Via Afrika Mathematics

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Our Teachers. Our Future.

Grade 11 Study Guide

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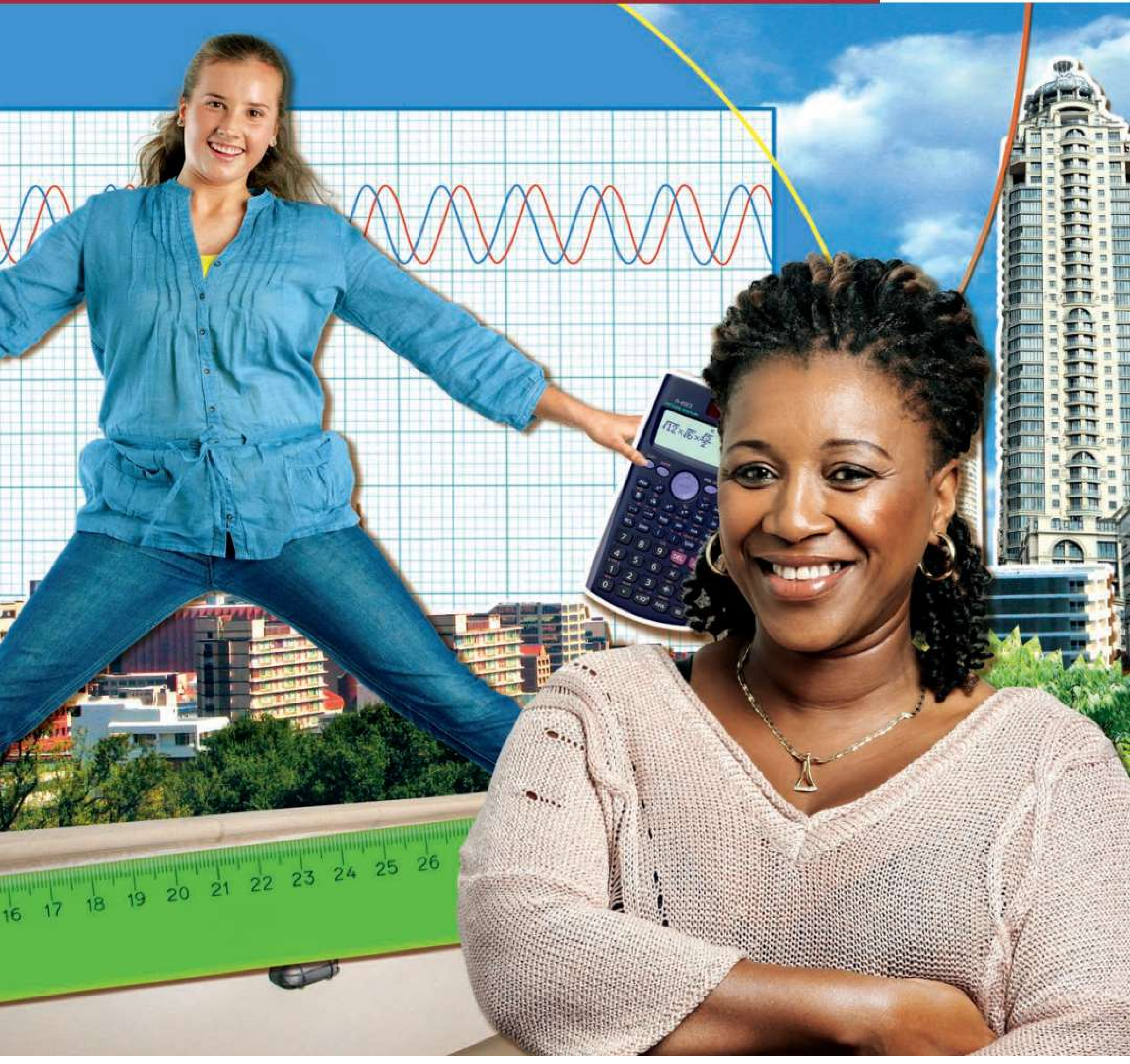


Chapter 4

Analytical geometry

Overview

Analytical Geometry is also known as Coordinate Geometry and combines Geometry and Algebra. In this chapter you will learn how to calculate the inclination of a line, how to find the equation of a straight line graph if certain coordinates and other information is given, and how to solve problems involving triangles.

1.1 Finding the gradient and inclination of a straight line

- The gradient of a line is its slope or steepness.
- The following formulae are useful to calculate different values:
  - The distance between points A and B:
    AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
  - The gradient of a line:
    $m = (y_2 - y_1)/(x_2 - x_1)$
  - The midpoint between two points:
    Midpoint = $(\frac{(x_1+x_2)}{2};\ \frac{(y_1+y_2)}{2})$
- The inclination of a line AB is the angle θ that is formed between the line and the positive x-axis.
- For acute angles $(0 < \theta \leq 90°)$ the gradient is positive and $\tan\theta$ is positive.
- For obtuse angles $(0 < \theta \leq 180°)$ the gradient is negative and $\tan\theta$ is negative.
- The inclination of AB $= \theta$ where $\tan\theta =$ gradient of AB (we denote this as m_{AB}).
- Two parallel lines with inclinations θ and α, have $m_1 = m_2$ and $\tan\theta = \tan\alpha$.
- Two perpendicular lines with inclinations θ and α have $m_1 \times m_2 = -1$ and $\tan\theta \times \tan\alpha = -1$.

Example 1

1 Determine the inclination of the line with gradient -2.

$\tan\theta = m$

$\therefore\ \tan\theta = -2$

$\therefore\ \theta = -63{,}43°$

2 Determine the inclination of line KY if K$(-12;\ 9)$ and Y$(6;\ 3)$.

$\tan\theta = m_{\text{KY}}$

$= (y_2 - y_1)/(x_2 - x_1)$

$= (3 - 9)/(-12 - 6)$ $= \frac{-6}{-18} = \frac{1}{3}$

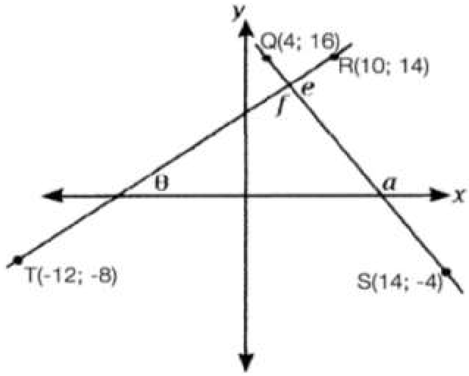
$\therefore\ \theta = 18{,}43°$

3 Calculate the gradient of the line with inclination $52{,}6°$.

$m = \tan\theta$ $\therefore\ m = \tan 52{,}6°$

$= 1{,}31$

4 Consider the sketch below and calculate the sizes of θ, α, e and f.



5 Determine whether LM is parallel or perpendicular to YZ in each case.

5.1 L(7; 6), M(−5; −1), Y(7; 2), Z(−5; −5)

5.2 L(−2; 5), M(−8; 2), Y(4; 3), Z(−2; 0)

5.3 L(6; 1), M(−1; −3), Y(6; 4), Z(−1; 0)

5.4 L(−6; 3), M(−3; −1), Y(7; 7), Z(3; 4)

5.5 L(−1; −6), M(3; −1), Y(7; −1), Z(2; 3)

The equation of a straight line

2.1 The gradient and the *y*-intercept

- When we are given the gradient and the y-intercept of a straight line, we use the equation $y = mx + c$ to find the equation with the given information. Remember that m is the gradient and c is the y-intercept.

2.2 The gradient and one point on the line

- When we are given the gradient and one point on the straight line, we use the equation $y - y_1 = m(x - x_1)$ to find the equation of the straight line with the given information.

2.3 Equation of a line going through two points

- When we are given two points on a line, we first have to calculate the gradient m and then use the same equation as in 2.2.

2.4 Equation of a line through one point and parallel or perpendicular to a given line

- To find the equation of a line such as the one described above, we need to follow the following simple steps:
  1. Write the equation of the given line in standard form to find m and c.
  2. Calculate the gradient of the line with the unknown equation by using the rules about parallel and perpendicular lines as discussed in Chapter 4, Unit 2 above.
  3. Substitute m and the coordinates of the given point into the standard equation for a straight line to find the equation.

Example 2

1 Determine the equation of the line with gradient -2 and y-intercept 17.

$y = mx + c$, therefore $y = -2x + 17$.

2 Determine the equation of the line with gradient 7 and that goes through point $(-2, 7)$.

$y - y_1 = m(x - x_1)$

$\therefore\ y - 7 = 7(x + 2)$

$\therefore\ y = 7x + 14 + 7$

$\therefore\ y = 7x + 21$

3 Find the equation of the line that goes through the points K$(3;\ -2)$ and L$(-4;\ -3)$.

First, we find the gradient:

$m_{KL} = (y_2 - y_1)/(x_2 - x_1)$

$= (-3 - (-2))/(-4 - 3)$

$= -1/-7$

$= \frac{1}{7}$

Second, we select any one of the two points. Let's use L$(-4;\ -3)$.

$y - y_1 = m(x - x_1)$

$\therefore\ y - (-3) = \frac{1}{7}(x - (-4))$

$\therefore\ y = \frac{1}{7}x + \frac{4}{7} - 3$

$= \frac{1}{7}x - \frac{17}{7}$

Useful information

The median of a triangle

- The median of a triangle bisects both the opposite side and the area of a triangle.
- If KL is the median of a triangle, we can calculate the coordinates of L by using the midpoint formula.
- We can use points K and L to find the equation of the median line.

The altitude of a triangle

- The altitude of a triangle is perpendicular to the opposite side.
- If we want to find the altitude PQ, we have to find the gradient of line ST that is perpendicular to it.
  We know $m_{ST} \times m_{PQ} = -1$, so we can calculate m_{PQ}. Now, we can use m_{PQ} and the coordinates of point P to find the equation of altitude PQ.

Perpendicular bisectors

- Two lines are perpendicular bisectors if they cross each other at an angle of $90°$ (perpendicular) and divide each other exactly in half (bisectors).
- To calculate the equations of two perpendicular bisectors:
  1. Find the coordinates of the point at the intersection using the midpoint formula.
  2. Calculate the gradient of one of the lines (let's call it line 1).
  3. Now, use the $m_1 \times m_2 = -1$ property to calculate m_2.
  4. Use the coordinates of the point of intersection and m_2 to find the equation of line 2.

Questions

Question 1

Determine the inclination of the lines given either the gradient or two points:

1.1	15	1.2	12/2
1.3	¾	1.4	−3
1.5	−0,125	1.6	Q(−4; −7) and R(−6; 2)
1.7	Q(8; −3) and R(5/7; 13)	1.8	Q(−3; −1) and R(1; −4)
1.9	Q(−2; −4) and R(−6; −1)	1.10	Q(5; −1) and R(−12; 7)

Question 2

Calculate the gradient of the line with inclination:

2.1	72°	2.2	250°
2.3	13°	2.4	−126°
2.5	−50,8°	2.6	185,15°

Question 3

Determine whether AB is parallel, perpendicular or neither, to CD in each case.

3.1 A(−1; 5), B(7; 8), C(5; −2), D(16; 3)

3.2 A(−5; −5), B(7; 2), C(−5; −1), D(7; 6)

3.3 A(−9; 6), B(−5; −7), C(9; −3), D(4; 7)

3.4 A(2; 5), B(3; 2), C(−9; 3), D(1; 3)

3.5 A(9; 1), B(8; 4), C(−4; 3), D(2; 5)

Question 4

Determine the equation of the line:

4.1 with gradient = 5, y-intercept = −17

4.2 with gradient = −2/5, y-intercept = 9

4.3 with gradient m = 7/3, passing through point (−13; 5)

4.4 with gradient m = −5, passing through point (−2; −1)

4.5 going through points (−4; 5) and (−1; −1)

4.6 going through points (13; −12) and (−5; 9)

4.7 BC if BC is parallel to 5y = x − 15 and passes through the point (−1; 4)

4.8 KT if KT is perpendicular to line BS, which has equation 2y = 6x − 7 and passes through point (4/3; 0,5)