Census@school

# Data Handling & Probability

## Grades 10, 11 and 12

Statistics South Africa

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4

# Grade 10 Probability

In this chapter you:

- Revise the language of probability
- Calculate theoretical probability of events happening
- Calculate the relative frequency of events happening
- Draw and interpret Venn diagrams
- Use Venn diagrams to determine the probability of events happening
- Define mutually exclusive events
- Use the addition rule for probability and the complementary rule to determine probabilities.

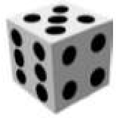
**WHAT YOU LEARNED ABOUT PROBABILITY IN GRADE 9**

*In Grade 9 you covered the following probability concepts for situations with equally probable outcomes:*

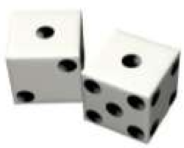
- *Determining probabilities for compound events using two-way tables and tree diagrams*
- *Determining the probabilities for outcomes of events and predicting their relative frequency in simple experiments*
- *Comparing relative frequency with probability and explaining possible differences*

## THE LANGUAGE OF PROBABILITY

✓ In this Study Guide we will use the term ***dice*** for both one dice or many dice.



**One dice**



**Two dice**

## a) What is Probability?

✓ Probability is a branch of mathematics that deals with calculating how likely it is that a given event occurs or happens. Probability is expressed as a number between 1 and 0. The words ***chance*** or ***likelihood*** are often used in place of the word probability.

- Tossing a coin is an ***activity*** or ***experiment***.
  If both Heads (H) and Tails (T) have an equal chance of landing face up, it is called a ***fair coin***

- Throwing a dice is an ***activity*** or ***experiment***.
  If each number on the dice has an equal chance of landing face up, it is called a ***fair dice***.

✓ When we talk about the probability of something happening, we call the something an **event**

- Getting tails when tossing a coin is an **event**.

## b) Listing Outcomes

✓ For any activity or probability experiment you can usually list all the ***outcomes***.

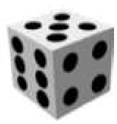
The set of ***all*** possible outcomes of a probability experiment is a ***sample space***.

An ***event*** consists of one or more outcomes and is a subset of the sample space. Outcomes of the event you are interested in are called the ***favourable outcomes*** for that event.

**EXAMPLE 1**
List the ***sample space, event*** and ***favourable outcomes*** of the following probability experiments:
a) Throw a dice and get a 6
b) Throw a dice and get an even number
c) Toss a coin and get a head (H)

**SOLUTION:**

a) The **activity** is *throw a dice*
The **sample space** is 1; 2; 3; 4; 5 and 6
The **event** you are interested in is ***get a 6***
The **favourable outcome** is 6.



b) The **activity** is *throw a dice*
The **sample space** is 1; 2; 3; 4; 5 and 6
The **event** you are interested in is ***get an even number***
The **favourable outcomes** are 2; 4 and 6.

**EXAMPLE 1 (continued)**

c) The **activity** is *toss a coin*
The **sample space** is heads (H) and tails (T)
The **event** you are interested in is ***get a head*** (H).
The **favourable outcome** is H.

## c) Probability Scales

✓ Some events ***always happen.*** We say that they are **certain** to happen and give them a probability of 1.

It is **certain** that the day after Monday is Tuesday
The probability that the day after Monday is Tuesday is 1.

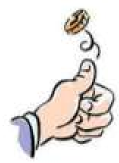
| |
|---|
| **Sunday**<br>**Monday**<br>**Tuesday** |

✓ Some events ***never happen.*** We say that they are **impossible** and give them a probability of 0.

If you throw an ordinary dice, it is **impossible** to get a 7.
The probability of getting a 7 when you throw an ordinary dice is 0.

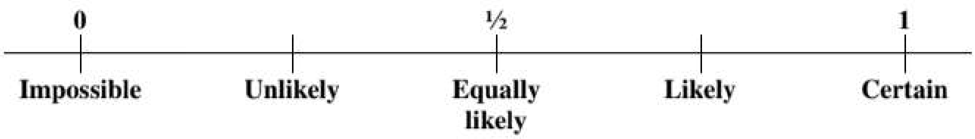
✓ Some events are ***not certain,*** but are ***not impossible*** either. They may or may not happen. These probabilities lie between 0 and 1.

If you toss a fair coin it may land on heads or it may not.
The chances are **equally likely.**
We say that there is a **50-50 chance** that it will land on heads.



✓ We can write probabilities **in words** or as **common fractions**, **decimal fractions** or **percentages**.

The following number line shows **words**:




To **compare probabilities**, we compare the sizes of the fractions, decimal fractions or percentages.

- The *less likely* an event is to happen, the *smaller* the fraction, decimal fraction or percentage.
- The *more likely* an event is to happen, the *larger* the fraction, decimal fraction or percentage.

The following number line shows **common fractions**:

| Impossible | | | | | | | | Certain |
|---|---|---|---|---|---|---|---|---|
| **0** | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{1}{2}$ | $\frac{5}{8}$ | $\frac{2}{3}$ | $\frac{7}{8}$ | 1 |

The following number line shows **decimal fractions**:

| Impossible | | | | | | | | | | Certain |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0,1 | 0,2 | 0,3 | 0,4 | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | 1 |

The following number line shows **percentages**:

| Impossible | | | | | | | | | | Certain |
|---|---|---|---|---|---|---|---|---|---|---|
| 0% | 10% | 20% | 30% | 40% | 50% | 60% | 70% | 80% | 90% | 100% |

Remember that 100% = 100 ÷ 100 = 1

# CALCULATING PROBABILITY

- ✓ The method you use to calculate probabilities depends on the type of probability you are dealing with. We can find ***theoretical probability*** (also called ***actual probability***) and ***relative frequency*** (also called ***experimental probability***).
- ✓ The probability that Event E will occur is written ***P(E)*** and is read ***the probability of Event E occurring***. The same terminology is used for both theoretical probability and relative frequency.

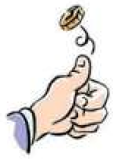
## a) Theoretical Probability (or Actual Probability)

- ✓ Theoretical probability is used when each outcome in a sample space is equally likely to occur.
- ✓ The theoretical probability for an Event E is given by:

***Probability of Event E happening***

$$= \frac{\textit{number of outcomes for Event E}}{\textit{total number of possible outcomes in the sample space}}$$

**EXAMPLE 2**
Calculate the probability of getting a head (H) when a fair coin is tossed.
Write the answer as a fraction in simplest form, as a decimal and as a percentage.



**SOLUTION:**
Because this is a fair coin, each outcome is equally likely to occur, so we can find the theoretical probability.

The **event** is *getting a head* (H).
The **possible outcomes** or **sample space (S)** are heads and tails (*H and T*).
The **total number of possible outcomes in the sample space = *n(S)*** = 2.

The **favourable outcome** is *heads (H)*.
The **number of favourable outcomes = *n(H)*** = 1.

We use the formula:

$$\textit{Probability of an event happening} = \frac{\textit{number of favourable outcomes for that event}}{\textit{total number of possible outcomes in the sample space}}$$

$$= \frac{\textit{number of heads}}{\textit{total number of possible outcomes in the sample space}}$$

$$P(H) = \frac{n(H)}{n(S)}$$

$$= \frac{1}{2}$$

$$= 0{,}5$$

$$= 50\%$$

***NOTE:***

- If P(E) stands for the probability of event E occurring then $\mathbf{0 \leq P(E) \leq 1.}$
- In other words, the probability of event E occurring is a rational number from 0 up to and including 1.

**EXAMPLE 3**

A regular six-sided fair dice is thrown once.

a) List the sample set.
b) How many elements are there in the sample set?
c) List all the favourable outcomes for getting a score of 3 or more.
d) How many favourable outcomes are there?
e) What is the probability of getting a score of 3 or more? Give your answer as a fraction in simplest form, as a decimal and as a percentage both correct to 2 decimal places.

- *You can make your own 6 - sided dice using the net on the last page of this chapter.*

**<u>SOLUTION</u>:**

There are 6 numbers on a dice and each number has an equal chance of landing face up. Because this is a fair dice, each outcome is equally likely to occur, so we can find the theoretical probability (also just called ***probability***).

a) The ***sample set***, S, is 1, 2, 3, 4, 5 and 6 or {1; 2; 3; 4; 5; 6}

b) $n(S) = 6$

c) The *favourable outcomes* for this event are the numbers that are 3 or more.
So the *favourable outcomes* are 3; 4; 5; 6 or {3; 4; 5; 6}

d) $n(3 \text{ or more}) = 4$

e) $Probability\ of\ an\ event\ happening = \dfrac{number\ of\ favourable\ outcomes\ for\ that\ event}{total\ number\ of\ possible\ outcomes\ in\ the\ sample\ set}$

$$\begin{aligned} P(3\ or\ more) &= \frac{n(3\ or\ more)}{n(S)} \\ &= \frac{4}{6} \\ &= \frac{2}{3} \\ &= 0{,}66666\ldots \\ &\approx 0{,}67 \\ &\approx 66{,}67\% \end{aligned}$$

## EXERCISE 4.1

Give each of the answers in this exercise

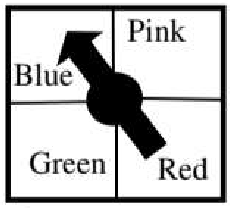
i) as a common fraction in simplest form,

ii) as a decimal fraction (correct to 2 decimal places)

iii) as a percentage (correct to 1 decimal place).

1) A fair dice is rolled once.
   a) List the elements of the sample space.
   b) What is the probability that you will get
      i) A six?
      ii) An odd number?
      iii) A seven?
      iv) More than 2?
      v) Less than 10?

2) The spinner alongside is spun.
   a) List the elements of the sample space
   b) What is the probability of the spinner (arrow) landing on
      i) Green?
      ii) Yellow?



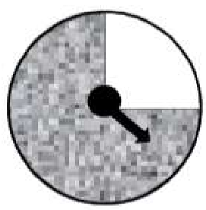

3) Each letter of the word **MATHEMATICS** is written on a separate piece of paper of the same shape and size and put in a box. Nomsa closes her eyes and takes one piece of paper out of the bag at random.
   a) List the elements of the sample space
   b) What is the probability that she takes a piece of paper with:
      i) An M on it?
      ii) A vowel on it?

***At random*** means you choose without method or without thinking about your choice.

4) Six counters in a bag are numbered 3 4 7 9 10 11. One counter is drawn at random from the bag.
   a) What does the sample space consist of?
   b) Calculate the probability that the number drawn is
      i) An odd number
      ii) A prime number
      iii) A square number

5) A learner is chosen at random from a group of 18 boys and 12 girls.
   a) Determine $n(S)$ where $S$ is the sample space.
   b) What is the probability that this learner is
      i) A boy?
      ii) A girl?

6) The spinner alongside is spun.
   a) Determine $n(S)$ where $S$ is the sample space.
   b) Calculate the probability of the spinner landing on the shaded area.



## b) Relative Frequency (or Experimental Probability)

✓ Sometimes we *calculate* probability and sometimes we *estimate* probability.

- Probability that is *calculated* is called ***theoretical probability*** or just ***probability***.
- Probability that is *estimated* is calculated after performing a very large number of trials of an experiment or conducting a survey involving a very large number of items, and is called ***relative frequency***.

**Examples of experiments** that can be used to calculate ***relative frequency:***

i) Tossing a coin 500 times and counting the number of times it lands on heads.

ii) Throwing a dice 200 times and counting the number of times it lands on an even number.

iii) Repeatedly taking a counter out of a bag containing ten counters numbered from 34 to 43. Recording the number on the counter, replacing the counter into the bag, and seeing how many times you get a multiple of 3 after taking out and returning a counter 1 000 times.

✓ When an experiment is repeated over and over, the relative frequency of an event approaches the theoretical or actual probability of the event.

- If you want to *estimate the probability* of an event by using an *experiment*, you need to perform *a very large number of trials* as a pattern often does not become clear until you observe a ***large*** number of trials
- If you want to *estimate the probability* of an event by using *the results of a* survey, the survey should involve a large number of items as a pattern often does not become clear until you observe a ***large*** number of items

✓ Relative frequency can be used even if each outcome of an event is not equally likely to occur.

✓ We find relative frequency using the following formula:

$$\textbf{\textit{Relative frequency of an event happening}} = \frac{\textbf{\textit{number of times the event happens}}}{\textbf{\textit{total number of trials in the experiment}}}$$

✓ Relative frequency is a fraction of the occurrences.

Like probability, $0 \leq$ ***relative frequency*** $\leq 1$

✓ The results of an experiment or of a survey are often shown in a table as in the following example.

## EXAMPLE 4

In the 2009 Census@School, learners were asked how they usually travel to school in the morning. The following table shows the responses from 443 590 learners who live less than 1 km from the school.

| How they get to school | Number of learners |
|---|---|
| Walk/foot | 399 220 |
| Car | 28 555 |
| Train | 782 |
| Bus | 3 052 |
| Bicycle | 1 415 |
| Scooter | 296 |
| Taxi | 10 270 |
| **TOTAL** | **443 590** |

a) Which are the 3 most popular ways of getting to school?
b) Determine $n(S)$ where $S$ is the sample set.
c) Estimate the probability (as a percentage correct to 2 decimal places) that one of these learners selected at random
   i) walks to school
   ii) comes to school by helicopter
   iii) comes to school by car and taxi
   iv) comes to school by car or taxi

**SOLUTION:**

a) The 3 most popular ways of getting to school are walk/foot, car and taxi.

b) 443 590 learners responded so $n(S) = 443\ 590$

c)

i) P(walks to school) = $\frac{n(walk\ to\ school)}{n(S)} = \frac{399\ 220}{443\ 590} \approx 90{,}00\%$

ii) P(comes to school by helicopter)

$$= \frac{n(come\ to\ school\ by\ helicopter)}{n(S)} = \frac{0}{443\ 590} = 0{,}00\%$$

iii) Nobody comes to school by car AND by taxi.
P(comes to school by car AND by taxi)

$$= \frac{n(come\ to\ school\ by\ car\ AND\ by\ taxi)}{n(S)} = \frac{0}{443\ 590} = 0{,}00\%$$

iv) 28 555 learners come by car and 10 270 learners come by taxi.
*n(come to school by car OR by taxi)* = 28 555 + 10 270 = 38 825.
P(comes to school by car or by taxi)

$$= \frac{n(come\ to\ school\ by\ car\ or\ by\ taxi)}{n(S)} = \frac{38\ 825}{443\ 590} \approx 8{,}75\%$$

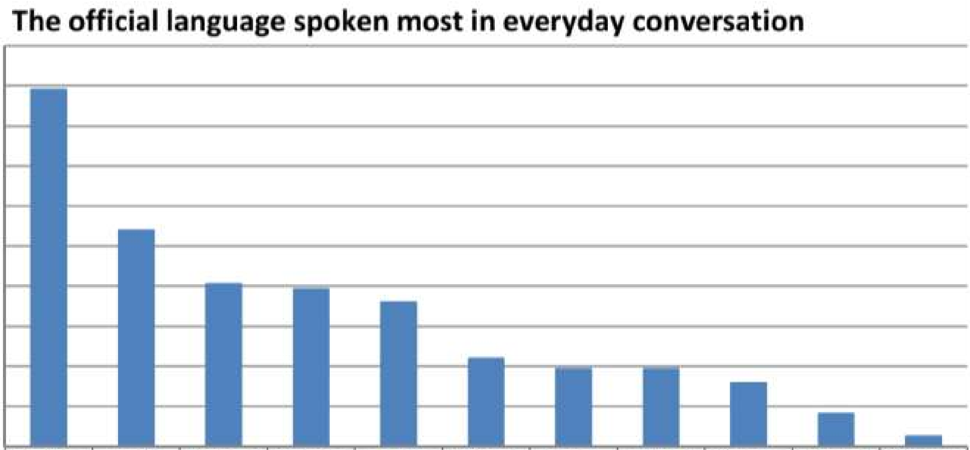
**EXERCISE 4.2**

1) This information comes from the table in the previous example giving the responses from 443 590 learners who live less than 1 km from the school.

| How they get to school | Number of learners |
|---|---|
| Bus | 3 052 |
| Bicycle | 1 415 |

a) Determine $n(S)$ if $S$ is the sample set
b) Estimate the probability (as a percentage correct to 2 decimal places) that one of these learners selected at random from the sample
   i) comes to school by bus
   ii) comes to school by bicycle
   iii) comes to school by bus or bicycle
c) You should find the probabilities in b) low. Why do you think this is so?

2) The bar graph below is taken from 2009 Census@School. A sample of all the learners in South Africa was asked which of the official languages they spoke most in everyday conversation. (*The language used in everyday conversation is the language you use most of the time when talking and listening to others).* The bar graph shows their answers.




| | English | isiZulu | Afrikaans | Setswana | Sesotho | isiXhosa | Sepedi | siSwati | Xitsonga | isiNdebele | Tshivenda |
|---|---|---|---|---|---|---|---|---|---|---|---|
| No: of learners | 445492 | 270062 | 202570 | 196259 | 180020 | 109596 | 96538 | 96538 | 79137 | 40961 | 12310 |

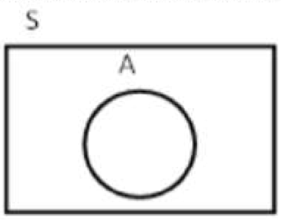
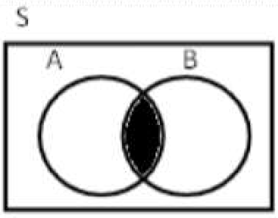
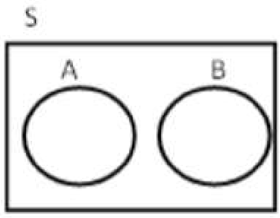
a) How many learners were surveyed?
b) Estimate the probability (as a percentage correct to 1 decimal place) that a learner selected at random from the sample
   i) Speaks mainly English in everyday conversation
   ii) Speaks mainly isiZulu OR Afrikaans in everyday conversation
c) In Census 2011 it was found that in South Africa, with a population of 51 770 560, 9,6% speak English and 36,2% speak isiZulu or Afrikaans in everyday conversation. Which results, 2009 Census@School or Census 2011, give better estimates? Give reasons for your answer.

# VENN DIAGRAMS

- ✓ ***Venn diagrams*** were introduced in 1880 by John Venn as a way of picturing relationships between different groups of items.
- ✓ Venn diagrams use overlapping circles or closed curves within an enclosing rectangle to represent the items that are common to the groups of items.
- ✓ You can use Venn diagrams to help you work out the probability of an event occurring.

## a) Drawing Venn diagrams

- ✓ We generally use a rectangle to represent a ***sample space (S)***. However, any closed shape could be used.
- ✓ The circles represent ***events*** within the sample space. However any closed shape could be used.

| | | |
|---|---|---|
| <br> | <br> | <br> |
| This Venn diagram shows Event A in sample space S. | This Venn diagram shows Events A and B which have common values.<br><br>The part where they overlap is called the ***intersection of A and B.***<br><br>The section that is shaded ***belongs to Event A and to Event B***. | This Venn diagrams shows Events A and B which have no common values.<br><br>A and B are called ***disjoint sets***. |

## EXERCISE 4.3

1) A sample space S consists of whole numbers from 20 to 29 inclusive.
   Event A consists of the multiples of 4 in S.
   Event B consists of the factors of 420 in S.
   Event C consists of the multiples of 5 in S.
   Event D consists of the multiples of 3 in S.
   a) List the elements in
      i) S    ii) A    iii) B    iv) C    v) D
   b) Draw Venn diagrams to show
      i) Sample space S and event A
      ii) Sample space S, event A and event B
      iii) Sample space S, event C and event D

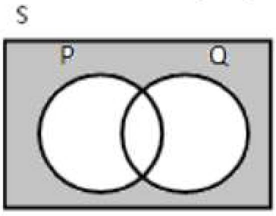
2) A fair eight-sided dice is rolled. [*You can make your own 8-sided dice using the net given on the last page of this chapter*].
   Event P is scoring a prime number.
   Event E is scoring a multiple of 2
   Event F is scoring more than 3
   Event G is scoring an even number
   Event H is scoring an odd number.
   a) List S, the possible outcomes of throwing a fair 8-sided dice.

   b) List the elements in
      i) P    ii) E    iii) F
      iv) G    v) H
      vi) The intersection of E and F
      vii) The intersection of G and H
   c) Draw Venn diagrams to illustrate each of the following in sample space S:
      i) Event P
      ii) Events E and F
      iii) Events G and H.

3) A sample space S consists of the letters of the alphabet.
   Event P consists of the letters of the word PROBABILITY.
   Event M consists of the letters of the word MATHEMATICS
   a) Which letters are common to the words PROBABILITY and MATHEMATICS?
   b) Draw a Venn diagram to show the information.
   *Hint: If a letter in Event P or Event M occurs more than once you must write it down each time it occurs.*

**EXAMPLE 6 (continued)**

e) Values in neither P nor Q = {4; 5; 7; 8}

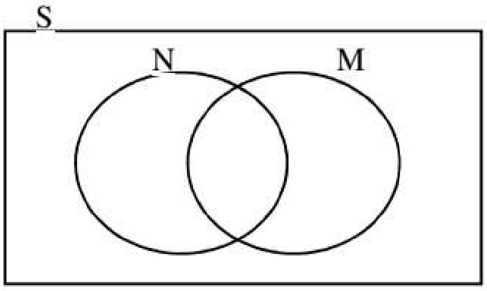



### EXERCISE 4.4

Draw six Venn diagrams like the one given.

On each one shade one of the following:

1) N
2) N *and* M
3) N *or* M
4) N but not M
5) M but not N
6) Neither M nor N



## c) Venn Diagrams Showing the Number of Outcomes in Events

✓ Sometimes there are too many outcomes to list in the Venn diagram. When this happens you write the ***number*** of outcomes in the Venn diagram.

**EXAMPLE 7**
150 people were asked which type of movie they like to watch.

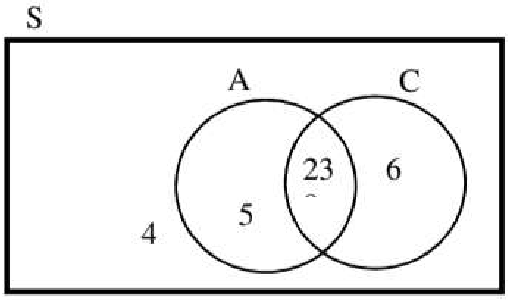
- 80 said they liked Action (A)
- 55 said they liked Comedy (C)
- 23 said they liked both.

Draw a Venn diagram to illustrate this information.

**<u>SOLUTION</u>:**

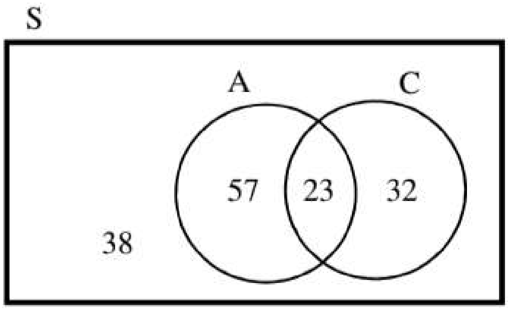
<u>Step1:</u>

- Because there are some people who like Action (A) *and* Comedy (C), we draw 2 intersecting circles in the sample space.
- Fill in 23 where the circles overlap (*in the intersection of A and B*) because 23 people like **both Action and Comedy**.




<u>Step 2:</u>

- The number of people who like ***A only*** $= 80 - 23 = 57$.
- The number of people who like ***C only*** $= 55 - 23 = 32$.
- The number of people who like ***neither*** $= 150 - (57 + 23 + 32) = 38$.
- Fill these values in on the Venn diagram.

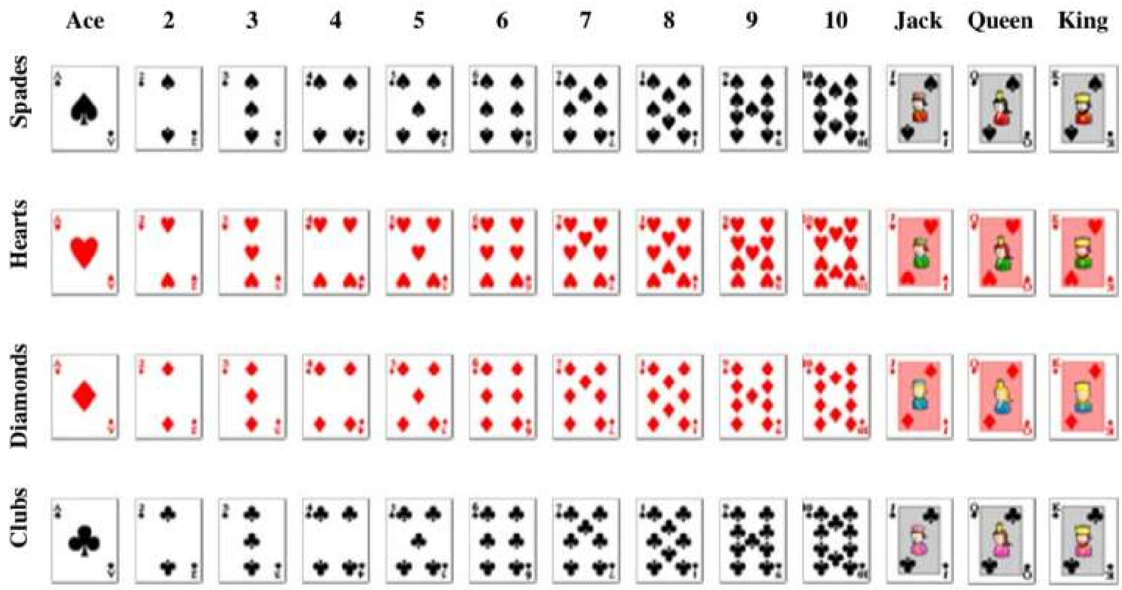



<u>Step 3:</u>
Check:

- o Are there 80 people altogether in A?
- o Are there 55 altogether in C?
- o Does 38 + 57 + 23 + 32 give 150?

✓ We often use playing cards in probability experiments

**A SET OF 52 PLAYING CARDS**



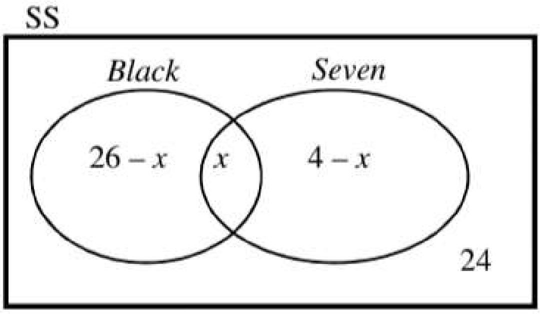

http://en.wikipedia.org/wiki/Playing_card

Notice that

- A set of playing cards contains 52 cards.
- Thirteen of the cards are marked with a black spade (♠)
- Thirteen of the cards are marked with a red heart (♥)
- Thirteen of the cards are marked with a red diamond (♦)
- Thirteen of the cards are marked with a black club (♣)
- Each set of thirteen cards is called a ***suit***
- Each suit consists of cards numbered 1 to 10, a card marked with a J (for Jack), Q (for Queen) and K (for King). The 1 is generally marked A and is called the Ace.

## EXERCISE 4.5

1) The Venn diagram illustrates the number of playing cards in a pack of playing cards which are black as well as the number of cards that are sevens. Use the Venn diagram to answer the following:




   a) How many cards are there in a pack of cards?
   b) How many black cards are there in a pack of playing cards?
   c) How many sevens are there in a pack of playing cards?
   d) How many cards are black *or* seven?
   e) Find the value of $x$; where $x$ is the number of black sevens in a pack of playing cards.
   f) Check your answers by substituting for $x$ and adding.

2) The 2009 Census@School was completed by 124 975 Grade 10 learners. 190 168 of the learners who completed it were 15 years old. 82 426 of the 15 year olds were in Grade 10.
   a) How many Grade 10s were not 15 years old?
   b) How many 15 year olds were not in Grade 10?
   c) Draw a Venn diagram to illustrate the situation.
   d) How many learners were in the sample set?
   e) Suppose one of the learners in the sample set was selected at random, what is the probability (written as a percentage correct to 2 decimal places) that this learner is in Grade 10 AND is 15 years old?

3) 24 learners in a class were invited to Adam and Nisha's birthday parties.
   13 learners decided to go to Adam's party.
   12 learners decided to go to Nisha's party.
   3 learners decided to not go to either of the parties.
   a) How many learners went to the two parties (either Adam's or Nisha's or both)?
   b) How many learners went to both Adam's party ***and*** to Nisha's party?
   c) Draw a Venn diagram to illustrate the situation.
   d) Suppose one of the 24 learners is selected at random, what is the probability (written as a decimal correct to 3 decimal places):
      i) That the learner only goes to Adam's party?
      ii) That the learner goes to both parties?
      iii) That the learner doesn't go to either of the parties?

## d) Finding A Relationship Between The Number Of Outcomes In Different Events

***Remember:***

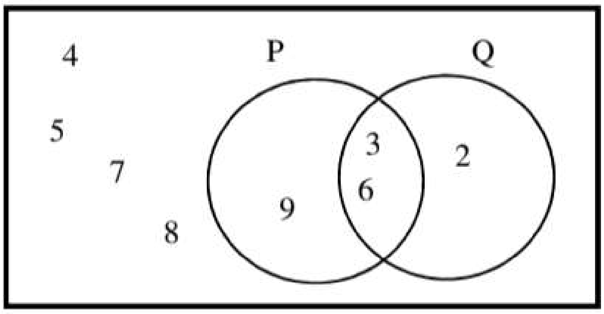
- To find $n(P)$ count the outcomes in $P$ only *and* in the intersection.
- To find $n(Q)$ count the outcomes in $Q$ only *and* in the intersection.

**EXAMPLE 8**

a) Use the given Venn diagram to calculate:

i) $n(P)$
ii) $n(Q)$
iii) $n(P\ and\ Q)$
iv) $n(P\ or\ Q)$
v) $n(P) + n(Q) - n(P\ and\ Q)$

S




b) Is $n(P) + n(Q) - n(P\ and\ Q) = n(P\ or\ Q)$?

**<u>SOLUTION</u>:**

a)

i) P consists of the elements 3, 6 and 9.
So $n(P) = 3$

ii) Q consists of the elements 2, 3 and 6
So $n(Q) = 3$

iii) P *and* Q consists of the elements in the intersection of P and Q.
So $n(P\ and\ Q) = 2$

iv) P *or* Q consists of the elements in P only, in P *and* Q and in Q only.
So P *or* Q consists of the elements 2, 3, 6 and 9 and $n(P\ or\ Q) = 4$

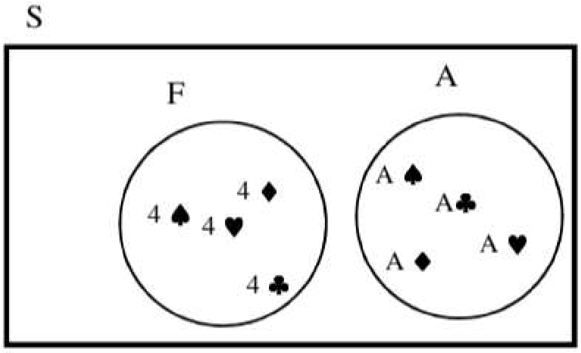
v) $n(P) + n(Q) - n(P\ and\ Q) = 3 + 3 - 2 = 4$

b) $n(P) + n(Q) - n(P\ and\ Q) = 4$
$n(P\ or\ Q) = 4$
So $n(P) + n(Q) - n(P\ and\ Q) = n(P\ or\ Q)$

**EXERCISE 4.6**

The Venn diagram illustrates the number of playing cards in a pack of playing cards that are fours as well as the number of cards that are aces.

Let F be the set of fours in a pack of playing cards
Let A be the set of aces in a pack of playing cards




1) Use the Venn diagram to find the following:
   a) $n(F)$
   b) $n(A)$
   c) $n(F\ and\ A)$
   d) $n(F\ or\ A)$
   e) $n(F) + n(A) - n(F\ and\ A)$
2) Is $n(F) + n(A) - n(F\ and\ A) = n(F\ or\ A)$?
3) You select a card at random from the pack of cards.
   a) Determine $n(S)$ where $S$ is the sample set.
   b) What is the probability (written as a fraction in simplest form) that this card is:
      i) A four?
      ii) An ace?
      iii) A four *and* an ace?
      iv) A four *or* an ace?
   c) Calculate $P(F) + P(A) - P(F\ and\ A)$
   d) Is $P(F\ or\ A) = P(F) + P(A) - P(F\ and\ A)$?

***NOTE:***

- The relationship between the number of outcomes in events A and B is:
  $n(A\ or\ B) = n(A) + n(B) - n(A\ and\ B)$
- This relationship is true
  - when the events have *common outcomes* as in EXAMPLE 8
  - when the events have *no outcomes in common* as in EXERCISE 4.6.
- The relationship is also true for probabilities. The relationship $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$ is called the ***addition rule for probability.***

## e) Venn Diagrams Showing The Probability Of Events Happening

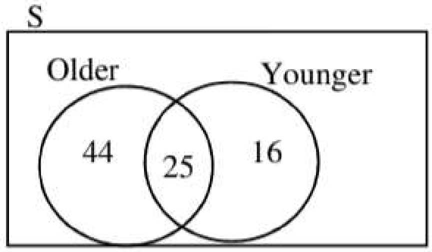
- ✓ You have worked with Venn diagrams
  - i) That list all the outcomes of the events
  - ii) That show the number of outcomes in the events.

- ✓ You can also show probabilities in Venn diagrams. The sum of the probabilities in the Venn diagram must be 1 or 100%.

**EXAMPLE 9**

A survey of 100 learners shows that 69 learners have an older sibling, 41 learners have a younger sibling and 25 learners have both. *(A sibling is a brother or a sister).*

The Venn diagram shows the results of the survey.




a) How many of the learners have no siblings?

b) Determine the probability (as a decimal correct to 2 decimal places) that a learner chosen at random has:
   - i) an older sibling
   - ii) a younger sibling
   - iii) an older sibling *and* a younger sibling
   - iv) no siblings

c) Redraw the Venn diagram to show the probabilities.

d) Use the Venn diagram to determine
   $P(an\ older\ sibling\ or\ a\ younger\ sibling)$

e) Is *P(older) + P(younger) – P(older **and** younger)*
   *= P(older **or** younger)?*

**SOLUTION:**

a) $n(no\ siblings) = 100 - (44 + 25 + 16) = 15$

b)

i) $P(older\ sibling) = \frac{n(older\ sibling)}{n(sample\ space)} = \frac{69}{100} = 0{,}69$

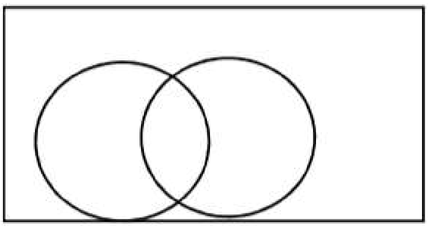
ii) $P(younger\ sibling) = \frac{n(younger\ sibling)}{n(sample\ space)} = \frac{41}{100} = 0{,}41$

iii) $P(older\ and\ younger\ sibling) = \frac{n(older\ and\ younger\ sibling)}{n(sample\ set)} = \frac{25}{100} = 0{,}25$

iv) $P(no\ siblings) = \frac{n(no\ siblings)}{n(sample\ set)} = \frac{15}{100} = 0{,}15$

**EXAMPLE 9 (continued)**

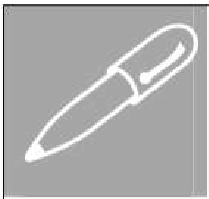
c)



d) $P(an\ older\ sibling\ or\ a\ younger\ sibling) = 0{,}44 + 0{,}25 + 0{,}16 = 0{,}85$

e) $P(older) + P(younger) - P(older\ and\ younger) = 0{,}69 + 0{,}41 - 0{,}25 = 0{,}85$

So $P(older) + P(younger) - P(older\ and\ younger) = P(older\ or\ younger)$.



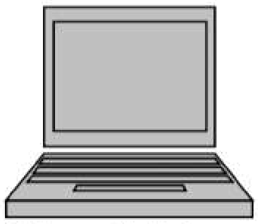
**EXERCISE 4.7**

1) Of the 420 Grade 10 learners at Farhana's high school, 126 do Geography (G) and 275 do Maths (M). 55 of the learners do both Geography and Maths.
   a) Draw a Venn diagram to illustrate this situation.
   b) What percentage (correct to 1 decimal place) of the learners do both Geography *and* Maths?
   c) How many learners do neither Geography nor Maths?

2) Two events A and B have the following probabilities:
   $P(A) = 0{,}2;\ P(B) = 0{,}4$ and $P(A\ and\ B) = 0{,}08$
   b) Draw a Venn diagram to illustrate the situation
   c) Determine $P(A\ or\ B)$
   d) Is $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$?

3) Results from Census 2011 show that of the 14 450 133 people in South Africa who have computer access, 16% connect to the internet using their cell phones and 9% connect to the internet using use their home computers. 3% use their cell phones to connect their home computers to the internet.

**16% use cellphone**

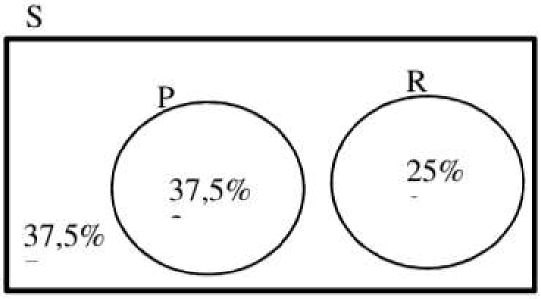


**9% use their home computer**

   a) Draw a Venn diagram to show this information.
   b) What is the probability that a person chosen at random uses neither a cell phone nor a home computer for internet access?
   c) How many South Africans do not use cell phones or home computers for internet access? Give your answer correct to the nearest ten thousand people.

**EXERCISE 4.7 (continued)**

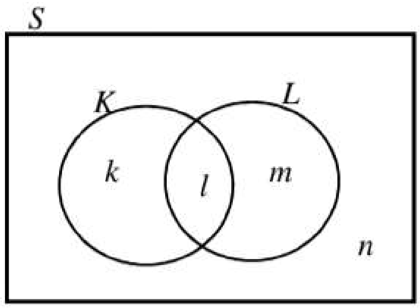
4) The following Venn diagram shows that the probability of Event P occurring is 37,5% and the probability of event R occurring is 25%.




a) Calculate the following probabilities:
   i) $P(P\ and\ R)$
   ii) $P(P\ or\ R)$

b) Show that $P(P) + P(R) - P(Pand\ R) = P(P\ or\ R)$

5) The Venn diagram shows the relationship between events $K$ and $L$.
Suppose one of the items in the events is selected at random, then the probability of the item being in event $K$ is $P(K) = \frac{n(K)}{n(S)} = \frac{k+l}{k+l+m+n}$




a) Use the Venn diagram to find:
   i) $P(L)$
   ii) $P(K\ and\ L)$
   iii) $P(K\ or\ L)$

b) Show that $P(K\ or\ L) = P(K) + P(L) - P(K\ and\ L)$

# MUTUALLY EXCLUSIVE EVENTS

✓ ***Mutually exclusive*** events are events that cannot happen at the same time.

***Examples of mutually exclusive events***:

- Turning left and turning right are mutually exclusive because you can't do both at the same time
- Taking a 4 and taking a 7 from a pack of cards are mutually exclusive because a card cannot be a 4 and a 7 at the same time.

***Examples of events that are NOT mutually exclusive:***

- Turning left and scratching your head can happen at the same time, so they are **NOT** mutually exclusive
- Taking a King (K) and taking a heart (♥) from a pack of cards in **NOT** mutually exclusive as the card could be the king of hearts (K♥).

✓ If events are **NOT** mutually exclusive, then $P(A \text{ and } B) \neq 0$ and $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

✓ Mutually exclusive sets are ***disjoint sets***. If events **ARE** mutually exclusive, then $P(A \text{ and } B) = 0$.

✓ If events are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$

**EXERCISE 4.8**

1) You randomly select a single card from a pack of playing cards.
   a) Can it be red and black at the same time?
   b) Can it be black and a 10 at the same time?
   c) Can it be a King (K) and a Queen (Q) at the same time?
2) Are the following events mutually exclusive or not? Explain your answer.
   a) Randomly drawing a red card and a black Jack (J) from a pack of playing cards.
   b) Randomly drawing a red card and a card marked with a diamond (♦) from a pack of playing cards.
   c) Tossing a coin and getting a heads (H) and getting a tails (T) at the same time.
   d) Rolling a fair dice and getting a 3 and a 4 at the same time.
   e) Eating a sandwich and eating jam.
   f) 'Living in Kwa-Zulu Natal' and 'speaking English at home'.

# PROBABILITY RULES

## a) The Addition Rule for Probability (often called the "OR LAW")

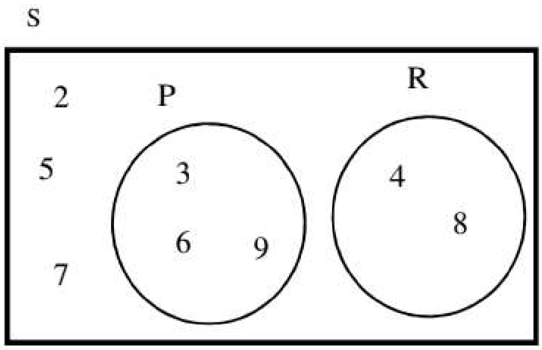
- ✓ If two events are ***NOT mutually exclusive***, then
  ***P(A or B) = P(A) + P(B) – P(A and B)***

- ✓ If two events are ***mutually exclusive***, then
  - ***P(A and B) = 0***
  - ***P(A or B) = P(A) + P(B)***

**EXAMPLE 10**

In the Venn diagram, Sample Space S contains the numbers from 2 to 9. The elements of Event P are the multiples of 3 in S.
The elements of Event R are the multiples of 4 in S.

a) Are events P and R mutually exclusive?
b) Are sets P and R disjoint?
c) Determine *n(S)* where S is the Sample Space.
d) Use the addition rule P(P or R) = P(P) + P(R) – P(P and R) to determine P(P or R).




**SOLUTION:**

a) Events P and R are *mutually exclusive* – they have no elements that are the same.
b) Sets P and R are *disjoint* – mutually exclusive events are represented as disjoint sets on a Venn diagram.
c) $N(S) = 8$
d) $P(P) = \frac{n(P)}{n(S)} = \frac{3}{8}$

   $P(R) = \frac{n(P)}{n(S)} = \frac{2}{8}$

   $P(P \text{ and } R) = \frac{n(P \text{ and } R)}{n(S)} = \frac{0}{8} = 0$

   So $P(P \text{ or } R) = P(P) + P(R) - P(P \text{ and } R) = \frac{3}{8} + \frac{2}{8} - 0 = \frac{5}{8}$

## b) The Complementary Rule:

- ✓ ***Complementary events*** are events that cannot occur at the same time.

- ✓ If the event A occurs, then the complement of A is ***not A***.

  ***Examples of complementary events*** are
  - Speaking English in everyday conversation or not speaking English in everyday conversation.
  - Walking to school on Monday morning or not walking to school on Monday morning.
  - Tossing a coin and getting a Head or not getting a Head.
  - Rolling a fair dice and scoring a 3 or not scoring a 3.

- ✓ The event ***A*** and its complement ***not A*** are mutually exclusive.

- ✓ We write: $\boldsymbol{P(A) + P(not\ A) = 1}$

  $\boldsymbol{or \quad P(not\ A) = 1 - P(A)}$

  $\boldsymbol{or \quad P(A) = 1 - P(not\ A)}$

**EXAMPLE 11**

a) The probability that a Grade 10 learner, chosen at random, will pass English is 80%. What is the probability that this learner will not pass English?

b) The probability of getting a white ball from a bag of balls is $\frac{1}{4}$. What is the probability of ***not*** getting a white ball?

c) A bag contains red and blue cards. The probability of taking a red card is $\frac{2}{5}$. What is the probability of taking a blue card?

<u>**SOLUTION**</u>**:**

a) You either pass English or you do not pass English, so the events are complementary.
$P(not\ pass\ English) = 1 - P(pass\ English) = 100\% - 80\% = 20\%$.
This learner has a 20% chance of not passing English.

b) $P(ball\ is\ not\ white) = 1 - P(ball\ is\ white) = 1 - \frac{1}{4} = \frac{3}{4}$

c) Taking a red card and taking a blue card from a bag of red and blue cards are complementary events.

$$\begin{aligned} P(taking\ a\ blue\ card) &= 1 - P(not\ taking\ a\ blue\ card) \\ &= 1 - P(taking\ a\ red\ card) \\ &= 1 - \frac{2}{5} \\ &= \frac{3}{5} \end{aligned}$$

**EXERCISE 4.9**

1) Out of a group of 27 girls, 10 play netball (N) and the rest do not play netball.
   a) How many in the group of girls ***do not*** play netball.
   b) Represent this information in a Venn diagram.
   c) Are the events play netball and do not play netball complementary?
   d) Determine *n(S)* where S is the sample set.
   e) One of the 27 girls is chosen at random. Determine the probability as a common fraction that:
      i) The chosen girl plays netball.
      ii) The chosen girl does not play netball.

2) A number is chosen at random from a set of numbers from 1 to 50, including 1 and 50. Event A is choosing a number that is a perfect square.
   a) Determine *n(S)* where S is the sample space.
   b) List the elements of A.
   c) Determine *P(A)*, written as a fraction in simplest form.
   d) Hence calculate the probability that the chosen number is ***not*** a perfect square.

3) In the 2009 Census@School, the Grade 10 to 12 learners were asked what type of home they stay in most of the time. 58,6% of the learners answered that they live in a house (H) and 14,8% live in a traditional dwelling (T).
   a) Are 'living in a house (H)' and 'living in a traditional dwelling (T)' mutually exclusive?
   b) Calculate the percentage of the Grade 10 to 12 learners who do not live in a house or traditional dwelling.
   c) Draw a Venn diagram to show the percentage of learners who live in a house (H), the percentage of learners who live in a traditional dwelling (T) and the percentage of learners who do not live in a house or in a traditional dwelling.
   d) Suppose one of the Grade 10 to 12 learners is selected at random. Determine
      i) *P(H)*
      ii) *P(T)*
      iii) *P(not H)*
      iv) *P(not T)*
      v) *P(H or T)*

## c) Using the probability rules to determine probabilities

✓ Venn diagrams and rules can be used to find unknown values.

**EXAMPLE 12**

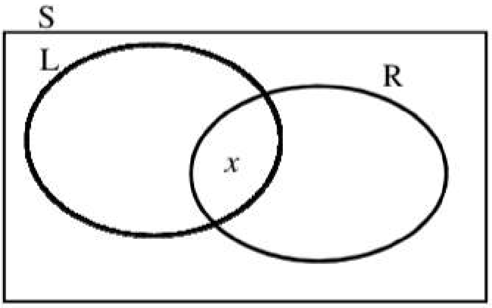
During the 2009 Census@School, 69 469 male learners were asked about whether they wrote with their right hand, left hand or both.
9 759 of the male learners said that they were left-handed (L).
61 368 of the male learners said that they were right-handed (R).
$x$ learners wrote with both their right and left hand.

a) Draw a Venn diagram to illustrate the information about the male learners who participated in this 2009 Census@ School project and determine the value of $x$.
b) Determine $n(S)$ where S is the sample set.
c) Calculate the probability (as a percentage correct to 1 decimal place) that a male learner chosen at random from this sample set writes with both hands.

**<u>SOLUTION</u>:**

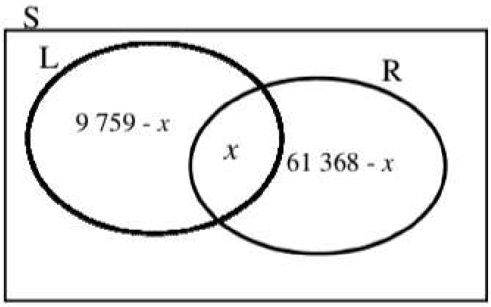
The events of writing with your left hand and writing with your right hand are not mutually exclusive because some people are able to write with both hands.

a) Draw 2 intersecting circles in the rectangle.




Fill in $x$ where the circles **intersect** because $x$ males write with both hands.

Use the information '9 759 are left handed' to determine the number of males who write with their left hand **only**
Number who write with their left hand only
$= 9\ 759 - x$
*Fill this in on the Venn diagram.*




Use the information '61 368 are right handed' to determine the number of males who write with their right hand **only**:

Number who write with their right hand only = $61\ 368 - x$
*Fill this in on the Venn diagram.*

Form an equation using the fact that: $n\,(L \text{ or } R) = n\,(L) + n\,(R) - n\,(L \text{ and } R)$

$$69\ 469 = 9\ 759 + 61\ 368 - x$$
$$69\ 469 = 71\ 127 - x$$
$$\therefore x = 1\ 658$$

So, 1 658 male learners write with both hands.

b) $n(S) = 69\ 469$

c) $P(writes\ with\ both\ hands)$
$= \frac{n(writes\ with\ both\ hands)}{n(S)} = \frac{1\ 658}{69\ 469} = 2{,}38\ ...\% = 2{,}4\%$

**EXERCISE 4.10**

1) At a fast food outlet 75 people were surveyed to find out what food they had eaten. 23 had eaten burgers (B), 30 had eaten fried chicken (F) and 7 had eaten both.
   a) Are these two events mutually exclusive?
   b) Calculate the number of people interviewed who had eaten neither fried chicken nor burgers.
   c) Draw a Venn diagram showing all the given information.
   d) Determine *n(S)* where S is the sample set.
   e) Calculate the probability that one of these people selected at random had eaten neither burgers nor fried chicken. Give your answer as a percentage correct to the nearest whole number.

2) A bag contains thirty five counters of the same shape and size; 10 counters are yellow and 25 counters are green. You draw one counter at random.
   a) Are these events mutually exclusive?
   b) Draw a Venn diagram to represent the information.
   c) Determine *n(S)* where S is the sample set.
   d) Suppose you put your hand in the bag and take out a counter. What is the probability (as a fraction in simplest form)
      i) That you take a yellow counter
      ii) That you take a green counter?
   e) Determine
      i) $P(Y\ and\ G)$
      ii) $P(Y\ or\ G)$

3) Given P(A) = 0,5; P(B) = 0,4 and P(A and B) = 0,3.
   a) Use the formula $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$ to determine P(A or B). Write the answer in a decimal form.
   b) Draw a Venn diagram showing these probabilities.

4) A card is drawn at random from a pack of 52 playing cards.
   What is the probability (giving your answer as a common fraction in simplest form) that this card is
   a) A black card?
   b) A seven?
   c) A black 7?
   d) Not a black 7?

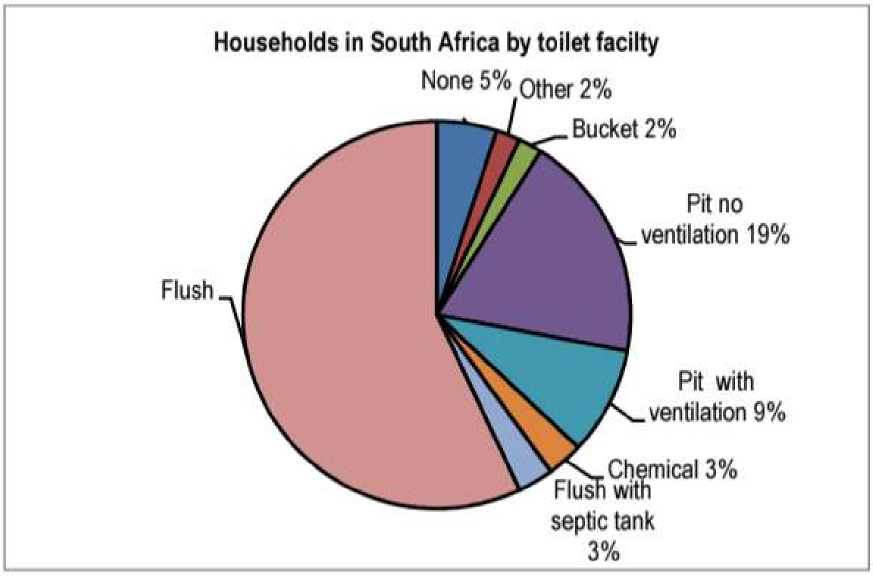
**EXERCISE 4.10 (continued)**

5) In a the 2009 Census@School survey of 15 to 19 year olds, learners were asked what sport they would like to take part in. Below is data adapted from the database:

| Age 15 -19 males and females sport | % |
|---|---|
| Athletics (A) | $x$ |
| Volleyball (V) | 21% |
| Athletics and Volleyball | 12% |
| Neither of these sports | 65% |

a) Draw a Venn diagram to illustrate the data given in the table.
b) Use the Venn diagram to determine the value of $x$.
c) Calculate the probability that a learner chosen at random:
   i) likes Athletics but not Volleyball
   ii) likes Athletics or Volleyball

6) Census 2011 gives the following results about the types of toilet facilities in South African homes:




a) What percentage of households has toilets that flush?
b) Assuming that households have only one type of toilet facility, calculate the probability that a household selected at random
   i) has a toilet that flushes or has a chemical toilet
   ii) has a pit toilet or a bucket toilet
c) Use your answers to b) to write down two conclusions about the types of toilets found in South African homes.

**EXERCISE 4.10 (continued)**

7) A set of cards of the same shape and size have either triangles or stars on them. A card is drawn at random from the set of cards.

| | |
|---|---|
| $\Delta\Delta\Delta$<br>$\Delta\Delta\Delta$<br>$\Delta\Delta\Delta$ | $***$<br>$***$<br>$***$ |

a) Is it possible to draw a card that has both triangles ***and*** stars on it?
b) There are 8 triangle cards ($\Delta$). Let the number of star cards ($*$) be $x$. Draw a Venn diagram to show the number of cards.
c) Determine the value of $x$ if the probability of drawing a star card = $P(*) = \frac{5}{7}$.

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