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Study Guide

Mathematics

Grade 10

Via Afrika Publishers

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Unit 1

Laws of exponents

1.1 What is an exponent?

In general, we define exponents as follows:

$x^n = x \times x \times x \times x \times \dots \times x,\ n \in \mathbb{N}$

This definition states that x is multiplied by itself n times. Here, x is the base, and n is the exponent.

1.2 The laws of exponents

- $b^m \times b^n = b^{(m+n)}$ (The product rule)
- $b^m \div b^n = b^{(m-n)}$ where $m \geq n$ and $b \neq 0$ (The quotient rule)
- $a^0 = 1$, when $a \neq 0$
- $(b^m)^n = b^{mn}$ (The power rule)
- $(ab)^n = a^n b^n$
- $a^{-1} = \frac{1}{a}$, when $a \neq 0$

Note that, for these rules, the base stays the same.

Unit 2

Simplifying exponents

In this unit, we revise how to use the laws of exponents to simplify expressions.

2.1 The product rule: $b^m \times b^n = b^{(m+n)}$

The product rule tells you that, when **multiplying** powers that have the **same base**, you can **add** the exponents.

Example

1 $x^5 \times x^3 = x^{5+3} = x^8$
2 $2^3 \times 2^{-2} = 2^{3+(-2)} = 2^1 = 2$
3 $2 \times 2^{3x} \times 2^2 = 2^{1+3x+2} = 2^{3x+3}$

2.2 The quotient rule: $b^m \div b^n = b^{(m-n)}$

The quotient rule tells you that you can **divide** powers with the **same base** by **subtracting** the exponents.

Example

1 $\frac{x^5}{x} = x^{5-1} = x^4$
2 $\frac{2^3}{2^5} = 2^{3-5} = 2^{-2}$
3 $\frac{12a^6b^6}{-4a \times^{-2} b^3} = -3a^{6-(-2)}b^{6-3} = -3a^8b^3$

An exponent is also called a power.

2.3 Rules for 0 and 1

Rules

$x^1 = x$	$1^n = 1$	$1^0 = 1$
$12^1 = 12$	$1^4 = 1 \times 1 \times 1 \times 1 = 1$	$x^0 = 1, x \neq 0$

Anything to the power zero is equal to 1, except 0^0, which is undefined.

Example

1 $16a^0, a \neq 0$
2 $16 \times 1 = 16$
3 $(2 + x)^0 = 1$
4 $(-3)^0 - 3^0 = 1 - 1 = 0$ $\quad [(-3)^0 = 1 \text{ and } 3^0 = 1]$
5 $\frac{3a^0}{2} + \left(\frac{b}{3a}\right)^0 = \frac{3}{2} + 1 = \frac{5}{2}$

Everything inside a bracket to the power of zero is always one.

2.4 Power rule: $(b^m)^n = b^{mn}$

The power rule tells us that to raise a power to a power, just **multiply the exponents**.

Example

1. $5(a^2)^3 = 5^{2\times 3} = 5^6$
2. $(27)^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^2 = 9$
3. $(a^{-3}.b^{\frac{1}{3}}.c)^3 = a^{-3\times 3}.b^{\frac{1}{3}} \times 3.c^{1\times 3} = a^{-9}.b.c^3 = \frac{bc^3}{a^9}$

Always express answers with positive exponents.

2.5 All inside brackets raised to a power: $(ab)^n = a^nb^n$ and $(\frac{a}{b})^n = \frac{a^n}{b^n}$

Example

1. $(5a^2)^3 = 5^3a^{2\times 3} = 125a^6$
2. $(2x^2y)^5 = 2^5x^{5\times 2}y^{1\times 5} = 32x^{10}y^5$
3. $\left(\frac{3a}{b^2}\right)^x = \frac{3^xa^x}{b^{2x}}$
4. $2^x.3^x = (2.3)^x = 6^x$
5. $(2a^6b)^3(3a^2b^3)^2 = (2^3a^{6(3)}b^3)(3^2a^{2(2)}b^{3(2)}) = (8a^{18}b^3)(9a^4b^6)$
   $= 72a^{18+4}b^{3+6} = 72a^{22}b^9$

Switch the denominator and numerator and multiply.

2.6 Negative power rule: $a^{-n} = \frac{1}{a^{-n}}$, provided that $a \neq 0$

Any non-zero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Example

1. $\frac{2^2}{3} = \frac{1}{2^2 - 3} = \frac{1}{12}$
2. $\frac{b}{a^5} = \frac{b}{\frac{1}{a^5}} = b \times \frac{a^5}{1} = a^5b$
3. $\frac{a^2x^2}{a^3x^1} = \frac{\frac{1}{a^2}x^2}{a^3\frac{1}{x}} = \frac{\frac{x^2}{a^2}}{\frac{a^3}{x}} = \frac{x^2}{a^2} + \frac{x}{a^3} = \frac{x^3}{a^5}$
4. $\frac{3x^3}{(3x)^2} = \frac{3}{9x^2.x^3} = \frac{1}{3x^5}$ $\quad \frac{4a^2b^3}{3ab^1} = \frac{4b^4}{3a^3}$

Unit 3

Solving equations that contain exponents

The key to solving equations that contain exponents is to ensure that the bases on both sides of the equation are the same. This enables us to compare the exponents, and so solve for the variable. The rule is:
If $a^x = a^n$, then $x = n$

Example

1 $3^x = 81 = 3^4$ [Rewrite 81 in exponential form: $81 = 3^4$]
Therefore, $x = 4$

2 $7^{x-3} = 49$ [Rewrite 49 in exponential form: $49 = 7^2$]
$7^{x-3} = 7^2$
$x - 3 = 2$ [Exponents are equal if bases are equal]
$x = 5$

3 $3.9^{x-1} = 27^{-x}$ [Make the bases the same by applying the laws of exponents]
$3.3^{2(x-1)} = 3^{3(-x)}$
$3^{1+2x-2} = 3^{-3x}$
$3^{2x-1} = 3^{-3x}$
$2x - 1 = -3x$
$2x + 3x = 1$
$5x = 1$
$x = \frac{1}{5}$

4 $2^x + 1 = 9$ [Move terms without x to the right side of the equation]
$2^x = 9 - 1$
$= 8$
$2^x = 2^3$ [$8 = 2^3$]
Therefore, $x = 3$ according to the principle of same bases.

3.1 Solving exponential equations using factorisation

Before we remove a common factor from an expression, we sometimes need to apply the laws of exponents in reverse.

Example

1 $2^{x+1} = 2^x = 24$ [Use the reverse of the product rule: $2^{x+1} = 2^x \times 2^1$]
$2^x.2 + 2^x = 24$ [Take out a common factor of $2x$]
$2^x(2 + 1) = 24$
$3.2^x = 24$
$2^x = \frac{24}{3}$

$2^x = 8$

$2^x = 2^3$

$x = 3$

2 $3^{x+1} = 3^{x-1} = 90$

$3^x(3^1 + 3^{-1}) = 90$ [Take out a common factor of $3x$]

$3^x(3 + \frac{1}{3}) = 90$

$3x\left(\frac{10}{3}\right) = 90$

$3x = 90 \quad \frac{3}{10}$

$3x = 27$ [The bases are the same, so exponents are equal]

$3x = 3^3$

$x = 3$

3 $2^{x+1} = 2^x + 4$ [Move terms containing x to the left side of the equation]

$2^{x+1} - 2^x = 4$

$2x(2 - 1) = 4$ [Take out a common factor of $2x$]

$2^x = 4(4 \div 1 = 4)$

$2^x = 2^2$

Therefore, $x = 2$

3.2 Exponents containing fractions

Fractions often make expressions look more complicated, but we use all the same rules and laws of exponents.

Example

1 $4^{\frac{5}{2}} = (2^2)^{\frac{5}{2}} = 2^5 = 32$

2 $(3^3x^{-12}) = 3^2x^{-8} = \frac{9}{x^8}$ [Remember to express exponents as positive numbers]

3 $\frac{1}{27}^{\frac{1}{3}} = (3^3)^{\frac{1}{3}} = 3^1 = 3$

4 $(27x^{12})^{\frac{2}{3}} = (3^3)^2.(x^{12})^2 = 3^2x^8 = \frac{9}{x^8}$

5 $\frac{2^ya^{2y}}{b^{\frac{y}{2}}} = \frac{2^{y\frac{2}{y}}.a^{2y\frac{2}{y}}}{b^{\frac{y}{2}\cdot\frac{2}{y}}} = \frac{2^2a^4}{b^1} = \frac{4a^4}{b}$

3.3 Three common mistakes to avoid when working with exponents

3.3.1 Brackets

The exponent next to a number only applies to that number. However, if the numbers are inside brackets, and the exponent lies outside the brackets, then the exponent applies to everything inside the brackets. For example:

$-3^2 \neq (-3)^2 \qquad -3 \times 3 \neq (-3)(-3)$

3.3.2 The product rule

1. The product rule only applies when the bases are the same. For example:
   $2^3 \times 3^2 \neq 6^5$
2. The product rule only applies when multiplying, not when adding. For example:
   $2^3 + 2^5 \neq 2^8$

3.3 Working backwards to go forwards

Remember, we need the bases to be the same before we can apply the laws of exponents. Therefore, first write all bases as a product of their **prime factors**, and then apply the laws of the exponents to those bases that are the same.

$27 = 3^3$ $81 = 3^4$ $32 = 2^5$ $36 = 2^2 \times 3^3$ $40 = 2^3 \times 5^1$

Example

$9 = 3^2$
$12 = 3 \times 4 = 3.2^2$
$6 = 2 \times 3$
$4 = 2^2$

Therefore:

$$\begin{aligned}\frac{9^n \times 12^{n+1}}{4 \times 6^n} &= \frac{(3^2)^n \times (3.2^2)^{n+1}}{2^2 \times (2.3)^n} \\ &= \frac{3^{2n} \times 3^{n+1} \times 2^{2n+2}}{2^2 \times 2^n \times 3n} \\ &= 3^{2n+n+1-n} \times 2^{2n+2-2-n} \\ &= 3^{2n+1} \times 2n\end{aligned}$$

Questions

1 Simplify the following expressions by applying the laws of exponents. All indices must be positive, and all variables represent positive rational numbers.

a $(2a^n - 2)^2$

b $5x^0 + 8^{\frac{-2}{3}} - \left(\frac{1}{2}\right)^{-2}.1x$

c $\frac{2a^{-2}3^a}{6^a}$

d $\frac{12x^2.x^4 - (-3x^3)^2}{3x^3}$

e $\frac{27^{x+1}.9^{x-1}}{3^{x-1}.81^{x+1}}$

f $\frac{2^{-3}x}{3^{-1}x^{-2}}$

g $\frac{4^{a-1}.8^{a+1}}{2^{a-2}.6a+1}$

h $\frac{1}{8^{\frac{-2}{3}}} - 3a^0 + 27^{\frac{1}{3}} - 1^{\frac{2}{3}}$

i $\frac{x^{2a-b}}{x^{b-2a}} \div \frac{x^{4a}}{x^{2b}}$

2 a Calculate b if $b = \frac{5^{n-3}}{5^{n+1}}$

b If $5x = f$, express 2.25^{x+1} in terms of f.

c If $3^m = 5$, find the value of

i 3^{m+2}

ii 9^{-2m}

d Remove the brackets and simplify: $(3^{2x} - 3)(3^{2x} + 3)$

3 Solve the following equations:

a $2^x + 1 = 2$

b $9^{x-2} = 27^{1-2x}$

c $5^{2x+1} = 0{,}04$

d $5^{x+1} + 5^x - 150 = 0$

4 An astronomer studying a region of space needs to determine the volume of a cubic region. The edges of the region measure 3×108 miles long. Find the volume.

Unit 1

Linear equations

1.1 Simplifying linear equations

Use the following steps to solve a linear equation:

Steps to follow	Example
Simplify by removing brackets and gathering like terms.	$3(5x - 2) = 2(4x + 11)$ $15x - 6 = 8x + 22$
Add and subtract terms on both sides until all variables are on the left of the equals sign and the constant terms are on the right.	$15x - 8x = 22 + 6$ $7x = 28$
Divide both sides by the coefficient of the variable (x).	$\frac{7x}{7} = \frac{28}{7}$ $\therefore x = 4$
Check the solution by substituting the value back into the original equation.	LHS: $3(5 \times 4 - 2) = 3(20 - 2) = 3(18) = 54$ RHS: $2(4 \times 4 + 11) = 2(16 + 11) = 2(27) = 54$ Since LHS = RHS, the solution is correct.

Special cases: Identities and false statements

The following statement is true for **all** values of x, and so we call it an **identity**.

$$\begin{aligned} 2(3x + 4) &= 6x + 8 \\ \therefore 6x + 8 &= 6x + 8 \\ \therefore 6x - 6x &= 8 - 8 \\ \therefore 0 &= 0 \\ \therefore x &\in \mathbb{R} \end{aligned}$$

The following statement can never be true, so we call it a **false statement**.

$$\begin{aligned} 2(3x + 4) &= 6x - 8 \\ \therefore 6x + 8 &= 6x - 8 \\ \therefore 6x - 6x &= -8 - 8 \\ \therefore 0 &= -16 \\ \therefore x &\notin \mathbb{R} \end{aligned}$$

1.2 Linear equations with fractions

When working with fractions, always remember that you cannot divide by 0.

Steps to follow	Example
Write down any restrictions (that is, values for x that will make the denominator 0): $x \neq \pm 3$	$\frac{3}{x^2 - 9} + \frac{2}{x + 3} = \frac{1}{x - 3}$
Multiply both sides of the equation by the LCM of all the denominators: $(x + 3)(x - 3)$	$\frac{3}{x^2 - 9} + \frac{2}{x + 3} = \frac{1}{x - 3}$ $\frac{3}{(x + 3)(x - 3)} + \frac{2}{x + 3} = \frac{1}{x - 3}$ $\therefore 3 + 2(x - 3) = x + 3$

Steps to follow	Example	
Simplify the equation (remove the brackets and collect the like terms together)	$\therefore 3 + 2x - 6 = x + 3$ $\therefore 2x - x = 3 - 3 + 6$ $\therefore x = 6$	
Check that the answer is allowed (by referring back to any restrictions)	Yes, $x \neq \pm 3$	
Check the solution by substituting it into the original equation.	LHS: $= \frac{3}{x^2 - 9} + \frac{2}{x + 3}$ $= \frac{3}{6^2 - 9} + \frac{2}{6 + 3}$ $= \frac{3}{36 - 9} + \frac{2}{9}$ $= \frac{3}{27} + \frac{2}{9}$ $= \frac{1}{3}$	RHS: $= \frac{1}{6 - 3}$ $= \frac{1}{3}$
	LHS = RHS, therefore the solution is correct.	

Example

1 $\frac{x + 2}{x^2 - 2x} = \frac{1}{x - 2}$ [Restriction: $x \neq 0$; $x \neq 2$]

$(x - 2)(x + 2) = 1(x^2 - 2x)$

$x^2 - 4 = x^2 - 2x$

$2x = 4$

$x = 2$

However, $x \neq 2$, therefore there is no solution.

2 $\frac{3}{a - 1} - \frac{2}{a + 1} = 0$ [Restriction: $a \neq 1$; $a \neq -1$]

$3(a + 1) - 2(a - 1) = 0$

$3a + 3 - 2a + 2 = 0$

$a = -5$

Check solution: $\frac{3}{-5 - 1} - \frac{2}{-5 + 1} = -\frac{1}{2} + \frac{1}{2} = 0$

3 $\frac{x - 3}{2x} + \frac{x - 2}{3x} = 1$ [Restriction: $x \neq 0$]

$3(x - 3) + 2(x - 2) = 6x$

$3x - 9 + 2x - 4 = 6x$

$-x = 13$

$x = -13$

Check solution: $\frac{-13 - 3}{-26} + \frac{-13 - 2}{-39} = \frac{16}{26} + \frac{15}{39} = \frac{8}{13} + \frac{5}{13} = 1$

1.3 Exponential equations

You learnt how to solve exponential equations in Chapter 2. In this section, we will show you how to solve an equation when you are not able to make the bases the same on both sides of the equation. In this case, you need to use the **trial-and-error method**, as shown in the following example.

<table>
<tr><th>Example</th><th>Help table</th></tr>
<tr><td>$2^{x-1} = 13$
so, $x - 1 = 3{,}7$
$x = 2{,}7$</td><td>
<table>
<tr><th>x</th><th>$x - 1$</th><th>Accuracy</th></tr>
<tr><td>5</td><td>16</td><td>Too big</td></tr>
<tr><td>4</td><td>8</td><td>Too small</td></tr>
<tr><td>3,5</td><td>11,313 …</td><td>Too small</td></tr>
<tr><td>3,6</td><td>12,125 …</td><td>Still too small</td></tr>
<tr><td>3,7</td><td>12,996 …</td><td>Very close</td></tr>
<tr><td>3,71</td><td>18,086</td><td>Too big</td></tr>
</table>
</td></tr>
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Unit 2

Quadratic equations

A quadratic equation has square term in it, for example, $x^2 + x - 12 = 0$. We say that the equation is of degree 2, or is a second-degree polynomial.

When you solve a quadratic equation, you may have 0, 1 or 2 solutions.

Steps to follow	Example
Remove any brackets and take all terms to the left of the equals sign so that there is only a 0 on the right. The equation will now be in the form: $ax^2 + bx + c = 0$	$2(2a^2 - 1) = 7a$ $4a^2 - 2 = 7a$ $4a^2 - 7a - 2 = 0$
Factorise the equation.	$(4a + 1)(a - 2) = 0$
We find the solution to the equation by letting each factor equal 0. Why? If $ab = 0$, then this statement will be true if $a = 0$ or if $b = 0$.	$(4a + 1) = 0$ or $(a - 2) = 0$ $\therefore a = -\frac{1}{4}$ or $a = 2$

Note: *Do not fall in the trap of trying to solve an equation without first removing the brackets and making the right-hand side of the equation equal to 0.*

Example

Solve for x: $x(x + 3) = 10$
Here, the solution is **not** $x = 10$ and $x + 3 = 10$.

The correct solution is:
$x^2 + 3x - 10 = 0$
$(x - 2)(x + 5) = 0$
$x = 2$ or $x = -5$

Unit 3

Literal equations

A literal equation is usually a formula, such as $E = mc^2$. Since there is more than one variable, we cannot solve a literal equation in the same way as an equation containing only one variable and numbers. What you are expected to be able to do is to change the subject of the formula. All this means is that you need to change the formula so that one variable appears on the left-hand side of the equation, and all the other variables and numbers are on the right-hand side of the equation.

Example

Steps to follow	Example
If needed, remove fractions by multiplying by the LCM (abc) of the denominators.	Solve for x: $\frac{x}{a} + \frac{1}{c} = \frac{x}{b}$ $bcx + ab = acx$
Write all the terms with the new subject on one side of the equals sign and all the other terms on the other side.	$bcx - acx = ab$
Factorise with the new subject (x) as a common factor.	$x(bc - ac) = ab$
Divide both sides by the coefficient ($bc - ac$) of the new subject.	$x = \frac{ab}{bc - ac}$

Let's work through a few more examples.

Steps to follow	Example
	Make b the subject of the formula: $3x = \sqrt{2b - y}$
If there is a square root (or cube root), square (or cube) both sides.	$(3x)^2 = (\sqrt{2b - y})^2$ $9x^2 = 2b - y$
Write all the terms with the new subject on one side of the equals sign, and all the other terms on the other side.	$2b = 9x^2 + y$
Divide both sides by the coefficient of the new subject.	$b = \frac{9x^2 + y}{2}$

Steps to follow	Example
	Make r the subject of the formula: $V = \frac{4}{3}\pi(R^3 - r^3)$
If needed remove fractions by multiplying by the LCM of the denominators.	$3V = 4\pi(R^3 - r^3)$
Multiply out the brackets to remove them.	$3V = 4\pi R^3 - 4\pi r^3$
Write all the terms with the new subject on one side of the equals sign, and all the other terms on the other side.	$4\pi r^3 = 4\pi R^3 - 3V$
Divide both sides by the coefficient of the new subject.	$r^3 = \frac{4\pi R^3 - 3V}{4\pi}$
Find the required root (cubed root) again, if necessary.	$r = \sqrt[3]{\frac{4\pi R^3 - 3V}{4\pi}}$

Steps to follow	Example
	Make h the subject of this formula: $T = 2\pi\sqrt{\frac{h}{g}}$
If there is a square root (or cube root), square (or cube) both sides after placing the root sign on one side by itself.	$T = 2\pi\sqrt{\frac{h}{g}}$ $\frac{T}{2\pi} = \sqrt{\frac{h}{g}}$ $\left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{\frac{h}{g}}\right)^2$ $\frac{T^2}{4\pi^2} = \frac{h}{g}$
Remove any fractions by multiplying both sides of the equation by the LCM of the denominators.	$4\pi^2 h = T^2 g$
Write all the terms with the new subject on one side of the equal sign and all the other terms on the other side. Divide both sides by the coefficient of the new subject.	$h = \frac{T^2 g}{4\pi^2}$

Unit 4

Simultaneous equations

Solving two equations simultaneously means finding one set of values for the variables that satisfy both equations. There are two ways to solve equations simultaneously (at the same time), namely the elimination method and the substitution method.

4.1 The elimination method

Steps to follow	Example
Rearrange the equations so that the coefficients of one of the variables are the same in both equations. **Tip: Number the equations** (The coefficient of y in both cases is now 4)	Solve the pair of equations simultaneously: $5x - 2y = 6$ .... (1) $3x + 4y = 14$ .... (2) Multiply equation 1 by 2: $10x - 4y = 12$ .... (3) [(1) × 2]
Add or subtract the equations to eliminate one of the variables.	Add (3) and (2): $13x = 26$
Solve for the unknown variable (x)	$x = 2$
Substitute this solution (x) back into either of the original equations to solve for the other variable (y).	Use $5x - 2y = 6$ .... (1) $5(2) - 2y = 6$ $-2y = 6 - 10$ $y = 2$ Therefore, $x = 2$ and $y = 2$.

4.2 The substitution method

Steps to follow	Example
	Solve the pair of equations simultaneously: $3x - y = 10$ .... (1) $3x - 4y = -8$ .... (2)
Rearrange at least one of the equations in the form $y = ...$ **Tip: Number the equations**	Use equation (1): $3x - y = 10$ $3x - 10 = y$ .... (3)
Substitute the value of y into the other equation.	Substitute (3) into (2): $3x - 4(3x - 10) = -8$
Solve for the unknown variable (x).	$3x - 12x + 40 = -8$ $3x - 12x = -8 - 40$ $9x = -48$ $x = \frac{48}{9}$ $x = 5\frac{1}{3}$
Substitute this solution (x) back into either of the original equations to solve for the other variable (y).	Use $3x - y = 10$: $3\left(\frac{16}{3}\right) - y = 10$ $16 - 10 = y$ $y = 6$ Therefore, $x = 5\frac{1}{3}$ and $y = 6$.

Unit 5

Linear inequalities

5.1 Illustrating inequalities

Linear inequalities have a range of solutions. As we will see later, we can represent the solution to an inequality graphically on a number line.

Example

Solve for x: $x + 3 < 2$

Even though we have an inequality rather than an equals sign, we use the same principles to solve the equation.

$x + 3 - 3 < 2 - 3$
$x < -1$

5.2 Solving inequalities

Note: *When multiplying or dividing an inequality by a negative number, you must reverse the direction of the inequality sign.*

Example

1 Solve for x: $\frac{x-4}{2} - \frac{5x+1}{3} \geq 1$
$3(x - 4) - 2(5x + 1) \geq 6$
$3x - 12 - 10x - 2 \geq 6$
$-7x \geq 6 + 12 + 2$
$-7x \geq 20$ [Divide by -7: reverse the inequality]
$x \leq \frac{20}{7}$

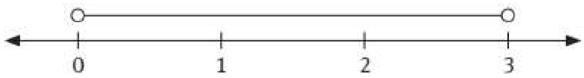
2 Solve for x: $3(x + 4) < 5x + 9$
$3x + 12 < 5x + 9$
$-2x < -3$ [Divide by -2: reverse the inequality]
$x > \frac{3}{2}$

As we mentioned earlier, we can show the solution of a linear inequality on a number line.

Example

Solve for n and indicate the solution on a number line: $-1 < 2 - n < 2$

$-1 - 2 < -n < 2 - 2$
$+3 > n > 0$
$0 < n < 3$



When the inequality is < or >, we show this as an open circle on the number line. When the inequality is ≥ or ≤, then we show this as a closed circle on the number line.

Questions

1 Solve the following equations and inequalities:

a $\frac{5a}{3} - 2 = \frac{a}{4} + 15$

b $\frac{m}{4} + 15 < \frac{5m}{3} - 2$

c $(2b + 1)(b + 8) = 27$

d $2 - \frac{1}{d-1} = \frac{3}{d+1}$

e $b^2 - 7b + 15 = 3$

f $\frac{5y-2}{3} + \frac{3y-1}{2} - \frac{y+1}{2} < -\frac{1}{3}$

g $\frac{3}{9h^2 - 16} - \frac{5}{3h^2 + 5h - 12} = \frac{2}{h(3h+4) + 3(4+3h)}$

2 Solve simultaneously: $2a = 24 + 7b$ and $3a + 5b = 5$

3 The formula for finding the volume, V, of material used to make a hollow sphere is given by $V = \frac{4}{3}\pi(R^3 - r^3)$, where R is the outer radius and r is the inner radius. Make R the subject of the formula, giving the expression in its simplest form.

4 Solve for in the following equations:

a $x^2 + 3(2x - 5) = x^2 - 2(7 + 3x)$

b $\frac{y - 3x}{x + z} = 2$

c $\frac{x+5}{6} + 1 = \frac{x - 5 - x}{3}$

d $\frac{3}{x^2 - 9} + \frac{2}{x+3} = \frac{1}{x-3}$

e $x - \frac{x-2}{3} \geq \frac{3}{2} + \frac{7x}{8}$

5. Solve for a and b:

$$7(a + 2) + 3(b - 5) = 34$$

and $3(a + 2) - 2(b - 5) = 8$