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Study Guide

Mathematics

Grade 10

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Chapter 7

Polygons

Overview

In this chapter, we will first revise what you have learnt previously about polygons and the concepts of similarity and congruence in triangles. Then, we extend this knowledge to quadrilaterals, including the kite, parallelogram, rectangle, rhombus, square and trapezium. Here, you will investigate and learn about some of the important geometric properties of each of these shapes.

Here, you will learn:

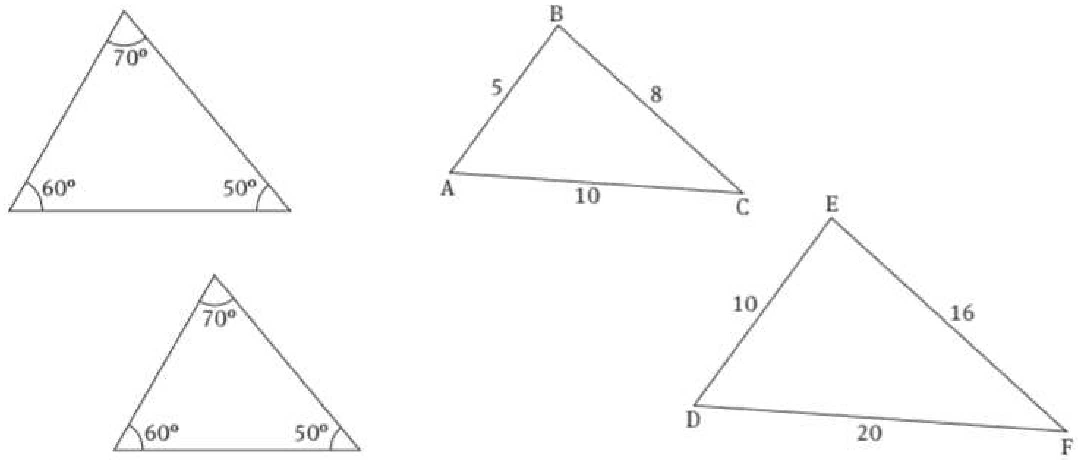
- how two triangles are similar if they are the same shape, but different sizes
- about corresponding angles and sides
- how two triangles are congruent if their corresponding sides and angles are the same size
- about different types of quadrilaterals (trapezium, parallelogram, rhombus, rectangle, square, kite)

Unit 1

Similar triangles

Similar figures have the same shape, but are different sizes. In the case of triangles:

- Corresponding angles are equal.
- Corresponding lengths are in the same ratio.



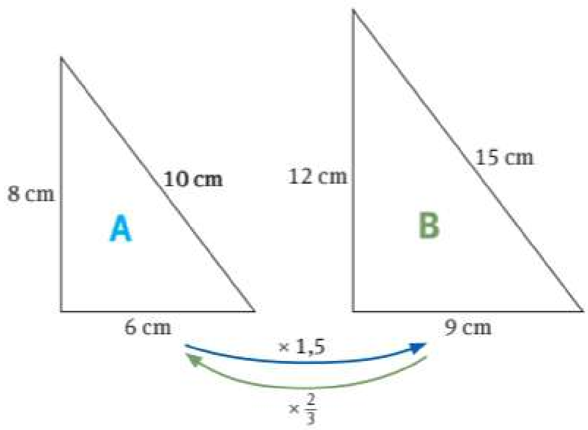
There are three ways to determine if two triangles are similar.

1. **AAA similarity**: If in two triangles, the corresponding angles are equal, the triangles are similar. (The 3rd set of angles will be equal, because of the sum of the interior angles of a triangle being 180°.)
2. **SSS similarity**: If the corresponding sides of two triangles are proportional the triangles are similar.
3. **SAS similarity**: If one angle of a triangle is equal to one angle of the other triangle and the sides containing these angles are proportional, the triangles are similar.

Finding the scale factor

We use the scale factor to find missing lengths of similar figures. Here, we can:

- Find a corresponding side in each shape when we know the length of both.
- Divide the length in the shape we are going to by the length in the shape we are coming from.



Example

To find the scale factor from A to B, divide the length in B by the length in A:
$9 \div 6 = 1{,}5$
The scale factor from A to B is 1,5.

Similarly, to find the scale factor from B to A, divide the length in A by the length in B:
$6 \div 9 = \frac{2}{3}$
The scale factor from B to A is $\frac{2}{3}$.

To prove that two triangles are similar, we have to show that **one** (not all) of the following statements is true:

- The three sides are in the same proportion.
- Two sides are in the same proportion, and their included angle is equal.
- The three angles of the first triangle are equal to the three angles of the second triangle.

Notes:

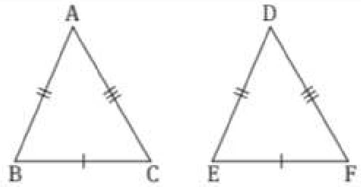
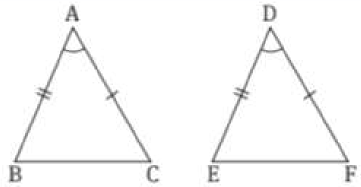
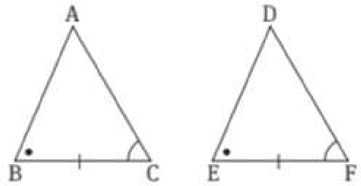
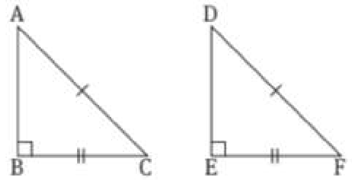
- The symbol for similarity is |||
- The order of the points in the names of the triangles is important. Equal angles of the two triangles must coincide.

Unit 2

Congruent triangles

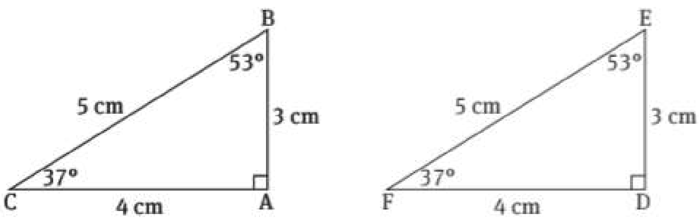
Congruent triangles have the same size *and* the same shape. The corresponding sides and the corresponding angles of congruent triangles are equal.

There are four methods to prove if two triangles are congruent.

1. The Side-Side-Side principle (SSS): The three corresponding sides of two triangles must be the same.	
2. The Side-Angle-Side principle (SAS): The two corresponding sides and the included angle of two triangles must be equal.	
3. The Angle-Side-Angle principle (ASA): The two corresponding angles and the included side of two triangles must be equal.	
4. The Right-Angle-Hypotenuse-Side principle (RHS): The hypotenuse and one corresponding side of the two right-angled triangles must be equal.	

Example

$\Delta ABC \equiv \Delta DFE$ because:



$\angle A = \angle D = 90°$
$\angle B = \angle E = 53°$
$\angle C = \angle F = 37°$

Notes:

- The symbol for congruency is ≡
- The order of the points in the names of the triangles is important. Equal angles of the two triangles must coincide.

Unit 3

Quadrilaterals

- A polygon with four sides is called a quadrilateral.
- The special types of quadrilaterals include the parallelogram, rectangle, rhombus, square, trapezoid and kite.
- It is important to understand the relationship of these figures and their properties in order to properly classify or identify a figure.
- A conjecture is an assertion that is likely to be true but has not been formally proven.

- To prove conjectures you need to complete a proof that will always be true.
- To show that a conjecture is false, show one counter example.

3.1 Parallelogram

Properties:
- opposite sides are parallel
- opposite angles are congruent
- opposite sides are congruent
- diagonals bisect each other
- consecutive angles are supplementary

Theorems:
1. If one pair of opposite sides of a quadrilateral is equal and parallel, then the quadrilateral is a parallelogram.
2. If both pairs of opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.
3. If both pairs of opposite angles of a quadrilateral are equal, then the quadrilateral is a parallelogram.
4. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

3.2 Rectangles

Properties:
- has all the properties of a parallelogram
- diagonals are congruent
- contains four right angles

Theorem:
1. If the diagonals of a parallelogram are equal, then the parallelogram is a rectangle.

3.3 Rhombus

Properties:

- has all the properties of a parallelogram
- four sides are equal in length
- diagonals are perpendicular
- diagonals bisect each pair of opposite angles

Theorems:

1. If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.
2. If each diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.

3.4 Square

Properties:

- has all the properties of a parallelogram
- diagonals are congruent and perpendicular
- is a rectangle with all sides congruent
- is a rhombus with four right angles

3.5 Kite

Properties:

- is a quadrilateral that has exactly two distinct pairs of adjacent congruent sides

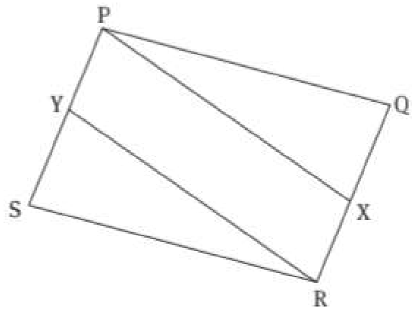
3.6 Trapezium

Properties:

- one pair of opposite sides that are parallel
- two parallel sides are called bases and the nonparallel sides are the legs
- isosceles trapezoid has one pair of congruent sides and congruent diagonals

Example

1 PQRS is a parallelogram with SY = QX.
Prove that PXRY is a parallelogram.



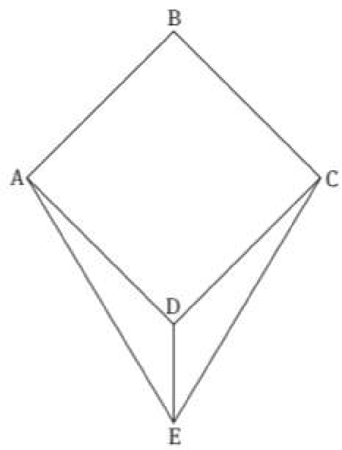
Steps to follow:

- Identify what is given and what to prove.
- Redraw the diagram, filling in the information given.
- Plan your proof using the drawing.
- Write up the proof explaining your reasoning using symbols and valid reasons in brackets.

QR = PS	(opp sides of parallelogram)
QX = YS = PY = XR	
And PY||XR	(given)

Therefore, PYRX is a parallelogram (one pair of opp. sides || and =)

2 ABCD is a rhombus and AE = CE.
Prove that ΔADE = ΔCDE.

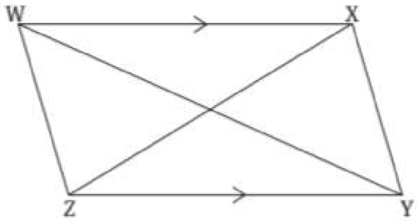


Use congruency to prove the required angles are equal. Remember, a rhombus is a parallelogram with all sides equal.

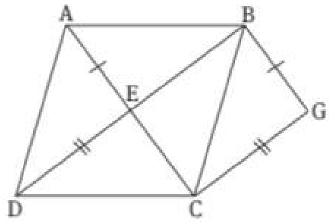
AE = CE	(given)
Then AD = DC	(all 4 sides equal)
DE = DE	(common side)
$\therefore$ ΔADE $\equiv$ ΔCDE	(SSS)
$\therefore$ ΔADE = ΔCDE	

Questions

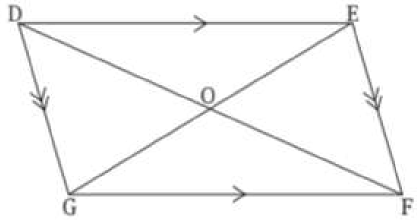
1 In the diagram, $WX \parallel ZY$ and $\Delta WXZ = \Delta WYZ$.
Prove that WXYZ is a parallelogram.



2 In the diagram alongside, ABCD is a rhombus with $BG = AE$ and $CG = DE$. Prove that EBGC is a rectangle.



3 Use the diagram alongside to prove that the diagonals of a parallelogram bisect each other.



4 In the diagram alongside, $AD = DB$ and $AE = EC$. Prove that:

a $\Delta ADE \mid\mid\mid \Delta ABC$

b $DE \parallel BC$

c $DE = BC$