Via Afrika Mathematics

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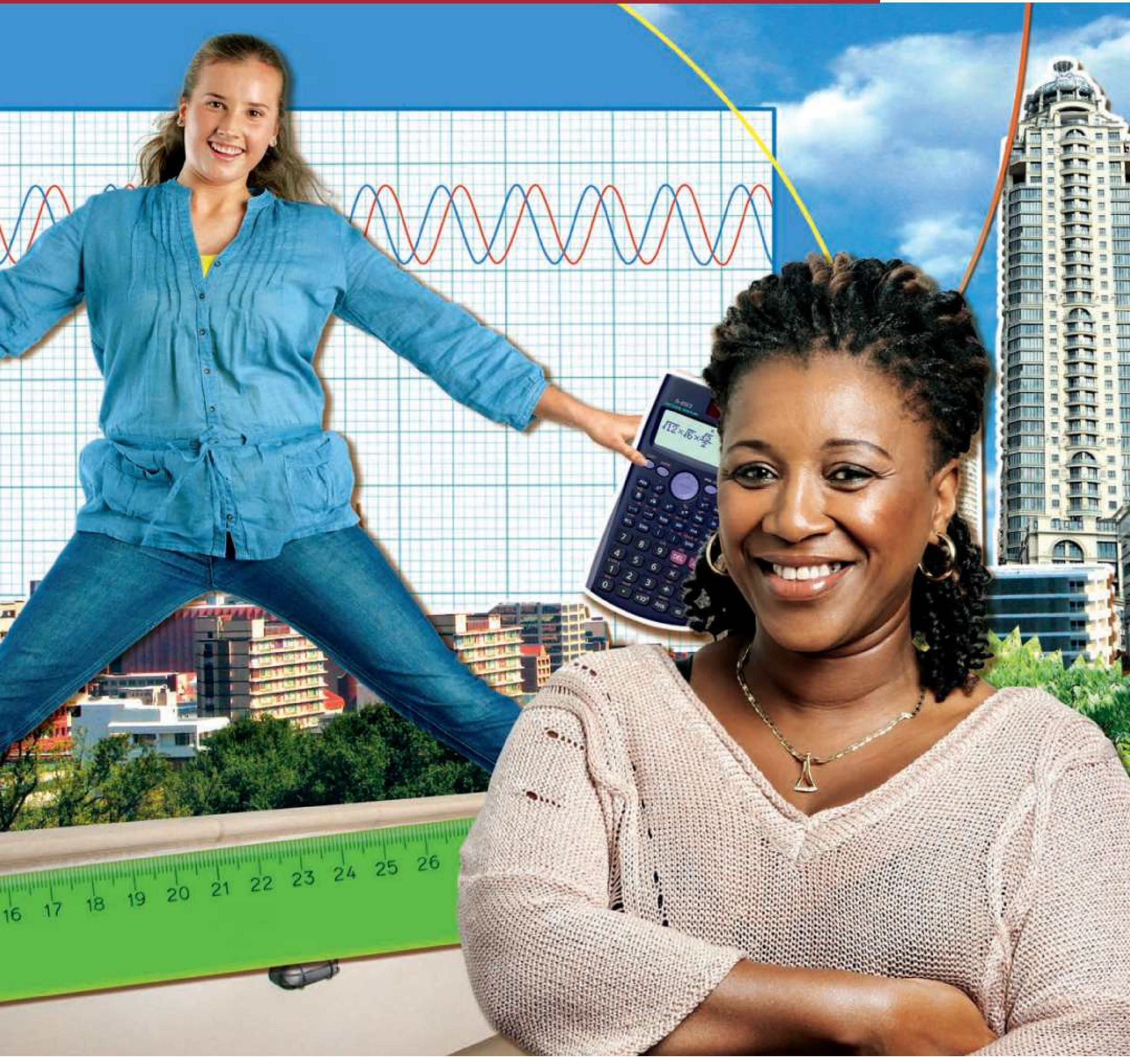
Grade 11 Study Guide

M. Pillay, L.J. Schalekamp, L.Bruce, G. du Toit,
C.R. Smith, L.M. Botsane, J. Bouman, A.D. Abbott



Chapter 2

Equations and inequalities

Overview

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In this chapter we will learn about factorisation, and how to complete perfect squares. We'll also have a look at the quadratic formula, and how to solve quadratic inequalities.

Solving quadratic equations by factorisation

1.1 Definitions

- A quadratic, or second-degree, equation has the standard form $ax^2 + bx + c$, where a, b and c are constants and $a \neq 0$.
- Its solutions are called roots, and to find the roots we use the zero-product principle:

  If $a \times b = 0$, then $a = 0$ or $b = 0$, or a and b are both 0.

1.2 Solving quadratic equations

- Solving quadratic equations involves the following steps:
  1. Simplify the equation.
  2. Write the equation in standard form.
  3. Factorise the equation.
  4. Apply the zero-product rule.

Example 1

Solve for x:

1 $-x^2 + x = 5 - 2x - 3x^2$

$\therefore 2x^2 + 3x - 5 = 0$

$\therefore (2x + 5)(x - 1) = 0$

$\therefore 2x + 5 = 0 \text{ or } x - 1 = 0$

$\therefore x = -\dfrac{5}{2} \quad \text{or } x = 1$

2 $(3x + 2)(x - 5) = 0$

$\therefore 3x + 2 = 0 \text{ or } x - 5 = 0$

$\therefore 3x = -2 \quad \text{or } x = 5$

$\therefore x = -\frac{2}{3} \quad \text{or } x = 5$

3 $3x^2 - 5x = 0$

$\therefore x(3x - 5) = 0$

$\therefore x = 0$ or $3x - 5 = 0$

$\therefore 3x = 5$

$\therefore x = \frac{5}{3}$

1.3 Quadratic equations involving fractions

- We solve quadratic equations involving fractions in exactly the same way as we did earlier.
- You have to remember one golden rule here, and that is *the denominator can never be zero*!
- This means that if the denominator contains a variable, then that variable has to be taken into account, e.g. if the denominator is $x - 9$ it means that $x - 9 \neq 0$, therefore $x \neq 9$.

$\frac{1}{x} \leftarrow$ The denominator is never = 0.

Example 2

Solve for x:

1 $(x - 3)/(x^2 + 3x + 2) - 4/(-x - 1) = 5/(x^2 - 4)$

$\therefore (x - 3)/(x^2 + 3x + 2) + 4/(x + 1) = 5/(x^2 - 4)$

[Multiply $(-x - 1)$ by -1]

$\therefore (x - 3)/(x + 2)(x + 1) - 4/(x + 1) = 5/(x^2 - 4)$

[Factorise denominator]

$\therefore x \neq -2$ and $x \neq 2$ and $x \neq -1$ [Sort out restrictions]

$\therefore (x - 3)(x - 2) + 4(x + 2)(x - 2) = 5(x + 1)$

[Divide denominator with LCD and multiply]

$\therefore (x - 3)(x - 2) + 4(x^2 - 4) = 5(x + 1)$ [Answer with numerator]

$\therefore x^2 - 5x + 6 + 4x^2 - 16 = 5x + 5$ [Multiply out the brackets]

$\therefore x^2 - 5x + 6 + 4x^2 - 16 - 5x - 5 = 0$ [Take everything to the LHS]

$\therefore 5x^2 - 10x - 15 = 0$ [Simplify to standard form]

$\therefore x^2 - 2x - 3 = 0$

$\therefore (x - 3)(x + 1) = 0$ [Factorise]

$\therefore x - 3 = 0$ or $x + 1 = 0$ [Zero-product principle]

$\therefore x = 3$ or $x = -1$, but $x \neq -1$ [Check for restrictions]

1.4 Quadratic equations involving square roots

- We remove the square root by squaring both sides of the equation.

Example 3

Solve for x:

1 $\sqrt{x - 1} - 1 = -x$

$\therefore \sqrt{x - 1} = -x + 1$ [Isolate the root]

$\therefore (\sqrt{x - 1})^2 = (-x + 1)^2$ [Square both sides]

$\therefore x - 1 = x^2 - 2x + 1$ [Multiply brackets out]

$\therefore x - 1 - x^2 + 2x - 1 = 0$ [Take everything to LHS]

$\therefore -x^2 + 3x - 2 = 0$ [Simplify to standard form]

$\therefore x^2 - 3x + 2 = 0$ [Multiply with a negative]

$\therefore (x - 1)(x - 2) = 0$ [Factorise}

$\therefore x - 1 = 0$ or $x - 2 = 0$ [Zero-product principle]

$\therefore x = 1$ or $x = 2$

1.5 Quadratic equations involving squares

- When squares are involved in quadratic equations, we take the square root on both sides to solve it.
- Remember to make sure that the expression containing the variable is on one side of the equation.

Example 4

Solve for x:

1 $(x - 2)^2 = 4$

$\therefore \sqrt{(x - 2)^2} = \pm\sqrt{4}$

$\therefore x - 2 = \pm 2$

$\therefore x = 2 \pm 2$

$\therefore x = 0$ or $x = 4$

2 $x^2 = 25$

$\therefore \sqrt{x^2} = \pm\sqrt{25}$

$\therefore x = \pm 5$

1.6 Solving quadratic equations using substitution

- When an equation seems too complicated to work with, look for a common factor in some of the expressions and substitute it with something simple, like the variable *k*.
- Now simplify the equation.
- Then substitute the common factor back in place of *k*, and solve the equation.
- The substitution method makes a complicated equation easier to work with.

Example 5

1 $2(x - 6)^2 - 5(x - 6) - 12 = 0$

Let $(x - 6) = m$

[Remember, you can use any variable here, as long as it is not already in the problem.]

Now we have $2m^2 - 5m - 12 = 0$ $\therefore (2m + 3)(m - 4) = 0$

$\therefore 2m + 3 = 0$ or $m - 4 = 0$

$\therefore m = \frac{3}{2}$ or $m = 4$, but $m = x - 6$

$\therefore x - 6 = \frac{3}{2}$ or $x - 6 = 4$

$\therefore x = \frac{3}{2} + 6$ or $x = 4 + 6$

$\therefore x = \frac{15}{2}$ or $x = 10$

Completing the square

2.1 Solve for *x* by completing the square

- A perfect square is a rational number (or an expression) that is equal to the square of another rational number (or expression).
- 16 is a perfect square: $4 \times 4 = 4^2 = 16$.
- $(x+5)^2$ is a perfect square: $(x+5)\times(x+5) = (x+5)^2 = x^2 + 10x + 25$.
- If an expression is not a perfect square we can make it one by adding the same term on both sides.

Example 6

Solve the following equations by completing the square.

1 $\quad x^2 - 8x - 6 = 0 \qquad x^2 - 8x = 6$

$\therefore x^2 - 8x + (\frac{b}{2})^2 = 6 + (\frac{b}{2})^2$

$\therefore x^2 - 8x + 16 = 6 + 16 = 22$

$\therefore (x-4)^2 = 22 \qquad \therefore \sqrt{(x-4)^2} = \pm\sqrt{22}$

$\therefore x - 4 = \pm\sqrt{22} \qquad \therefore x = 4 \pm \sqrt{22}$

$\therefore x = -0{,}6904$ or $x = 8{,}6904$

2 $\quad 2x^2 + 8x - 6 = 0$

$x^2 + 4x - 3 = 0 \qquad x^2 + 4x = 3$

$\therefore x^2 + 4x + (\frac{b}{2})^2 = 3 + (\frac{b}{2})^2$

$\therefore x^2 + 4x + 4 = 7$

$\therefore (x+2)^2 = 7$

$\therefore \sqrt{(x+2)^2} = \pm\sqrt{7}$

$\therefore x + 2 = \pm\sqrt{7}$

$\therefore x = \pm\sqrt{7} - 2$

$\therefore x = -4{,}6458$ or $x = 0{,}6458$

Unit 3

The quadratic formula

3.1 Solve for *x* by using the quadratic formula

- The quadratic formula is a general formula that we can derive. It gives us the roots of *any* quadratic equation. We derive it by completing the square:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Example 7

Solve for x by using the quadratic formula:

1 $\quad 2x^2 + 9x - 6 = 0 \qquad a = 2; \quad b = 9; \quad c = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore x = \frac{-9 \pm \sqrt{81 - 4(2)(6)}}{2(2)}$$

$$\therefore x = \frac{-9 \pm \sqrt{33}}{4}$$

$$\therefore x = -7{,}3723 \quad \text{or} \quad x = -1{,}6277$$

2 $\quad x^2 + 7x = 5$

$$\therefore x^2 + 7x - 5 = 0 \qquad a = 1; \quad b = 7; \quad c = -5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore x = \frac{-7 \pm \sqrt{49 - 4(1)(-5)}}{2(1)}$$

$$\therefore x = \frac{-7 \pm \sqrt{69}}{2} \therefore x = -7{,}6533 \quad \text{or} \quad x = 0{,}6533$$

Quadratic inequalities

4.1 Factorising inequalities

- The only difference between inequalities and equations is the sign, and everything you can do to an equation, you can also do to an inequality.
- But, remember that inequalities have additional rules.
- When dividing by a negative number to solve an inequality, the sign changes.
- Solutions to inequalities are represented on a number line.

Example 8

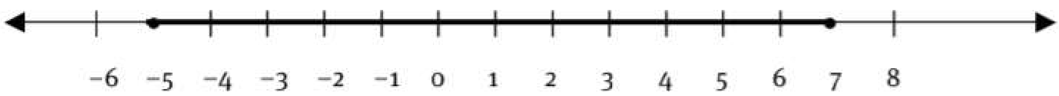
Solve for x:

1 $x^2 + 2x - 35 \leq 0$

$\therefore\ (x+7)(x-5) \leq 0$ ($x = -7$ and $x = 5$ are the critical values)

For $x + 7$:		For $x - 5$:	
$x - 7 = 0$	if $x = 7$	$x + 5 = 0$	if $x = -5$
$x - 7 < 0$	if $x < 7$	$x + 5 < 0$	if $x < -5$
$x - 7 > 0$	if $x > 7$	$x + 5 > 0$	if $x > -5$

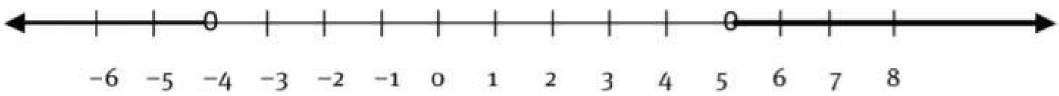
We can represent the result on a number line:



Thus the solution to $(x - 7)(x + 5) \leq 0$ is $-5 \leq x \leq 7$

2 $x^2 - x - 20 > 0$

$(x+4)(x-5) > 0$



The solution is $x < -4$ or $x > 5$

5.1 Solve for x and y by solving the equations simultaneously

- When you see two equations with two variables, you can solve for both the variables by solving the equations simultaneously.
- When these equations are drawn on a set of axes, the intersection(s) of the graphs will give you the value(s) of the variables.
- When you draw a parabola (a quadratic equation) and a straight line on the same set of axes, you could have either no intersections, or one intersection or two intersections. This means the variables x and y will have zero, one or two solutions.
- We solve simultaneous equations algebraically as follows:
  1. Write down both equations and number them: (1) for the straight line and (2) for the parabola.
  2. Make y the subject of the straight line equation, number it (3), and substitute (3) into (2).
  3. Use the value for x calculated above (remember, you may find zero, one or two values) and substitute it into (1) to solve for y.

Example 9

Solve for x and y in each case:

1 $3x - y = -9$ (1)

$x^2 + 2x - y = 3$ (2)

From (1): $y = 3x + 9$ (3)

Substitute (3) into (2):

$x^2 + 2x - (3x + 9) = 3$

$x^2 + 2x - 3x - 9 - 3 = 0$

$x^2 - x - 12 = 0$

$(x + 3)(x - 4) = 0$

$x = -3$ or $x = 4$

Substitute the x-values into (3):

For $x = -3$ For $x = 4$

$y = 3(-3) + 9$ $y = 3(4) + 9$

$y = 0$ $y = 21$

For $x = -3,\ y = 0$ For $x = 4,\ y = 21$

$(-3;\ 0)$ $(4;\ 21)$

2 $y - 6x = 12$ (1)

$x^2 + 4x - 9 = 5y$ (2)

From (1): $y = 6x + 12$ (3)

Substitute (3) into (2):

$x^2 + 4x - 9 = 5(6x + 12)$

$x^2 + 4x - 9 = 30x + 60$

$x^2 + 4x - 9 - 30x - 60 = 0$

$x^2 - 26x - 69 = 0$

$x^2 + 26x + 69 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$= -26 \pm \sqrt{676 - 4(1)(69)}/2$

$= (-26 \pm \sqrt{400})/2$

$x = -23$ or $x = -3$

Substitute the x-values into (3):

For $x = -23$ For $x = -3$

$y = 6(-23) + 12$ $y = 6(-3) + 12$

$y = -126$ $y = -6$

For $x = -23;\ y = -126$ For $x = -3;\ y = -6$

$(-23;\ -126)$ $(-3;\ -6)$

6.1 Solving word problems

- When real-life problems that need to be solved mathematically are given in words, the problems are called word problems.
- The four steps to solve word problems:
  1 Understand the problem.
  2 Make a plan (write the problem down in Mathematical terms).
  3 Work your plan (solve the problem).
  4 Assess your answer to the problem (check if your answer holds).

Example 10

Nomsa owns a spaza shop in Pretoria. She buys packets of peanuts for R1 500. She gives 20 of the packets away to loyal customers, and she sells the rest of the packets at R4 more (each) than what she paid for them. She makes a profit of R1 860 on the packets of peanuts. Determine how many packets she bought, the cost price and her selling price.

Step 1: Bought packets of peanuts ⇢ gave 20 away ⇢ sold the balance

Selling price = R4 more than cost price; Profit = R1 860

Step 2: Let x be the number of packets. Then:

Cost price $= 1\,500/x$ Packets sold $= x - 20$

Total money made $=$ R1 500 $+$ R1 860 $=$ R3 360

Selling price $=$ R3 $360/(x-20)$; Selling price $=$ cost price $+$ R4

Step 3: $\frac{3\,360}{x-20} = (\frac{1500}{x}) + 4$

$3\,360x = 1\,500(x-20) + 4x(x-20)$

$3\,360x = 1\,500x - 30\,000 + 4x^2 - 80x$

$4x^2 - 1940x - 30\,000 = 0$

$x^2 - 485x - 7\,500 = 0$ $a = 1$; $b = -485$; $c = -7\,500$

$x = (485 \pm \sqrt{(-485)^2 - 4(1)(-7\,500)})/2(1)$

$x = (485 \pm 515)/2$ $x = -15$ or $x = 500$

Step 4: Since Nomsa cannot buy a negative number of packets, -15 is invalid. Therefore, she bought 500 packets of peanuts.

Cost price $= 1\,500/500 =$ R3 Selling price $=$ R3 $+$ R4 $=$ R7

Unit 7

The nature of roots

7.1 The roots of an equation

- The roots of an equation are the values of the variables that satisfy that equation, for example

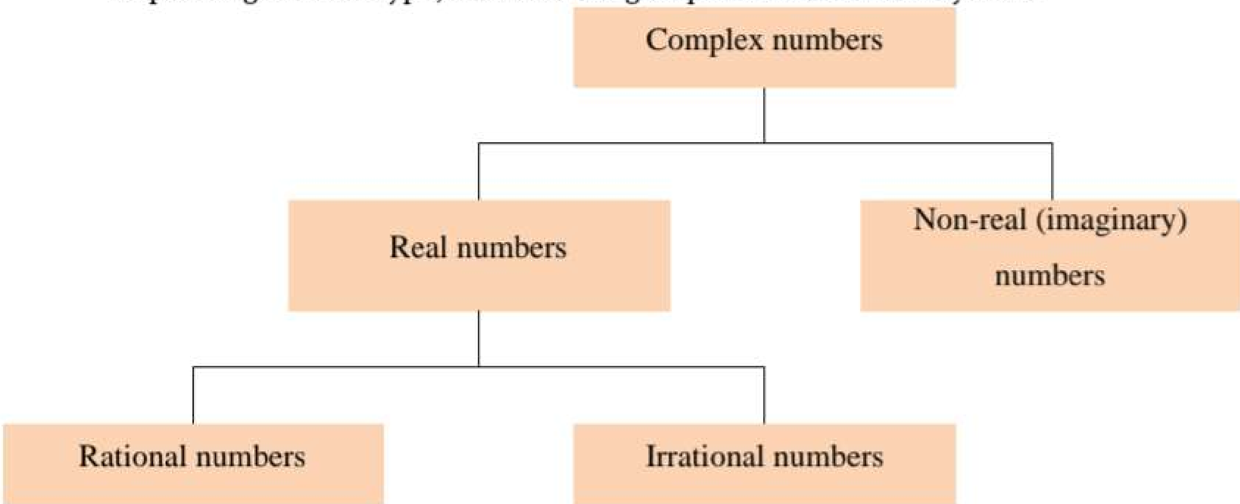
  $x^2 - 4 = 0$

  $(x + 2)(x - 2) = 0$

  Therefore $x = -2$ and $x = 2$ are the roots of the equation.

7.2 Quadratic theory

- Depending on their type, numbers are grouped into a number system:



- When we work out the roots of quadratic equations, we want to find out if the roots are real or non-real numbers, rational or irrational numbers and whether they are equal or unequal.
- We do this by using the value under the square root in the quadratic formula, $b^2 - 4ac$. This value is called the discriminant and is denoted by Δ(delta).
- Use the discriminant to define the roots.

  If:
  - $\Delta < 0$: the roots are non-real
  - $\Delta > 0$: the roots are real
  - $\Delta = 0$: the roots are equal
  - $\Delta =$ perfect square: the roots are rational
  - $\Delta \neq$ perfect square: the roots are irrational
- We can use the facts above to prove the nature of any quadratic equation's roots

  .

7.3 The nature of roots

- By using the formula for Δ as described above, we can find the roots of any quadratic equation.
- By calculating the value of Δ, we can calculate the value of an unknown variable in an equation if the nature of the roots is given, e.g. $kx^2 - 6x + 4 = 0$, where k is the unknown variable.
- Δ also gives us the ability to prove that the nature of the roots of a quadratic equation is of a certain type.

Example 11

1 If $x = 3$ is one of the roots of $x^2 + 3x + k = 0$, determine the value of k and the other root.

Substitute $x = 3$ into the equation to find k:

$3^2 + 3(3) + k = 0$

$\therefore 9 + 9 + k = 0$

$\therefore k = -18$

Now substitute $k = -18$ into the equation and factorise to find the other root.

$x^2 + 3x - 18 = 0$

$(x - 3)(x + 6) = 0$

$\therefore x = 3 \quad \text{or} \quad x = -6$

$\therefore$ The other root is -6.

2 Find the nature of the roots of:

2.1 $x^2 + 6x - 9 = 0$

$\Delta = b^2 - 4ac$

$= 36 - 4(1)(-9)$

$= 36 + 36$

$= 72$

Δ is positive, but not a perfect square, $\therefore$ roots are real, irrational and unequal.

2.2 $x^2 + 8x + 16 = 0$

$\Delta = b^2 - 4ac$

$= (8)^2 - 4(1)(16)$

$= 64 - 64$

$= 0$

Δ is positive, a perfect square and zero, $\therefore$ roots are real, rational and equal.

3 For which values of m will the equation $mx^2 - 4x + 5 = 0$have :

3.1 equal roots? 3.2 real roots? 3.3 non-real roots?

$mx^2 - 4x + 5 = 0$

$\Delta = b^2 - 4ac$ $= (-4)^2 - 4(m)(5)$

$= 16 - 20m$

3.1 For equal roots, make $\Delta = 0$:

$16 - 20m = 0$

$\therefore\ 20m = 16$

$\therefore\ m = \frac{4}{5}$

3.2 For real roots, make $\Delta \geq 0$:

$16 - 20m \geq 0$

$\therefore 16 \geq 20m$

$\therefore\ m \leq \frac{4}{5}$

3.3 For non-real roots, make $\Delta \leq 0$:

$16 - 20m \leq 0$

$\therefore 16 \leq 20m$

$\therefore\ m \geq \frac{4}{5}$

4 Prove that the roots of $-2x^2 + (a + b)x + 4 = 0$ are real for all real values of a and b.

$a = -2;$ $b = (a + b);$ $c = 4$

$\Delta = b^2 - 4ac$

$= (a + b)^2 - 4(-2)(4)$

$= (a + b)^2 + 32$

Now we must prove that Δ is greater than 0.

Proof: Any number squared is positive or 0. If we add a positive number to a positive number, the answer must always be positive.

$(a + b)^2 \geq 0$

$32 > 0$

$\therefore (a + b)^2 + 32 > 0$

$\therefore \Delta > 0$; $\therefore$ the roots are real.

Questions

Question 1

Solve the following quadratic equations:

1.1 $(x - 2)(x + 7) = 0$

1.2 $(2y - 3)(y + 5) = 0$

1.3 $x^2 + 21x + 10 = 0$

1.4 $3k(k + 4) = 0$

1.5 $9x^2 - 5x = 0$

1.6 $3k(1 - k) + 5(k + 1) = 0$

1.7 $(3p - 2)(p + 1) + 2 = 0$

1.8 $b(b + 5) = 6$

1.9 $(x - 2)(x + 2) = 6(3x + 5)$

1.10 $4(x - 1)(x + 1) = 3(2 - x) + 5$

Question 2

Solve for x:

2.1 $5/(x - 1) = x/(x + 1)$

2.2 $3/(2x - 6) + x/(x - 3) = 0$

2.3 $(x + 2)/(x - 3) = 7 + 2/(x - 3)$

2.4 $(x - 1)/(x^2 - 9) = 2/(4(x + 3)) - (1 - x)/(x + 1)$

2.5 $1/(x^2 + 2x + 3) + 3/(x^2 + x + 2) = 2/(x^2 + 2x + 3) - 2/(2 - x^2)$

2.6 $\frac{3}{4} + (x + 3)/(2x + 3) = (4x + 3)/(x + 5)$

Question 3

Solve for x:

3.1 $\sqrt{6x + 5} = x$

3.2 $\sqrt{x - 5} = x - 2$

3.3 $\sqrt{2x-3}-x=0$

3.4 $\sqrt{x-6}+x+4=0$

3.5 $2=\sqrt{x^2-27}$

3.6 $\sqrt{x-1}=\sqrt{4x-2}$

Question 4

Solve for x:

4.1 $x^2=81$

4.2 $x^2=27$

4.3 $x^2-16=0$

4.4 $-x^2+49=0$

4.5 $(x+4)^2=48$

4.6 $5(x+5)^2=125$

4.7 $3(x+4)^2-12=0$

4.8 $x^2=(2x-3)^2$

4.9 $3(x-2)^2-16=2$

4.10 $-x=(\frac{1}{2}x+2)^2-6$

Question 5

Solve for x:

5.1 $(x^2-3x)^2-2(x^2-3x)-5=0$

5.2 $(x^2-x)^2=14(x^2-x)+15$

5.3 $\sqrt{x-2}+4=5/\sqrt{x-2}$

5.4 $(\frac{3}{x}+x)^2+\frac{3}{x}-x=19$

5.5 $5(2x^2+x-1)=20(2x^2+x-1)+8$

5.6 $2(x+3)^2-3(x+3)-4=2$

5.7 $24/(3(x-2)=7/(9(x+6))-3$

Question 6

6.1 Solve for p:

$p^2-p-12=0$

6.2 Hence solve for x:

$(x^2+3x)^2-(x^2+3x)-12=0$

Question 7

Given $(a+3)(b+4)=0$, solve for b if $a=-7$.

Question 8

If $(y-2)(x^2+25x-6)=0$, determine y if:

8.1 $x=-7$

8.2 $x=12$

Question 9

Solve for x by completing the square:

9.1 $x^2+2x+4=0$

9.2 $x^2-5x+15=0$

9.3 $x^2-x+20=0$

9.4 $x^2+6x=-5$

9.5 $13x=10+x^2$

9.6 $x^2-6x=3$

Question 10

Solve for y:

10.1 $3y(y-3)-3=0$

10.2 $8y^2-2y+5=0$

10.3 $-y^2-3y=-2$

10.4 $6=3y(y+4)$

10.5 $\frac{1}{8}y^2-2y+5=0$

10.6 $(y+4)^2+(y-4)(y+3)=2$

10.7 $(y-3)(4y+2)-9=0$

10.8 $y(y+1{,}5)+3y=3y(y-9)+0{,}3$

10.9 $5(y+3)=3y^2-5$

10.10 $2/(y(y+2))=3/(y+2)-y/(y+1)$

Question 11

Solve for x:

11.1 $x^2+7x-14>0$

11.2 $2x^2-9x-17<0$

11.3 $x^2>3x-7$

11.4 $-5x\geq -3x^2+8$

11.5 $x^2\leq 6x-9$

11.6 $3x+2<5x-6x^2$

Question 12

Solve for each of the variables in the following equations:

12.1 $9x - y = 12$
$13y^2 + 4xy - x = 0$

12.2 $y + 6 - 13x = 0$
$xy = -13$

12.3 $y + x - 5 = 0$
$x^2 - 4 = y - 6x$

12.4 $x + y = 5$
$3x^2 + 5x - 19 = 0$

12.5 $3a + 5b = 17a - 6$
$(2a - 4b)(a + b) = 0$

12.6 $n + m = -13$
$n = 3m^2 + 4m - 6$

Question 13

13.1 The sum of two numbers is 15, while the product of the same numbers is 36. Find the two numbers.

13.2 Two fruit pickers are picking fruit. Fruit picker A picks 750 pieces of fruit in 4 hours, while fruit picker B picks 750 pieces of fruit in 3 hours. How long will it take them to pick 750 pieces of fruit if they work together?

13.3 Two trains travel 1 600 km from Cape Town to Pretoria. Train A is 15 km per hour faster than train B, and arrives at Pretoria Station 2 hours ahead of train B. Determine the speed of train B.

13.4 Nomsa and Emily own a home cleaning business. Nomsa takes 2,5 hours longer than Emily to clean a home. If they work together, they take 6 hours to clean a home. How long will it take each of them to clean a home on their own?

Question 14

14.1 Given that -6 is one of the roots of the equation $x^2 + mx - 30 = 0$

14.1.1 Determine the value of m.

14.1.2 Now determine the other root of the equation.

14.2 If $-\frac{7}{2}$ is one of the roots of $x^2 + kx - 7 = 0$, determine the value of k and the other root.

14.3 Determine the value of p that will make the following expression a perfect square: $x^2 - 10x + p$

14.4 Determine the value of n that will make this equation a perfect square: $3x^2 - 3x + n$

Question 15

Find the value of the discriminant and describe the nature of the roots without solving the equation.

15.1 $3x^2 - 6x + 4 = 0$

15.2 $x^2 + 64 = 12x$

15.3 $3x = 5x^2 - 6$

15.4 $x - 3 = -12x^2$

Question 16

For which values of k will $3x^2 - 3x + k$ have equal roots?

Question 17

Find the values of r for which $x^2 - 3rx + r = 0$ has real roots.

Question 18

Prove that $x^2 + (k-1)x + k = 0$ has rational roots for all rational values of k.

Question 19

Show that the roots of $x^2 + m = (m+2)x$ will be real for all real and unequal values of m.