education

Department:
Education
PROVINCE OF KWAZULU-NATAL

KZN DEPARTMENT OF EDUCATION

MATHEMATICS
JUST IN TIME MATERIAL
GRADE 11

TERM 2 – 2020

This document has been compiled by the FET Mathematics Subject Advisors together with Lead Teachers. It seeks to unpack the contents and to give more guidance to teachers.

CONCEPTS TO BE RE-INFORCED WITH LEARNERS

1. **Subsititution**

   e.g if $f(x) = 2x^3 + 3x - 3$, determine (i) $f(-2)$; (ii) $f(a)$; (iii) $f(p+1)$

2. **Graph interpretation**

- Interval notation and algebraic notation:

  ( ) ; $<$ or $>$ denotes excluding critical / end values.

  [ ] $\leq$ or $\geq$ denotes including critical / end values.

  On the side where there is $\pm\infty$ we use **only** "open" brackets ( ).
- $f(x) > 0$: where the graph is above the $x -$ axis.
- $f(x) < 0$: where the graph is below the $x -$ axis.
- $f(x).g(x) \geq 0$: where both graphs are either below or above the $x -$ axis.
- $f(x) = g(x)$: where the graphs of f and g intersect.
- $f(x) > g(x)$: where the graph of f is above the graph of g.
- $f(x) < g(x)$: where the graph of f is below the graph of g.
- $\frac{f(x)}{g(x)} \geq 0$: also, where both graphs are either below or above the $x -$ axis. But in this case, always remember that $g(x) \neq 0$.

3. **Transformations**

- $f(x-2)$: a horizontal shift of the graph of f two units to the right.
- $f(x+2)$: a horizontal shift of the graph of f two units to the left.
- $f(x)+2$: a vertical shift of the graph of f two units upwards.
- $f(x)-2$: a vertical shift of the graph of f two units downwards.
- $f(x-2)+3$: a horizontal shift of the graph of f two units to the right and 3 units upwards.
- $f(-x)$: reflection of the graph of f about the y-axis (the line $x = 0$).
- $-f(x)$: reflection of the graph of f about the x-axis (the line $y = 0$).

BACKGROUND KNOWLEDGE THAT LEARNERS NEED

1. FUNCTION AND MAPPING NOTATION

In Grade 10 learners were introduced to different ways of representing functions.
The different notations are summarised below:

- $y = ...$ equation notation
- $f(x) = ...$ function notation
- $f: x \to .....$ mapping notation

2. INTERCEPTS WITH THE AXES

To determine the $x-$ intercept(s), substitute $y = 0$.
For example: If $f(3) = 0$, then the function has an x intercept at $(3;\ 0)$.

To determine the $y-$ intercept(s), substitute $x = 0$.
For example: If $f(0) = 4$, then the function has a y intercept at $(0;\ 4)$.

3. GRAPH INTERPRETATION

3.1 Axes of symmetry:

If a function has a line of symmetry, it means that the function is a mirror image of itself about that line. In other words, if the graph was folded along the line of symmetry, it would duplicate itself on the other side of the line.

3.2 Asymptotes:

Asymptotes are imaginary lines that a graph approaches, but never touches or cuts.

3.3 Domain and range:

Domain: The domain refers to the set of possible $\boldsymbol{x-}$ **values** for which a function is defined.

Range: The range refers to the set of possible $\boldsymbol{y-}$ **values** that the function can assume.

4. BASELINE ACTIVITY

For each of the following functions
(I) Sketch the graph of the function.
(II) Determine the domain and the range of each function.
(III) Determine the equation of the axes of symmetry and asymptotes, where applicable.

Straight Line

1.1 $y = x + 3$
1.2 $y = (x - 2)$
1.3 $y = -x$
1.4 $y = -2x - 3$

Parabola

2.1 $y = x^2 + 3$
2.2 $y = (x - 2)^2$
2.3 $y = -x^2$
2 .4 $y = -2x^2 - 3$

Hyperbola

3.1 $y = \frac{6}{x} + 3$
3.2 $y = \frac{6}{x-2}$
3.3 $y = -\frac{6}{x}$
3.4 $y = \frac{-2}{x} - 3$

Exponential graph

4.1 $y = 2^x + 3$
4.2 $y = 2^{x-2}$
4.3 $y = 2^{-x}$
4.4 $y = -(2)^x - 3$

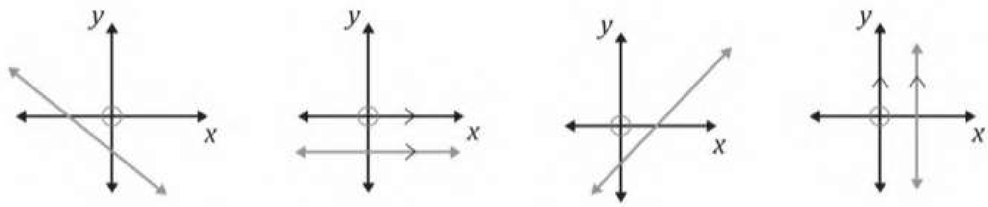
BASIC INFORMATION ON THE DIFFERENT TYPES OF GRAPHS

A. STRAIGHT LINE

General representation or equation:

$y = ax + q$ or $f(x) = mx + c$, a or m is the gradient, and q or c is the $y-$intercept .

Also note the shape of the following linear functions:



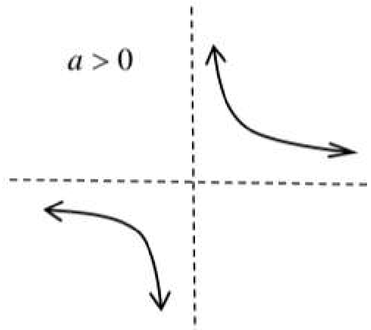
$a < 0$	$a = 0$	$a > 0$	a is undefined
$q < 0$	$y = q$	$q < 0$	there is no q-value

For all the linear functions, except horizontal and vertical lines, the domain is $x \in R$, and the range is $y \in R$.

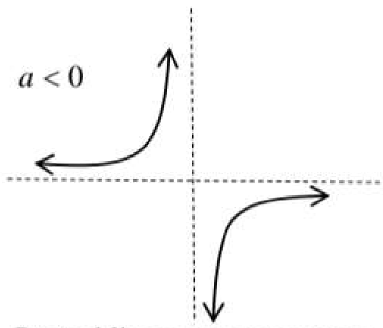
B. HYPERBOLA

General representation or equation:

$$y = \frac{a}{x+p} + q$$



Dotted lines are asymptotes



Dotted lines are asymptotes

- The value of q represents the vertical translation (shift) from the x-axis.
- The value of p represents the horizontal translation (shift) from the y-axis.
- In the case of $y = \frac{a}{x}$, $p = 0$ and $q = 0$. The **vertical** asymptote is the y-axis ($x = 0$) and the **horizontal** asymptote is the x-axis ($y = 0$). The **axes of symmetry** are $y = x$ (+ve gradient) and $y = -x$ (-ve gradient).
  The **domain** is $x \in R,\ x \neq 0$; and the **range** is $y \in R, y \neq 0$.
- In the case of $y = \frac{a}{x} + q$, $p = 0$. The **vertical** asymptote is the y-axis ($x = 0$) and the **horizontal** asymptote is $y = q$. The **axes of symmetry** are $y = x + q$ (+ve gradient) and $y = -x + q$ (-ve gradient). The **domain** is $x \in R,\ x \neq 0$; and the **range** is $y \in R,\ y \neq q$.

- In the case of $y=\frac{a}{x+p}+q$, the **vertical** asymptote is $x=-p$ and the **horizontal** asymptote is $y=q$. The **axes of symmetry** are $y=\pm(x+p)+q$. The d**omain** is $x\in R,\ x\neq -p$ and the **range** is $y\in R, y\neq q$.
- Alternative method to determine the equations of the axes of symmetry:
  In all cases the one axis of symmetry has a gradient of $+1$ and the other a gradient of -1. Therefore the equations of the axes of symmetry are $y=x+c$ and $y=-x+c$. In all cases the value of c may be determined by simply substituting the coordinates of the point of intersection of the two asymptotes into the above equations – since the axes of symmetry always pass through this point.

Example no. 1:

Given : $f(x)=\frac{3}{x-2}+1$

1.1. Write down the equations of the asymptotes of f.

1.2. Determine the coordinates of B, the x-intercept of f.

1.3. Determine the coordinates of D, the y-intercept of f.

1.4. Determine the domain and the range of f.

1.5. Determine the equations of the two axes of symmetry of f.

1.6. Draw a sketch graph of f.

Solution:

1.1 For the vertical asymptote:

$$x-2=0$$
$$x=2$$

Horizontal asymptote:

$$y=1$$

1.2 For the x – intercept, substitute $y=0$:

$$\frac{3}{x-2}+1=0$$
$$\frac{3}{x-2}=-1$$
$$-1(x-2)=3$$
$$x=-1$$

1.3 For the y – intercept, substitute $x=0$:

$$y=\frac{3}{-2}+1=\frac{3-2}{-2}=-\frac{1}{2}$$

1.4 Domain is $x\in R;\ x\neq 2$

Range is $y\in R;\ x\neq 1$

1.5 Point of intersection of asymptotes: (2 ; 1)

Axis of symmetry with positive gradient:

Substitute (2 ; 1) into $y=x+c$:

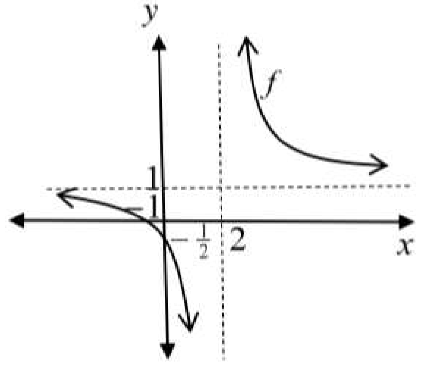
$$1=2+c$$
$$c=-1$$
$$y=x-1$$

Axis of symmetry with negative gradient:

Substitute (2 ; 1) into $y=-x+c$:

$$1=-2+c$$
$$c=3$$
$$y=-x+3$$

1.6)



C. PARABOLA

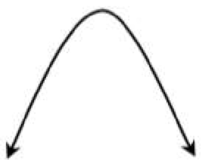
Defining Equation:

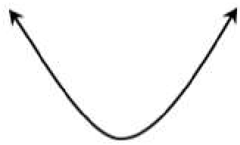
$y = a(x+p)^2 + q$ or $y = ax^2 + bx + c$ or $y = a(x - x_1)(x - x_2)$

Sketching a parabola:

for $a < 0$ for $a > 0$

Shape





For $y = ax^2 + bx + c$, the **turning point** is $\left(\frac{-b}{2a} ; f\left(\frac{-b}{2a}\right)\right)$ and the **y-intercept** is $y = c$.

Given: $y = ax^2 + bx + c$

***y*-intercept** : $(0 ; c)$

Turning point (TP) :

$x = \dfrac{-b}{2a}$ (the axis of symmetry)

Substitute this value into the equation to find the *y*-coordinate of the TP, i.e. the minimum or maximum value.

Given: $y = a(x+p)^2 + q$

Multiply out the expression to get it in the form $y = ax^2 + bx + c$

***y*-intercept**:$(0 ; c)$

Turning Point (TP): $(-p ; q)$

If there are ***x*-intercepts**: Let $y = 0$ and solve for x (factorise or use the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$).

- If $a < 0$, the function has a **maximum** value, represented by the ***y*** value of the turning point.
- If $a > 0$, the function has a **minimum** value, represented by the ***y*** value of the turning point.
- The equation of the axis of symmetry is given by $x = \frac{-b}{2a}$, (is the x value of the turning point)
- The **domain** is $x \in R$
- The **range: If $a > 0$** then $y \geq$ minimum value ; **If $a < 0$** then $y \leq$ maximum value.

To determine the equation of a parabola:

<u>Given:</u> TP and one other point

Use $y = a(x+p)^2 + q$

- TP is $(-p ; q)$; substitute that in above equation.
- Substitute the other point for x and y.
- Solve for a.
- Rewrite the equation with the values for a, p and q.
- If required, rewrite in the form $y = ax^2 + bx + c$.

<u>Given:</u> *x*-intercepts and one other point

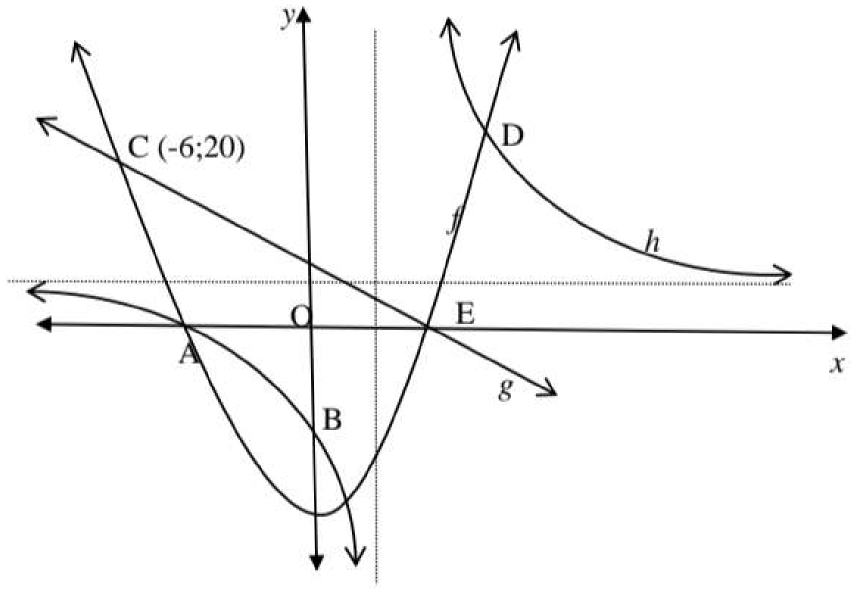
Use $y = a(x - x_1)(x - x_2)$

- Substitute the x-intercepts for x_1 and x_2.
- Substitute the other point for x and y.
- Solve for a.
- Rewrite the equation with the values for a, x_1 and x_2.
- If required, rewrite in the form $y = ax^2 + bx + c$.

Example no. 2:

Sketched below are the graphs of: $g(x)=-2x+8$; $f(x)=x^2+k$; and $h(x)=\dfrac{6}{x-2}+1$.

A is an x-intercept and B a $y-$intercept of h. C $(-6;20)$ and E are the points of intersection of f and g.



2.1 Determine the coordinates of A, B and E.

2.2 Show that the value of $k=-16$

2.3 Determine the domain and the range of f.

2.4 Write down the values of x for which $g(x)-f(x)\geq 0$.

2.5 Determine the equation of the axis of symmetry of h that has a negative gradient.

2.6 Write down the range of s, if $s(x)=f(x)+2$.

2.7 Write down the range of t, if $t(x)=h(x)+2$.

Solution:

2.1 At A, substitute $y=0$:

$$\frac{6}{x-2}+1=0$$
$$6=-x+2$$
$$\therefore x=-4$$

Thus: $\mathrm{A}(-4;0)$

At B, substitute $x=0$:

$$y=\frac{6}{-2}+1$$
$$y=-3+1$$
$$\therefore y=-2$$

Thus: $\mathrm{B}(0;-2)$

E is the x-intercept of the straight line and the parabola. It is easy and straight-forward to use the equation of the straight line to get the coordinates of E.

At E, substitute $y=0$, $\therefore 0=-2x+8$

$$x=4$$

Thus: $\mathrm{E}(4;0)$

2.2 C(-6; 20) is on f and g.

Substitute C into $f(x)=x^2+k$

$$20=(-6)^2+k$$
$$k=-16$$

2.3 Domain is $x\in R$

Range is $y\geq -16;\ y\in R$

2.4 These are the values of x for which the graphs of g and f intersect **or** where f is below g.
It occurs from C(-6 ; 20) and E(4 ; 0).
That is $-6 \leq x \leq 4$.

2.5 Point of intersection of asymptotes: $(2;1)$
For axis of symmetry with negative gradient:
$$y = -x + c$$
Substitute $(2;1)$: $1 = -2 + c$
$$c = 3$$
$$y = -x + 3$$

2.6 The "+ 2" implies a shift vertically upwards by 2 units. The new minimum value will now be -14. The range of s is $y \geq -14$.

2.7 The " + 2" implies a shift vertically upwards by 2 units.
The range of t is $y \neq 1 + 2;\ y \in R$
$$y \neq 3;\ y \in R$$

D. EXPONENTIAL GRAPH

Defining equation: $y = ab^{x+p} + q$.

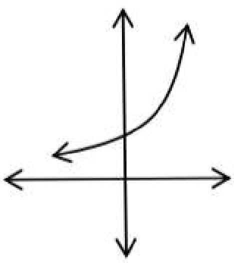
If $q = 0$ and $p = 0$ then $y = ab^x$.

If $p = 0$ then $y = ab^x + q$.

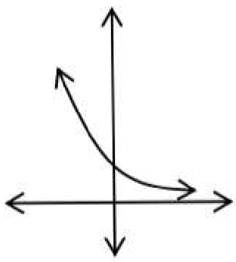
The restriction is $b > 0; b \neq 1$

Shape:

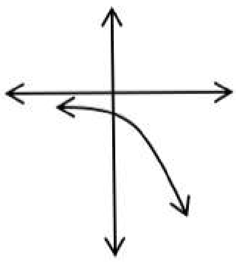
for $a > 0$ *and* $b > 1$



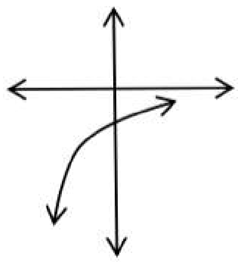
for $a > 0$ *and* $0 < b < 1$



for $a < 0$ *and* $b > 1$



for $a < 0$ *and* $0 < b < 1$



- For $y = ab^x$, the **asymptote** is $y = 0$ and the **y-intercept** is $y = a$.
- For $y = ab^x + q$, the **asymptote** is $y = q$ and the **y-intercept** is $y = a + q$.
- For $y = ab^{x+p} + q$, the **asymptote** is $y = q$ and the **y-intercept** is $y = ab^p + q$.

Example no. 3:

Given: $f(x) = 3^{-x+1} - 3$

3.1 Write $f(x)$ in the form $y = ab^x + q$

3.2 Draw the graph of f, showing all the intercepts with the axes and the asymptote.

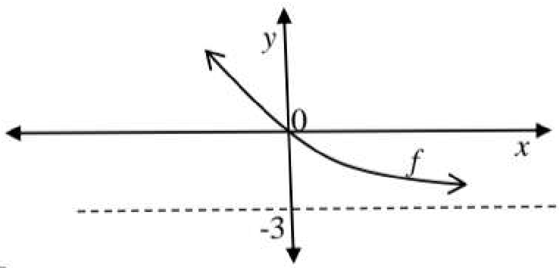
3.3 Write down the domain and the range of f.

Solution**:**

3.1 $y = 3^{-x+1} - 3 = 3^{-x}.3 - 3 = 3.3^x - 3 = 3\left(\frac{1}{3}\right)^x - 3$

3.2 The asymptote is $y = -3$.

For the x-intercept, let $y = 0$: $3\left(\frac{1}{3}\right)^x - 3 = 0$
$$\left(\frac{1}{3}\right)^x = 1$$
$$x = 0$$



3.3. The domain is $x \in R$, and the range is $y > -3;\ y \in R$.

QUESTIONS FROM PAST PAPERS ON FUNCTIONS AND GRAPHS

QUESTION 5 (GR. 12 DBE NOVEMBER 2010)

Consider the function $f(x) = 4^{-x} - 2$

5.1 Calculate the coordinates of the intercepts of *f* with the axes. (4)

5.2 Write down the equation of the asymptote of *f*. (1)

5.3 Sketch the graph of *f*. (3)

5.4 Write down the equation of g if *g* is the graph of *f* shifted 2 units upwards. (1)

5.5 Solve for x if $f(x) = 3$. (You need not simplify your answer.) (3)

[12]

QUESTION 5 (Gr. 12 DBE MARCH 2011)

Consider the function $f(x) = \dfrac{3}{x-1} - 2$.

5.1 Write down the equations of the asymptotes of f. (2)

5.2 Calculate the intercepts of the graph of f with the axes. (3)

5.3 Sketch the graph of f. (3)

5.4 Write down the range of $y = -f(x)$. (1)

5.5 Describe, in words, the transformation of f to g if $g(x) = \dfrac{-3}{x+1} - 2$. (2)

[11]

QUESTION 5 (GR. 12 DBE MARCH 2010)

Given: $f(x) = \dfrac{2}{x-3} + 1$

5.1 Write down the equations of the asymptotes of *f*. (2)

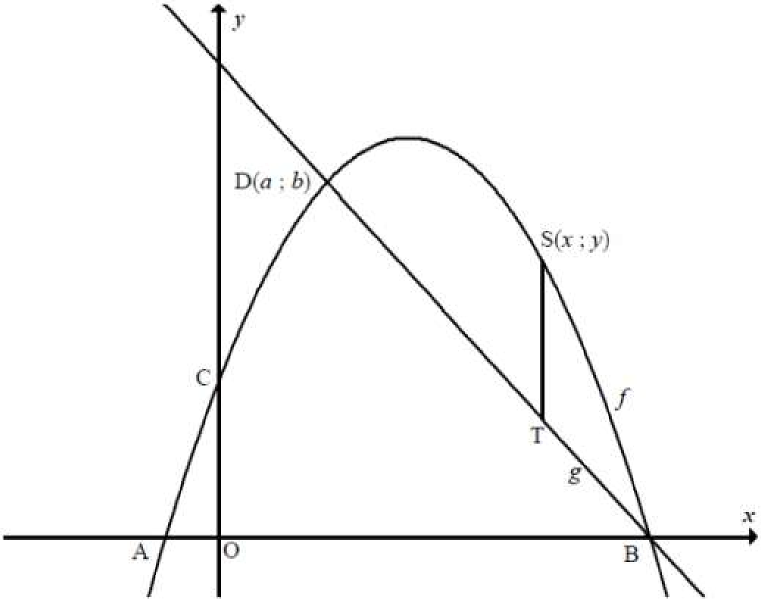
5.2 Calculate the coordinates of the *x*- and *y*-intercepts of *f*. (3)

5.3 Sketch *f*. Show all intercepts with the axes and the asymptotes. (3)

[8]

QUESTION 6 (GR. 12 DBE MARCH 2010)

The graphs of $f(x) = -x^2 + 7x + 8$ and $g(x) = -3x + 24$ are sketched below.
f and g intersect in D and B. A and B are the x-intercepts of f.



6.1 Determine the coordinates of A and B. (4)

6.2 Calculate a, the x-coordinate of D. (4)

6.3 S(x ; y) is a point on the graph of f, where $a \leq x \leq 8$. ST is drawn parallel to the y-axis with T on the graph of g. Determine ST in terms of x. (2)

6.4 Calculate the maximum length of ST. (2)

[12]

QUESTION 4 (GR. 12 DBE MARCH 2015)

Given: $g(x) = \frac{6}{x+2} - 1$

4.1 Write down the equations of the asymptotes of g. (2)

4.2 Calculate:

4.2.1 The y – intercept of g (1)

4.2.2 The x – intercept of g (2)

4.3 Draw the graph of g, showing clearly the asymptotes and the intercepts with the axes. (3)

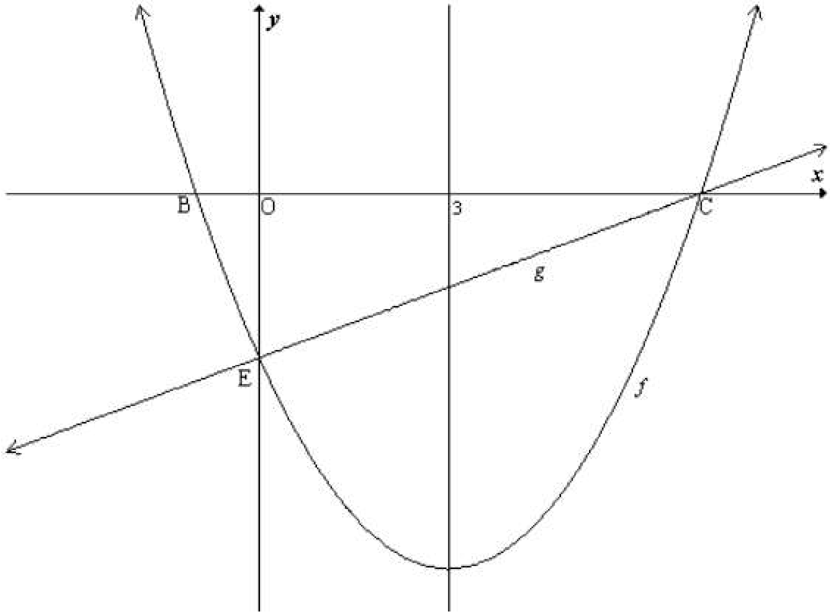
4.4 Determine the equation of the line of symmetry that has a negative gradient, in the Form $y = \ldots$ (3)

4.5 Determine the value(s) of x for which : $\frac{6}{x+2} - 1 \geq -x - 3$ (2)

[13]

QUESTION 6 (GR. 12 DBE MARCH 2011)

A parabola f intersects the x-axis at B and C and the y-axis at E. The axis of symmetry of the parabola has equation $x = 3$. The line through E and C has equation $g(x) = \frac{x}{2} - \frac{7}{2}$.



6.1 Show that the coordinates of C are (7 ; 0). (1)

6.2 Calculate the x-coordinate of B. (1)

6.3 Determine the equation of f in the form $y = a(x-p)^2 + q$. (6)

6.4 Write down the equation of the graph of h, the reflection of f in the x-axis. (1)

6.5 Write down the maximum value of $t(x)$ if $t(x) = 1 - f(x)$. (2)

6.6 Solve for x if $f(x^2 - 2) = 0$. (4)

[15]

QUESTION 4 (GR. 12 DBE NOVEMBER 2015)

Given: $f(x) = 2^{x+1} - 8$

4.1 Write down the equation of the asymptote of f. (1)

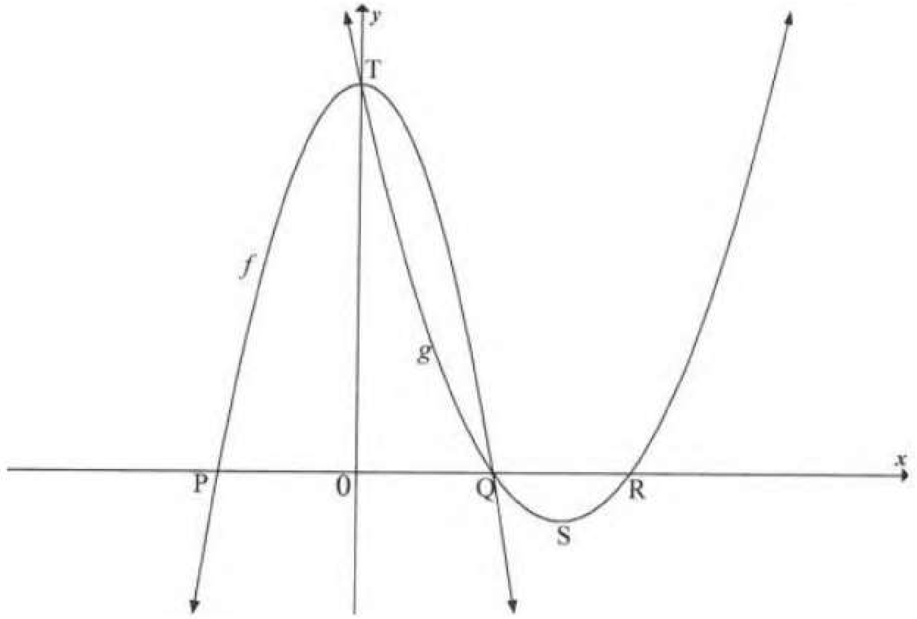
4.2 Sketch the graph of f. Clearly indicate ALL intercepts with the axes as well as the asymptote. (4)

4.3 The graph of g is obtained by reflecting the graph of f in the y – axis. Write down the equation of g. (1)

[6]

QUESTION 6 (GR. 12 DBE NOVEMBER 2015)

6.1 The graphs of $f(x) = -2x^2 + 18$ and $g(x) = ax^2 + bx + c$ are sketched below. Points P and Q are the x – intercepts of f. Points Q and R are the x – intercepts of g. S is the turning point of g. T is the y – intercept of both f and g.



6.1.1 Write down the coordinates of T. (1)

6.1.2 Determine the coordinates of Q. (3)

6.1.3 Given that $x = 4{,}5$ at S, determine the coordinates of R. (2)

6.2 The function defined as $y = \frac{a}{x+p} + q$ has the following properties:

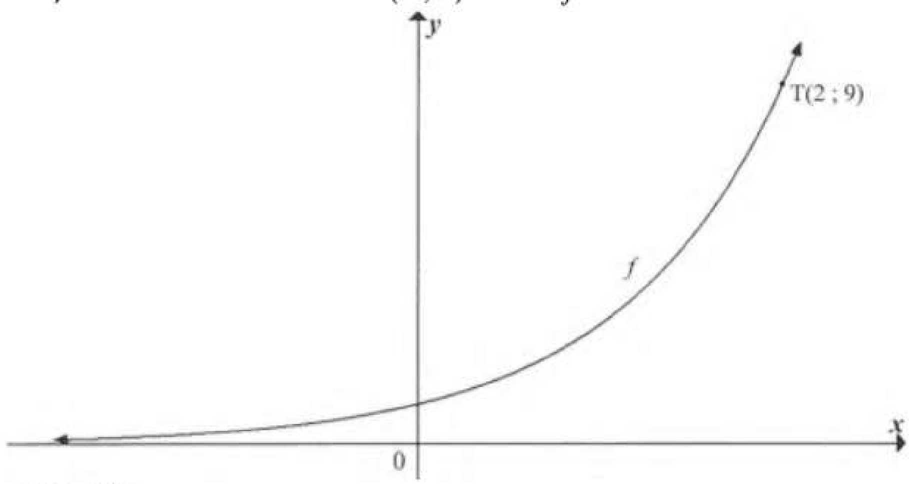
- The domain is $x \in R,\ x \neq 2$.
- $y = x + 6$ is an axis of symmetry.
- The function is increasing for all $x \in R,\ x \neq 2$.

Draw a neat sketch graph of this function. Your sketch must include the asymptotes, if any. (4)

[10]

QUESTION 5 (GR. 12 DBE MARCH 2015)

The graph of $f(x) = a^x, a > 1$ is shown below. T(2 ; 9) lies on f.



5.1 Calculate the value of a. (2)

5.2 Determine the equation of $g(x)$ if $g(x) = f(x)$ (1)

5.3 Determine the value(s) of x for which $f^{-1}(x) \geq 2$. (2)

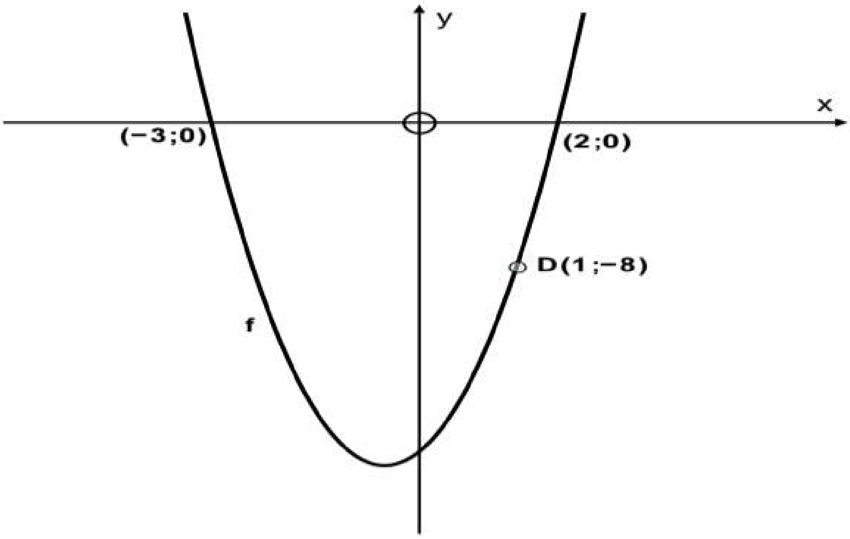
5.4 Is the inverse of f a function? Explain your answer. (2)

QUESTION 5 (GR. 12 DBE MARCH 2015 – adapted for gr. 11)

The graph of $f(x) = ax^2 + bx + c\ ; a \neq 0$ is drawn below.

D(1 ; – 8) is a point on f.

f intersects the x – axis at (– 3 ; 0) and (2 ; 0).



6.1 For which value(s) of x is $f(x) \leq 0$? (2)

6.2 Determine the values of a, b and c. (5)

6.3 Determine the coordinates of the turning point of f. (3)

6.4 Write down the equation of the axis of symmetry of h if $h(x) = f(x-7) + 2$. (2)

[12]

QUESTION 5 (GR. 12 DBE NOVEMBER 2011)

5.1 Consider the function: $f(x) = \frac{-6}{x-3} - 1$

5.1.1 Calculate the coordinates of the y – intercept of f. (2)

5.1.2 Calculate the coordinates of the x – intercept of f. (2)

5.1.3 Sketch the graph of f in your ANSWER BOOK, showing clearly the asymptotes and the intercepts with the axes. (4)

5.1.4 For which values of x is $f(x) > 0$? (2)

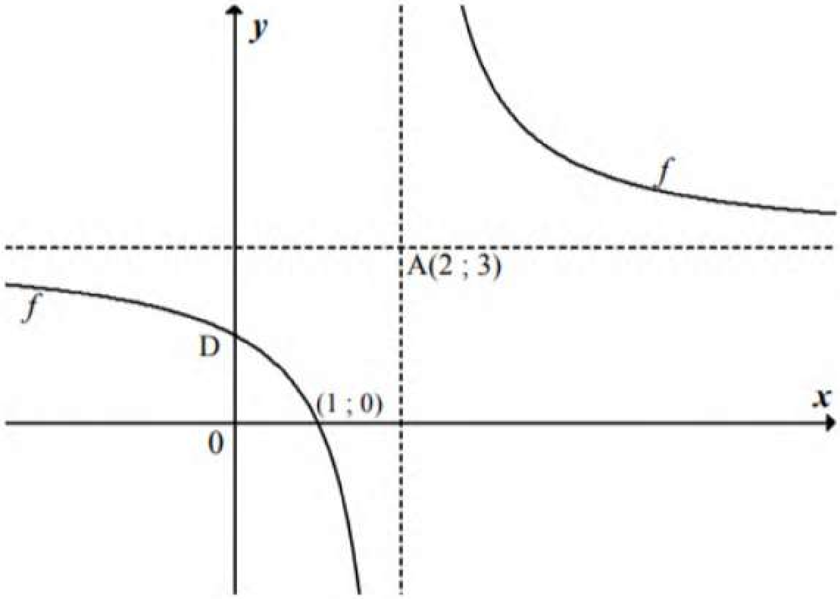
5.1.5 Calculate the average gradient of f between $x = -2$ and $x = 0$. (4)

5.2 Draw a sketch graph of $y = ax^2 + bx + c$, where $a < 0,\ b < 0,\ c < 0$ and $ax^2 + bx + c = 0$ has only ONE solution. (4)

[19]

QUESTION 4 (GR. 12 DBE NOVEMBER 2010)

Given: $f(x) = \frac{a}{x-p} + q$.
The point A(2; 3) is the point of intersection of the asymptotes of f.
The graph of f intersects the $x -$ axis at (1; 0).
D is the $y -$ intercept of f.



4.1 Write down the equations of the asymptotes of f. (2)

4.2 Determine an equation of f. (3)

4.3 Write down the coordinates of D. (2)

4.4 Write down an equation of g if g is the straight line joining A and D. (3)

4.5 Write down the coordinates of the other point of intersection of f and g. (4)
[14]