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Study Guide

Mathematics

Grade 10



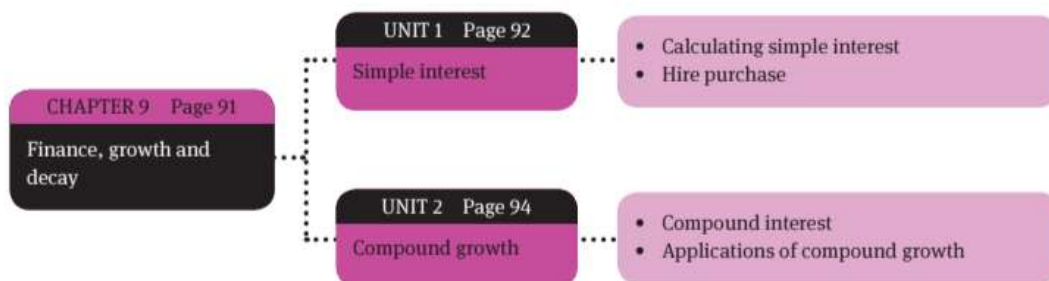
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Finance, growth and decay

Overview

Interest is a fee paid to borrow money. It is usually charged or paid as a percentage of the total amount borrowed or invested. An interest rate is the cost stated as a percentage of the amount borrowed/invested per period of time, usually one year.

Simple interest and compound interest are examples of how interest can be calculated. The compound interest formula can also be used in other areas in which compound growth occurs, such as inflation, exchange rates and population growth.



Simple interest

Simple interest is calculated as a constant percentage based only on the amount of money invested/borrowed. Here, we do not receive or pay interest on any interest accrued during the period of the loan or investment.

1.1 Calculating simple interest

- Simple interest is calculated on the original principal amount only.
- The formula is: simple interest = $P \times i \times n$, where
 - P = principal (original amount borrowed or loaned)
 - i = interest rate for one period
 - n = number of periods

Example

- 1 You borrow R10 000 for 3 years at 5% simple interest, paid annually.
 $P = \text{R}10\,000$; $n = 3$; $i = 0,05$ $(5\% = \frac{5}{100} = 0,05)$
 interest = $P \times i \times n = 10\,000 \times 0,05 \times 3 = \text{R}1\,500$
- 2 a You borrow R10 000 for 60 days at 5% simple interest per year (assume a 365 day year).
 $P = \text{R}10\,000$; $n = (60 \div 365)$; $i = 0,05$
 interest = $P \times i \times n = 10\,000 \times 0,05 \times (60 \div 365) = \text{R}82,19$
Note: interest is calculated per year, but the loan is only for 60 days. This is a fraction part of the year. Therefore, $n = (60 \div 365)$.
- b After the 60 days, you have to pay the money back, with interest.
 $A = P + \text{simple interest}$
 $= \text{R}10\,000 + \text{R}82,19$
 $= \text{R}10\,082,19$

$$A = P + P \times i \times n \text{ (take out a common factor)}$$

$$A = P(1 + in)$$

Example

An amount, b , is invested at 7,5% p.a. for 3 years to yield simple interest amounting to R500. Find the value of b .

$P \times i \times n$ = Simple interest

$$P = b; i = \frac{7,5}{100} = 0,075; n = 3$$

$$\frac{b \times 7,5 \times 3}{100} = 500$$

$$b = \text{R}2\,222,22$$

1.2 Hire purchase

Hire purchase is a system in which a buyer pays for something in regular instalments while enjoying the use of it. During the repayment period, the buyer does not own whatever was bought. The buyer only takes ownership once the loan has been paid in full.

Example

Jean wants to buy a motorbike for R10 500. He pays a deposit of R2 000. He wishes to pay the balance using a hire purchase agreement over 3 years. The interest charged on the loan is 18% per annum. Included in the agreement, is an insurance cost of 2% per annum on the purchase price of the motor bike. Calculate his monthly instalment.

Balance owing after deposit = R8 500

Insurance = (2% of R10 500) \times 3 years = R630

Interest on R8 500 = $P \times i \times n$

$$= \text{R}8\,500 \times 18 \times 3$$

$$= \text{R}4\,590$$

$$\text{Monthly Payment} = \frac{8\,500 + 630 + 4\,590}{36}$$

$$= \text{R}381,11$$

Compound growth

2.1 Compound interest

- Compound interest is calculated each period on the original principal and all interest accumulated during past periods.
- Although the interest may be stated as a yearly rate, the compounding periods can be yearly, semi-annually, quarterly, or even continuously.
- You can think of compound interest as a series of back-to-back simple interest contracts. The interest earned in each period is added to the principal of the previous period to become the principal for the next period.

For example, you borrow R10 000 for three years at 5% annual interest compounded annually:

- interest year 1 = $P \times i \times n = 10\,000 \times 0,05 \times 1 = 500$
- interest year 2 = $(P_2 = P_1 + i_1) \times i \times n = (10\,000 + 500) \times 0,05 \times 1 = 525$
- interest year 3 = $(P_3 = P_2 + i_2) \times i \times n = (10\,500 + 525) \times 0,05 \times 1 = 551,25$
- Total interest earned over the 3 years = $500 + 525 + 551,25 = 1\,576,25$

Compare this to 1 500 paid over the same period using simple interest.

The formula for compound interest is $A = P(1 + i)^n$, where

- A = the amount at the end of a loan/investment period
- P = the principle (initial amount borrowed/invested)
- i = the interest rate (expressed as a decimal number)
- n = the period of the loan/investment

Example

An amount of R1 500,00 is deposited in a bank paying an annual interest rate of 4,3%, compounded *quarterly*. What is the balance after 6 years?

Using the compound interest formula, we have:

$$P = 1\,500, i = 0,043, n = 6$$

However, because the interest is calculated quarterly, we divide i by 4 and multiply n by 4. Therefore:

$$A = 1\,500 \left(1 + \frac{0,043}{4} \right)^{4(6)} \approx \text{R}1\,938,84$$

So the balance after 6 years is approximately R1 938,84

2.2 Applications of compound growth

2.2.1 Inflation

Inflation refers to an ongoing general increase in prices.

Use the compound growth formula if you need to work out how much something will cost in future: $A = P(1 + i)^n$

Inflation means that we can buy less in future, so if you need to work out what is something worth in future, just an adjusted formula: $A = P(1 - i)^n$

Example

If the average rate of inflation for the past few years was 7,3%, and your family's water and electricity bill is on average R1 425, what can you expect to pay in 6 years' time?

$$A = P(1 + i)^n$$

2.2.2 Exchange rates

- An exchange rate gives the relationship between two countries' currencies.

Example

- 1 Ruff wants to import a TV from England at a price of £507, plus a delivery cost of 15% and import duty of 20%. The equivalent TV locally costs R13 000. The exchange rate is R14,08 to the pound.

Show, with necessary calculations, whether Ruff should import or buy locally.

$$£507 \times 14,08 = R7\,138,56$$

$$\text{Delivery cost} = R7\,138,56 \times 15\% = R1\,070,78$$

$$\text{Import duty} = R7\,138,56 \times 20\% = R1\,427,71$$

$$\text{Total cost} = R9\,637,05$$

Conclusion: It costs less to import the TV

- 2 If the exchange rate is 1 euro = R8,1671 and 1 pound sterling = R12,1668, determine the exchange rate between the euro and the pound.

$$1 \text{ euro} = R8,1671$$

$$1 \text{ rand} = 1 \div 8,1671 = 0,1224 \text{ euro}$$

$$1 \text{ pound sterling} = R12,1668$$

$$1 \text{ rand} = (£1 \div 12,1668) = £0,0822$$

$$0,1224 \text{ euro} = £0,0822$$

$$1 \text{ euro} = £0,6713$$

2.2.3 Population growth

- Since we are referring to growth, we use the formula $A = P(1 + i)^n$.
- Here, i = rate of population growth and P = initial size of the population.

Example

There are 12 500 people in a small town. The population of the town increases every year by 5,5%. What will the population of the town be after 5 years?

$$A = 12\,500(1 + 0,055)^5$$

$$A \approx 16\,337$$

Questions

- 1 Mr du Toit decided to invest an amount of money at an interest rate of 9% p.a. compounded annually. He would like R70 000 after six years. How much money must he invest now? (4)
- 2 A person invests R30 000 in a savings account for eight years at a simple interest rate of 6% p.a. Calculate the accumulated amount. (4)
- 3 A motor vehicle is purchased for R105 000. Of this, 20% is paid in cash and the balance is paid using a hire purchase agreement for seven years by means of equal monthly payments. The bank decides to charge an interest rate of 14,5% p.a. for the seven-year period. Calculate:
 - a The loan amount (4)
 - b The full amount including interest (2)
 - c The monthly repayments (3)
 - d The total amount paid if the insurance on the car per month is R780 (4)
- 4 A mother decides to buy a baby's pram from Kuwait. The price of the pram is 80 Kuwaiti dinars. Calculate how much she will pay if the exchange rate is 1 Kuwaiti dinar to R26,09. (4)
- 5 A tourist from the United Kingdom buys computers from South Africa at a cost of R530 000. He wants to pay for these computers in British pounds. If the exchange rate is $1\text{£} = \text{R}11,72$, how much will he pay in pounds? (To the nearest pound) (4)
- 6 Mr Dube wants to buy a car. So he borrows R420 000 and agrees to settle this amount in eight years together with simple interest charged at 12,5% p.a. How much will he have to pay in eight years from now? (4)
- 7 Ronnie wants to receive an amount of R180 000 after 10 years. How much must he invest now at an interest rate of 7,25% p.a. compounded annually (to the nearest rand)? (4)
- 8 The population of Phoenix is growing at a rate of 4% p.a. compounded annually. If the population is now 50 000, what will be the percentage growth in five years time? (Give your answer correct to two decimal places) (4)
- 9 Oil costs \$76 a barrel. The exchange rate is 1\$ is equal to R7,23. If South Africa were to import 40 000 barrels, how much will the oil cost (in rand)? (4)
- 10 Matthew invests R30 000. The institution offers Matthew two investment options:
Option 1: 8 years at 12,5% p.a. simple interest.
Option 2: 8 years at 12,5% p.a. compound interest.
Which option should Matthew choose? Motivate your answer. (9)