Chapter 3

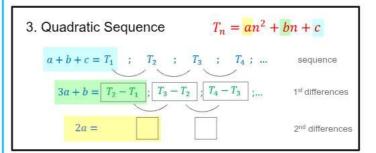
Number Patterns



Number Patterns

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Types of Number Patterns 1. Arithmetic $T_n = a + (n-1)d$ (linear) sequence $a : 1^{\text{st}} \text{ term } (T_1)$ d : constant difference(common difference) 2. Geometric $T_n = a. \, r^{n-1}$ (exponential) sequence $a : 1^{\text{st}} \text{ term } (T_1)$ r : common ratio



NB!

n: number of terms

 T_n : term "n" (the nth term)

 T_4 : term 4



(I.) Arithmetic (linear) sequence

$$* T_n = a + (n -)a$$

*
$$T_n = \alpha + (n -)d$$

• $d = T_2 - T_1$
• $T_2 - T_1 = T_3 - T_2$

$$T_1$$
 T_2 T_3 T_4

E.g.
$$\frac{1}{3}$$
; $\frac{5}{5}$; $\frac{7}{7}$; ...

Constant difference: +2

*
$$T_n = \alpha + (n-1)d$$

$$T_n = 1 + (n-1)(2)$$

$$T_n = 1 + 2n - 2$$

$$T_n = 2n-1$$



2.) Geometric (exponential) sequence

• r =
$$\frac{T_2}{T_1}$$

$$\cdot \frac{\mathsf{T}_2}{\mathsf{T}_1} = \frac{\mathsf{T}_3}{\mathsf{T}_2}$$

E.g. $T_1 \quad T_2 \quad T_3 \quad T_4$ $\times 2 \quad \times 2 \quad \times 2$

Common ratio: x2

$$\alpha = 3$$

*
$$T_n = \alpha \cdot r^{n-1}$$

$$T_n = 3 \cdot (2)^{n-1}$$

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3. Quadratic sequence

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$$* T_n = \alpha n^2 + b n + c$$

- 1st difference form an arithmetic sequence
- · 2nd differences remain constant

: quadratic sequence

a + b + c = 2; 4; 8; 14 3a + b = +2 + 4 + 6 1st differences 2a = +2 + 2 2nd differences

•
$$2a = 2$$

•
$$3a + b = 2$$

$$3(1) + b = 2$$

$$3 + b = 2$$

•
$$a + b + c = 2$$

$$1 + (-1) + c = 2$$

$$T_n = an^2 + bn + c$$

$$\therefore T_n = \ln^2 - \ln + 2$$

$$T_n = n^2 - n + 2$$

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Mixed sequences * consists of a combination of 2 sequences Eg. $\frac{1}{2}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{7}{8}$; ... (1); 3; (4); 6; (7); 12; (6); O 1;4;7;10;... separate sequences ⊙ bottom O 3; 6; 12; ... Calculate T₅₅: Find T_n : 1;4;7;10;...is Tn of top section: $T_1; T_3; T_5; \dots$ of the · d = 2 original sequence. 3;6;12;...is :. $T_n = 1 + (n-1)(2)$ T_2 ; T_4 ; T_6 ; ... of the $T_n = 1 + 2n - 2$ original sequence. $T_n = 2n - 1$ 8 O Copyright, Maths 4 Africa



Tn of the bottom section:

$$T_n = 2 + (n-1)(2)$$

$$T_n = 2 + 2n - 2$$

$$T_n = 2n$$

Therefore:

Tn of pattern:

$$T_n = \frac{2n-1}{2n}$$

In which of the two sequences will T55 be?

The term T_{55} has an uneven number.

Therefore we need to find term $(55 + 1) \div 2$ of this sequence:

$$a=1$$

 $d=3$ Arithmetic

• d = 3
• n = 28
$$(55+1) \div 2 = 28$$

$$T_n = \alpha + (n-1)d$$

$$T_n = 1 + (n-1)(3)$$

$$T_n = 1 + 3n - 3$$

$$T_n = 3n - 2$$



T₂₈ of "pink" sequence:

$$T_{28} = 3(28) - 2$$
= 82

 $\therefore T_{55}$ of mixed sequence :

= 82



Consider the following sequence: 8;5;2;...

- (a) Find the nth term.
- (b) Find the 25th term.
- (c) Which term is equal to -109?

Solutions:

$$\cdot d = -3$$

$$\therefore T_n = \alpha + (n-1)d$$

$$T_n = 8 + (n-1)(-3)$$

$$T_n = 8 - 3n + 3$$

$$T_n = -3n + 11$$

(b)
$$T_n = -3n + 11$$
 with $n = 25$

$$T_{25} = -3(25) + 11$$

$$T_{25} = -75 + 11 = -64$$

(c)
$$T_n = -3n + 11$$
 with $T_n = -109$

$$-109 = -3n + 11$$

$$3n = 11 + 109$$

$$3n = 120$$

$$n = 40$$



Consider the number pattern: -2; 3; 10; 19;...

- (a) Find the general term for this sequence.
- (b) Find the value of term 50.
- (c) Which term will have a value of 138?

Solutions:

(a)
$$-2;3;10;19;...$$

+5+7+9 \Rightarrow 1st differences not constant
+2+2 \Rightarrow 2nd differences are constant

: quadratic sequence

$$a+b+c = -2$$
; 3; 10; 19; ...

 $3a+b = +5+7+9$
 $2a = +2 + 2$

• $2a = +2$

• $a+b+c = -2$

•

$$T_n = an^2 + bn + c$$

$$\therefore T_n = \ln^2 + 2n - 5$$

$$\therefore T_n = n^2 + 2n - 5$$



(b) Find the value of term 50.

$$T_n = n^2 = 2n - 5$$
 with $n = 50$
 $T_n = (50)^2 + 2(50) - 5$
= 2595

(c) Which term will have a value of 138?

$$T_{n} = n^{2} + 2n - 5 \quad \text{with } T_{n} = 138$$

$$138 = n^{2} + 2n - 5$$

$$0 = n^{2} + 2n - 143$$

$$n = \frac{-b \pm \sqrt{b^{2} - 4\alpha c}}{2\alpha} \quad \cdot \alpha = +1$$

$$\cdot b = +2$$

$$\cdot c = -143$$

$$n = \frac{-(2) \pm \sqrt{(2)^{2} - 4(1)(-143)}}{2(1)}$$

$$n = -13 \quad \text{or} \quad n = 11$$

$$n/\alpha$$

(because term number is always positive)

: term II



Consider the sequence:

- (a) Find the general formula for the 1st differences.
- (b) Find the general formula for the sequence.

Solutions:

Sequence formed by 1st differences:

$$\cdot a = 8$$

 $\cdot d = -2$ Arithmetic sequence

$$T_n = a + (n-1)d$$

$$T_n = 8 + (n-1)(-2)$$

 $T_n = 8 - 2n + 2$

$$T_n = 8 - 2n + 2$$

$$T_n = -2n + 10$$

(b) Find the general formula for the sequence.

$$a+b+c = 20$$
; 28; 34; 38; 40; ...

 $3a+b=+8+6+4+2$
 $2a=-2-2-2$

•
$$2\alpha = -2$$

∴ $\alpha = -1$
• $\alpha + b + c = 20$
• $-1 + 11 + c = 20$
IO + C = 20
• $3\alpha + b = 8$
3(-1) + b = 8
• $-3 + b = 8$

$$\therefore T_n = -n^2 + IIn + IO$$

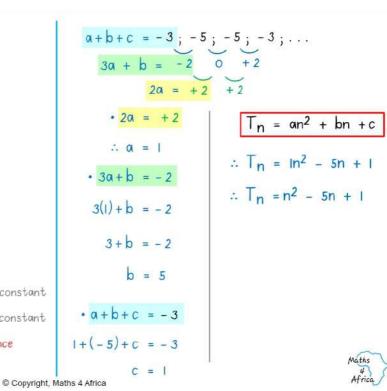


Consider the number pattern:

- (a) Find the general term for this sequence
- (b) Between which two terms of the quadratic sequence will the first difference be 72?

Solutions:

16



(b) Between which two terms of the quadratic sequence will the first difference be 72?

T₁ T₂ T₃ T₄

$$-3; -5; -5; -3; \dots \Rightarrow \underline{\text{Quadratic sequence}}$$

$$T_1 \quad T_2 \quad T_3$$

$$-2 \quad 0 \quad +2$$

$$+2 \quad +2 \quad \Rightarrow \underline{\text{Sequence formed by 1st}}$$

$$\underline{\text{differences}}$$

Sequence formed

by 1st differences:
$$-2$$
 0 $+2$; ... $+2$ $+2$ \Rightarrow constant

: Arithmetic sequence

•
$$\alpha = -2$$

· d = 2

$$T_n = \alpha + (n-1)d$$

= $-2 + (n-1)(2)$

$$= -2 + 2n - 2$$

$$T_n = 2n - 4$$

$$72 = 2n - 4$$

$$76 = 2n$$

$$38 = n$$

 \therefore T_{38} of sequence formed by 1st differences

 \therefore Between T38 and T39 of quadratic sequence

A quadratic number pattern has a constant second difference of 4 and $T_4=T_{14}=18$. Find the general term for this sequence.

$$T_n = an^2 + bn + c$$

$$2a = 4$$

$$\therefore \alpha = 2$$

$$T_n = 2n^2 + bn + c$$
 with $T_4 = 18$

$$T_4 = 2(4)^2 + b(4) + c$$

$$18 = 32 + 4b + c$$

$$-14 - 4b = c$$

$$T_n = 2n^2 + bn + c$$
 with $T_{14} = 18$

$$T_{14} = 2(14)^2 + b(14) + c$$

$$18 = 392 + 14b + c$$

Substitute (1) into (2):
$$-374 = 14b + (-14-4b)$$

$$-374 = 14b - 14 - 4b$$

$$-360 = 10b$$

$$-36 = b$$



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Now substitute b = -36 into \bigcirc :

$$c = -14 - 4b$$

$$T_n = 2n^2 - 36n + 130$$



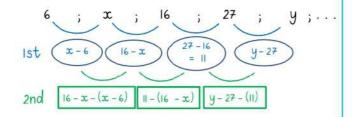
Find the values of x and y if

is a quadratic sequence.

Solution:

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Quadratic ⇒ 2nd differences constant



2nd differences remain constant:

Therefore we can equate the 2nd differences.

•
$$|16-x-(x-6)| = |1|-(16-x)$$

$$|6-x-x+6| = |1-|6+x|$$

$$22 - 2x = -5 + x$$

$$27 = 3x$$

$$9 = x$$

$$II - (16 - x) = y - 27 - (11)$$

$$11 - 16 + x = y - 38$$

$$33 + x = y$$

Now substitute x = 9:

$$33+9=y$$

$$42 = y$$

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