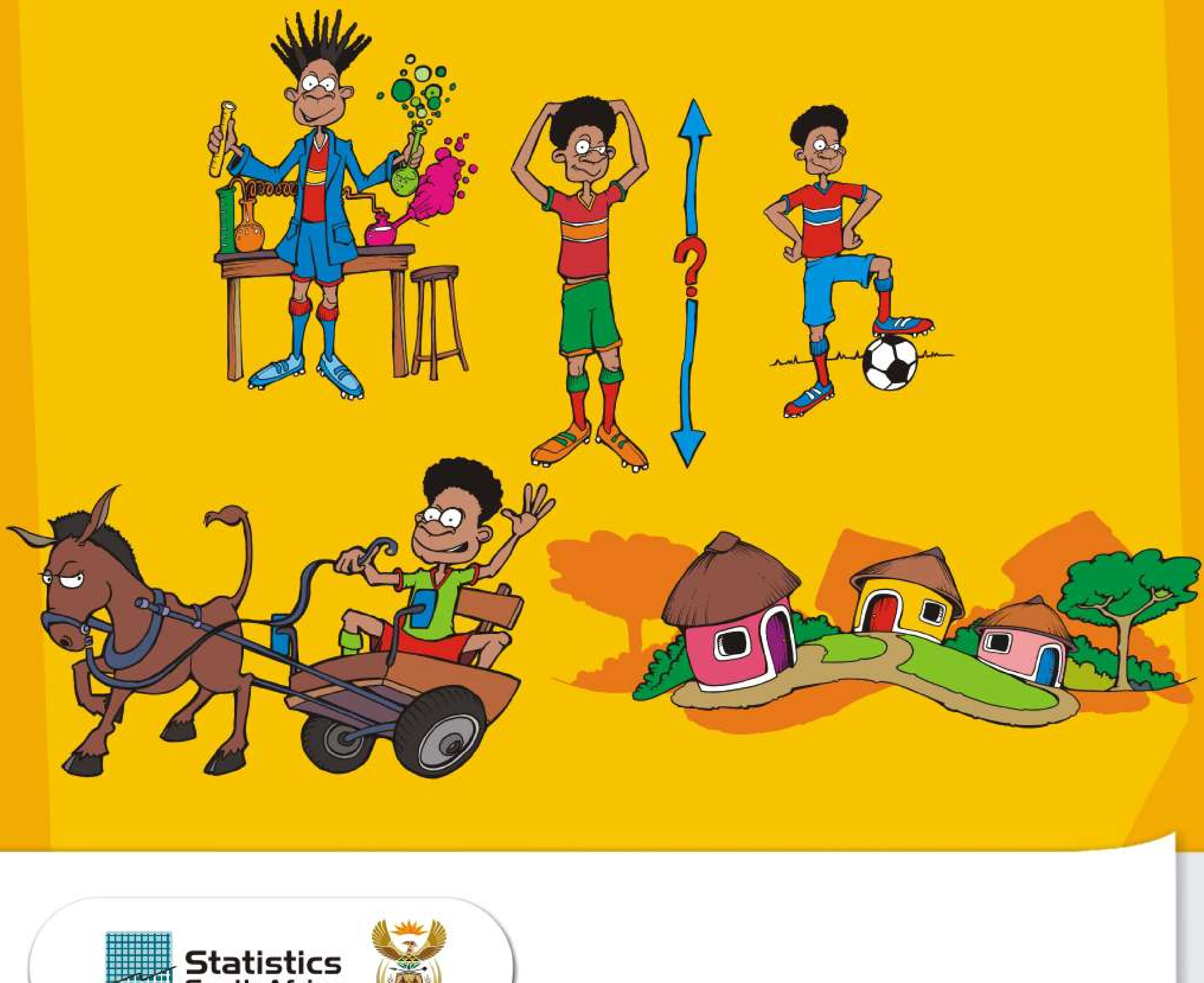
Data Handling & Probability
Grades 10, 11 and 12



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Grade 11 Probability

In this chapter you:

- Revise the language of probability
- Use Venn diagrams showing two events to determine probabilities
- Identify independent and dependent events
- Use 2-way tables to determine probabilities of events happening
- Investigate the multiplication law for probability
- Use tree diagrams to determine probabilities of events happening
- Use Venn Diagrams showing three events to determine probabilities.

WHAT YOU LEARNED ABOUT PROBABILITY IN GRADE 10

In Grade 10 you covered the following probability concepts:

- *The use of probability models to compare the relative frequency of events with the theoretical probability*
- *The use of Venn diagrams to solve probability problems*
- *Rules for any **two events** A and B in sample space S:*
    - $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
    - *A and B are mutually exclusive if* $P(A \text{ and } B) = 0$
    - *A and B are complementary if they are mutually exclusive*
      And $P(A) + P(B) = 1$
      Or $P(B) = P(\text{not } A) = 1 - P(A)$
      Or $P(A) = P(\text{not } B) = 1 - P(B)$

THE LANGUAGE OF PROBABILITY

- ✓ A ***statistical experiment*** is one in which there are a number of possible outcomes and we have no way of predicting which outcome will actually occur.

- ✓ A ***sample space*** is the set of all the possible outcomes in an experiment.

- ✓ An ***event*** is any set of possible outcomes of an experiment.

  For example:
  If we roll a dice, the ***sample space*** consists of the numbers 1, 2, 3, 4, 5 and 6. Getting an even number is an ***event***, and is a subset of the sample space.

- ✓ $The\ probability\ of\ any\ event\ A\ occurring$
  $$= \frac{number\ of\ outcomes\ in\ A}{total\ number\ of\ outcomes\ in\ the\ sample\ space}$$

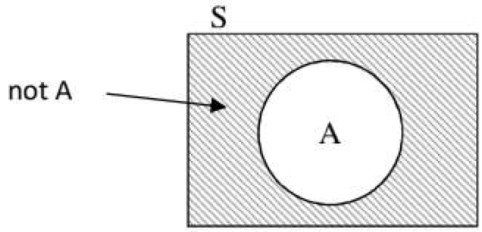
  We write $P(A) = \frac{n(A)}{n(S)}$ where $0 \leq P(A) \leq 1$.

  - This means the probability cannot be negative or greater than 1
  - If the event A is ***impossible*** then its probability is 0.
  - If the event A is ***certain***, then its probability is 1.

- ✓ ***Complementary events:***

  The events 'A' and 'not A' cannot happen at the same time. We say that they are ***complementary***.

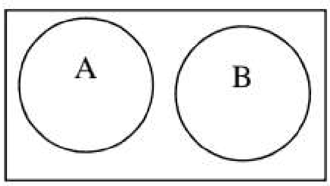
  So $P(not\ A) = 1 - P(A)$
  or
  $P(A) + P(not\ A) = 1$
  or
  $P(A) = 1 - P(not\ A)$



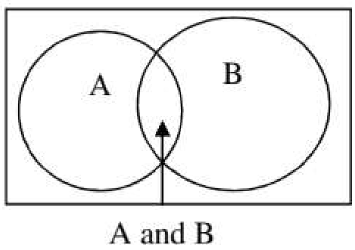
- ✓ ***Mutually exclusive events***

  Two events are ***mutually exclusive*** if they cannot happen at the same time. If one of them happens, it excludes the other.

  - If events are mutually exclusive then $P(\text{A and B}) = 0$.
  - Mutually exclusive sets are ***disjoint sets.***



A and B are mutually exclusive



A and B are NOT mutually exclusive

- ✓ The ***addition law for probability:***

  - If the events A and B are ***NOT*** mutually exclusive, the probability that A *or* B will occur is given by: $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$
  - If the events A and B ***ARE*** mutually exclusive, then the rule can be simplified to $P(A\ or\ B) = P(A) + P(B)$

VENN DIAGRAMS SHOWING TWO EVENTS

✓ A ***Venn diagram*** is a simple representation of a sample space.

- Usually a rectangle is used to represent a Sample Space (S).
- Circles are used to represent events within the sample space.

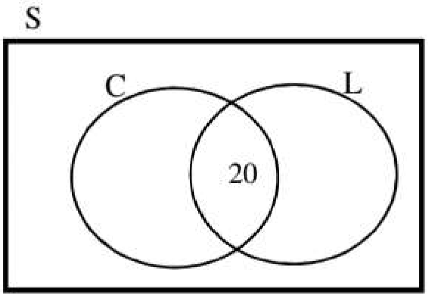
✓ We can use Venn diagrams to assist us to work out the probability of an event happening.

EXAMPLE 1
In the 2009 Census@School learners were asked to indicate whether they have access to a telephone or a cellphone at home.
290 Grade 11 learners in one high school responded as follows:
150 have access to a cellphone
80 have access to a telephone at home
20 have access to both cellphone and to a telephone at home
Draw a Venn diagram to represent the situation.

SOLUTION:
Let C be the event that a learner has access to a cellphone and L be the event that a learner has access to a telephone at home.

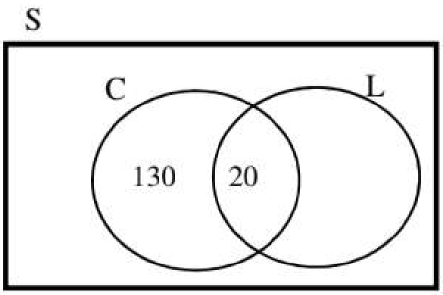
STEP 1:
- Draw a rectangle to represent the Sample Space. Inside the rectangle draw two intersecting circles
- Start at the intersection:
  20 learners have access to both a cellphone and a landline phones at home i.e. n(C and L) = 20.

Write 20 in the intersection.



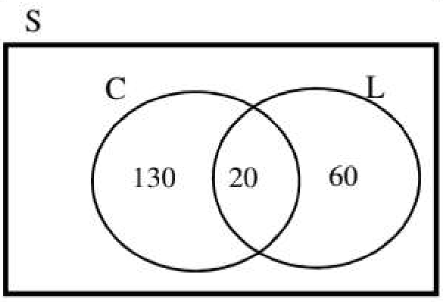
STEP 2
- 150 have access to cellphone i.e. n(C) = 150
- But 20 has already been entered in the intersection, so $150-20=130$ must be entered into the other part of circle C.

Write 130 into the remaining part of C



STEP 3
- 80 learners have access to a telephone at home i.e. n(L) = 80
- But 20 of the 80 have been entered into the intersection, which means that $80-20=60$ must be entered into the remaining part of circle L.

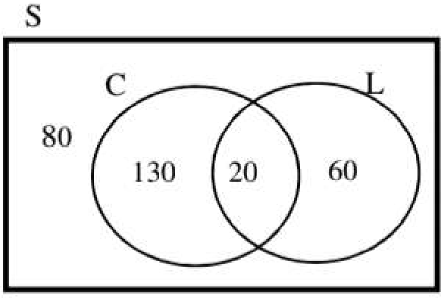
Write 60 into the remaining part of L



EXAMPLE 1 (continued)

STEP 4:

- The total number of learners is 290 i.e. n(S)=290
- The total number of learners who neither have access to a Cellphone nor to a telephone at home = 290 – (130 + 20 + 60) = 80



EXAMPLE 2

Use the Venn diagram to answer the following.

a) Determine *n(S)*

b) Determine the following probabilities as a percentage correct to 1 decimal place

i) P(C)

ii) P(L)

iii) P(C and L)

iv) P(C or L)

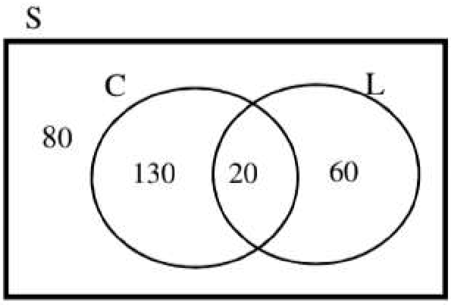
v) P(not C)

vi) P(not L)

vii) P(not (C or L))

viii) P(not (C or L)

ix) P(at least one type of phone)



SOLUTION:

a) $n(S) = 80 + 130 + 20 + 60 = 290$

b)

i) $P(C) = \frac{n(C)}{n(S)} = \frac{130+20}{290} = \frac{150}{290} \approx 51{,}7\%$

ii) $P(L) = \frac{n(L)}{n(S)} = \frac{20+60}{290} = \frac{80}{290} \approx 27{,}6\%$

iii) $P(C \text{ and } L) = \frac{n(C \text{ and } L)}{n(S)} = \frac{20}{290} \approx 6{,}9\%$

iv) P(C or L) = P(C) + P(L) – P(C and L) ≈ 51,7% + 27,6% – 6,9% = 72,4%

v) P(not C) = 1 – P(C) ≈ 100% – 51,7% = 48,3%

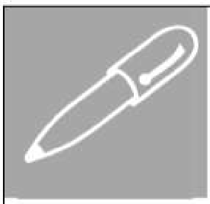
vi) P(not L) = 1 – P(L) ≈ 100% – 27,6% = 72,4%

vii) P(not (C or L)) = 1 – P(C or L) ≈ 100% – 72,4% = 27,6%

viii) $P(L \text{ only}) = \frac{60}{290} \approx 20{,}7\%$

ix) P(at least one type of phone) = P(C or L) = 72,4%

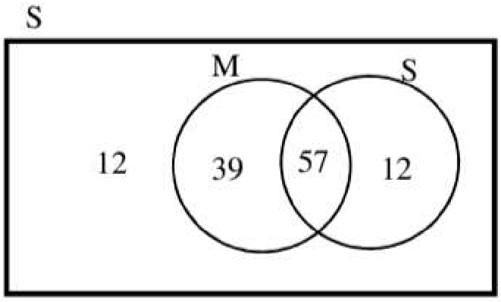
Remember that $1 = 100\% = \frac{100}{100}$



EXERCISE 5.1

1) A survey was conducted with the Grade 11 learners at a certain school to find out which subjects they are taking this year. It was found that
   - 96 learners were taking Maths (M),
   - 69 were taking Physical Sciences (S),
   - 57 were taking Maths and Physical Sciences
   - 12 were taking neither Maths nor Physical Sciences.

   The results are shown in the Venn diagram.



   a) How many learners were interviewed?
   b) A learner is selected at random from the learners who were interviewed. Determine the following as a percentage:
      i) P(M)
      ii) P(S)
      iii) P(M and S)
      iv) P(M or S)
      v) P(not M)
      vi) P(not S)
      vii) P(not (M or S))

2) During 2009 Census@School, learners were asked to indicate their favourite subjects. 1 320 learners in a certain school responded as follows:
   - 474 learners like Maths (M)
   - 894 learners like English (E)
   - 105 learners like both Maths and English

   a) Draw a Venn diagram to show the results of the survey
   b) How many learners like neither Maths nor English
   c) What is the probability (as a decimal fraction correct to 2 decimal places and as a percentage correct to 1 decimal place) that a randomly picked learner in the school likes
      i) both Maths and English
      ii) Maths only
      iii) English only?

INDEPENDENT AND DEPENDENT EVENTS

✓ ***Independent events*** are events where the outcome of the second event is not affected by the outcomes of the first event.

For example

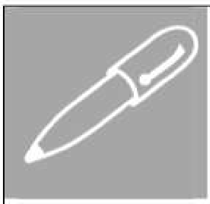
a) Suppose a dice is thrown twice. The second throw of the dice is not affected by the first throw of the dice. Thus, ***the two events are independent***.

b) Suppose you toss a coin and throw a dice. The number that you throw on the dice is not dependent on whether you get heads or tails on the coin. Thus, ***the two events are independent***

c) If you randomly select a card from a pack of playing cards, ***replace it*** then randomly select a second card, ***the events are independent.***

✓ ***Dependent events*** are events that are not independent. The probabilities of dependent events DO affect each other.

For example,
If you select a card from a pack of playing cards and then randomly select another card ***without replacing the first card*** these events are ***dependent events.***



EXERCISE 5.2

Decide whether the events described below are independent or dependent. Give a reason for your answer.

1) Tossing a coin and taking a card from a pack of playing cards.
2) Throwing a dice and spinning a spinner.
3) Taking a card and spinning a spinner.
4) Getting 7 on the spin of a spinner, and then getting 3 when it is spun a second time.
5) Driving over 120 km/h, and causing a car accident.
6) Exercising frequently and having a low resting heart rate.
7) Randomly selecting a ball numbered from 1 to 40 from a box, and then selecting a second numbered ball from the remaining balls in the box.
8) Selecting a ball numbered from 1 and 52 from a box, and then selecting a second numbered ball between 1 and 52 from a second box.

TWO-WAY TABLES

✓ In some probability problems we look at the combined or compound outcomes of several activities.

These activities are often 'games' such as

- Spinning a spinner and taking a card
- Throwing two dice
- Tossing a coin three times.

✓ To find out the possible outcomes (the sample set), you can make a list, draw up a table, or use a tree diagram. In order to make sure that we don't miss any of the combined outcomes, it is best to use a table or a tree diagram.

✓ For two or more activities, a table is often the easiest way to list the outcomes.

We draw up a ***two-way table*** as follows:

- Write one activity down the side of the table
- Write the other activity along the top.
- Write the combined result from the two activities in each cell of the table.

For example:

The following table shows the results obtained when two coins are tossed.

		Coin 2	
		H	**T**
Coin 1	**H**	H ; H	H ; T
	T	T ; H	T ; T

A two-way table

EXAMPLE 3

Bag A contains 2 red balls (R) and 1 white ball (W).
Bag B contains 2 red balls (R) and 1 white ball (W).
A ball is taken at random for each bag.

a) Copy and complete the table to show all possible pairs of colours.

		Bag A		
		R	**R**	**W**
Bag B	**R**			
	R			
	W			

b) Determine n(S) where S is the sample set.

c) A ball is selected at random from each of the two bags. Determine the following as a fraction in simplest form:

i) P(R; R)
ii) P(R; W)
iii) P(W; R)
iv) P(W; W)

b) Determine the probability that the two balls are the same colour

SOLUTION:

a)

		Bag A		
		R	**R**	**W**
Bag B	**R**	R; R	R; R	R; W
	R	R; R	R; R	R; W
	W	W; R	W; R	W; W

b) n(S) = 9

c)

i) $P(R;R) = \frac{n(R;R)}{n(S)} = \frac{4}{9}$

ii) $P(R;W) = \frac{n(R;W)}{n(S)} = \frac{2}{9}$

iii) $P(W;R) = \frac{n(W;R)}{n(S)} = \frac{2}{9}$

iv) $P(W;W) = \frac{nW;W)}{n(S)} = \frac{1}{9}$

d) Two balls of the same colour were taken out of the two bags five times – four times the two balls were red and one time the two balls were white.

$P(two\ balls\ are\ the\ same\ colour) = \frac{5}{9}$

EXAMPLE 4

A coin is tossed and a dice is thrown

a) Are the events 'toss a coin' and 'throw a dice' independent?
b) Use a two-way table to determine all the possible outcomes
c) Determine n(S) where S is the sample set.
d) Suppose a coin and a dice are thrown together and the results noted. Determine the following probabilities, writing them as fractions in simplest form:
   i) P(H)
   ii) P(3)
   iii) P(H and 3)
e) Determine P(H) × P(3)
f) What do you notice about the answers P(H and 3) and P(H) × P(3)

SOLUTION:

a) The two events are independent as the number on the dice does not depend on whether you get heads or tails on the coin and getting heads or tails on a coin does not affect the number that comes up on a dice.

b)

		DICE					
		1	**2**	**3**	**4**	**5**	**6**
COIN	**H**	H; 1	H ; 2	H ; 3	H ; 4	H ; 5	H ; 6
	T	T ; 1	T ; 2	T ; 3	T ; 4	T ; 5	T ; 6

c) n(S) = 12

d)

i) $P(H) = \frac{n(events\ having\ H\ as\ one\ of\ its\ outcomes)}{n(S)} = \frac{6}{12} = \frac{1}{2}$

ii) $P(3) = \frac{n(events\ having\ 3\ as\ one\ of\ its\ outcomes)}{n(S)} = \frac{2}{12} = \frac{1}{6}$

iii) $P(H\ and\ 3) = \frac{n(H\ and\ 3)}{n(S)} = \frac{1}{12}$

e) $P(H) \times P(3) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

f) The answers to P (H and 3) and P (H) ×P (3) are the same.

EXERCISE 5.3

1) A coin and a dice are used in a game. The counter has two faces, one marked 1 and the other one marked 2. The counter and dice are thrown together. The numbers obtained are multiplied together.
   a) Copy and complete this table to show all the possible outcomes.

×		Number on dice					
		1	2	3	4	5	6
Number on counter	1						
	2						

   b) Determine n(S) where S is the sample space
   c) The counter and the dice are thrown together and the results noted. Determine the following as as a fraction in simplest form:
      i) P(2)
      ii) P(5)
      iii) P(2 or 5)
      iv) P(even number)

2) A supermarket runs a competition. Each customer is given a card with two whole numbers printed on it. Each of these whole numbers are chosen at random from the numbers 1; 2; 3; 4; 5; 6; 7; 8; 9. The customer has to add her two whole numbers together to find her total.

   a) Copy and complete the table to show all the totals of the pairs of numbers.

+		Second number								
		1	2	3	4	5	6	7	8	9
First number	1									
	2									
	3									
	4									
	5									
	6									
	7									
	8									
	9									

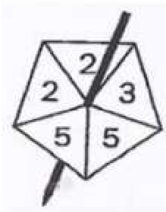
   b) How many pairs of numbers give a total of 14?
   c) Determine the probability (as a percentage of the sample space, correct to 1 decimal place) that these numbers would win the prize:
      i) 14
      ii) 15
   d) The supermarket decides to make 2 and 10 the lucky numbers of the day. Which one of these two numbers has a greater chance of being chosen? Why.

<u>EXERCISE 5.4</u> (continued)

3) A coin is tossed and a dice is thrown. All the possible outcomes are recorded in the two-way table.
   a) Are these two events independent?
   b) Determine n(S) where S is the sample set.
   c) Determine the following, writing each one as a fraction in simplest form:
      i) P(T)
      ii) P(6)
      iii) P(T) × P(6)
   d) Is P (T and 6) = P (T) × P (6)?

		DICE					
		1	**2**	**3**	**4**	**5**	**6**
COIN	**H**	H; 1	H ; 2	H ; 3	H ; 4	H ; 5	H ; 6
	T	T ; 1	T ; 2	T ; 3	T ; 4	T ; 5	T ; 6

   e) Determine the following, writing them as fractions in simplest form.
      i) P(T and even)
      ii) P(even)
      iii) P(T) × P(even)
   f) Is P (T and even) = P (T) × P (even)?
   g) Determine the following, writing them as fractions in simplest form.
      i) P(H)
      ii) P(H and a multiple of 3)
      iii) P(multiple of 3)
      iv) P(H) ×P(multiple of 3)
   h) Is P (H) × P (multiple of 3) = P (H and multiple of 3)?
4) A spinner has the shape of a regular pentagon. The five sections of the spinner are numbered 2; 2; 3; 5; 5. When the spinner is used, it is equally likely to stop on any one of its five edges.



   a) In a game, Nomsa spins the spinner twice and notes the results. Is spinning the spinner twice two independent events?
   b) Draw a two-way table to show all the possible results for the two spins.
   c) Determine the following as fractions in simplest form
      i) P(the first number is a 2)
      ii) P(the second number is a 5)
      iii) P(the first number is a 2 and the second number is a 5)
      iv) P(the first number is a 2) × P(the second number is a 5)
   d) Is P(the first number is a 2 and the second number is a 5) = P(first number is a 2) × P(second number is a five)?
5) Suppose you have two independent events M and N. Complete the following:

   P(M and N) = ……………………………………

THE MULTIPLICATION LAW FOR PROBABILITIES

✓ The ***multiplication law*** for probability only holds for ***independent events.***

If A and B are independent events, then P(A and B) = P(A) × P(B).

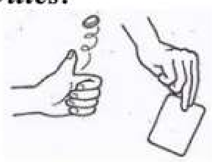
✓ The multiplication law is often called the ‘***and law***’. It gives the probability that one event **AND** another happens

✓ This multiplication law can be used for ***more than two*** independent events.
If A, B and C are independent events, then

P(A and B and C) = P(A) × P(B) × P(C)

✓ The multiplication law is a quick way to find the probability that one event ***and*** another event happens. It saves you having to list all the possible combined results for the two events and then picking out the favourable results for a combined event.

However, before you use it, ***make sure that the events to be combined are <u>independent</u>***!

DIFFERENT ACTIVITIES	REPEATED ACTIVITIES
When two events result from ***two completely separate and different activities***, it is clear that they must be ***independent***. Here are some examples of ***different activities:***	***Repeating an activity in exactly the same way and under the same circumstances*** results in independent events. Here are some examples of ***repeated activities:***
Tossing a coin and taking a card 	Tossing a coin twice OR tossing two coins 
Throwing a dice and spinning a spinner. 	Throwing a dice several times OR throwing several dice 

Using two-way tables to show probabilities

EXAMPLE 5

Two dice are thrown.

a) Draw a tree diagram to show all the totals when the dice are added.

b) Determine
   i) n(S)
   ii) P(7)
   iii) P(4)

c) Are P(7) and P(4) independent events?

d) Use a two-way table to find
   i) The probability of getting a sum of 7 on the first throw of the two dice **OR** a sum of 4 on the second throw.
   ii) The probability of getting a sum of 7 on the first throw of the two dice **AND** a sum of 4 on the second throw.

e) Which is greater: the probability of getting a 7 ***or*** a 4, OR the probability of getting a 5 ***and*** a 4.

<u>SOLUTION</u>:

a) The two-way table shows the totals when the numbers on the two dice are ***added***

+		**DICE 2**					
		1	**2**	**3**	**4**	**5**	**6**
DICE 1	1	2	3	4	5	6	**7**
	2	3	4	5	6	**7**	8
	3	4	5	6	**7**	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b)
   i) There are 36 outcomes, so $n(S) = 36$
   ii) Six of the outcomes give 7 as the total: $P(7) = \frac{n(7)}{n(S)} = \frac{6}{36} = \frac{1}{6}$
   iii) Three of the outcomes give 4 as the total: $P(4) = \frac{n(4)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

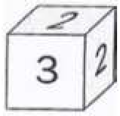
c) The probability of getting a sum of 7 on the first throw of 2 dice and the probability of getting a sum of 4 on the second throw are ***independent events*** because getting a 7 has no effect on getting a 4

d)
   i) $P(7 \text{ or } 4) = P(7) + P(4) = \frac{1}{6} + \frac{1}{12} = \frac{2+1}{12} = \frac{3}{12} = \frac{1}{4}$
   ii) $P(7 \text{ and } 4) = P(7) \times P(4) = \frac{1}{6} \times \frac{1}{12} = \frac{1}{72}$

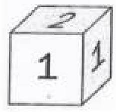
e) So the probability of getting a 7 or a 4 is greater than getting a 7 and a 4.

EXAMPLE 6

A fair BLUE dice has its faces marked with the numbers 2; 2; 2; 2; 3; 3



A fair RED dice has its faces marked with the numbers 1; 1; 2; 2; 2; 3



The two dice are thrown together.

a) Are these two events independent? Explain.

b) Copy and complete this probability table for this situation

		RED DICE		
		P(1) =	P(2) =	P(3) =
BLUE DICE	P(2) =			
	P(3) =			

c) Use your table to find:

i) The probability of getting a 2 on the blue die and a 1 on the red dice

ii) The probability of getting a 3 on the blue dice and a 2 on the red dice

iii) The probability of getting a double 2

SOLUTION:

a) These two events are independent. A number thrown on the blue dice is independent of a number thrown on the red dice.

b) On the BLUE dice, $P(2) = \frac{4}{6} = \frac{2}{3}$ and $P(3) = \frac{2}{6} = \frac{1}{3}$

On the RED dice, $P(1) = \frac{2}{6} = \frac{1}{3}$; $P(2) = \frac{3}{6} = \frac{1}{2}$ and $P(3) = \frac{1}{6}$

As the two events are independent, we can use the multiplication law.

		RED DICE		
		$P(1) = \frac{2}{6}$	$P(2) = \frac{3}{6}$	$P(3) = \frac{1}{6}$
BLUE DIE	$P(2) = \frac{4}{6}$	P(2 and 1) $= \frac{4}{6} \times \frac{2}{6} = \frac{2}{9}$	P(2 and 2) $= \frac{4}{6} \times \frac{3}{6} = \frac{1}{3}$	P(2 and 3) $= \frac{4}{6} \times \frac{1}{6} = \frac{1}{9}$
	$P(3) = \frac{2}{6}$	P(3 and 1) $= \frac{2}{6} \times \frac{2}{6} = \frac{1}{9}$	P(3 and 2) $= \frac{2}{6} \times \frac{3}{6} = \frac{1}{6}$	P(3 and 3) $= \frac{2}{6} \times \frac{1}{6} = \frac{1}{18}$

c)

i) P(2 and 1) $= \frac{2}{9}$

ii) P(3 and 2) $= \frac{1}{6}$

iii) P(double 2) = P(2 and 2) $= \frac{1}{3}$

EXERCISE 5.4

1) Bernice takes two cards without looking at them from a full pack of 52 cards. She replaces the first card before taking the second card.
   a) Calculate the probability that the first card taken is
      i) Red (R)
      ii) Black (B)
   b) Calculate the probability that the second card taken is
      i) A picture card (P)
      ii) Not a picture card (not P)

   A picture card is a playing card which has a picture on it (usually the Jack, Queen and King).

   c) Copy and complete the probability table
   d) Use your table to find the probability that Bernice takes
      i) First a red card and then a picture card
      ii) First a black card and then not a picture card.

		Second card	
		P(P) =	**P(not P) =**
First card	**P(R) =**		
	P(B) =		

2) In a soccer match, a team can win, draw or lose. United and City are due to play a match next week. Kenneth estimates the probabilities of the two teams winning, losing and drawing and writes them in a table.

		UNITED		
		P(w) = $\frac{1}{2}$	**P(d) =** $\frac{1}{6}$	**P(*l*) =** $\frac{1}{3}$
CITY	**P(W) =** $\frac{7}{10}$	P(W; w) =	P(W; d) =	P(W; *l*) =
	P(D) = $\frac{1}{5}$			
	P(L) = $\frac{1}{10}$			

   a) Copy and complete the table to show the probability of each combined event.
   b) What is the probability that City wins *and* United wins?
   c) What is the probability that City loses *and* United draws?
   d) Calculate the following for next week's match:
      i) P(both teams win or both teams lose)
      ii) P(City wins)
      iii) P(only one team loses)
      iv) P(only one team draws)

TREE DIAGRAMS

- ✓ When listing the combined outcomes of 2 or more activities, drawing a ***tree diagram*** is often the easiest method to use.

- ✓ We can use a ***tree diagram*** to work out combined outcomes of events that are either independent or dependent.

EXAMPLE 7

Suppose a family has three children. There are many combinations of boys (B) and girls (G) that can make up these three children.

a) Draw a tree diagram to find all the possible combinations of three children in the family.

b) Determine n(S) where S is the sample space.

c) Use the tree diagram to work out:

i) The probability that all three children are girls

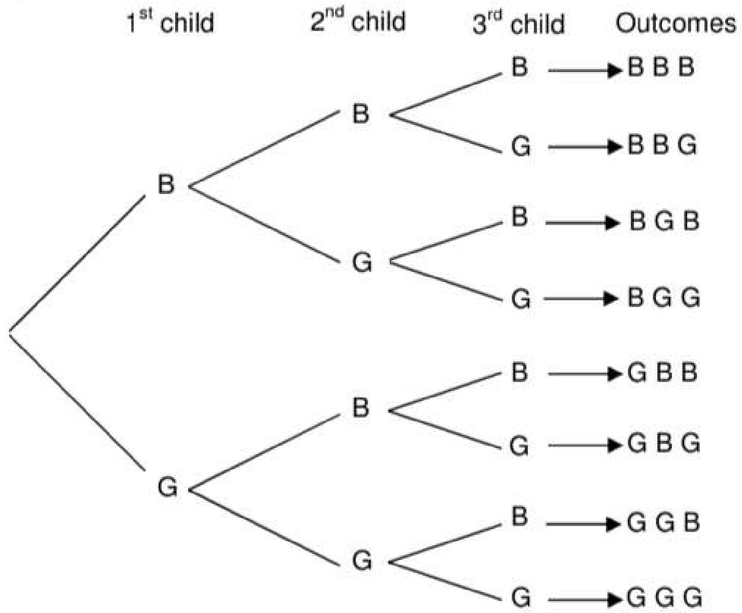
ii) The probability that all three children are of the same gender

iii) The probability that at least two of the three children are boys

iv) The probability that one of the three children is a girl

<u>SOLUTION:</u>

a)



a) n(S) = 8

b)

i) So, P(GGG) = $\frac{n(GGG)}{n(S)} = \frac{1}{8}$

ii) P(same gender) = P(BBB or GGG) = $\frac{2}{8} = \frac{1}{4}$

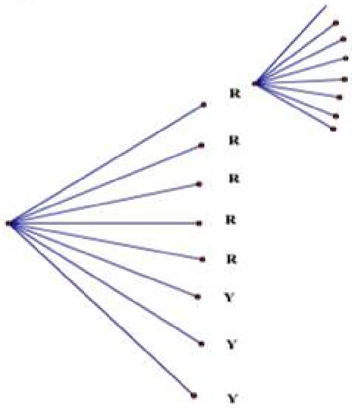
iii) P(at least two children are boys) = P(BBB or BBG or BGB or GBB) = $\frac{4}{8} = \frac{1}{2}$

iv) P(one child is a girl) = P(BBG or BGB or GBB) = $\frac{3}{8}$

a) Events without replacement

- ✓ In some situations we work with probabilities where ***object are replaced*** before being selected again.
- ✓ When objects are replaced we can often end up with tree diagrams that can become very complicated (like the one in the following example)

Example:
Suppose a bag contains 5 red counters and 3 yellow counters of same shape and size.
A counter is taken from the bag, replaced and then another counter is taken.



- ✓ To simplify the drawing of a tree diagram like this, we can show probabilities are along the branches of the tree diagram, as shown in the next example.

EXAMPLE 8

A bag contains five red counters (R) and three yellow counters (Y) of the same shape and size.
A counter is picked at random and then **replaced** in the bag. Another counter is picked and replaced in the bag.

a) Are these events of picking a red counter (R) and picking a yellow counter (Y) independent? Explain.
b) Are these events of picking a red counter (R) and picking a yellow counter (Y) mutually exclusive?
c) Draw a tree diagram and use it to calculate the probability of each event occurring
d) Determine
   i) The probability that a red counter was picked first followed by a yellow counter
   ii) The probability that a yellow counter was picked first followed by another yellow counter
   iii) The probability that a yellow counter was picked first followed by a red counter
   iv) The probability that a red counter was picked and then a yellow OR a yellow counter and then a red

<u>EXAMPLE 8</u> (continued)

<u>SOLUTION</u>:

a) Picking a red counter and picking a yellow counter are independent as the second event is not affected by the outcome of the first event. There are the same numbers of counters to choose from each time because the counters are replaced after being taken out.

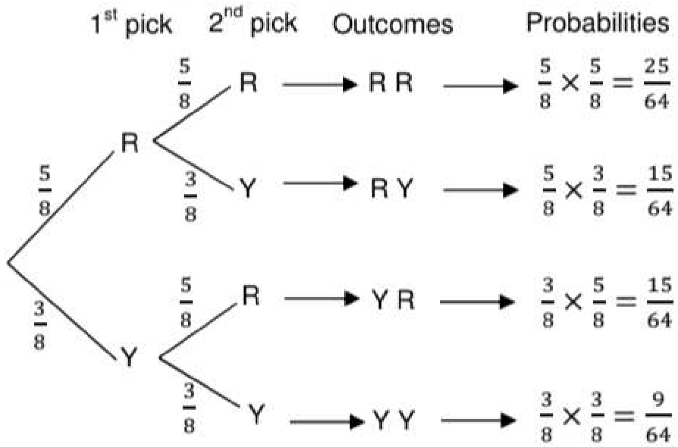
b) The events picking a red counter and picking a yellow counter are mutually exclusive – only one counter is picked at a time so the events cannot happen at the same time.

c) You can have one branch of the tree diagram for each counter, ***or*** we can write the probability of getting each colour counter on each branch.

→ There are 8 counters for each draw, five red and three yellow, so $P(R) = \frac{5}{8}$ and $P(Y) = \frac{3}{8}$. Write these probabilities along the branches.

→ To find out the probability of choosing a red and then choosing another red counter, work out P(R and R). To do this we follow the route along the branches and get $P(\text{R and R}) = \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$

→ We find the rest of the probabilities in the same way.



→ The four combined results cover all the possible outcomes, so their probabilities must add up to 1: $\frac{25}{64} + \frac{15}{64} + \frac{15}{64} + \frac{9}{64} = \frac{64}{64} = 1$. ***This is a useful check to make sure that you have worked out the probabilities correctly.***

d)

i) $P(\text{R and Y}) = \frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$

ii) $P(\text{Y and Y}) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$

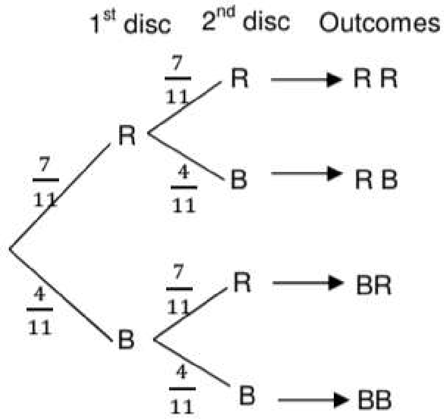
iii) $P(\text{Y and R}) = \frac{3}{8} \times \frac{5}{8} = \frac{15}{64}$

iv) $P\big((\text{R and Y})\ or\ (\text{Y and R})\big) = P(\text{R and Y}) + P(\text{Y and R})$

$= \frac{15}{64} + \frac{15}{64} = \frac{30}{64}$

EXERCISE 5.5

1) A coin is flipped and a dice is rolled.
   a) Are the two events independent? Explain.
   b) Draw a tree diagram to illustrate the situation where the first set of outcomes is H and T, and the second set of outcomes is 1, 2, 3, 4, 5 and 6.
   c) Manuel flips a coin and rolls a dice. Determine the probability as a common fractions in simplest form that Manuel gets:
      i) a H and a 4
      ii) a T and a 6
      iii) a H and an odd number
2) A jar contains 7 red discs and 4 blue discs.
   Two discs are selected, and then replaced. This means that the first disc is returned to the jar before the second disc is selected.
   a) Are these two events independent? Why or why not?
   b) A tree diagram is drawn to show the probabilities.



   Calculate the probability as a common fraction in simplest form that
      i) The two discs are red
      ii) The first disc is red and the second disc is blue
      iii) Only one of the discs is red
      iv) At least one of the discs is red.
3) A bag contains 3 red balls, 2 blue balls and 5 white balls.
   A ball is selected, its colour noted, and then it is replaced.
   A second ball is selected, its colour noted and then it is replaced.
   a) Are the two events independent? Explain.
   b) Draw a tree diagram to illustrate the situation. Fill in the probabilities along the branches of the tree diagram
   c) Find the probability as a common fraction in simplest form of:
      i) Selecting 2 blue balls
      ii) Selecting a blue ball and then a white ball
      iii) Selecting a red ball and then a blue ball
      iv) Selecting two balls of the same colour
      v) Selecting at least one blue ball

b) Events without replacement

✓ In some situations, we need to be able to work out probabilities when ***there is no replacement.***

EXAMPLE 9

Consider a jar containing seven red balls and four blue balls.

Two balls are selected and are NOT replaced.

This means that the first ball is NOT returned to the jar before the second ball is selected.

a) Draw a tree diagram to illustrate the situation. Fill in the probabilities along the branches.

b) Use the tree diagram to find the probability that

   i) Both of the balls are red.

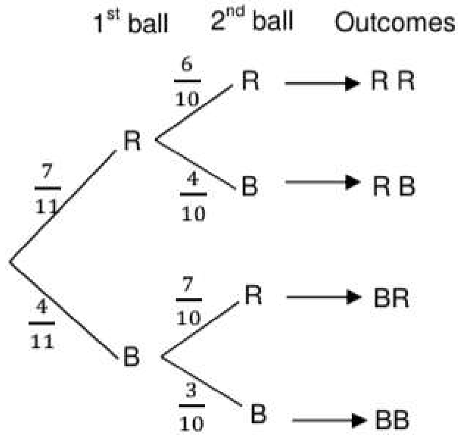
   ii) The first ball is red and the second ball is blue

   iii) At least one of the balls is red

SOLUTION:

a) There are seven red balls and four blue balls in the jar.

- The probability of choosing a red ball the first time = $\frac{7}{11}$ and the probability of choosing a blue ball the first time = $\frac{4}{11}$
- The balls are NOT replaced, so ***if a red ball is chosen the first time***, there are ten balls left in the jar with six of them being red and four of them blue.
  So the probability of choosing a red ball the second time = $\frac{6}{10}$ and the probability of choosing a blue ball the second time = $\frac{4}{10}$.
- ***If a blue ball is chosen for the first time***, there are ten balls left in the jar with seven of them being red and three of them being blue.
  So the probability of choosing a red ball the second time = $\frac{7}{10}$ and the probability of choosing a blue ball the second time = $\frac{3}{10}$.



b)

i) $P(\text{R and R}) = \frac{7}{11} \times \frac{6}{10} = \frac{42}{110} = \frac{21}{55}$

ii) $P(\text{Rand B}) = \frac{7}{11} \times \frac{4}{10} = \frac{28}{110} = \frac{14}{55}$

iii) P(at least one R) = P((R and R) or (R and B) or (B and R))

$= P(\text{R and R}) + P(\text{R and B}) + P(\text{B and R})$

$= \left(\frac{21}{55}\right) + \left(\frac{14}{55}\right) + \left(\frac{4}{11} \times \frac{7}{10}\right) = \frac{21}{55} + \frac{14}{55} + \frac{14}{55} = \frac{49}{55}$

<u>OR</u> P(at least one R) = $1 - P(\text{B and B}) = 1 - \left(\frac{4}{11} \times \frac{3}{10}\right) = 1 - \frac{6}{55} = \frac{49}{55}$

EXERCISE 5.6

1) Nomvula owns a collection of 30 CDs, of which 5 are gospel music (G). She selects one CD at random and then selects another CD ***without replacing the first CD***.
   a) How many CDs are NOT gospel music (NG)?
   b) Draw a tree diagram to illustrate DRAW 1: select a CD without replacing it and DRAW 2: select a second CD. Add in the probabilities along the branches.
   c) Nomvula selects two CDs at random. Determine the probability as a percentage correct to 1 decimal place:
      i) That both CDs are gospel music.
      ii) That the first CD is gospel music and the second one is not gospel music.

2) A bag contains one blue cube (B), two green cubes (G) and three red cubes (R). One cube is taken at random from the bag. Its colour is recorded and ***it is not replaced***. A second cube is then taken at random.
   a) Draw a tree diagram that represents all the possible outcomes in this situation. Fill in the probabilities along the branches.
   b) Use the tree diagram to determine probabilities as a common fraction in simplest form that
      i) two blue cubes are taken
      ii) two red cubes are taken
      iii) at least one green cube is taken
      iv) at most 1 red cube is taken.

3) Patrice enters both the 100 m sprint (S) and the long jump (J) at a school sports competition. The probability that he wins the 100 m sprint (SW) is 0,4. The probability that he wins the long jump (JW) is 0,7. The two events are independent.
   a) What is the probability (written as a decimal to 2 decimal places) that Patrice:
      i) Does not win the 100 m sprint (SL)
      ii) Does not win the long jump (JL)?
   b) Draw a probability tree diagram to show the situation.
   c) Use your tree diagram to determine the probability that, at the school event:
      i) Patrice wins both competitions
      ii) Patrice wins the 100 m sprint but does not win the long jump
      iii) Patrice does not win either competition
      iv) Patrice wins only one of the competitions.

VENN DIAGRAMS SHOWING THREE EVENTS

- ✓ We can solve probability problems involving ***three events*** using Venn diagrams.

EXAMPLE 10

During a survey at a certain school, 350 learners were asked about their favourite sport. It was found that

180 like Tennis (T)
180 like Rugby (R)
240 like Cricket (C)
78 like all three of these sports
60 like Tennis and Cricket but not Rugby
54 like Rugby and Cricket but not Tennis
30 like Tennis and Rugby but not Cricket

a) Calculate how many learners:
   i) Like Tennis only
   ii) Like Cricket only
   iii) Do not like any of the three sports.
b) Represent the given information in a Venn diagram
c) Calculate the probability (as a percentage correct to 2 decimal places) that one of these learners, chosen at random
   i) Likes rugby only
   ii) Likes none of these sports

SOLUTION:

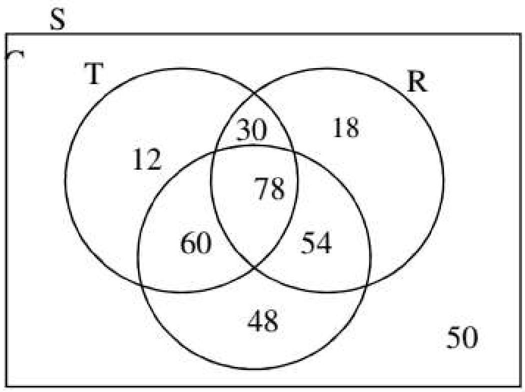
a)

i) 180 like Tennis, 78 like all 3 sports, 60 like playing Tennis and Cricket but not Rugby, and 30 like Tennis and Rugby but not Cricket.
So number of learners who like Tennis only = 180 – (78 + 60 + 30)
= 12

ii) 240 like Cricket, 78 like Cricket, Rugby and Tennis, 60 like Tennis and Cricket, 54 like Rugby and Cricket but not Tennis.
So number of learners who like Cricket only = 240 – (78 + 60 + 30) = 48

iii) The number of learners who do not like any of the three sports
= 350 – (12 + 30 + 18 + 78 + 54 + 48 + 60) = 50

b)



c)

i) P(Rugby only) = $\frac{n(Rugby\ only)}{n(sample\ set)} = \frac{18}{350}$ = 0,051 ... ≈ 0,05

ii) P(likes none of the 3 sports) = $\frac{n(likes\ none\ of\ the\ 3\ sports)}{n(sample\ set)} = \frac{50}{350}$ = 0,142 ... ≈ 0,14

EXAMPLE 11

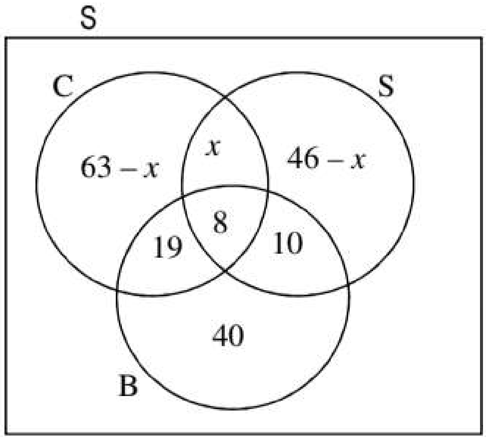
During the 2009 Census@School, 174 learners were asked about their favourite sport. It was found that:

90 prefer chess (C)
64 prefer swimming (S)
77 prefer boxing (B)
8 prefer all three sports: chess, swimming and boxing
18 prefer swimming and boxing
27 prefer chess and boxing
26 do not like any of the three sports
x prefer chess and swimming but don't like boxing

a) Draw a Venn diagram to illustrate the results of this survey
b) Calculate the value of x, the number of learners who prefer chess and swimming but don't like boxing.
c) What is the probability that a randomly selected learner prefers two types of sporting codes? Write your answer as a percentage correct to 1 decimal place.

SOLUTION:

a) STEP 1:
Fill 8 into the intersection of all 3 events.
STEP 2:
18 prefer swimming and boxing, so 10 prefer swimming and boxing but don't like chess
STEP 3:
27 prefer chess and boxing, so 19 prefer chess and boxing but don't like swimming
STEP 4:
x prefer chess and swimming but do not like boxing



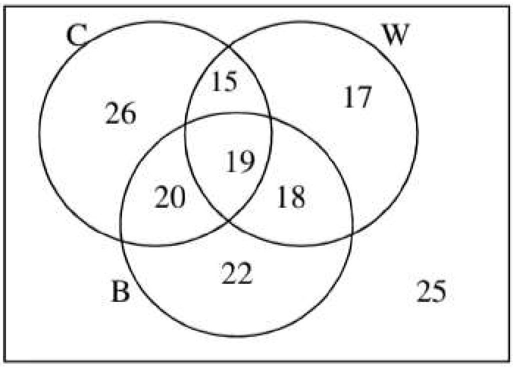
STEP 5:
77 prefer boxing, so the number who prefer boxing only = 77 – (19 + 8 + 10) = 40
64 prefer swimming, so the number who prefer swimming only
$= 64 - (x + 8 + 10) = 46 - x$
90 prefer chess, so the number who prefer chess only $= 90 - (x + 8 + 19) = 63 - x$

b) Add up all the values and set them equal to the total number of learners surveyed.
$(63 - x) + x + (46 - x) + 19 + 8 + 10 + 40 = 174$
$186 - x = 174$
$x = 12$
So 12 learners play chess and swim but do not box.

c) Number of learners who prefer two types of sporting codes = 19 + 8 + 12 + 10 = 49
P(prefer two types) = $\frac{n(prefer\ two\ types)}{n(sample\ set)} \frac{49}{174} \approx 0{,}281\ 6\ldots \approx 28{,}2\%$

EXERCISE 5.7

1) The Venn diagram shows the numbers of learners who came to school using different modes of transport. Some learners walk (W), others ride a bicycle (B) and others come by car (C).
   a) Are these three events independent?
   b) Determine n(S) if S is the sample set
   c) Use the information on the Venn diagram to find the probability (as a common fraction in simplest form) that one of these learners, picked at random
      i) Walks
      ii) Comes by car
      iii) Uses a bicycle
      iv) Walks only
      v) Does not walk
      vi) Walks or uses a bicycle



2) A survey of 80 people at a local food outlet indicated their meal preferences.
   - 44 like Pap and Wors (P)
   - 33 like Burgers (B)
   - 39 like Fried Chicken (FC)
   - x people like Pap and Wors and Burgers but not Fried Chicken
   - 23 like Pap and Wors as well as Fried Chicken
   - 19 like Burgers and Fried Chicken
   - 9 like Pap and Wors, Burger and Fried Chicken
   - 69 like at least one of these meals

   a) How many people did not like any of these meals?
   b) Draw the Venn diagram to represent the meal preferences of these learners.
   c) Determine the value of x.
   d) What is the probability, written as a percentage, that one of these people, chosen at random,
      i) Like Burgers?
      ii) Likes Fried Chicken only?
      iii) Like Burgers and Fried Chicken?
      iv) Likes Pap and Wors or likes Fried Chicken?

REFERENCES

Banks T and Alcorn D (2004) *Mathematics for AQA GCSE*, Causeway Press
Charles, E. (1986) *Advanced Mathematical Concepts*. Merrill Publishing Co.
Graham D and C (1996) *Mainstream Maths for GCSE*, Macmillan
The Answer Series Grade 12 Mathematics Paper 3
Speed, B. (1997) *Higher Mathematics for GCSE*, Collins Educational
Statistics South Africa (2010); *Census@School Results 2009*
2011 Census in South Africa
www.onlinemathlearning.com/probability-venn-diagram.html
http://www.haesemathematics.com.au/samples/ibmyp4_10.pdf