the most “slack” in classification
  - Don’t care about easy cases, care about borderline cases!
    - Focus on the margin
  - Maximize the “margin of error” between two classes

Support Vectors

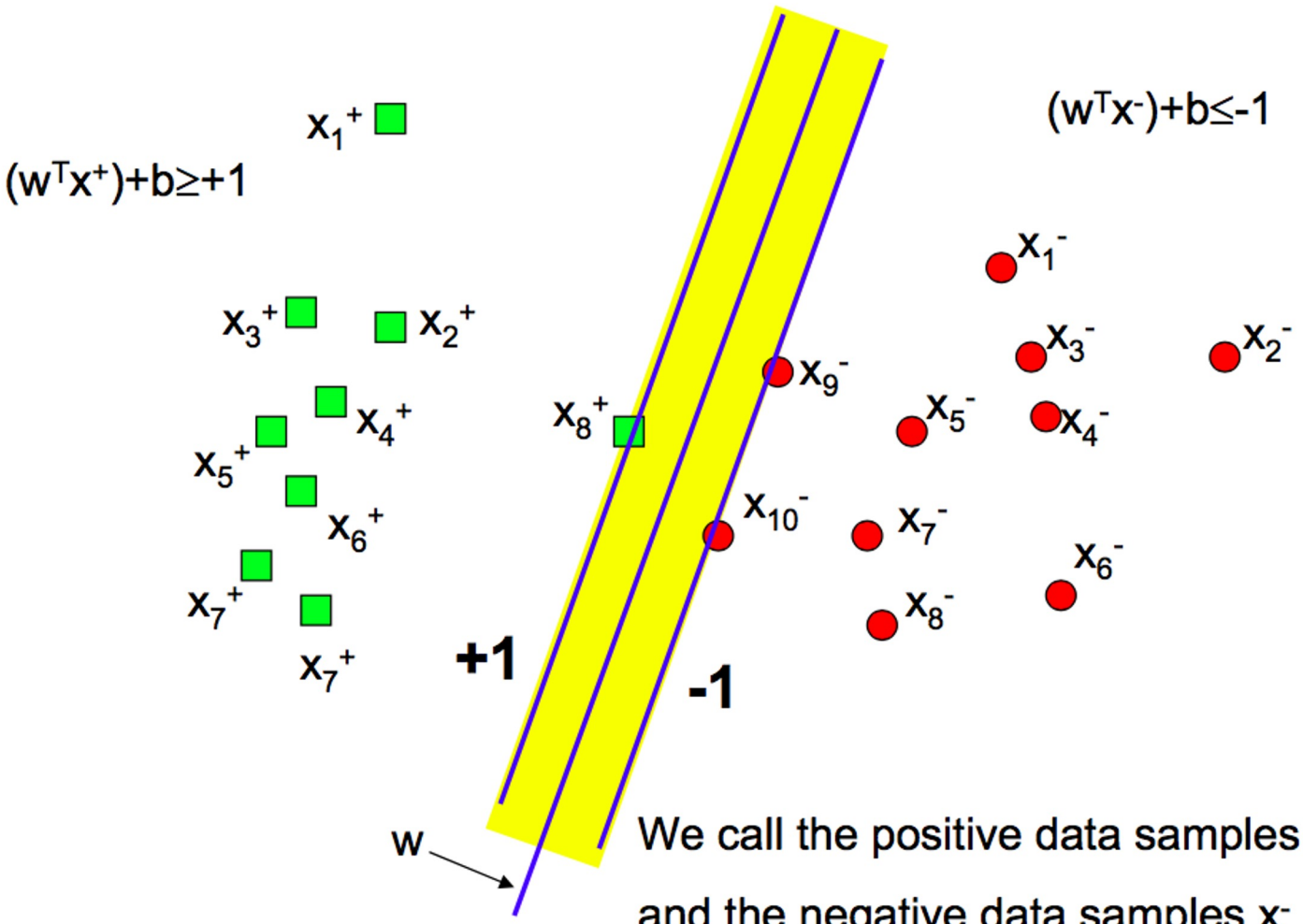


Optimal decision boundary with maximum margin of separation.

Support Vectors



Support Vectors

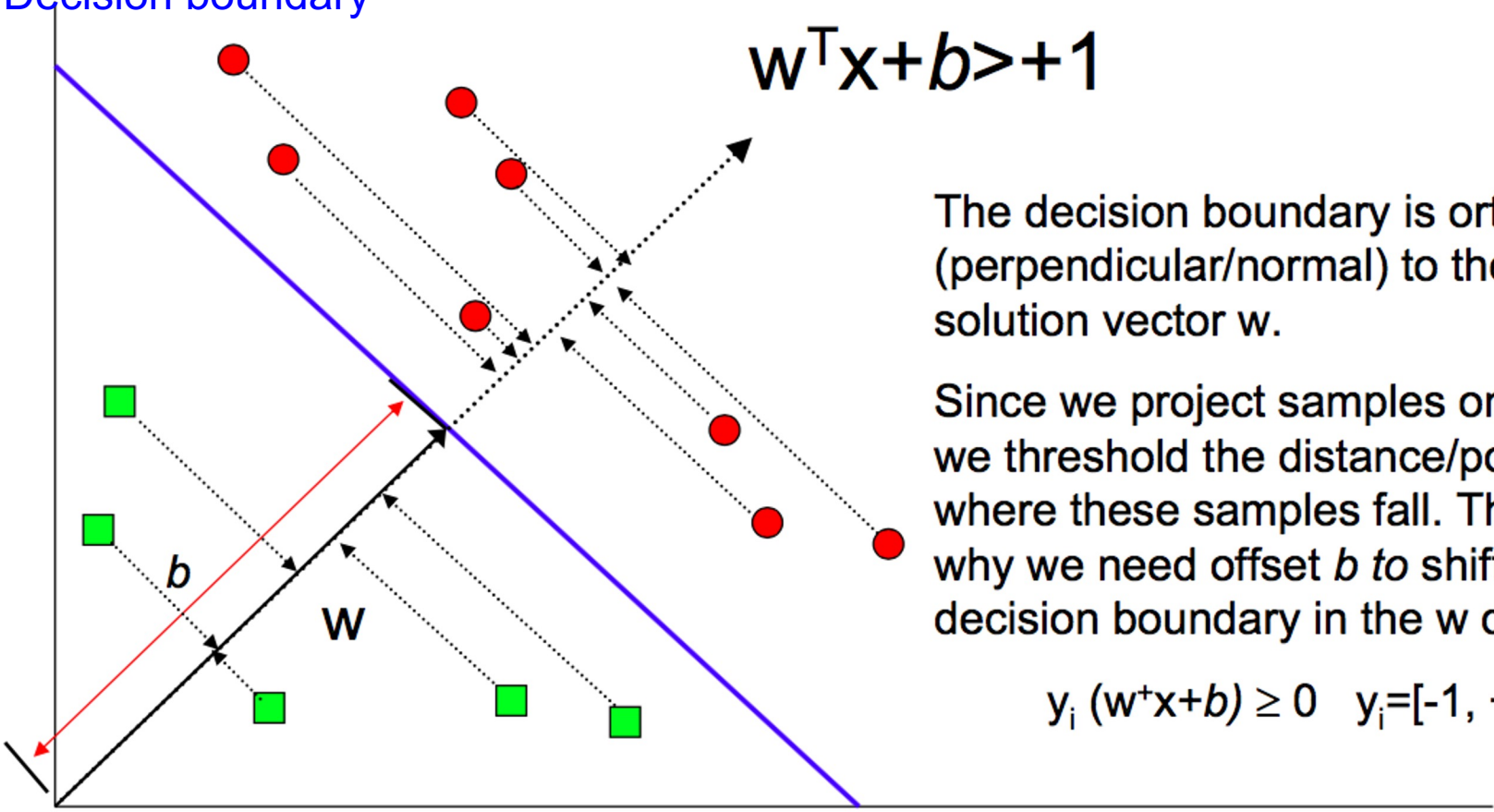


We call the positive data samples x^+ i.e. $(w^T x^+)+b \geq +1$

and the negative data samples x^- i.e. $(w^T x^-)+b \leq -1$

Geometric interpretation

Decision boundary

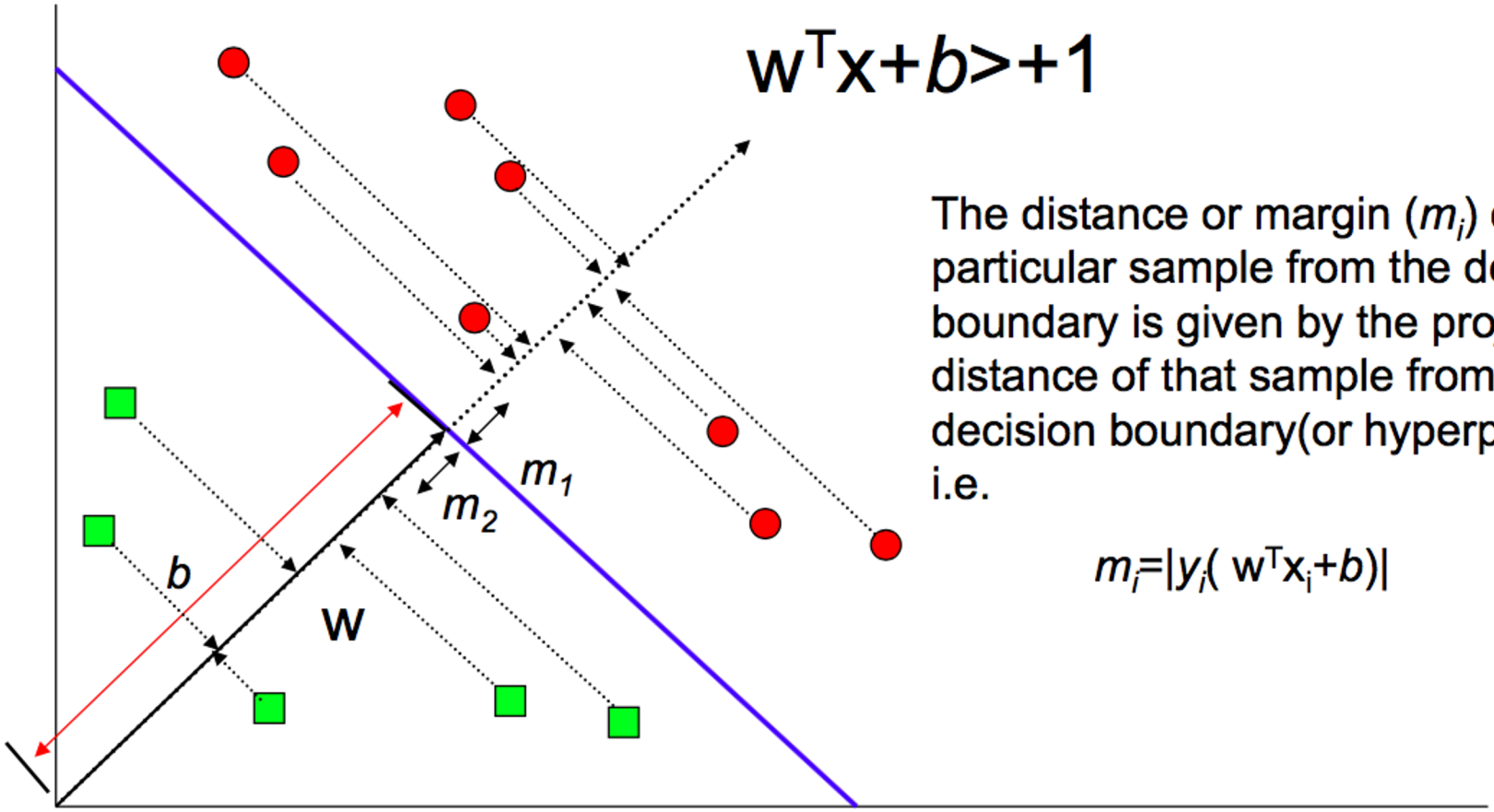


The decision boundary is orthogonal (perpendicular/normal) to the solution vector w.

Since we project samples onto w, we threshold the distance/position where these samples fall. That is why we need offset *b to* shift the decision boundary in the w direction.

$y_i (w^+x+b) \geq 0 \quad y_i=[-1, +1]$

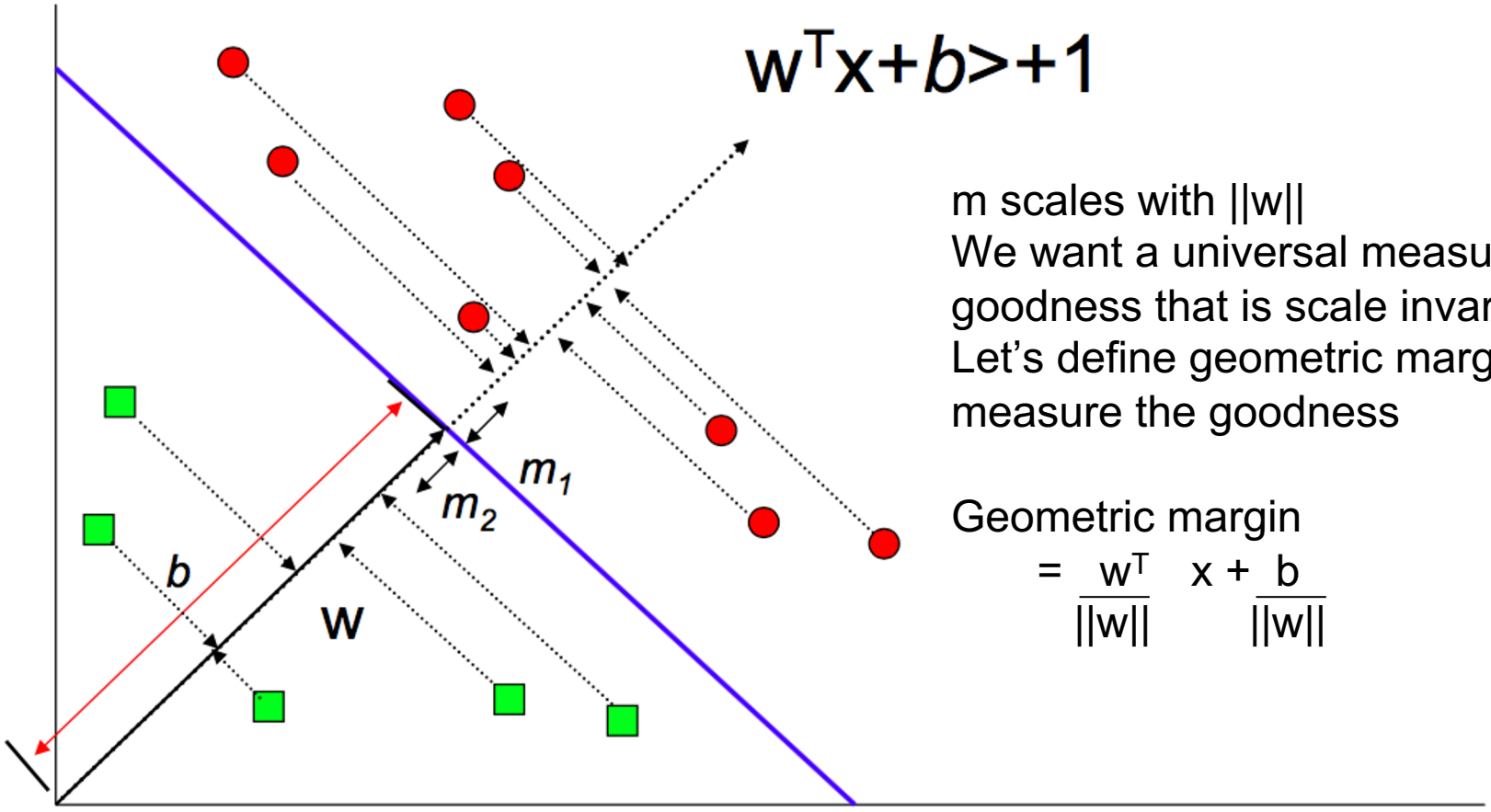
Functional Margin



The distance or margin (m_i) of a particular sample from the decision boundary is given by the projected distance of that sample from the decision boundary(or hyperplane) i.e.

$$m_i=|y_i(w^T x_i+b)|$$

Geometric Margin

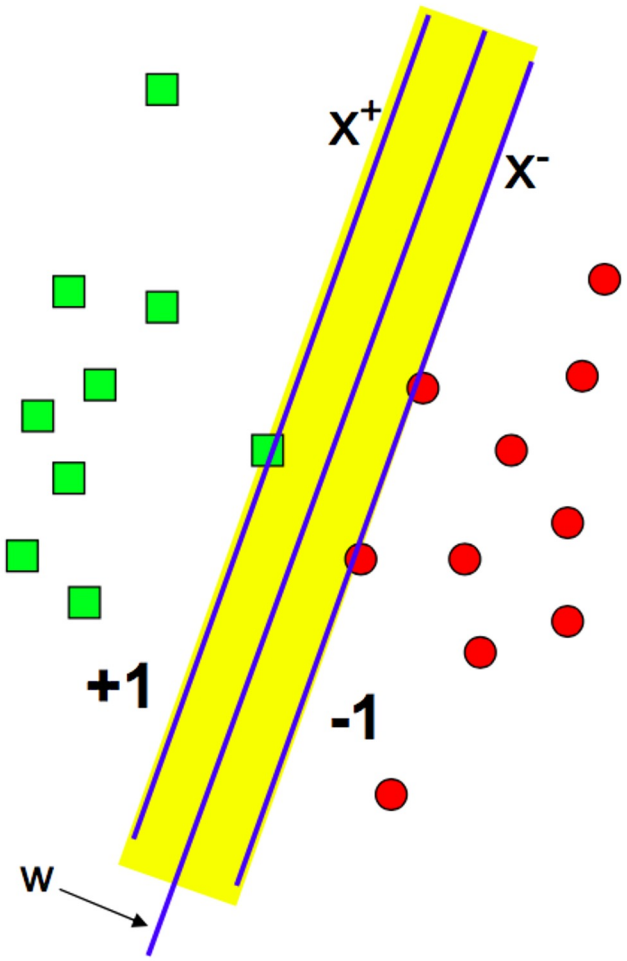


m scales with ||w||
We want a universal measure of goodness that is scale invariant.
Let's define geometric margin to measure the goodness

Geometric margin

$$= \frac{w^T}{||w||} x + \frac{b}{||w||}$$

Support Vectors



Let x^+ denote a positive point with functional margin of 1 and x^- denote a negative point respectively.

This implies:

$$w^T x^+ + b = +1$$

$$w^T x^- + b = -1$$

The difference between the two geometric margins is

$$m = \left(\left\langle \frac{\mathbf{w}}{\|\mathbf{w}\|}, \mathbf{x}^+ \right\rangle - \left\langle \frac{\mathbf{w}}{\|\mathbf{w}\|}, \mathbf{x}^- \right\rangle \right)$$

$$= \frac{1}{\|\mathbf{w}\|} \left(\langle \mathbf{w}, \mathbf{x}^+ \rangle - \langle \mathbf{w}, \mathbf{x}^- \rangle \right)$$

$$= \frac{2}{\|\mathbf{w}\|}$$

< > denotes dot product

Max margin

- We want to maximize the margin
  - Maximize $\frac{2}{\|\mathbf{w}\|}$
- Same as minimize $\langle \mathbf{w}, \mathbf{w} \rangle = \mathbf{w}^{\mathrm{T}}\mathbf{w}$

SVM objective function

- Minimize $\mathbf{w}^T\mathbf{w}$
- Subject to
  - $y_i(<\mathbf{w}, \mathbf{x}_i> + b) = y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 1$

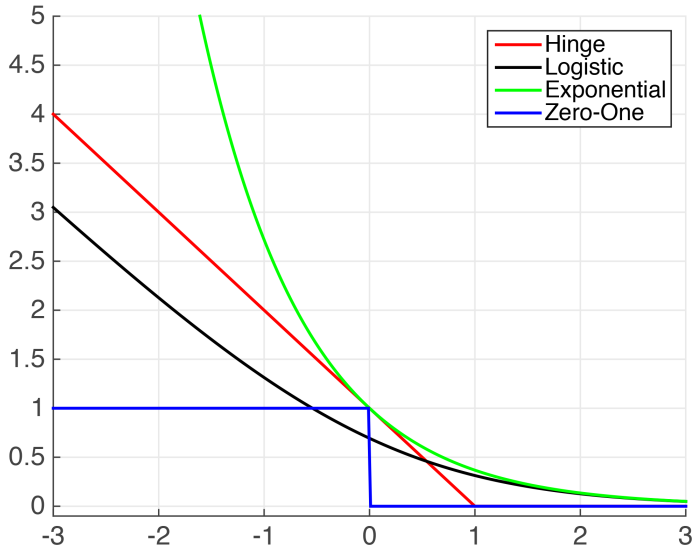
- $y_i = \{+1,-1\}$ depending on the binary class
  - Positive class must fall on the positive side of the boundary
  - Negative class must fall on the negative size

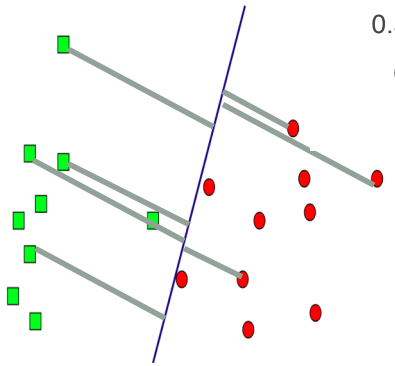
- Convex optimization (No local minimas)
- Can be solved by Quadratic Programing (QP)

Notes on the Losses

- Logistic regression optimizes for the log loss (binary cross entropy loss) https://www.analyticsvidhya.com/blog/2020/11/binary-cross-entropy-aka-log-loss-the-cost-function-used-in-logistic-regression

- SVM optimize for the hinge loss
  - $y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 1$
  - Or $0 \geq 1\text{-}y_i(\mathbf{w}^T\mathbf{x}_i + b)$
  - We don't want this inequality to be broken so our effective loss is
    - $\max(0, 1\text{-}y_i(\mathbf{w}^T\mathbf{x}_i + b))$

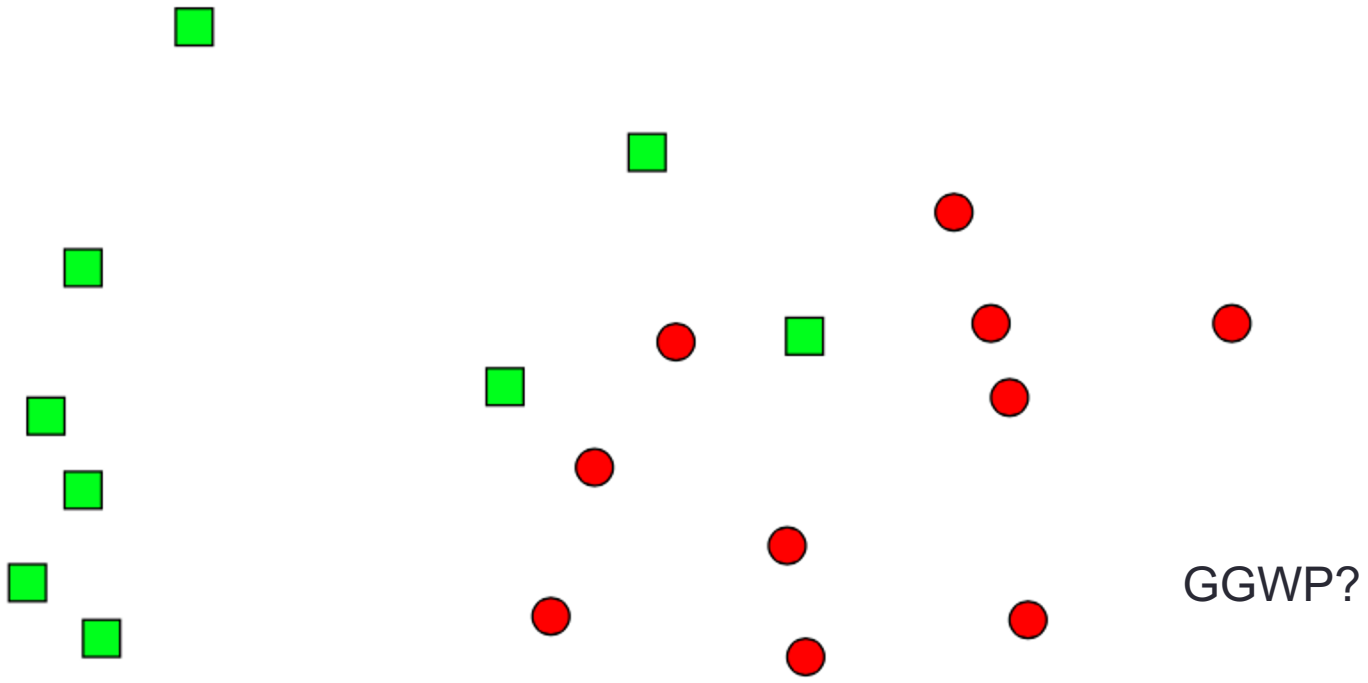




https://www.cs.cornell.edu/courses/cs4780/2017sp/lectures/lecturenote10.html

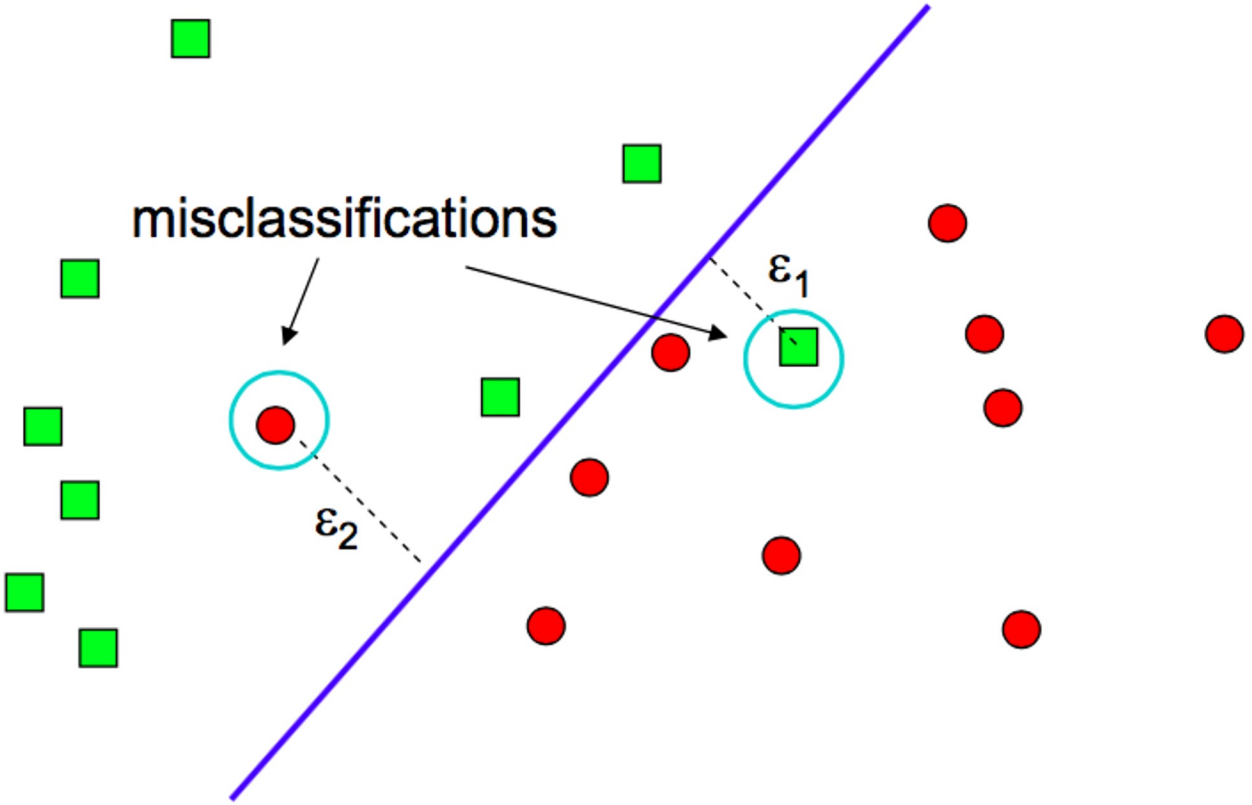
Linearly non-separable

- What happens when you cannot separate the two classes with a linear boundary



Introducing an error term ε

- Aim for a hyperplane that tries to maximize the margin while minimize total error $\Sigma\varepsilon_i$



lack variables

- We call these error terms “Slack variables”
- Give SVM some slack so that the SVM can do its job.

SVM objective function

- Minimize $\mathbf{w}^T\mathbf{w}$
- Subject to
  - $y_i(<\mathbf{w}, \mathbf{x}_i> + b) = y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 1$

- $y_i = \{+1,-1\}$ depending on the binary class
  - Positive class must fall on the positive side of the boundary
  - Negative class must fall on the negative size

SVM objective with slack

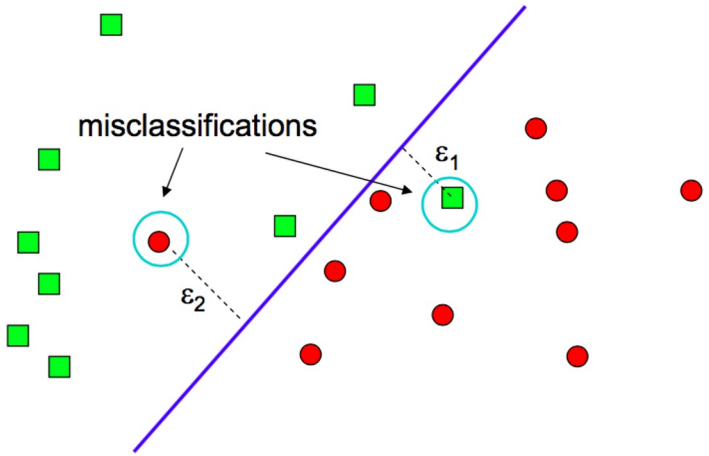
- Minimize $\mathbf{w}^T\mathbf{w} + C\Sigma\varepsilon_i$
- Subject to

C is a weight parameter, how much we care about slack

$$\mathbf{w}^T\mathbf{x}_i + b \geq 1 - \varepsilon_i \quad for \quad +ve \quad class$$

$$\mathbf{w}^T\mathbf{x}_i + b \leq -1 + \varepsilon_i \quad for \quad -ve \quad class$$

$$\varepsilon_i \geq 0 \quad \forall i$$



Notes about slacks

- Even if the problem has linear separability we might want some slack still.
  - Miss labeled points near the boundaries, noise in the data set etc.
  - In this case, we trade-off classifier bias for classifier variance.

  - A form of regularization!
  - What is regularization?

Regularization in one slide

- What?
  - Regularization is a method to lower the model variance (and thereby increasing the model bias)
- Why?
  - Gives more generalizability (lower variance)
  - Better for lower amounts of data (reduce overfitting)
- How?
  - Introducing regularizing terms in the original loss function
    - Can be anything
      - $\mathbf{w}^T\mathbf{w} + C\sigma\varepsilon_i$
      - L1 and L2 regularization
      - MAP estimate is MLE with regularization (the prior term)

Famous types of regularization

- L1 regularization: Regularizing term is a sum

$$\frac{1}{2}\Sigma_{i=1}^{m}(y_i - \theta^T \mathbf{x_i})^2 \; + \Sigma_j \mid \theta_j \mid$$

- L2 regularization: Regularizing term is a sum of squares

$$\frac{1}{2}\Sigma_{i=1}^{m}(y_i - \theta^T \mathbf{x_i})^2 \; + \Sigma_j \mid \theta_j \mid^2$$

(Not to be confused with L1 and L2 losses)

http://www.chioka.in/differences-between-l1-and-l2-as-loss-function-and-regularization/

Numerical example

Training data x = [3, 2, 1] y = 10, regression task

Objective: $10 = w_1*3 + w_2*2 + w_3*1$ Find w_1, w_2, w_3

w_1	w_2	w_3	L1 loss	L2 loss
3	0.25	0.5	3.75	9.31
5	-2	-1	8	30
3.33	0	0	3.33	11.11
2.14	1.42	0.71	4.29	7.14

L1 does feature selection (makes most numbers 0)
L2 spreads the numbers (no 0)

SVM objective with slack

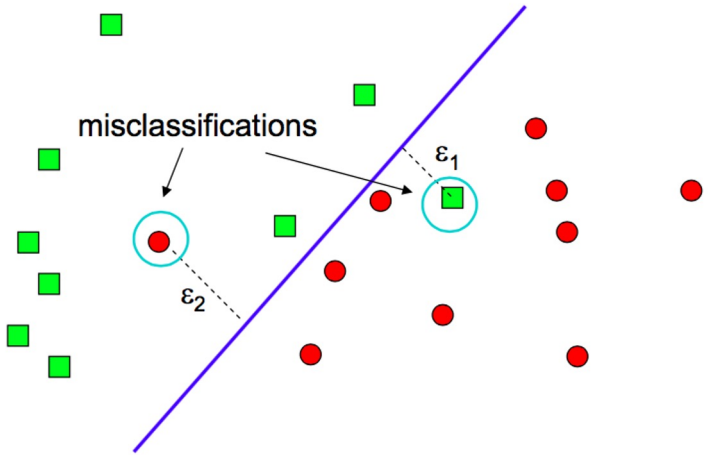
- Minimize $\mathbf{w}^T\mathbf{w} + C\Sigma\varepsilon_i$
- Subject to

C is a weight parameter, how much we care about slack

$$\mathbf{w}^T\mathbf{x}_i + b \geq 1 - \varepsilon_i \quad for \quad +ve \quad class$$

$$\mathbf{w}^T\mathbf{x}_i + b \leq -1 + \varepsilon_i \quad for \quad -ve \quad class$$

$$\varepsilon_i \geq 0 \quad \forall i$$



Primal form – Dual form

- In optimization, many problems can be framed in two ways
  - Original version: Primal form
  - Transformed version: Dual form
- Both yield the same solution (under some conditions), but sometimes solving one method is a lot easier than the other.

SVM objective function – Primal form

- Minimize $\mathbf{w}^T\mathbf{w}$
- Subject to
  - $y_i(<\mathbf{w}, \mathbf{x}_i> + b) = y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 1$

- $y_i = \{+1,-1\}$ depending on the binary class
  - Positive class must fall on the positive side of the boundary
  - Negative class must fall on the negative size

Primal Lagrangian Form

$$L(\mathbf{w}, b, \alpha) = \frac{1}{2}\mathbf{w}^{\mathbf{T}}\mathbf{w} - \sum_{i=1}^{N} \alpha_i \left[y_i(\mathbf{w}^{\mathbf{T}}\mathbf{x}_i + b) - 1 \right]$$

margin

constraints

- Where α_i are the lagrange multipliers $\alpha_i \geq 0$
- We want to optimize this function

Primal form **w**

- Primal form:

$$L(\mathbf{w},b,\alpha)=\tfrac{1}{2}\mathbf{w}^{\mathrm{T}}\mathbf{w}-\sum_{i=1}^{N}\alpha_i\left[y_i(\mathbf{w}^{\mathrm{T}}\mathbf{x}_i+b)-1\right]$$

- Differentiate with respect w to find

$$\frac{L(\mathbf{w},b,\alpha)}{\partial\mathbf{w}}=\mathbf{w}-\sum_{i=1}^{N}\alpha_i y_i\mathbf{x}_i=\mathbf{0}$$

$$\mathbf{w}=\sum_{i=1}^{N}\alpha_i y_i\mathbf{x}_i$$

Primal form b

- Primal form:

$$L(\mathbf{w},b,\alpha) = \tfrac{1}{2}\mathbf{w}^{\mathbf{T}}\mathbf{w} - \sum_{i=1}^{N}\alpha_i\left[y_i(\mathbf{w}^{\mathbf{T}}\mathbf{x}_i + b) - 1\right]$$

- Differentiate with respect b to find

$$\frac{L(\mathbf{w},b,\alpha)}{\partial b} = \sum_{i=1}^{N}\alpha_i y_i = 0$$

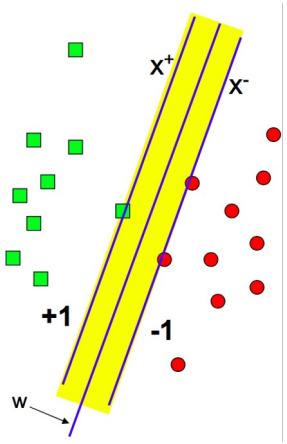
$$\sum_{i=1}^{N}\alpha_i y_i = 0$$

Primal form solution

- w is a linear combination of our training data

$$\mathbf{w} = \sum_{i=1}^{N} \alpha_i y_i \mathbf{x}_i \qquad \sum_{i=1}^{N} \alpha_i y_i = 0 \qquad \alpha_i \geq 0$$

- Which training data depends on whether that training data is a support vector (the vector on the boundary) or not



α_i will be 0 for non support vectors

Dual form

- Primal form:

$$L(\mathbf{w},b,\alpha)=\tfrac{1}{2}\mathbf{w}^{\mathbf{T}}\mathbf{w}-\sum_{i=1}^{N}\alpha_i\left[y_i(\mathbf{w}^{\mathbf{T}}\mathbf{x}_i+b)-1\right]$$

- Substitute $\mathbf{w}=\sum_{i=1}^{N}\alpha_i y_i \mathbf{x}_i$ back into above Eq.

$$L(\mathbf{w},b,\alpha)=\tfrac{1}{2}\mathbf{w}^{\mathbf{T}}\mathbf{w}-\sum_{i=1}^{N}\alpha_i\left[y_i(\mathbf{w}^{\mathbf{T}}\mathbf{x}_i+b)-1\right]$$

$$=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}y_i y_j\alpha_i\alpha_j\left\langle x_i,x_j\right\rangle-\sum_{i=1}^{N}\sum_{j=1}^{N}y_i y_j\alpha_i\alpha_j\left\langle x_i,x_j\right\rangle+\sum_{i=1}^{N}\alpha_i$$

$$=\sum_{i=1}^{N}\alpha_i-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}y_i y_j\alpha_i\alpha_j\left\langle x_i,x_j\right\rangle$$

Dual form

Dual form optimization

- Dual form:

$$\sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} y_i y_j \alpha_i \alpha_j \left\langle x_i, x_j \right\rangle$$

- Subject to the constraints $\sum_{i=1}^{N} \alpha_i y_i = \mathbf{0}$ and $\alpha_i \geq 0$

- Again this is solvable with QP.

What does the dual form give us?

- Dual form

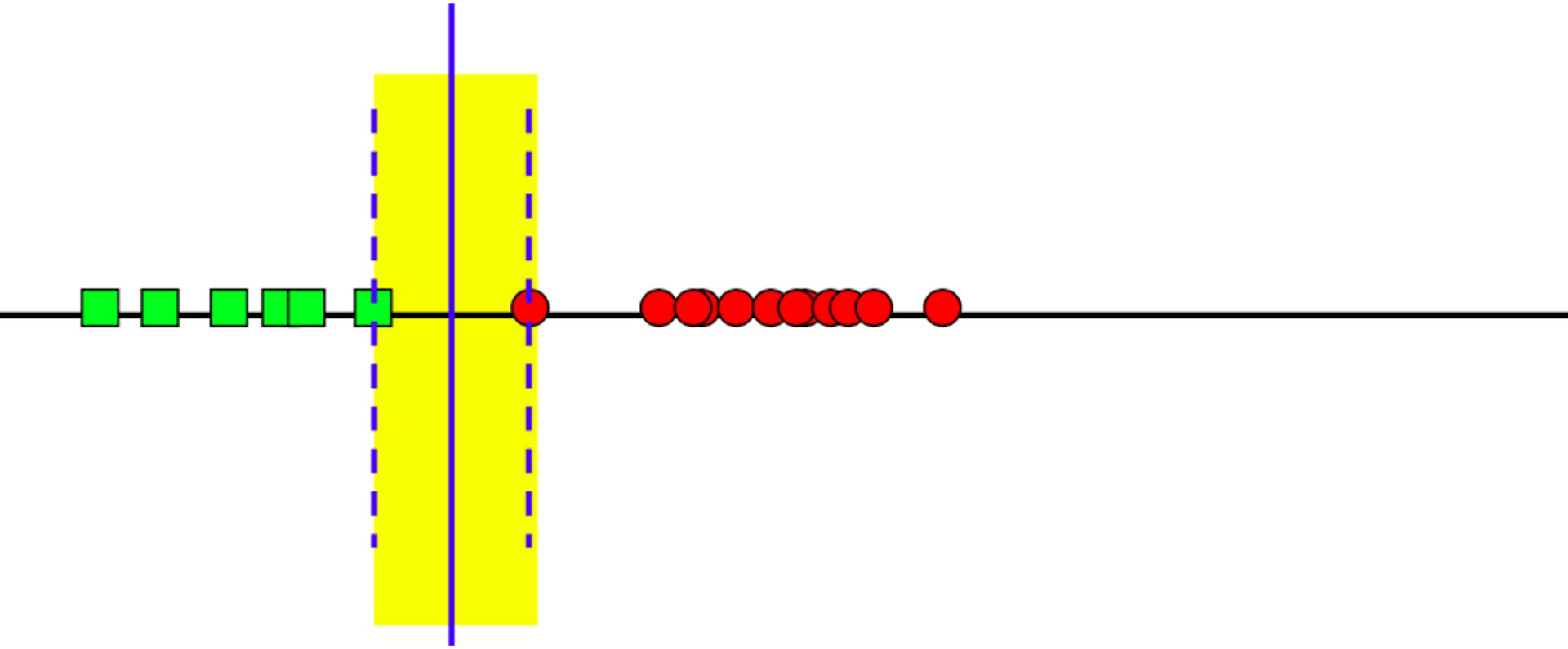
$$\sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} y_i y_j \alpha_i \alpha_j \langle x_i, x_j \rangle$$

- Optimize using pairwise inner product of inputs instead of inputs
- Gram matrix (matrix of inner product between inputs)

- How is this useful?

Example SVMs

- Easy



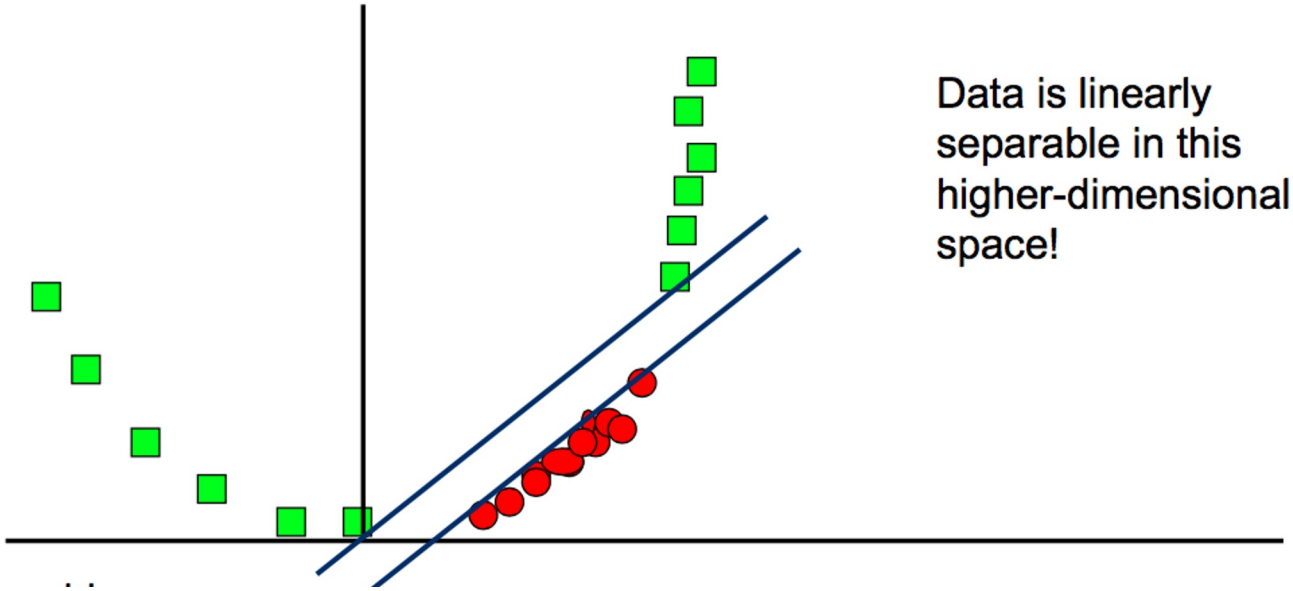
Example SVMs

- ??????

Adding features (non-linear transformation)

- Remember we add non-linear features to linear regression to do non-linear fitting
- Consider as a non-linear transformation to higher dimensional space

$F(x) \rightarrow (x, x^2)$



Data is linearly separable in this higher-dimensional space!

What about curse of dimensionality?

- Didn't we say higher dimension sucks?

- In this case our data is NOT separable in the original space, so we want to map to higher dimensions
  - Just high enough so the data is separable

- However, this will require higher compute because of dimensionality
  - Dual form will help with this!

Mapping functions

$$\phi : X \rightarrow F$$

- A mapping function that maps to higher dimensional space
- Our solutions become

$$\mathbf{w} = \sum_{i=1}^{N} \alpha_i y_i \Phi(\mathbf{x}_i)$$

- And if we want to classify a new sample

$$\mathbf{w}^{\mathrm{T}} \Phi(\mathbf{x}) = \sum_{i=1}^{N} \alpha_i y_i \left\langle \Phi(\mathbf{x}_i), \Phi(\mathbf{x}\ ) \right\rangle$$

Mapping function dual form

- In the dual form we solve

$$\sum_{i=1}^{N}\alpha_i - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} y_i y_j \alpha_i \alpha_j \left\langle \Phi(x_i), \Phi(x_j) \right\rangle$$

Inner product of the higher space

- Claim: sometimes inner product of the higher space can be solved directly without mapping to the higher space and compute the inner product

Kernel function

- We define the inner product in the mapped space as a kernel function K(x,y)

$$K(\mathbf{x},\mathbf{y}) = \langle \Phi(\mathbf{x}), \Phi(\mathbf{y}\ ) \ \rangle$$

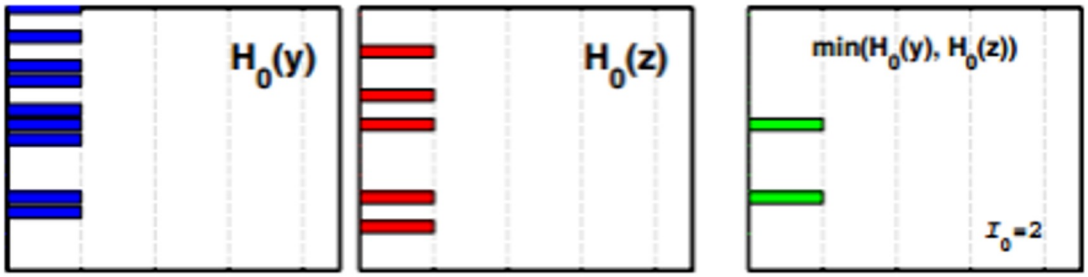
- Kernel of x -> (x, x^2)
  - $xy + x^2y^2$
- Kernel of x -> (x, x^2,x^3)
  - $xy + x^2y^2+x^3y^3$

Kernel functions

- Sometimes we don't even know what the mapping function is, but we "dream up" a kernel
  - A kernel is legitimate if it satisfies "Mercer's Condition"
  - Mercer's condition guarantees existence of a higher dimensional space that yields the dot product, but we just don't know what space

Histogram intersection kernels

- Given input features which are histograms
  - Histogram of first data $H_0(y)$. Histogram of second data $H_1(z)$
- The Kernel that counts the intersection of the histograms is a valid kernel.
  - E.g. Sum of $\min(H_0(y), H_1(z))$ for all histogram bins
- (One of the most used kernels in computer vision)



Radial Basis Kernels

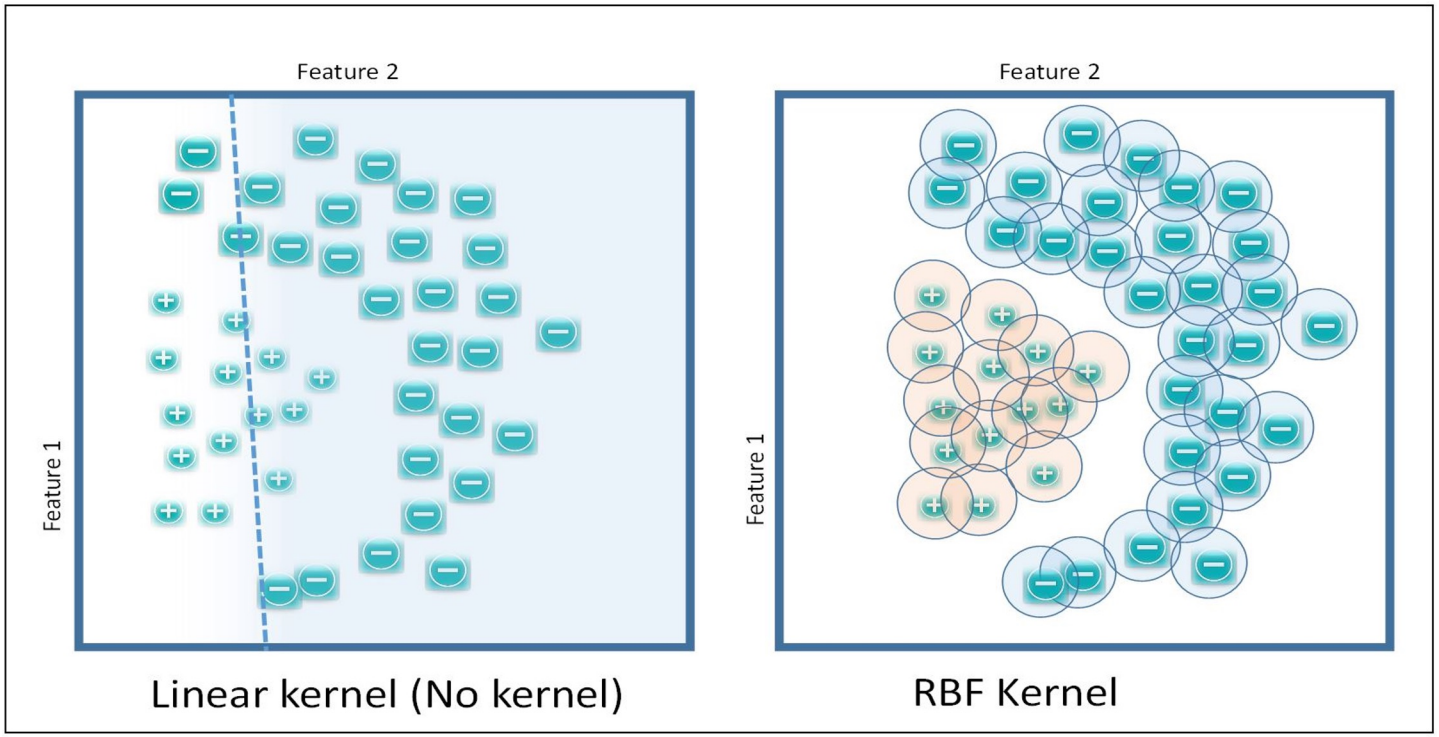
- Most powerful general-purpose kernel

$$K(\mathbf{x}, \mathbf{x}') = \exp\left(-\frac{\|\mathbf{x} - \mathbf{x}'\|^2}{2\sigma^2}\right)$$

- Pretty much a Gaussian with mean x' and variance σ^2
  - Variance is a parameter to select
- This kernel comes from a space that has infinite dimensions

RBF kernels

- Think of RBF as putting Gaussians onto the support vectors



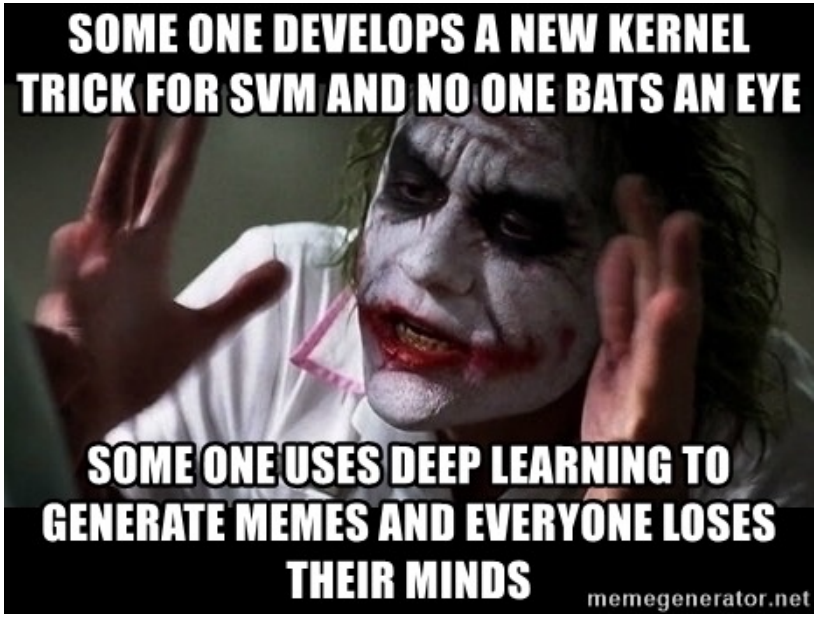
https://www.intechopen.com/books/epilepsy-topics/nonlinear-epilepsy-forewarning-by-support-vector-machines

Design your own kernel

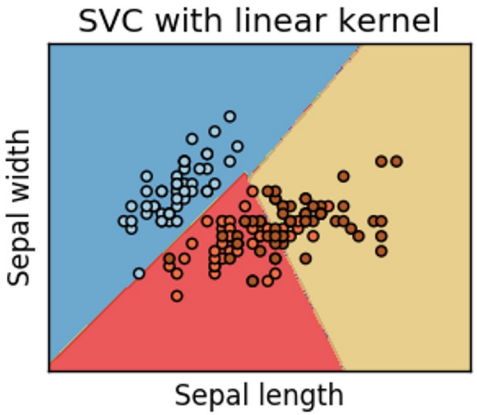
- A kernel is valid if (Mercer's condition)
  - It's symmetric K(x,y) = K(y,x)
  - The matrix of K where $K_{ij} = K(x_i,x_j)$ is positive definite (for any x_i, x_j)

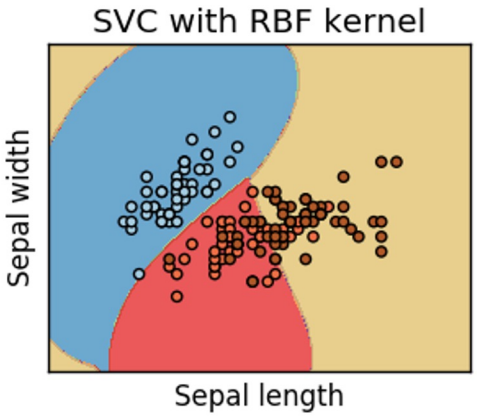
- Build from existing kernels
  - If K1 K2 are valid kernels
    - K = aK1 + bK2
    - K = K1*K2
    - K = K1^(K2)

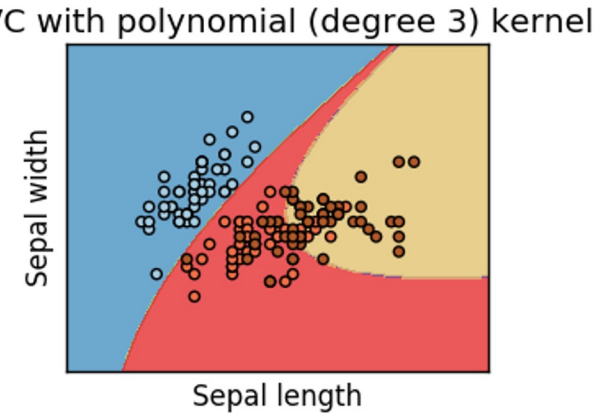
    are valid kernels



SVM examples





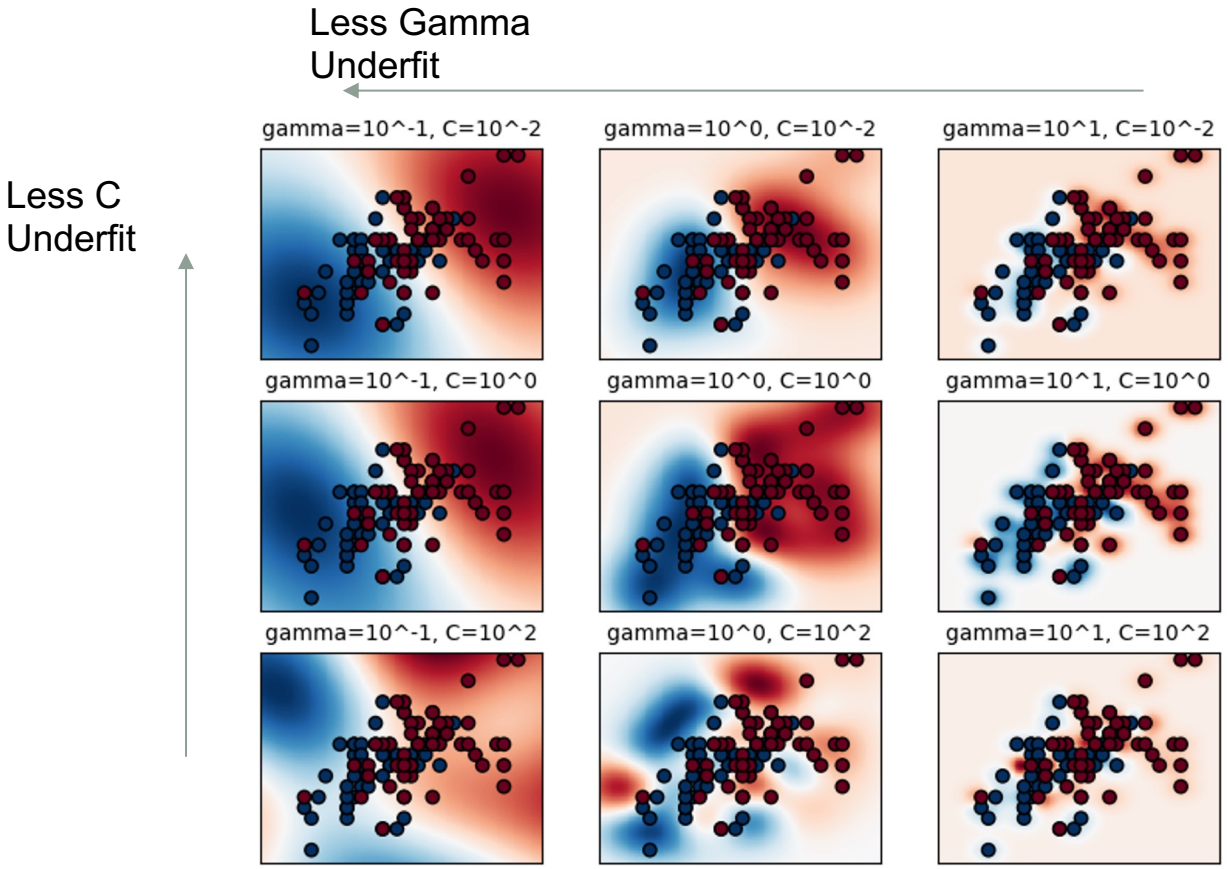


Think of SVM with RBF as K-NN with “support vectors”

http://lijiancheng0614.github.io/scikit-learn/modules/svm.html

RBF SVM and sci-kit learn

- Gamma is the inverse of the variance
- C is the inverse slack variable weight



http://scikit-learn.org/stable/auto_examples/svm/plot_rbf_parameters.html

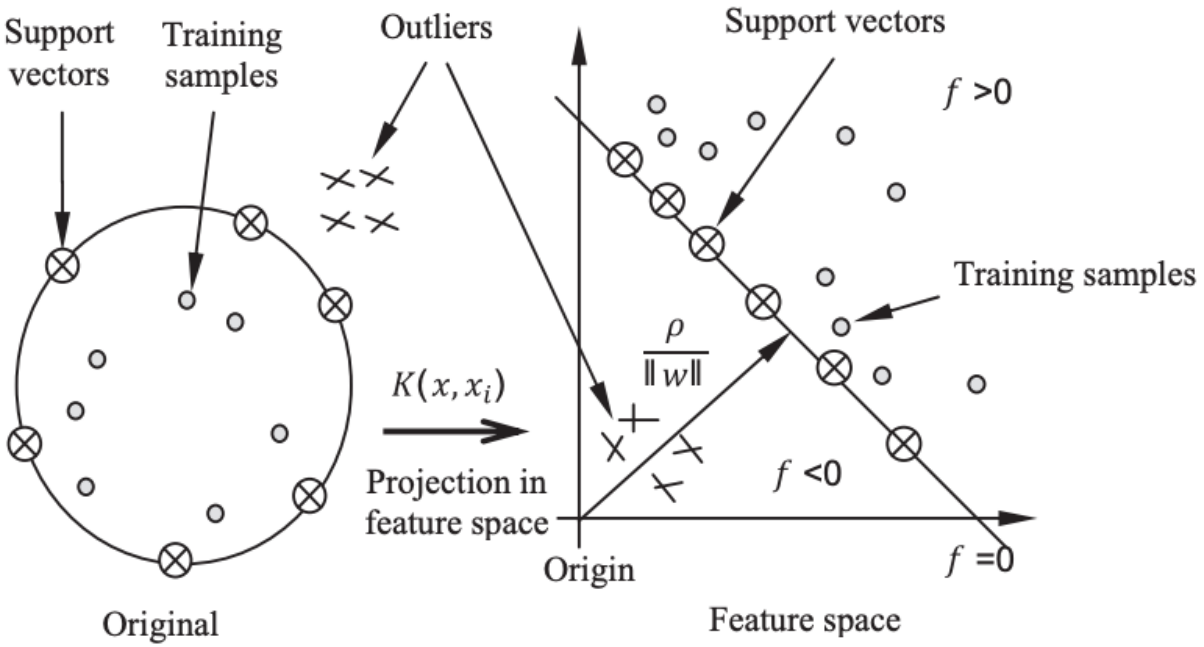
One class SVMs

- Sometimes it is easy to get positive examples but hard to acquire all possible negative examples
  - Email spam filter
    - We kind of know what a good email looks like. And we have lots of examples
    - Hard to model what a spam is. Spammer can change the format and evade detection.

- Solution: train on just the positive class
  - Model what that class looks like
  - Anything that deviates too much from it is considered negative examples

How?

- Separates the data from the “origin” (in mapped space)
- Maximize the distance between data points and the origin



https://www.sciencedirect.com/science/article/abs/pii/S0031320314002751

http://rvlasveld.github.io/blog/2013/07/12/introduction-to-one-class-support-vector-machines/

SVM objective with slack

- Minimize $\mathbf{w}^T\mathbf{w} + C\Sigma\varepsilon_i$
- Subject to

C is a weight parameter, how much we care about slack

$$\mathbf{w}^T\mathbf{x}_i + b \geq 1 - \varepsilon_i \quad for \quad +ve \quad class$$

$$\mathbf{w}^T\mathbf{x}_i + b \leq -1 + \varepsilon_i \quad for \quad -ve \quad class$$

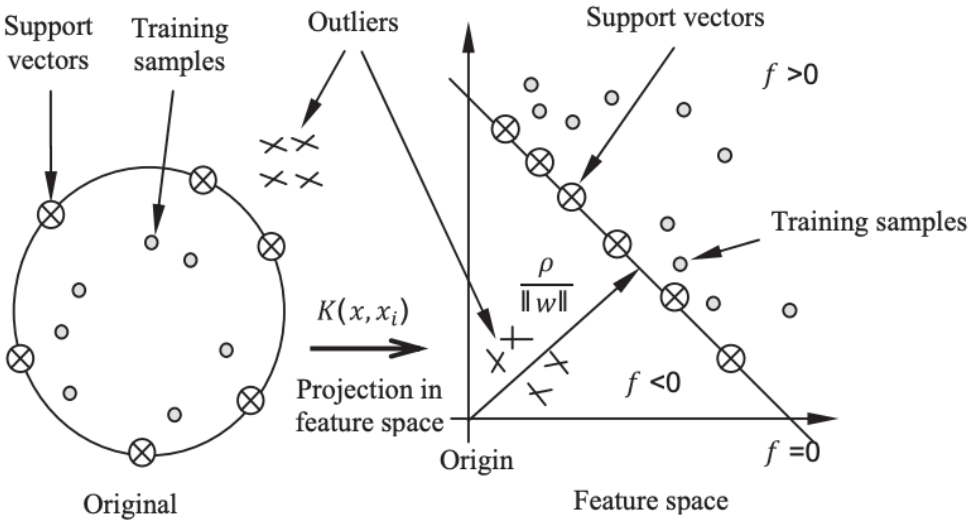
$$\varepsilon_i \geq 0 \quad \forall i$$

One class SVM with slack

- Minimize $\mathbf{w}^T\mathbf{w} + 1/(vn)\ \Sigma\varepsilon_i - \rho$
- Subject to ρ is the radius of the ball that we also need to optimize

$$\mathbf{w}^T\mathbf{x}_i + b \geq \rho - \varepsilon_i$$

$$\varepsilon_i \geq 0 \qquad \forall i$$



The hyper parameter v (nu, greek letter n) sets the upper bound of the fraction of training examples to be regarded negative (even though we only put in positive examples). Also called nu-svm

SVM Notes

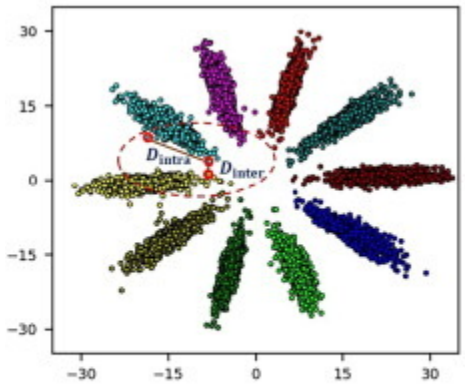
- One of the strongest models and hard to overfit
  - With proper tuning of hyperparameters (kernel, gamma, etc.)
- Convex optimization – no need to try multiple initializations
- Easiest to do one-class setups
- Does not scale well to large datasets
  - Need to compute Gram matrix (sometimes does not fit in memory if too many data points)
  - Inference time and memory requirement during inference depends on the number of support vectors

$$\mathbf{w} = \sum_{i=1}^{N} \alpha_i y_i \Phi(\mathbf{x}_i)$$

Margin-based today

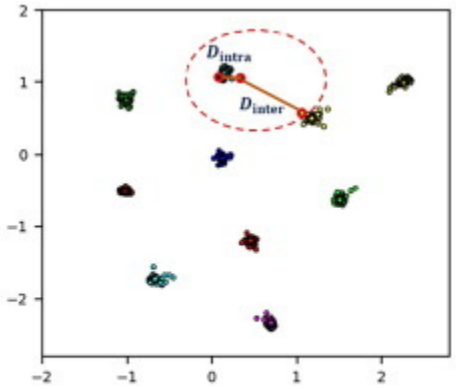
- Many deep learning models now incorporate some kind of margin in the loss function
  - Used in face recognition, embedding-based models

Regular loss without margin



(a) Softmax loss

Loss with margin term



(b) I2CS loss

https://www.sciencedirect.com/science/article/pii/S0925231220318178

Ways to group machine learning models

How do you acquire training data?

- Supervised
- Unsupervised
- Reinforcement

What are you outputting?

- Regression
- Classification/Clustering

How are you modeling? New!

- Discriminative
- Generative

Generative Models

- Naïve Bayes, Bayes classifiers are generative models
- Learn the model for each class y given input features x p(x|y). (The likelihood probability)

- To do classification we want to solve for the best y given input feature x (the posterior)

$$y^* = argmax_y P(y|x)$$

- We can use Bayes' rule

$$y^* = argmax_y \frac{P(x|y)P(y)}{P(x)}$$

Generative Models

$$y^* = argmax_y \frac{P(x|y)P(y)}{P(x)}$$

- P(y) is called the prior probability
- P(x) is ignored since we only care for argmax wrt. Y

- Can we use P(y|x) instead?

$$y^* = argmax_y P(y|x)$$

Discriminative models

- Discriminative models model P(y|x) directly

$$y^* = argmax_y P(y|x)$$

- P(y|x) is called the posterior probability
- Generally, P(y|x) can be any function h(x,y) that gives a score for each class
  - Logistic regression
  - SVM
  - Neural networks

Discriminative vs Generative

- Model the posterior P(y|x)
- Care about how to *discriminate* between different classes
- Usually outperforms generative models in classification tasks
- Need to retrain the whole model

- Model the likelihood P(x|y)
  Learns about how x is *generated* from y
- Worse classification performance.
  Mismatch between training objective
- Easy to add a new class y' – train P(x|y = y')
  Example: Adding a new class in Naïve Bayes

Notes on Generative modeling

- Some people say generative models model the joint probability, p(x,y).
- This is also true because when we model p(x|y), we also model the prior p(y).
  - p(x,y) = p(x|y)p(y)

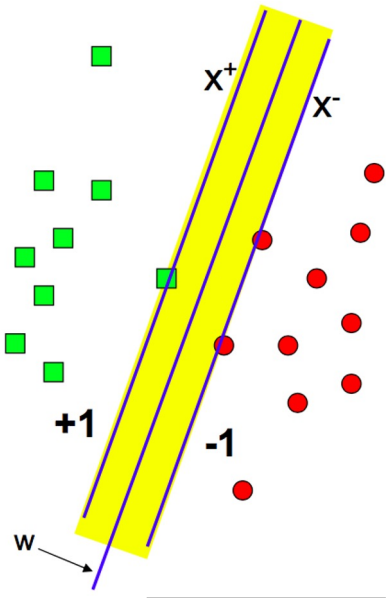
- With this view,
  - Generative models use the joint probability p(x,y)
  - Discriminative models use the conditional probability p(y|x)

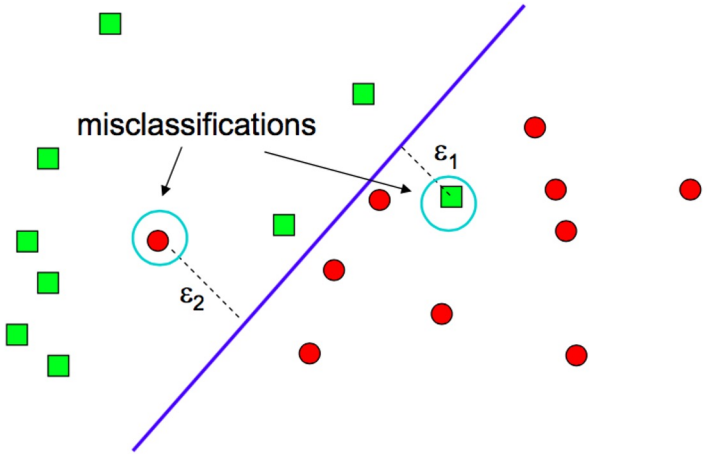
Generating data from generative model?

- We have the joint p(x,y) so we can sample from the distribution for a new (x,y) pair.
- This generates a new data sample x
  - You cannot sample from the posterior p(y|x) because you do not know p(x).

Summary

- SVMs
  - Max margin loss
  - Slack
  - Dual-primal
    - Kernel (inner product of higher space)
  - RBF kernels
  - One class SVM





NEURAL NETWORKS

Deep learning = Deep neural networks = neural networks

DNNs (Deep Neural Networks)

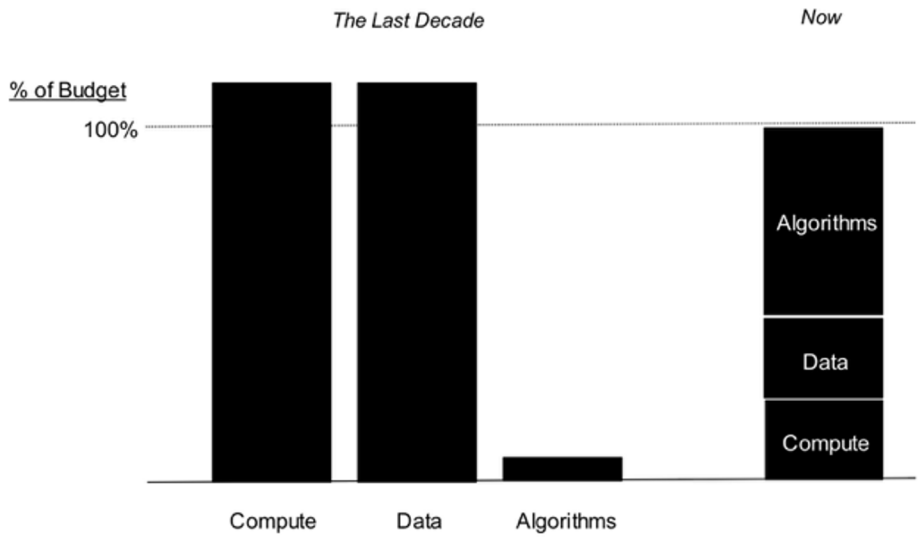
- Why deep learning?
- Greatly improved performance in ASR and other tasks (Computer Vision, Robotics, Machine Translation, NLP, etc.)
- Surpassed human performance in many tasks

Task	Previous state-of-the-art	Deep learning (2012)	Deep learning (2019)
TIMIT	24.4%	20.0%	13.8%
Switchboard	23.6%	16.1%	5.0%
Google voice search	16.0%	12.3%	4.9%
MOOC (Thai)	38.7%		19.6%

Geoffrey Hinton, Deep Neural Networks for Acoustic Modeling in Speech Recognition, 2012

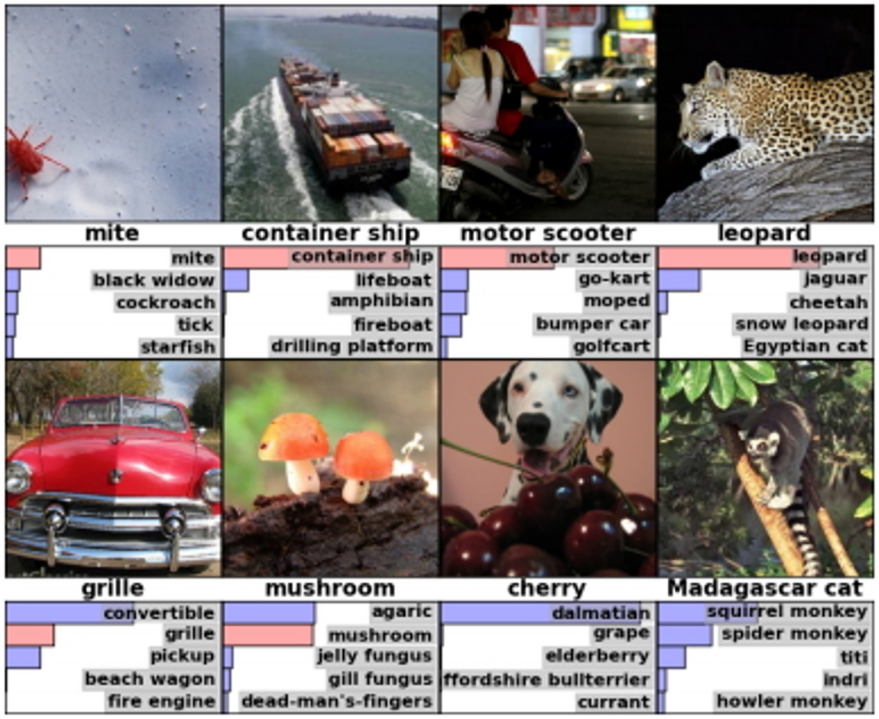
Why now

- Neural Networks has been around since 1990s
- Big data – DNN can take advantage of large amounts of data better than other models
- GPU – Enable training bigger models possible
- Deep – Easier to avoid bad local minima when the model is large



http://www.kdnuggets.com/2017/06/practical-guide-machine-learning-understand-differentiate-apply.html

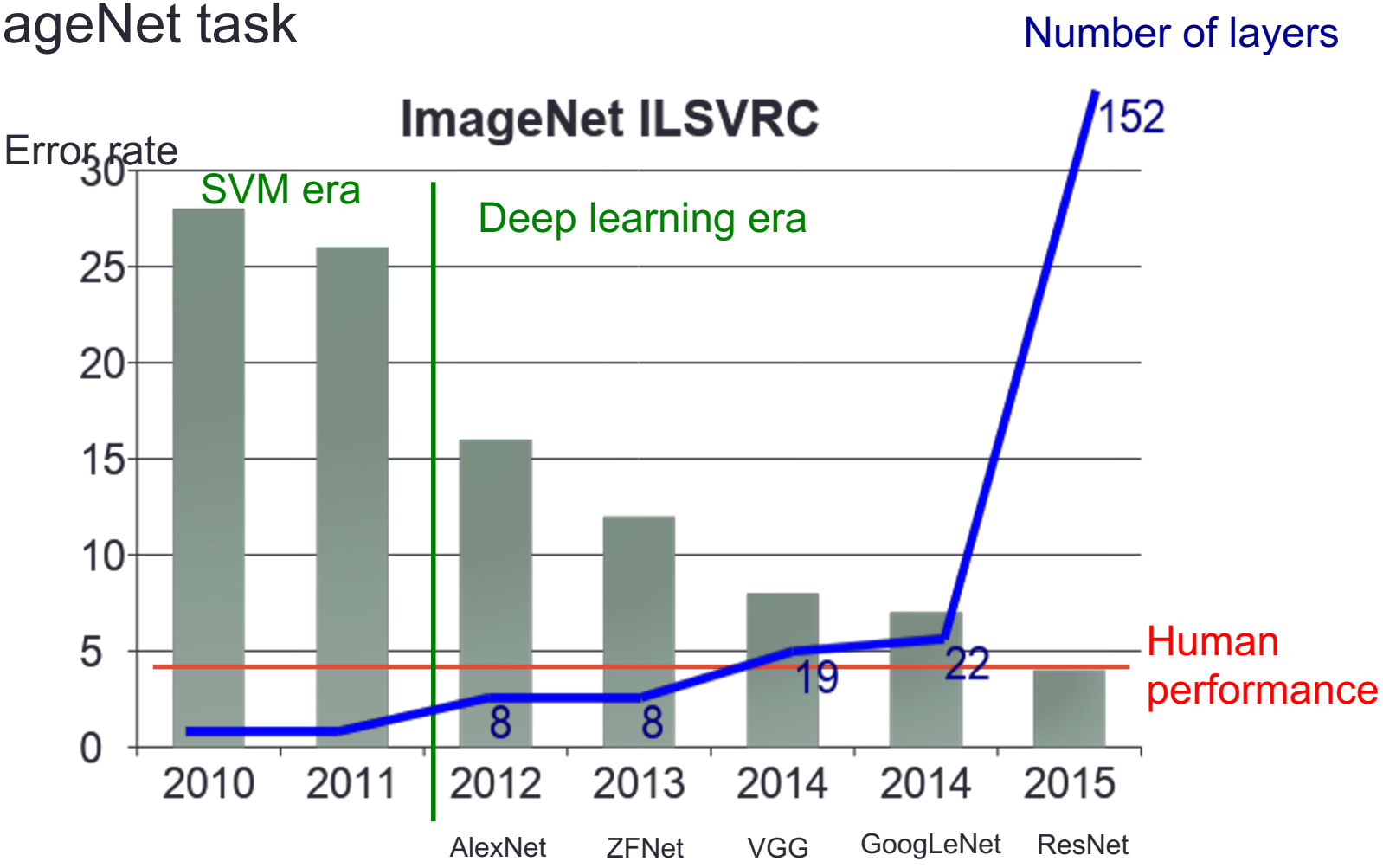
ImageNet - Object classification



Alex, Krizhevsky, Imagenet classification with deep convolutional neural networks, 2012

Wider and deeper networks

- ImageNet task



Olga Russakovsky, ImageNet Large Scale Visual Recognition Challenge, 2014 https://arxiv.org/abs/1409.0575

Statistics > Machine Learning

Dynamical Isometry and a Mean Field Theory of CNNs: How to Train 10,000-Layer Vanilla Convolutional Neural Networks

Lechao Xiao, Yasaman Bahri, Jascha Sohl-Dickstein, Samuel S. Schoenholz, Jeffrey Pennington

(Submitted on 14 Jun 2018)

In recent years, state-of-the-art methods in computer vision have utilized increasingly deep convolutional neural network architectures (CNNs), with some of the most successful models employing hundreds or even thousands of layers. A variety of pathologies such as vanishing/exploding gradients make training such deep networks challenging. While residual connections and batch normalization do enable training at these depths, it has remained unclear whether such specialized architecture designs are truly necessary to train deep CNNs. In this work, we demonstrate that it is possible to train vanilla CNNs with ten thousand layers or more simply by using an appropriate initialization scheme. We derive this initialization scheme theoretically by developing a mean field theory for signal propagation and by characterizing the conditions for dynamical isometry, the equilibration of singular values of the input-output Jacobian matrix. These conditions require that the convolution operator be an orthogonal transformation in the sense that it is norm-preserving. We present an algorithm for generating such random initial orthogonal convolution kernels and demonstrate empirically that they enable efficient training of extremely deep architectures.

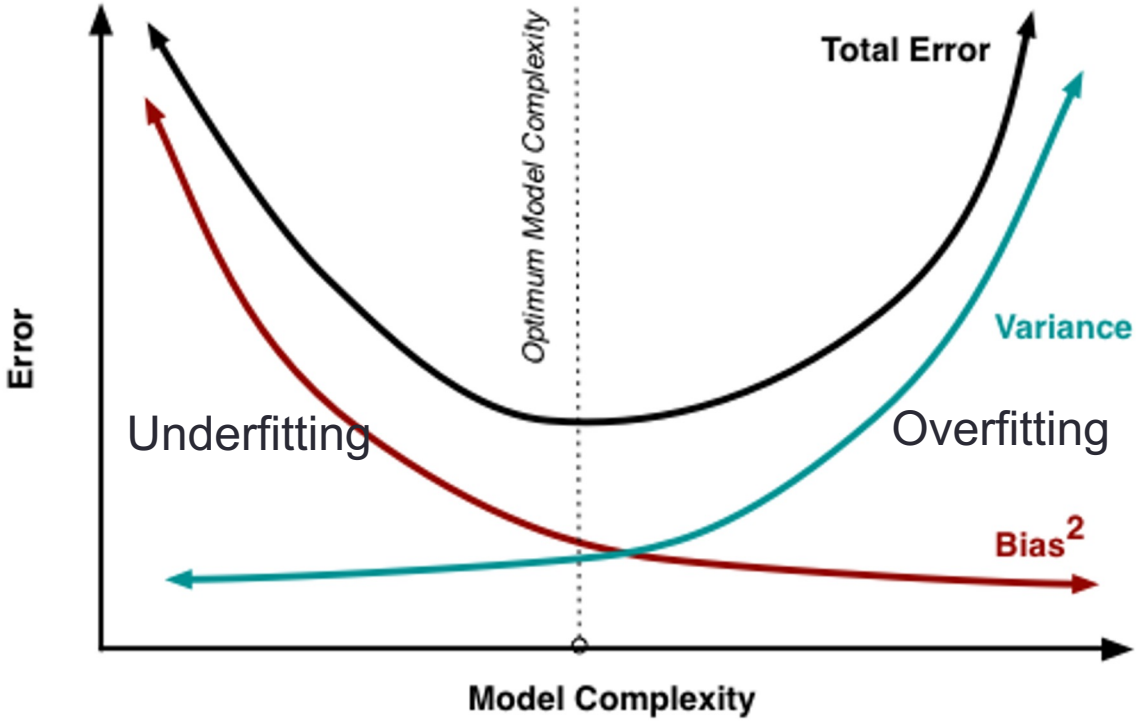
Comments: ICML 2018 Conference Proceedings
Subjects: **Machine Learning (stat.ML)**; Machine Learning (cs.LG)
Cite as: arXiv:1806.05393 **[stat.ML]**
(or **arXiv:1806.05393v1 [stat.ML]** for this version)

Submission history

From: Samuel Schoenholz [view email]
[v1] Thu, 14 Jun 2018 07:04:15 GMT (6734kb,D)

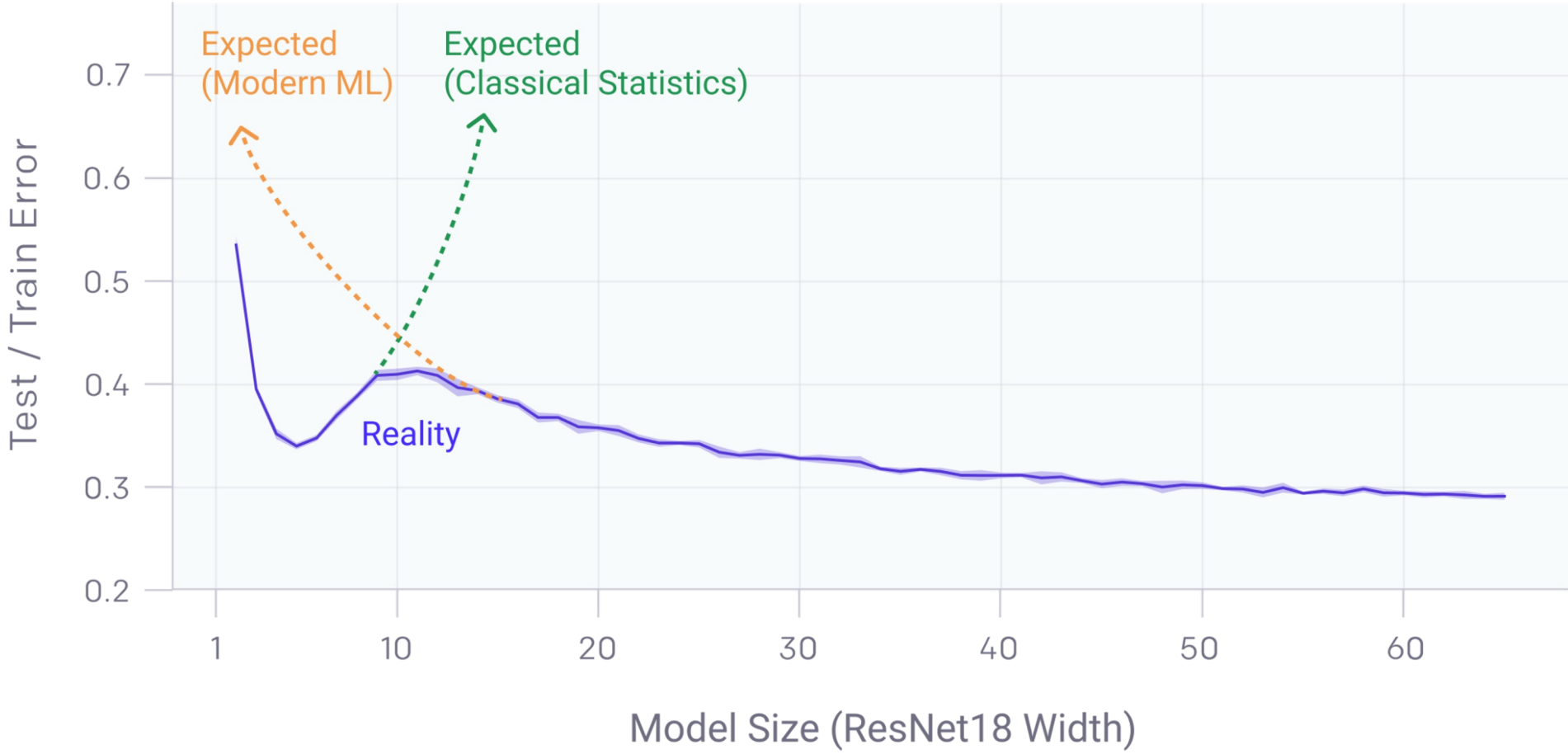
Bias-Variance Underfitting-Overfitting

- Usually if you try to reduce the bias of your model, the variance will increase, and vice versa.
- Called the bias-variance trade-off



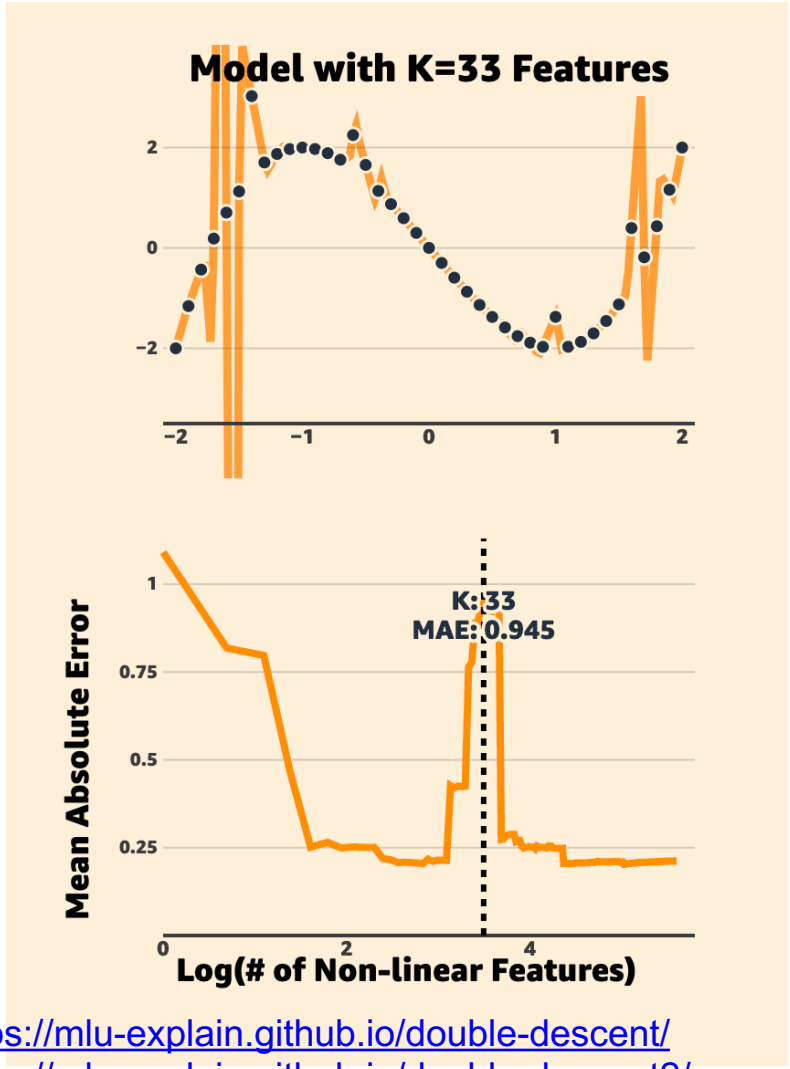
http://www.cs.cornell.edu/courses/cs4780/2017sp/lectures/lecturenote11.html

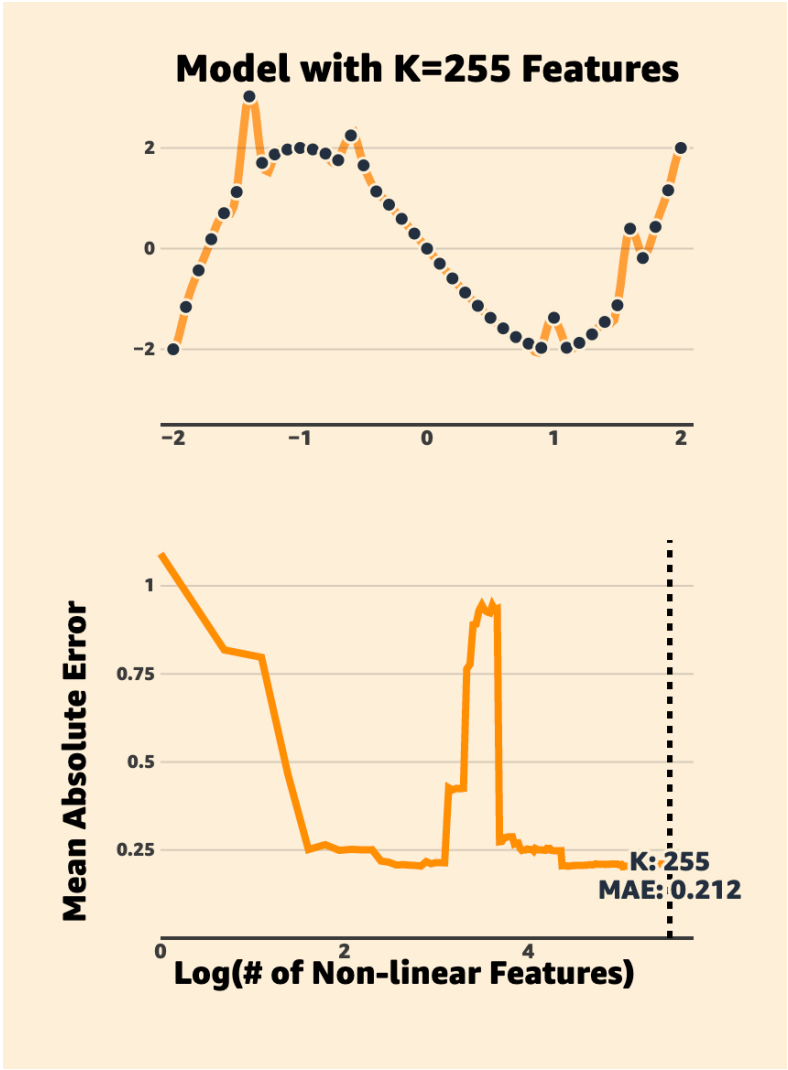
The double descent problem



https://openai.com/blog/deep-double-descent/

The double descent problem





https://mlu-explain.github.io/double-descent/
https://mlu-explain.github.io/double-descent2/

Inductive bias and ML

- The **inductive bias** of a learning algorithm is a set of assumptions that the algorithm used to generalize to new inputs.

http://www.cs.cmu.edu/~tom/pubs/NeedForBias_1980.pdf

- Your choice of model forces a certain type of behavior
  - Tells the model how to "overfit" the training data
- Putting inductive bias into deep learning model is easier than other models
  - ~~Domain knowledge to construct features~~
  - Domain knowledge to encourage certain learning behaviors

Traditional VS Deep learning