CAPS

Mind the Gap!

Physical Science Part 1
Physics
Study Guide

Grade 12

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

Mechanics: Force and Newton's Laws

Summary

This section covers important aspects from the Grade 11 work. Make sure that you revise it well so that you can apply this knowledge to your Grade 12 work. In order to be successful in this section, you need to revise trigonometry and Pythagoras' theorem.

You must know how to:

- Draw a sketch of parallel and perpendicular vectors.
- Determine the resultant vector graphically using the head to tail method as well as by calculation.
- Resolve a vector into its parallel and perpendicular components.

You must remember:

- Newton's Laws and how to apply them.
- Different kinds of forces.
- Force diagrams and free body diagrams.

Definitions and Laws you must remember:

1. A **force** is a push or pull upon an object resulting from the object's interaction with another object.
2. **Gravitational force** is the force of **attraction** that objects exert on other objects in virtue of having mass. It is the force that makes all things fall and causes tides in the ocean. The greater the mass of an object, the greater its gravitational pull.
3. The **normal force** is a **perpendicular** force that a surface exerts on an object with which it is in contact.
4. The **resultant (net) force** acting on an object is the **vector sum** of all the forces acting on the object. **The vector sum** is the sum of all vectors (all the forces added up, taking their directions into account).
5. **Newton's First Law of Motion:** An object will remain at rest or continue moving at a constant velocity (or at constant speed in a straight line) unless acted upon by a non-zero **external** resultant force.
6. **Newton's Second Law of Motion: If a resultant (net) force** acts on an object, the object will accelerate in the direction of the resultant force. The acceleration is directly proportional to the resultant force and inversely proportional to the mass of the object.
7. **Newton's Third Law of Motion:** When object A exerts a force on object B, object B simultaneously exerts a force on object A, which is **of equal magnitude** but **opposite in direction.**
8. **Newton's Law of Universal Gravitation:** A force of gravitational attraction exists between any two particles or objects anywhere in the universe. The magnitude of this force is **directly proportional** to the product of the objects' masses and is **inversely proportional** to the square of the distance between their centres.

hint

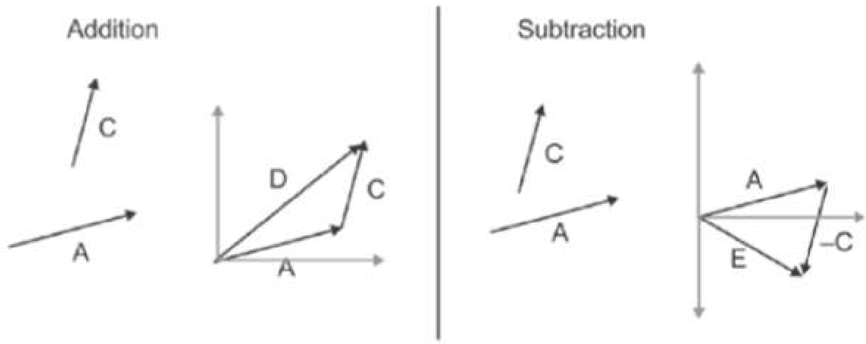
"Universal" means the statement is valid for any object in the universe.

1.1 Revision: Vectors

A vector is a quantity that has both **magnitude** (size) and **direction.**

- We can use bold type to represent a vector, **R**, or an arrow above the letter $\vec{R}$.
- Vectors may be added or subtracted graphically by laying them head to tail / head to head on a set of axes.

Worked example 1



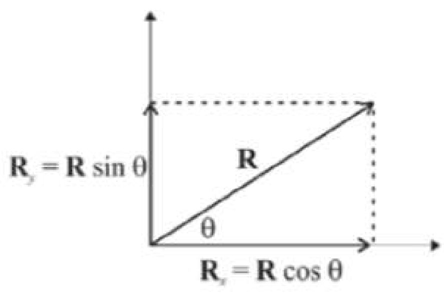
$\vec{A} + \vec{C} = \vec{D}$

$\vec{A} - \vec{C} = \vec{E}$

- Just as one vector is the sum of two vectors, we can also find two vectors to make up one vector.
- In mechanics, it is often useful to break up a vector into two component vectors, one horizontal and the other vertical. We use basic trigonometry to find the components.

Worked example 2

Example: Vector $\vec{R}$ makes an angle θ with the x-axis. $\vec{R}$ is broken into component vectors $\vec{R}_x$ and $\vec{R}_y$.



$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{R_x}{R}$

$\therefore R\cos\theta = R_x$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{R_y}{R}$

$\therefore R\sin\theta = R_y$

$\vec{R}_x = \vec{a}\cos\theta$ and $\vec{R}_y = \sin\ \vec{R}_y$, where $\vec{R}_y$ is the angle between $\vec{a}$ and the x-axis ($\vec{R}_x$).

e.g. Worked example 3

If vector $\vec{R}$ has a magnitude of 5 and is at an angle of $\theta = 36{,}86°$, the components are $\vec{F}_x = 5 \cos 36{,}86° = +4$ and $\vec{R}_y = 5 \sin 36{,}86° = +3$.

1.2 What is force?

When objects interact with each other, they exert forces on each other.

If a force acts on an object, it can cause a change to the object. Some of the possible changes are:

- the shape of an object
- the object's state of rest
- the velocity of the object
- the direction in which the object moves
- the object's acceleration.

NB

DEFINITION

Repulsion: a force between objects that tends to separate them

Attraction: a force between objects that brings them together

Force ($\vec{F}$) is a **vector quantity**. This means it has magnitude and direction.

- It may be represented by an arrow in a vector diagram. The length of the arrow shows its **magnitude** and the angle shows its **direction**.
- It is measured in the SI unit **newton (N)**.

We show the force vector using $\vec{F}$.

F *without* an arrow represents the size of the force vector only.

Example

$\xrightarrow[\text{12 N}]{}$ represents a force ($\vec{F}$) of 12 N to the right.

- Objects exert push (**repulsion**) or pull (**attraction**) forces on each other.
- A force can be classified as either a ***contact force*** or a ***non-contact force***
- Objects can exert a force on each other when they are in contact (touching each other) e.g. friction and normal forces

OR

- Objects can exert a force on each other when they are **not in contact** (i.e. are apart from each other) e.g. magnetic, electrostatic and gravitational forces.

1.3 Different types of forces

We study these different forces:

1. Gravitational force or weight ($\vec{F}_g$ or $\vec{w}$)
2. Normal forces ($\vec{F}_N$ or $\vec{N}$)
3. Frictional forces ($\vec{F}_f$)
4. Applied forces (push or pull)
5. Tension ($\vec{F}_T$ or $\vec{T}$)

1. **Gravitational force ($\vec{F}_g$ or $\vec{a}$):**
   - Gravitational force is the force of **attraction** that the Earth exerts on an object above its surface.
   - Gravitational force acts **downwards** towards the centre of the Earth.
   - The **weight** ($\vec{w}$) of an object is the same as the gravitational force ($\vec{F}_g$) on the object, so $\vec{F}_g = \vec{w}$
   - The **weight** of an object is the product of the **mass** and the **gravitational acceleration** of the Earth.
     so $\vec{w} = m\vec{g}$ where m is mass and $\vec{g}$ is the acceleration due to gravity.

$\therefore \vec{F}_g = \vec{w} = m\vec{g}$

where $\vec{F}_g$ is gravitational force

$\vec{w}$ is the weight of an object

$m\vec{g}$ is mass × gravitational acceleration

2. **Normal force ($\vec{F}_N$ or $\vec{N}$):**

   When an object rests on a surface, the surface exerts a force on the object, called a **normal force**.

   It is a **contact** force that acts at a right angle (90°) upwards from the surface.

   In the diagrams below, you will see a free body diagram and a force diagram. In a force diagram, you show the object that is experiencing forces. The forces act on the body at its "centre of gravity". In a free body diagram, you do not show the object that is experiencing forces; i.e. you treat the object as a single point.

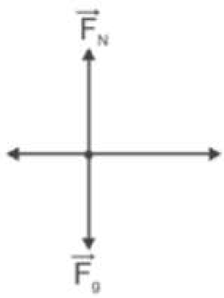
NB

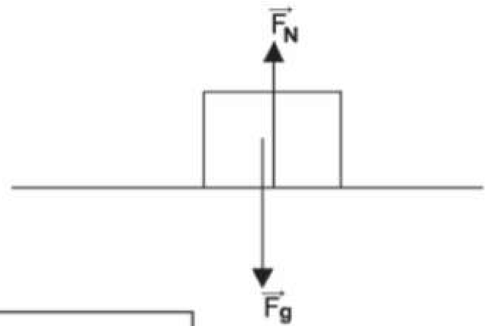
DEFINITION

A "centre of gravity" is a point from which the weight of a body or system may be considered to act. In uniform gravity it is the same as the center of mass.

2.1. **When an object is resting or moving on a horizontal surface the normal force will have the same magnitude, but an opposite direction to the weight of the object or gravitational force.**

<u>An object resting on a horizontal surface</u>

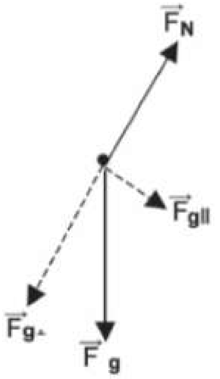




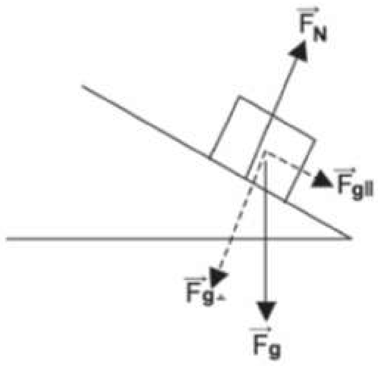
So $\vec{F}_N = -\vec{F}_g$ and $\vec{F}_N = -m\vec{g}$

2.2. **When an object is resting or moving on an inclined plane (surface), the normal force will have the same magnitude, but an opposite direction to the perpendicular component of the weight of the object or gravitational force.**

An object resting on an inclined plane (surface)



So $\vec{F}_N = -\vec{F}_{g\perp}$



3.1 Frictional force ($\vec{F}_f$ or $\vec{f}$):

- Frictional force opposes motion. So it works against the movement of an object.
- Frictional force acts in the **opposite direction** to an object's motion or intended motion.
- The rougher the surface, the more friction there is between the object and the surface.

The less rough the surface, the less friction there is between the object and the surface.

This means that the greater the magnitude of the normal force acting on the object, the greater the magnitude of the frictional force. Think of grinding something here. The harder you press, the more "normal" (perpendicular) force there is. Hence, when you are grinding something, e.g. crushing maize for making pap, it experiences strong normal (perpendicular) forces and thus strong frictional forces; hence it is ground up.

- If an object is at rest, then there is a **static** frictional force.
- If the object is moving, then there is **kinetic** frictional force.

3.2 The coefficient of friction (μ)

The **coefficient of friction** depends on the material of the two surfaces that are in contact.

Examples
- Steel on wet ice has a low coefficient of friction (slides easily).
- Rubber on tar has a higher coefficient of friction (more grip, less sliding).

- When an object is at rest on a horizontal surface and **no force is applied** to it, then there is no static friction.
- When a small force is applied to an object at rest, then the force of static friction increases as the applied force increases.

- As the force increases, the static friction continues to increase.
- This continues until the static friction reaches a maximum value – it cannot increase further. Eventually maximum static friction force is exceeded and the object moves.
- The friction then decreases to a smaller value called the kinetic friction ($\vec{f}_k$).
- The kinetic friction remains constant while the object moves at a constant speed.
- The kinetic friction remains smaller than the maximum static friction. $f_s \leq \mu_s F_N$ and $f_k = \mu_k F_N$

NB

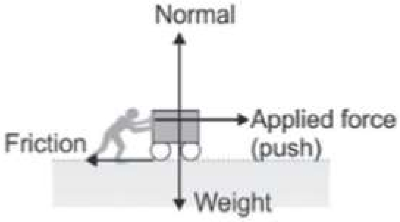
When an object moves along a surface inclined at an angle θ, the normal force is multiplied by the kinetic coefficient of friction to find the frictional force.

The kinetic coefficient is calculated using cos θ:

$\vec{F}_N = \vec{F}_{g\perp} = m\vec{g}\cdot\cos\theta$ $\quad\quad f_k = \mu_k \vec{F}_N$

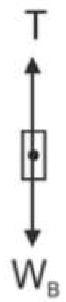
4. Applied forces

An applied force is a force that a person or object applies to another object. If a person is pushing a cart along the ground, then there is an applied force acting upon the object.



5. Tension ($\vec{F}_T$ or $\vec{T}$):

When an object is pulled by a rope (or string or cable), or hanging from a ceiling, the rope applies a force on the object. This force is called **tension**. It is a **contact** force and acts in the opposite direction to the 'pull'. If an object hangs from a rope, the direction of the tension is always **upwards** in the rope. This force complies with Newton's Third Law, i.e. it is the reaction to the action of the pulling.



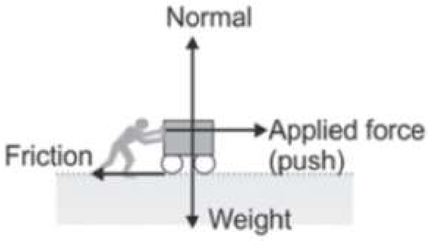
1.4 Force diagrams and free body diagrams

NB

1. In force and free body diagrams we consider forces acting on ONE (the same) object
2. When you answer questions about force, you must:
   - name the forces
   - state which object exerts a force on which object
   - state the directions of the forces.

Example

A man pushes a loaded trolley along a horizontal floor.



We can identify the following forces that are acting on the trolley:

- the weight or $\vec{F}_g$ of the trolley and load, i.e. the force exerted downwards on the trolley by gravity/ the earth
- the normal force, $\vec{F}_N$, exerted upwards on the trolley by the floor
- the applied force that the man exerts on the trolley, which acts in a forwards direction
- the frictional force, $\vec{F}_f$, in the opposite direction to the motion.

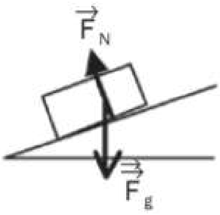
Forces acting on an object can be represented by **force diagrams** or by **free body diagrams**.

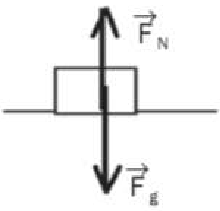
Force diagram

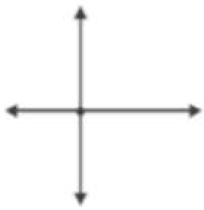
- The object is represented as a block and the forces as vectors.
- The vectors start at the point of application.
- Weight is drawn from the object's centre of gravity, downwards.

Free body diagram

- The object is represented as a dot and the forces as vectors.
- The vectors start at the dot and they all point away from the dot.
- If the object is on an inclined surface, the weight vector can be resolved into two component vectors.







Force symbols in diagrams

We use these symbols to help represent forces in force diagrams and free body diagrams:

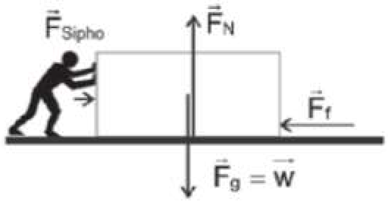
- $\vec{F}$ or $\vec{F}_{applied}$: applied force, in the direction applied
- $\vec{F}_f$ or $\vec{F}$: friction force, surface on object, opposite to direction of motion
- $\vec{F}_g$ or $\vec{w}$: gravitational force or weight, force exerted by the earth on object, downwards
- $\vec{F}_N$ or N: normal force, surface on object; perpendicularly upwards from the surface
- $\vec{F}_T$ or $\vec{T}$: tension, cable or rope on object, in direction of motion.

Steps for drawing force or free body diagrams

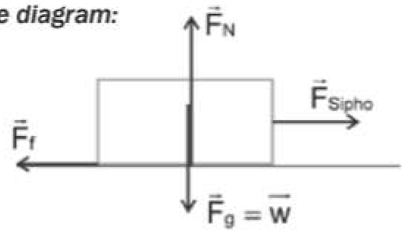
Follow the steps in this example.

Example: An object on a horizontal surface (plane):

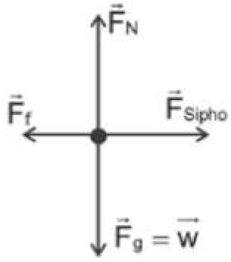
Sipho exerts a force to the right while pushing a car along a rough, flat road. Draw a force diagram and a free body diagram to represent the situation.



Force diagram:



Free body diagram:



Step 1. If there is a surface, draw a line to represent it.

Step 2. Force diagram: draw a block to represent the object.

Step 3. Free body diagram: draw a dot to represent the object.

Step 4. Draw a vector to represent the weight ($\vec{F}_g$) of the object.

Step 5. If the body rests on a surface draw an arrow to represent the normal force, upwards from and perpendicular to the surface. ($\vec{F}_N$)

Step 6. Draw an arrow to represent each applied force.

Step 7. Draw an arrow to represent friction (if there is friction).

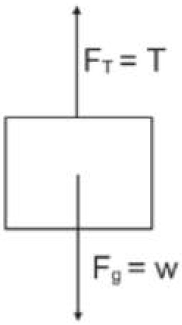
Step 8. If you are drawing a free body diagram, erase the line representing the surface.

Worked example 4

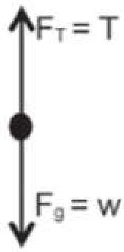
Draw a force diagram and a free body diagram for an object hanging from a rope or a cable.

Solution

Force diagram:



Free body diagram:



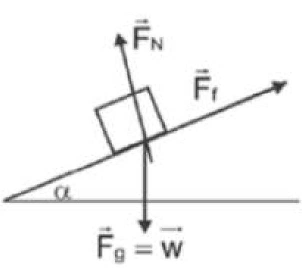
Component vectors in free body diagrams

Free body diagrams are useful for showing all the forces involved in a situation.

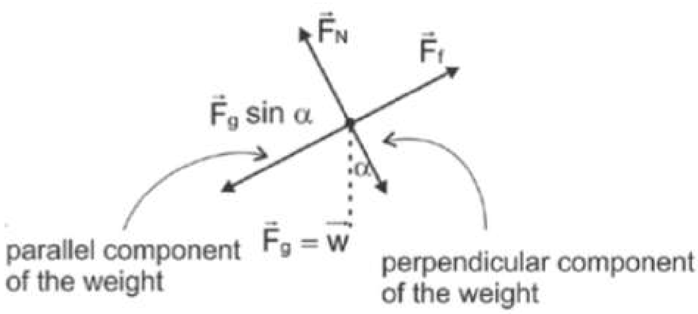
When an object rests on an inclined plane, the force due to gravity may be shown by two vectors:

- one representing the component parallel to the surface
- the other representing the component perpendicular to the surface.

Force diagram:

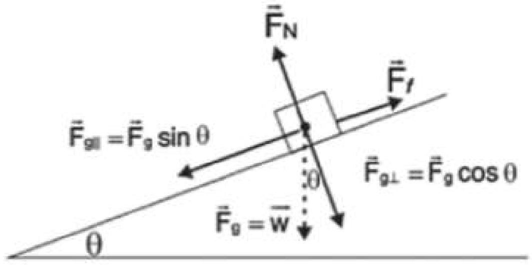


Free body diagram:



Worked example 5

If an object with a mass of 40 kg slides down a surface which has a coefficient of kinetic friction μk = 0,14, and a slope of 15°, what is the net force on the object as it slides down the surface? Use the diagram to help you.



Solution

Weight component down the slope

$= \vec{F}_g \sin\theta = mg\sin 15° = 40 \times 9{,}8 \times 0{,}26 = 101{,}92$ N.

Frictional force up the slope

$\vec{F}_g \cdot \mu_k \cos\theta = 40 \times 9{,}8 \times 0{,}14 \times 0{,}96 = 52{,}68$ N

∴ Net force down the slope is:

$101{,}92 + (-52{,}68) = 49{,}24$ N.

NB

Direction of frictional force:

The friction acts against the object to prevent it from sliding down the slope so it acts upwards parallel to the slope.

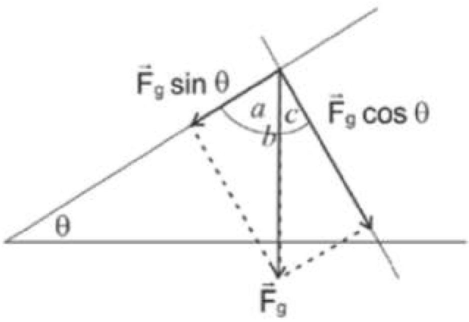
Worked example 6

Calculate the components of the weight of an object when it is resting on a surface which slopes at an angle of θ.

Solution

$\theta + a = 90°$ and $a + c = 90°$. So $c = \theta$.

The component of the weight perpendicular to the surface with a slope of θ is $\vec{F}_{g\perp} = \vec{F}_g \cos\theta$ and parallel to the surface is $\vec{F}_{g\parallel} = \vec{F}_g \sin\theta$.



1.5 Resultant (net) force

When a number of forces act on an object, we need to determine the resultant or net force acting on the object.

NB

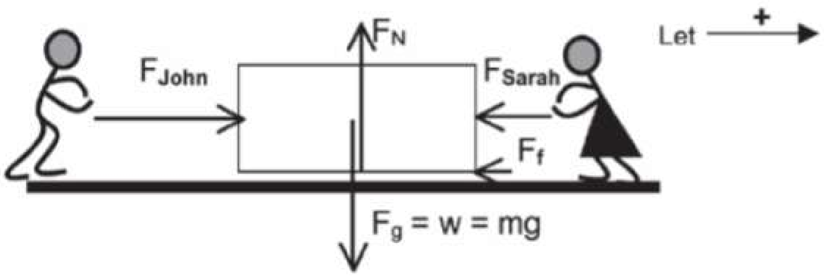
The **resultant (net) force** acting on an object is the **vector sum** of all the forces acting on the object.

$$\vec{F}_{net} = \vec{F}_{res} = \Sigma\vec{F} = \vec{F}_1 + \vec{a}_2 + \ldots$$

$\Sigma\vec{F}$ is the sum of all the forces acting on the object

e.g. Worked example 7

John exerts a force of 100 N to the right on a box resting on a rough, horizontal surface. Sarah exerts a force of 50 N to the left on the box. The friction between the box and the surface is 5 N. Draw a force diagram and calculate the resultant force acting on the box.



Solution

$\vec{F}_{net} = \Sigma\vec{F} = (+\vec{F}_{John}) + (-\vec{F}_{Sarah}) + (-\vec{F}_f)$

$= \vec{F}_{John} - \vec{F}_{Sarah} - \vec{F}_f$

$= 100 - 50 - 5 = 45$ N $\therefore \vec{F}_{net} = 45$ N to the right

Remember:

- Force is a vector.
- Indicate the direction of the force with a + or – sign.
- **Interpret the answer in words as the final step in your solution**

Now consider a situation where a box slides down a slope. The force that makes the box slide down the slope is the component of the box's weight that acts parallel to the slope.

$\therefore \vec{F}_{g\parallel}$ = **mg·sin α** where α is the angle between the slope and the horizontal. **Always** calculate this force first.

Activity 1

A box of mass 100 kg slides down a rough slope which forms an angle of 30° to the horizontal. The friction that acts on the box is 20 N. Draw a **free body diagram** representing all the forces acting on the object and calculate the resultant force acting on the box and causing it to slide. Perpendicular forces may be ignored. (12)

Solution

Let the direction down the slope be positive.

Then $\vec{F}_f = -20\text{N}$ ✓

$\vec{F}_{net} = \Sigma\vec{F} = (+\vec{F}_{g\parallel}) + (\vec{F}_f)$ ✓ ✓

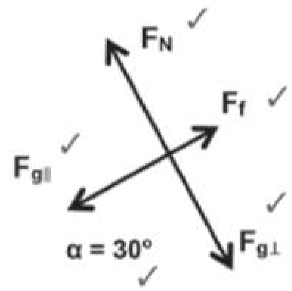
$= \vec{F}_{g\parallel} + \vec{F}_f$

$= mg\cdot\sin\alpha + \vec{F}_f$ ✓

$= (100)(9{,}8)(\sin 30°) + (-20)$ ✓ ✓

$= 490 - 20 = 470$ N

$\therefore$ 470 N, down the slope ✓



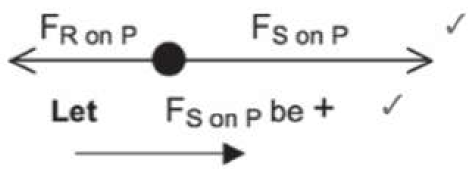
[12]

Activity 2

R and S are two positively charged spheres. P is a negatively charged sphere. Sphere R exerts an electrostatic force of 0,2 N on P and sphere S exerts a force of 0,6 N on sphere P.

Draw a free body diagram for sphere P and then calculate the resultant force on sphere P. (5)

Solution



$\vec{F}_{net} = \Sigma\vec{F} = (\vec{F}_{S\text{ on }P}) + (\vec{F}_{R\text{ on }P})$ ✓

$= 0{,}6 + (-0{,}2)$ ✓

$= 0{,}4$ N $\therefore$ 0,4 N towards S ✓

Note: $\vec{F}_R = -0{,}2$ N

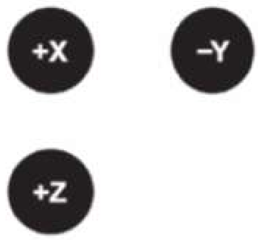
[5]

Activity 3

Three identical spheres X, Y and Z are in the same horizontal plane. Spheres X and Z are both positive and sphere Y is negative. Sphere Y exerts an electrostatic force of 450 N on sphere X and sphere Z exerts an electrostatic force of 350 N on sphere X.

1. Draw a free body diagram for sphere X and indicate the electrostatic forces acting on it. (2)
2. Calculate the magnitude of the resultant electrostatic force on sphere X. (8)

[10]

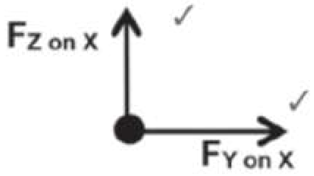


hint

X and Z are positive
∴ Z **repels** X and
Y is negative and Z is positive
∴ Y **attracts** X

Solutions

1. Free body diagram:

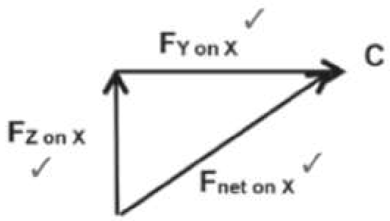


(2)

2. $\vec{F}^2_{net} = \vec{F}^2_{Z\ on\ X} + \vec{F}^2_{Y\ on\ X}$ (pythagoras)

$= 350^2 + 450^2 = 325\ 000$

$= \therefore \vec{F}_{net} = \sqrt{325\ 000} = 570{,}09\ \text{N}$



(8)

[10]

1.6 Newton's First Law of Motion (Law of Inertia)

DEFINITION

Resists:
opposes, prevents, works against

Inertia

- **Inertia** is the property of an object that resists any change in the state of rest or uniform motion.
  If the object is at rest, it resists any change to a state of motion.
  If it is in motion, it resists any change to the speed and direction of its motion.
- Inertia is determined by the object's mass. The greater an object's mass, the greater its inertia.

Example

A box lying in the boot of a car will move forwards when the car brakes.

The box's inertia resists the change in movement and allows the box to continue moving in the direction in which the car was moving before it stopped. This is why you must wear seatbelts!

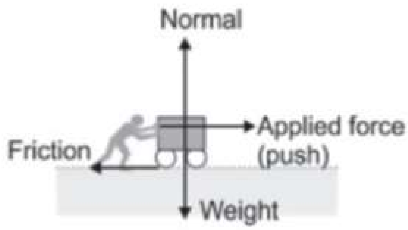
Newton's 1st Law of Motion

An object will remain at rest or continue moving at a constant velocity (in a straight line) unless acted upon by a non-zero external resultant force.

$F_{net} = 0\ \text{N}\ \therefore\ \vec{a} = 0\ \text{m·s}^{-2}$

Worked example 8

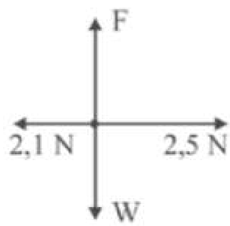
The diagram shows the forces on a trolley moving with constant velocity.



1. A man pushes a loaded trolley with constant velocity along a horizontal floor. The trolley and load have a mass of 56 kg and the friction of the moving trolley is 2,1 N. Calculate the force the man exerts to push the trolley along the floor.
2. If he then pushes the trolley with a force of 2,5 N to the right, calculate the acceleration of the trolley.

Solutions

1. The acceleration = 0, so the net force is equal to zero. The force the person pushes with is equal and opposite to the force of kinetic friction on the trolley.
   $F_{push} = F_{trolley} = 2{,}1\ \text{N}$
2. The diagram shows the free body diagram.
   The net force is 0,4 N in the forward direction.

   The trolley accelerates forward:
   $\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{0{,}4}{56} = 7{,}14 \times 10^{-3}\ \text{m·s}^{-2}$
   forward / to the right.

1.7 Velocity and acceleration: Revision

- Velocity (v) is the rate of change in position (displacement). It is a vector. *Speed* is a scalar.

  $\vec{V}_{average} = \frac{\Delta \vec{x}}{\Delta t}$ ... Δx is the displacement; rate is shown by change in time Δt

- Acceleration ($\vec{a}$) is the rate of change of velocity.

  $\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$ ... Δv is change in velocity: final velocity (v_f) – initial velocity (v_i)

- Equations of motion: In Grade 10 you learnt these equations that describe the relationships between velocity, acceleration, displacement and time:
  - $\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$
  - $\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\Delta x$
  - $\Delta \vec{x} = \vec{v}_i \Delta t + ½\Delta t^2$

1.8 Newton's Second Law of Motion in terms of acceleration

When the resultant force acting on an object is NOT zero, the object's state of motion will change.

It may:

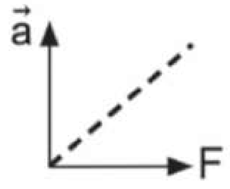
- start moving (then $\vec{v}_i = 0\ m{\cdot}s^{-1}$ and $\vec{v}_f \neq 0\ m{\cdot}s^{-1}$);
- stop moving (come to rest, then $\vec{v}_f = 0\ m{\cdot}s^{-1}$);
- move faster (accelerate); move slower (decelerate); or
- the direction in which it moves will change.

Newton's Second Law of Motion

If a **resultant (net) force** acts on an object, the object will accelerate in the direction of the resultant force. The acceleration produced is directly proportional to the resultant force and inversely proportional to the mass of the object. In other words, acceleration is the amount of change in speed (or velocity), per second, hence, it is metres per second change per second, or $m \cdot s^{-2}$

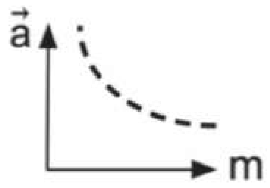
For any object $\vec{a} \propto \vec{F}_{net}$ and $\vec{a} \propto \frac{1}{m}$ $\therefore \vec{F}_{net} = m\vec{a}$

where $\vec{a}$ is acceleration ($m \cdot s^{-2}$), $\vec{F}$ is force (N) and m is mass (kg)

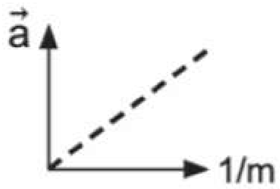


$\vec{a} \propto \vec{F}_{net}$

$\therefore$ straight line through origin



When $\vec{a} \propto m$ the graph is a hyperbola with $\vec{a}$ and m on the axes.



When $\vec{a} \propto \frac{1}{m}$ the graph is a straight line with $\vec{a}$ and $\frac{1}{m}$ on the axes.

If *different* forces are applied to the same object and its mass stays constant, then $\vec{a} \propto \sum \vec{F}$. The bigger the net resultant force acting on the object, the more the object will accelerate.

e.g. Worked example 9

A resultant force $\vec{F}$ is applied to an object of mass m and the object accelerates at $\vec{a}$. What will the object's acceleration be if the resultant force acting on the object is tripled?

Solution

m is constant $\therefore \vec{a} \propto \vec{F}$ and if the force is tripled (from $\vec{F}$ to $3\vec{F}$), the acceleration will also triple $\therefore$ the object will accelerate at $3\vec{a}$.

NOTE:

If a constant non-zero resultant force is applied to **two** objects, then $\vec{a} \propto \frac{1}{m}$. The object with the smaller mass will accelerate more than the object with the bigger mass. Think about it: it's easier to make a lighter object move further and faster.

Worked example 10

A constant resultant force $\vec{F}$ is applied to objects of masses m and 2 m. If the object of mass m accelerates at $\vec{a}$, what will the acceleration of the other object be?

Solution

$\vec{F}$ is constant $\therefore \vec{a} \propto \frac{1}{m}$

If the mass doubles (from m to 2 m), the acceleration will halve

$\therefore$ the object of mass 2 m accelerates at $\frac{1}{2}\vec{a}$.

Steps to solve problems on Newton's Laws

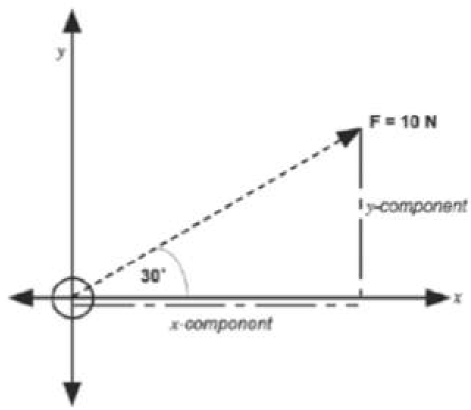
Step 1: Read the problem as many times as you need.

Step 2: Sketch the situation if it is necessarily.

Step 3: Draw a force diagram for the situation.

Step 4: Draw a free body diagram; you must resolve the forces into components on the Cartesian plane if necessary.

Consider this example. You are told that the force F acts at an angle of 60˚ to the normal or 30˚ to the horizontal plane. What are its vertical and horizontal components?



Well, the y-component is opposite the angle, and the hypotenuse (10 N) is known, so since sine is O/H, sin 30˚ × 10 N = the y-component: 5 N. Likewise, the x-component is adjacent to the angle, so since cosine is A/H, cos 30˚ × 10 N = 8,67 N. So your x-component is 8,67 N and your y-component is 5 N.

Step 5: List all the given information and convert the units if necessary.

Step 6: Determine which physical principle (law) can be applied to solve the problem.

Step 7: Use the principle (law) to answer the question, often by substituting numerical values into an appropriate equation.

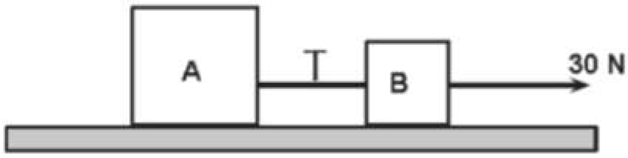
Step 8: Check that the question has been answered and that the answer makes sense.

hint

Sometimes the forces you are given in a diagram are not at right angles to each other, yet when you draw a force diagram on the Cartesian Plane, the forces must be drawn at right angles to each other. In order, then, to find out what the vertical and horizontal parts of a force are, when that force is at an angle, we have to use trigonometry.

Activity 4

Two boxes, **A** and **B**, are lying on a table and are connected by a piece of string. The mass of box **A** is 3 kg and the mass of box **B** is 2 kg. Assume that the mass of the string is very small, so we can ignore it. A 30 N pulling force, pointing to the right, is applied to box **B**, causing the two boxes to move. The surface acts with a frictional force of 5,9 N on box A and 4,1 N on box **B**.

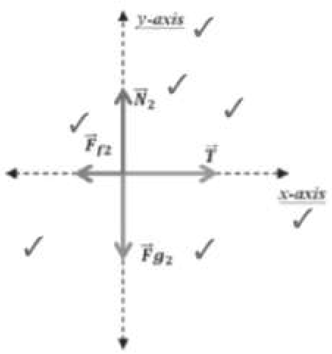


1. Calculate the acceleration of boxes A and B. (14)
2. Calculate the magnitude of the tension on the string. (5)

[19]

Solution

1. We are going to take the whole system as a unit.



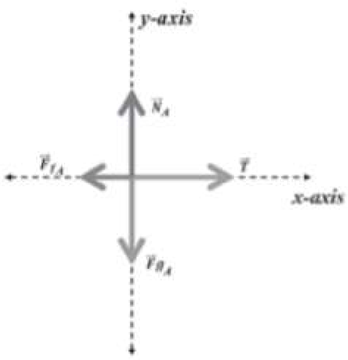
<u>Data</u>:
$m_A = 3$ kg, $m_B = 2$ kg ✓✓
$m_B = 2$ kg $m_T = m_A + m_B = 3$ kg $+ 2$ kg $= 5$ kg
$F_A = 30$ N to the right
$F_{fA} = 5{,}9$ N to the left
$F_{fB} = 4{,}1$ N to the left
$F_{fT} = F_{fA} + F_{fB} = 5{,}9 + 4{,}1 = 10$ N to the left
$a = ?$ to the right
$T = ?$

(9)

$F_{RTx} = m_T a$ (from $F = ma$) ✓ ✓
$F_A + F_{fT} = m_T a$
$(30) + (-10) = 5a$
$a = 4$ m/s² to the right ✓ (5)

2. To calculate the tension you may use box A **or** box B

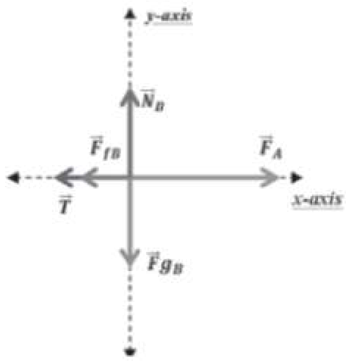
<u>Tension using box **A**</u>



$F_{RAx} = m_A a$ ✓
$T + F_{fA} = m_A a$ ✓
$T - 5{,}9 = (3)(4)$ ✓✓
$T = 12 + 5{,}9$
$T = 17{,}9$ N ✓

(5)

Tension using box B



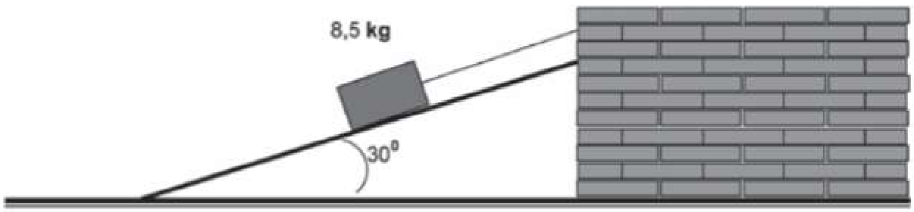
$F_{RBx} = m_B a$ ✓
$F_A + T + F_{fB} = m_B a$ ✓
$30 - T - 4{,}1 = (2)(4)$ ✓✓
$-T + 25{,}9 = 8$
$-T = -17{,}9$
$T = 17{,}9$ N ✓

(5)

[19]

Activity 5

The sketch below shows a block of 8,5 kg at equilibrium on an inclined (sloping) plane (surface).



Calculate:

1. The magnitude of the tension in the cord. (12)
2. The magnitude of the normal force acting on the block. (6)
3. The magnitude of the block's acceleration, If the cord is cut. (4)

[22]

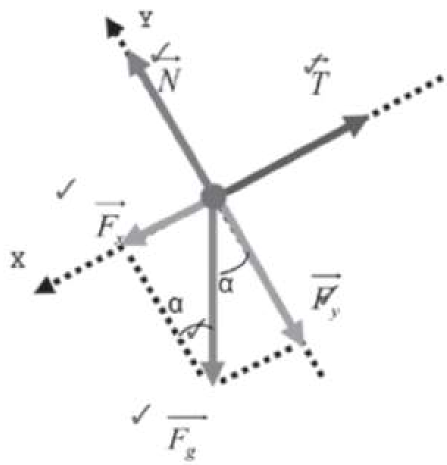
Solution

1. **Data**

   m = 8,5 kg ; $\alpha = 30^0$; $v_i = 0$; $a_i = 0$

   Let's make the free body diagram of forces.

(6)



The gravitational force is not in the direction of any axis then we have to determine its components on the x-axis and y-axis.

1. Applying Newton's First Law

$\sum \vec{F} = 0$

$\vec{F}_g + \vec{T} = 0$ ✓

Working with the projections of the forces on the x-axis we get:

$F_{gx} - T = 0$

$F_g \cdot \sin\alpha - T = 0$ ✓

$m \cdot g \cdot \sin 30° - T = 0$ ✓

$\overset{\checkmark}{8{,}5} \times \overset{\checkmark}{9{,}8} \times 0{,}5 - T = 0$

$41{,}65 - T = 0$

$T = 41{,}65\ N$ ✓ (6)

2. Working on the y-axis

$N - F_{gy} = 0$

$N - (F_g \cdot \cos\alpha) = 0$ ✓

$N - (m \cdot g \cdot \cos 30^0) \overset{\checkmark}{=} 0$

$N - (\overset{\checkmark}{8{,}5} \times \overset{\checkmark}{9{,}8} \times \overset{\checkmark}{0{,}866}) = 0$

$N - 73{,}1 = 0$

$N = 73{,}1\ N$ ✓ (6)

3. Applying Newton's Second Law.

$\sum \vec{F}_x = m\vec{a}_x$ ✓

If the cord is cut there is no tension force acting on the block and there is only one force acting on the direction of the x-axis, causing acceleration to the block.

Working with the projections

$F_{gx} = m \cdot a$

$F_g \cdot \sin\alpha = m \cdot a$

$mg \cdot \sin 30° = m \cdot a$ ✓

Simplifying:

$g \cdot \sin 30° = a$

$a = g \cdot \sin 30°$

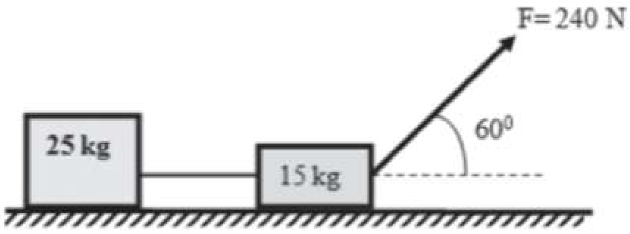
$a = 9{,}8 \times 0{,}5$ ✓

$a = 4{,}9\ m \cdot s^{-2}$ ✓ (4)

[22]

Activity 6

Two blocks of 25 kg and 15 kg are connected by a light string on a horizontal surface. Assume that the string cannot stretch. A force of magnitude 240 N is applied to the block of 15 kg forming an angle of 60° with the horizontal as shown in the sketch below. The coefficient of kinetic friction is 0,20.



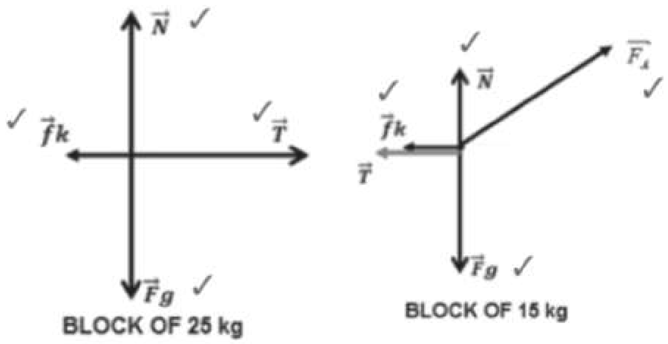
1. State Newton's Second Law of Motion in words. (7)
2. Draw a free body diagram for each block. (8)
3. Calculate the magnitude of the acceleration of the blocks. (14)

[29]

Solutions

1. If a resultant force ✓ acts on a body, it will cause the body to accelerate ✓ in the direction of the resultant ✓ force. The acceleration of the body will be directly ✓ proportional to the resultant ✓ force and inversely ✓ proportional to the mass ✓ of the body. (7)

2.



(8)

3.

Option 1

Taking the objects as a system

$F_{Rx} = ma$ ✓

$F_x + F_{fT} = ma_x$ ✓

$F_x - F_{fT} = m_T a_x$

$F_x - (F_{f1} + F_{f1}) = (m_1 + m_2)a_x$ ✓

$F_x - (\mu N_1 + \mu N_2) = (m_1 + m_2)a_x$

Note: In this series of solutions we have omitted (taken out) the vector arrow above F and a; this is just to make it easier to read the solution.

We have to calculate the normal force for both blocks

$N_1 = F_g = m_1g$

$N_2 = m_2g - F{\cdot}\sin 60^\circ$ ✓

$F{\cdot}\cos 60^\circ - [\mu m_1g + \mu(m_2g - F{\cdot}\sin 60^\circ)] = (m_1 + m_2)a_x$ ✓ ✓

$(240{\cdot}\cos 60^\circ) - [(0{,}2)(25)(9{,}8) + (0{,}2)[(15)(9{,}8)] - 240{\cdot}\sin 60^\circ]]$ ✓ ✓

$= (25+15)a_x$ ✓ ✓ ✓

$120 - [49 + (0{,}2)(147 - 207{,}85)] = 40a_x$ ✓

$83{,}17 = 40a_x$ ✓

$a_x = 2{,}08\ m/s^2$ ✓

Option 2

Applying Newton's Second Law of motion to each object individually

$\mathbf{F_R = m_1a}$

For object 1:

$T = F_{f1} = m_1a_x$ ✓

$T - F_{f1} = m_1a_x$

$T - \mu m_1g = m_1a_x$ ✓

For object 2:

$F_{R2x} = m_2a$ ✓

$F_x + T + F_{f2} = m_2a_x$ ✓

$F_x - T - F_{f2} = m_2a_x$ ✓

$F{\cdot}\cos 60^\circ - T - \mu N_1 = m_1a_x$

$F{\cdot}\cos 60^\circ - T - \mu(m_2g - F{\cdot}\sin 60^\circ) = m_2a_x$ ✓

Adding equation (1) and (2).

$T - \mu m_1g + F{\cdot}\cos 60^\circ - T - \mu(m_2g - F{\cdot}\sin 60^\circ) = m_1a_x + m_2a_x$

Taking out T and a_x: $-\mu m_1g + F{\cdot}\cos 60^\circ - \mu(m_2g - F{\cdot}\sin 60^\circ) = (m_1 + m_2)a_x$

$[-(0{,}2)(25)(9{,}8)] + [240{\cdot}\cos 60^\circ - (0{,}2)[(15)(9{,}8) - (240{\cdot}\sin 60^\circ)] =$ ✓ ✓

$(25 + 15)a_x$ ✓ ✓

$(-49 + 120) - (0{,}2)(147 - 207{,}85) = 40a_x$ ✓ ✓

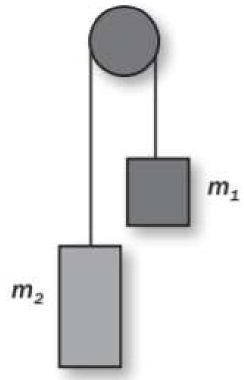
$71 + 12{,}17 = 40a_x$ ✓

$83{,}17 = 40a_x$

$a_x = 2{,}08\ m/s^2$ ✓ (14)

[29]

Activity 7



The sketch below shows two blocks connected by a string of negligible mass that passes over a frictionless pulley also of negligible mass. The arrangement is known as *Atwood's machine*. One block has mass m_1= 2 kg and the other has mass m_2= 4 kg.

The blocks have just this instant been released from rest.

1. Draw a free body diagram of all the forces acting on each block. (6)
2. Calculate the magnitude of the acceleration of the system. (7)
3. Calculate the magnitude of the tension in the string. (4)
4. Compare the magnitude of the net force on m_1 with the net force on m_2. (1)

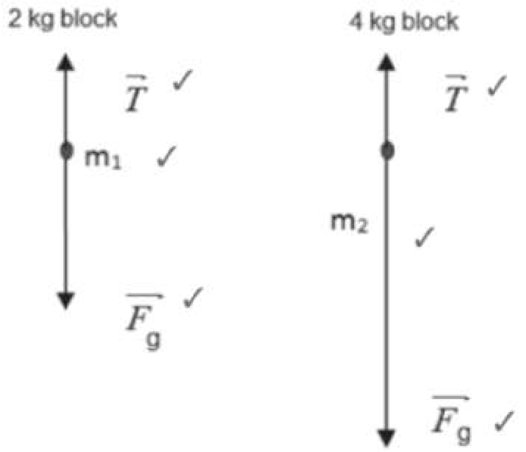
Write down only GREATER THAN, SMALLER THAN or EQUAL TO.

5. Will the pulley rotate *clockwise* or *anticlockwise*? (1)

[19]

Solutions

1. See diagram below:



Note: In this series of solutions we have omitted (taken out) the vector arrow above F and a; this is just to make it easier to read the solution.

(6)

2. $\Sigma\vec{F}_{net} = m\vec{a}$ ✓

For the 2 kg block (+ upwards)

$T - F_{g1} = m_1a$

$T - m_1g = m_1a$ ✓

$T - 2 \times 9{,}8 = 2a$ ✓

For the 4 kg block (+ downwards)

$-T + m_2g = m_2a$

$-T + 4 \times 9{,}8 = m_2a$ ✓

Solving the system of equations

$T - 2 \times 9{,}8 - T + 4 \times 9{,}8 = (2 + 4)a$ ✓ ✓

$2 \times 9{,}8 = 6\ a$

$a = +\ 3{,}27\ m{\cdot}s^{-2}$ (upwards)

$a = 3{,}27\ m{\cdot}s^{-2}$ ✓ (7)

3. **Option 1**

$T - \left(\frac{2m_1m_2}{m_1 + m_2}\right) \times g$ ✓✓

$T = \left(\frac{2 \times 2 \times 4}{2 + 4}\right) \times 9{,}8$

$T = 26{,}13$ N ✓✓

Option 2

$T - F_{g1} = m_1a$ ✓ OR $T = m_1(a+g)$ ✓

$T = 2\ (3{,}27 + 9{,}8)$ ✓

$T = 26{,}14$ N ✓

Option 3

$-T + m_2g = m_2a$ ✓ OR $-T = m_2a - m_2g$ ✓ OR $T = -m_2a + m_2g$ ✓

$T = 4(-3{,}27 + 9{,}8)$ ✓ ✓

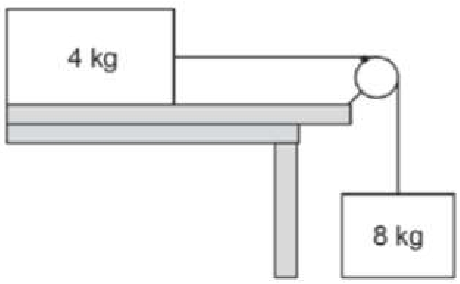
$T = 26{,}12$ N ✓ (4)

4. Smaller than. ✓ (1)
5. Anticlockwise. ✓ (1)

[19]

Activity 8

A 4 kg block on a horizontal, rough surface is connected to a 8 kg block by a light string that passes over a frictionless pulley as shown below. Assume that the string cannot stretch. The coefficient of kinetic (dynamic) friction between the block of 4 kg and the surface is 0,6.

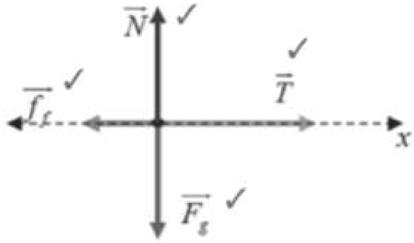


1. Draw a free body diagram of all the forces acting on both blocks. (6)
2. Calculate the acceleration of the system. (10)
3. Calculate the magnitude of the tension in the string. (3)
4. Calculate the magnitude of the frictional force that acts on the 4 kg block. (4)
5. Calculate the apparent weight of the 8 kg block. (4)
6. How does the apparent weight of the 8 kg block compare with its true weight? Write down only GREATER THAN, EQUAL TO or LESS THAN. (1)
7. How does the apparent weight of the 4 kg block compare with its true weight? Write down only GREATER THAN, EQUAL TO or LESS THAN. (1)

[29]

Solutions

1. BLOCK OF 4 kg



Positive to the right

BLOCK OF 8 kg

Positive downwards

(6)

Let's take the direction of motion as positive

2. **Let's apply Newton's Second Law of Motion to each block.**

$\sum \vec{F} = m\vec{a}$ ✓

Block of 4 kg (A)

In the x direction (horizontal)

$T - f_f = m_A a$

$T - \mu N = m_A a$ ✓

$T - \mu m_A g = m_A a$ (call this Equation 1)

> **Note:** In this series of solutions we have omitted (taken out) the vector arrow above F and a; this is just to make it easier to read the solution.

Block of 4 kg (A)

In y direction (up/down)

$N - F_g = 0$ (not moving horizontally)

$N = F_g = mg$ ✓

Block of 8 kg (B)

In the x direction (horizontal)

$-T + F_g = m_B a$ ✓

$-T + m_B g = m_B a$ (call this Equation 2)

Solving the system of equations (adding Equation 1 and 2)

$T - \mu m_A g - T + m_B g = m_A a + m_B a$ ✓ ✓

Removing T and isolating a:

$-\mu m_A g + m_B g = (m_A + m_B)a$

$-(0,6)(4)(9,8) + (8)(9,8) = (4 + 8)a$ ✓ ✓ ✓

$54,88 = 12a$

$a = 4,57 \text{ m/s}^2$ ✓ (10)

3. **Using Equation 2**

$-T + m_B g = m_B a$

$-T = 8 \times 4,57 - (8 \times 9,8)$ ✓ ✓

$T = 41,84 \text{ N}$ ✓ (3)

Using Equation 1

$T - \mu m_A g = m_A a$

$T - (0{,}6)(4)(9{,}8) = (4)(4{,}57)$ ✓ ✓

$T = (0{,}6)(4)(9{,}8) + (4)(4{,}57)$

$T = 41{,}8\ \text{N}$ ✓ (3)

4. $f_r = \mu N$

$N = mg$

$f_r = \mu mg$ ✓

$f_r = 0{,}6 \times 4 \times 9{,}8$ ✓ ✓

$f_r = 23{,}52\ \text{N}$ ✓ (4)

5. $-T + m_B g = m_B a$ ✓ ✓

$-T = -8 \times 4{,}57 + (8 \times 9{,}8)$ ✓

Apparent weight = T = 41,84 N ✓ (4)

6. Less than ✓ (1)

7. Equal to ✓ (1)

[29]

1.9 Newton's Third Law of Motion

For a **third law forces pair:**

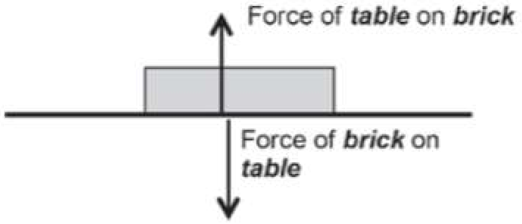
- the forces are **equal in magnitude**
- the forces act in the same straight line but in **opposite directions** on **different objects**
- the forces do *not* cancel each other, as they act on **different objects.**

When pairs of objects interact they exert forces on each other. If object A exerts a force on object B, object B will exert an equal force on object A but in the opposite direction.

For any two objects A and B;
$\vec{F}_{A\ on\ B} = -\vec{F}_{B\ on\ A}$

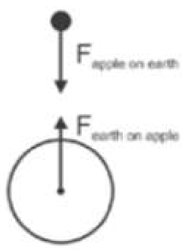
Example

The force diagram shows the pair of forces when a brick rests on a table. (Note: these are the *contact* forces)



Example

The **reaction force** of the weight of an object is the force that the object exerts on the earth, upwards. (These are not contact forces, they act at a distance.)



1.10 Newton's Law of Universal Gravitation

DEFINITION

Universal means that the statement is valid anywhere in the universe.

Newton's Law of Universal Gravitation states that:

Each body in the universe **attracts every other body** with a force that is **directly proportional** to the product of their masses and is **inversely proportional** to the square of the distance between their centres.

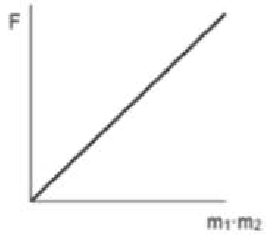
For any two objects: $F \propto m_1 \cdot m_2$ and $F \propto \frac{1}{r^2}$ $\therefore F = G\frac{m_1 m_2}{r^2}$

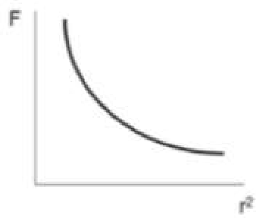
F: magnitude of force (N)

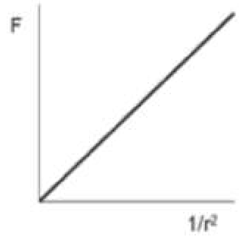
m: mass (kg)

r: distance between centres of the objects (m)

G: universal gravitation constant ($6{,}67 \times 10^{-11}$ $N \cdot m^2 \cdot kg^{-2}$)



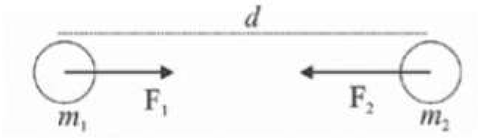




1) $F \propto m_1.m_2$
$\therefore$ straight line through the origin

2) $F \propto \frac{1}{r_2}$
$\therefore$ hyperbola

Worked example 11



A **force of gravitational attraction** exists between the earth with mass m_1 and a person with mass m_2. The force on m_1 is $\vec{F}_1$ and the force on m_2 is $\vec{F}_2$.

Compare the magnitudes (sizes) of these forces and state the name of the law which explains your answer.

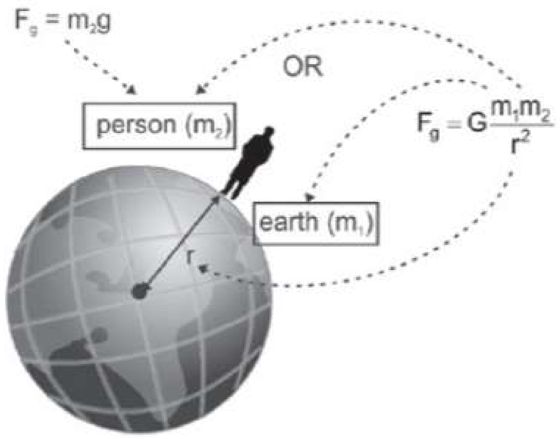
Solution

$\vec{F}_1 = -\vec{F}_2$ according to Newton's Third Law of Motion:

The force between the earth (m_1) and a person (m_2) standing on its surface:

$\therefore m_2 g = G\frac{m_1 m_2}{r^2}$

$\therefore g = G\frac{m_1}{r^2}$



The gravitational acceleration on earth (or on any planet) is:

- **dependent** on the **mass of the earth (planet) (m_1)**
- **dependent** on the **distance** between the centre of the object and the centre of the earth (planet)
- **independent** of the **mass of the object** on the planet on which the force acts. On a different planet, acceleration due to gravity is different.

Application of the law of Universal Gravitation: This law enables us to calculate the size (mass) of astronomical bodies like planets, stars, etc.

1.11 The difference between mass and weight

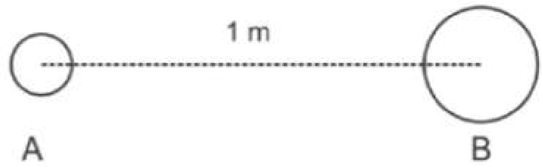
NB

It is important to understand and be able to compare mass and weight.

<u>Mass</u>	<u>Weight</u>
• Mass is the amount of **matter** in an object. • Mass determines the object's **inertia**. • Mass remains **constant**. • Mass is measured in **kilograms** (kg). • Mass is a **scalar** quantity (with magnitude, but not direction).	• Weight is determined by the **force of attraction** the earth exerts on the object. • Weight depends on the object's **distance** from the centre of the earth. • Weight depends on the **masses** of the earth (planet) and the object. • Weight is measured in **Newton** (N). • Weight is a **vector** quantity, so it has magnitude and direction. $\vec{F}g = m\cdot\vec{g}$ or $= m\cdot\vec{g}$ *where* $\vec{g}$ = gravitational acceleration ($9{,}8\ m\cdot s^{-2}$ on earth).

e.g. Worked example 12

The diagram shows a ball A of mass 0,01 kg which is 1 m (measured from centre to centre) from another ball B of mass 520 g. Calculate the magnitude of the force of ball A on ball B.



Solution

$$F = G\frac{m_1 m_2}{r^2} = \frac{6{,}67 \times 10^{-11} \times 0{,}01 \times 0{,}52}{1^2}$$

$$= 3{,}57 \times 10^{-14}\ N$$

e.g. Worked example 13

An object weighs 720 N on earth. It orbits the Earth in a satellite at a height equal to the earth's diameter, above the surface of the Earth. What does the object weigh on the satellite?

Hint: diameter = 2 × radius

Step by step

Step 1. Determine the number of radii from the centre of the Earth. → On the Earth's surface, the object is 1 radius from the Earth's centre.

Step 2. Determine how many times the distance between the object and the centre of the Earth has increased. → In orbit, the object is 1 diameter = 2 radii above the surface.

Step 3. Square this number (multiply it by itself). → So the object is 3 radii from the centre. $(3)^2 = 9$

Step 4. The force has decreased this number of times because $\vec{F} \propto \frac{1}{r^2}$ → ∴ the gravitational force on the object has decreased 9 times because $\vec{F} \propto \frac{1}{r^2}$

Step 5. Divide the value of the force (or weight) by the value calculated in step 3. → ∴ its weight on the satellite is

$720 \text{ N} \div 9 = 80 \text{ N}$

Gravitational acceleration on planets other than earth

Newton's universal law of gravitation can be used to calculate the acceleration due to the force of gravity on any planet.

If the mass and radius of a planet are known, we can calculate $\vec{g}$ for that planet.

e.g. Worked example 14

The Mars Rover is an automated vehicle that has been sent to explore the surface of the planet Mars.

If the value of acceleration due to gravity on the planet Mars is $\vec{g}_{Mars} = 3{,}7\ m{\cdot}s^{-2}$.

Calculate the weight of the Mars rover on Mars if it has a mass of 174 kg.

Solution

$\vec{w}_{Mars} = \vec{g}_{Mars} \times m_{object} = 3{,}7 \times 174 = 643{,}8$ N towards the centre of the planet Mars.