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Study Guide

Mathematics

Grade 10

Via Afrika Publishers

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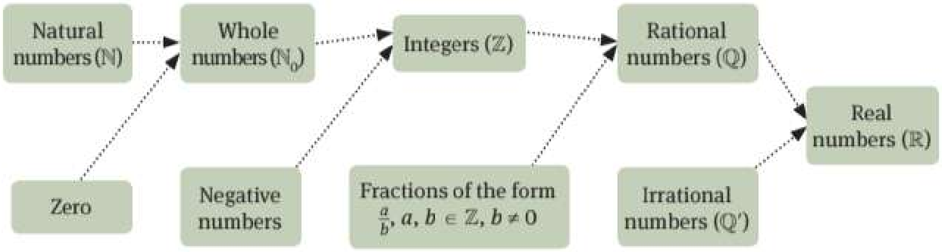
Unit 1

The number system

1.1 Real numbers

- Real numbers are divided into rational and irrational numbers.



- We can write a rational number as a fraction, $\frac{a}{b}$, where a and $b \in Z$, and $b \neq 0$.
- We cannot express an irrational number as a fraction.

1.2 Surds

- A surd is a root of an integer that we cannot express as a fraction.
- Surds are irrational numbers.
- Examples of surds are $\sqrt{3}$ and $\sqrt{2}$.

1.3 Rounding off

- If the number after the cut-off point is 4 or less, then we leave the number before it as it is.
- If the number is equal to 5 or more, then increase the value of the number before it by 1.

Unit 2

Multiplying algebraic expressions

2.1 Multiply integers and monomials by polynomials

- Each term inside the bracket is multiplied by the term in front of the bracket.

$$-\tfrac{1}{6}a^2b(6abc - 12ac + 18b) = (-\tfrac{1}{6}a^2b \times 6abc) - (-\tfrac{1}{6}a^2b \times 12ac) + (-\tfrac{1}{6}a^2b \times 18b)$$
$$= -a^3b^2c + 2a^3bc - 3a^2b^2$$

2.2 The product of two binomials

- Each term inside the first set of brackets is multiplied by each term inside the second set of brackets.

$$\begin{aligned}(a - 3b)(a + 7b) &= (a \times a) + (a \times 7b) - (3b \times a) - (3b \times 7b)\\ &= a^2 + 7ab - 3ab - 21b^2\\ &= a^2 + 4ab - 21b^2\end{aligned}$$

- When squaring a binomial:

$$\begin{aligned}(m + n)^2 &= (m + n)(m + n)\\ &= (m \times m) + (m \times n) + (n \times m) + (n \times n)\\ &= m^2 + mn + nm + n^2\\ &= m^2 + 2mn + n^2\end{aligned}$$

A common error is to think that $(m + n)^2 = m^2 + n^2$

The square of any binomial produces the following three terms:

1 The square of the first term of the binomial: m^2
2 Twice the product of the two terms: $2mn$
3 The square of the second term: n^2

2.3 Multiplying a binomial by a trinomial

Multiply each term in the first set of brackets by each term in the second set of brackets. Then we simplify by collecting the like terms.

$$\begin{aligned}(8 - 3y)(12 - 2y + 8y^2 - 4y^3) &= (8 \times 12) + (8 \times -2y) + (8 \times 8y^2) + (8 \times -4y^3) + (-3y \times 12)\\ &\quad + (-3y \times -2y) + (-3y \times 8y^2) + (-3y \times -4y^3)\\ &= 96 - 16y + 64y^2 - 32y^3 - 36y + 6y^2 - 24y^3 + 12y^4\\ &= 96 - 52y + 70y^2 - 24y^3 + 12y^4\end{aligned}$$

2.4 The sum and difference of two cubes

- The expression in the following form gives the **difference** of two cubes:

$$(a - b)(a^2 + ab + b^2) = a^3 + a^2b + \times ab \times {}^2 - a^2b - \times ab \times {}^2 - b^3$$
$$= a^3 - b^3$$

- The expression in this form gives the **sum** of two cubes:

$$(a + b)(a^2 - ab + b^2) = a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$$
$$= a^3 + b^3$$

Unit 3

Factorisation

Factorisation is the opposite process to the one you learnt in the previous unit when multiplying out algebraic expressions.

$$\begin{tikzcd} y(x+5) \arrow[r, bend left, "\text{expand}"] & xy+5y \arrow[l, bend left, "\text{factorise}"] \end{tikzcd}$$

3.1 Common factors

When finding a common factor, we find and take out a factor that can divide into each term in the expression. For example, in the expression $6h^3 + 18h^2$, $6h^2$ can divide into each term, so it becomes the common factor:

$$6h^3 \div 6h^2 = h$$
$$18h^2 \div 6h^2 = +3$$

Therefore:

$$6h^3 + 18h^2 = 6h^2(h + 3)$$

3.2 Difference between two squares

A difference of two squares occurs when we have two perfect squares separated by a minus sign. An expression in the form $a^2 - b^2$ has two factors, $(a - b)(a + b)$. For example:

$$4a^2 - 81b^4 = (2a + 9b^2)(2a - 9b^2)$$

$\sqrt{4a^2} = 2a$
$\sqrt{81b^4} = 9b^2$

3.3 Perfect squares

In Unit 2 you learnt that $(x + y)^2 = x^2 + 2xy + y^2$. Therefore, a trinomial will factorise into a perfect square if:

- the first and last terms are perfect squares (e.g. $x \times x = x^2$)
- the middle term is equal to 2 × first term × last term.

For example: $x^2 + 10x + 25 = (x + 5)(x + 5) = (x + 5)^2$

3.4 Trinomials of the form $x^2 + bx + c$

When factorising a trinomial:

- If the third term of the trinomial is positive, we use the sum of the factors of the third term to give us the middle term.
- If the third term of the trinomial is negative, we use the difference between the factors of the third term to give us the middle term.

Use the following rules to decide on the signs in the binomials:

- If the third term of the trinomial is positive, then the signs between the terms in the binomials are the same (both positive or negative).
- If the third term of the trinomial is negative, then the signs between the terms of the binomials are different.

For example: $x^2 + 3x + 2 = (x + 2)(x + 1)$

3.5 Trinomials of the form $ax^2 + bx + c$

In this case, the rules are the same as in the previous section, except that now we need to consider the factors of the coefficient of x^2 and the last term.

For example: $6x^2 + x - 15$

- Multiply the coefficient of the squared term (6) by the last term $(-15) = -90$.
- Look for the factors of -90 that will give $+1$ (the coefficient of the middle term) when added:
  $-9 \times 10 = -90$ and $-9 + 10 = 1$
- Rewrite $6x^2 + x - 15$ as $6x^2 + 10x - 9x - 15$.
- Group the terms: $(6x^2 + 10x) - (9x + 15)$
- Take out common factors from each set of brackets:
  $2x(3x + 5) - 3(3x + 5) = (3x + 5)(2x - 3)$

Remember to change the sign inside the brackets when you divide by -1.

Therefore:

$$\begin{aligned} 6x^2 + x - 15 &= 6x^2 + 10x - 9x - 15 \\ &= 2x(3x + 5) - 3(3x + 5) \\ &= (3x + 5)(2x - 3) \end{aligned}$$

3.6 Factorising by grouping

If an expression has four or more terms, but has no factor common to all of them, we can often group the terms, factorise each group, and then remove a common factor.

$x^3 - 3x^2 + 2x - 6 = (x^3 - 3x^2) + (2x - 6)$ (Group terms together using brackets)
$= x^2(x - 3) + 2(x - 3)$ (Factor out the common factors from each group)
$= (x - 3)(x^2 + 2)$

3.7 Factorising the sum and difference of cubes

- The **difference** between two cubes factorises as: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
  - The first factor is: $(\sqrt[3]{\text{first term}}) - (\sqrt[3]{\text{second term}})$
  - The second factor is: (Square of the first term) (Opposite sign) (Product of the two terms) + (Square of the last term)
    For example: $y^3 - 64 = (y - 4)(y^2 + 4y + 16)$
- The **sum** of two cubes factorises as: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
  For example: $y^3 + 64 = (y + 4)(y^2 - 4y + 16)$

Unit 4

Algebraic fractions

4.1 Simplifying fractions

Special fractions:	Negative fractions:
$\frac{a}{1} = a$ $\frac{1}{a} = \frac{1}{a}$ $\frac{a}{a} = 1$ $\frac{0}{a} = 0$ *But*: $\frac{a}{0}$ is undefined	$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$ $\frac{-a}{-b} = \frac{a}{b}$ *But*: $-\frac{a}{b} \neq \frac{-a}{-b}$

To simplify a fraction:

1 Factorise the expressions in the numerator and denominator

2 Cancel the terms common to both.

For example: $\frac{(x^2 - 4)}{(x - 2)} = \frac{(x + 2)(x - 2)}{(x - 2)} = (x + 2)$

4.2 Products of algebraic fractions

Multiplying and dividing algebraic fractions works in the same way as ordinary fractions that contain numbers:

- Factorise each expression.
- Cancel any like terms.
- Multiply what is left.

$$\frac{(4n^2 - 9)}{(2n + 3)} \times \frac{(2n^2 - n - 3)}{(n + 1)} = \frac{(2n + 3)(2n - 3)}{(2n + 3)} \times \frac{(2n - 3)(n + 1)}{(n + 1)}$$

$$= \frac{(2n - 3)}{1} \times \frac{(2n - 3)}{1}$$

$$= (2n - 3)^2$$

Multiply each term in the first set of brackets by each term in the second set of brackets. Then we simplify by collecting the like terms.

4.3 Adding and subtracting algebraic fractions

- As always, factorise and simplify where possible.
- Find a common denominator.
- Express each fraction in terms of that denominator.
- Add and subtract the like terms.

$\frac{3p-q}{2p}+\frac{2p-3q}{4q}$ $=\frac{2q(3p-q)+p(2p-3q)}{4pq}$ $=\frac{6pq-2q^2+2p^2-3pq}{4pq}$ $=\frac{2p^2+3pq-2q^2}{4pq}$ $=\frac{(2p-q)(p+2q)}{4pq}$	$\frac{2x+5}{5}-\frac{x-2}{3}$ $=\frac{3(2x+5)-5(x-2)}{15}$ $=\frac{6x+15-5x+10}{15}$ $=\frac{x+25}{15}$

Questions

1 Multiplying expressions:

a Square the binomial $(3x - 4)$.

b Expand: $(2x - 5)(2x + 1)$

c Expand: $(2a + b)(4a^2 - 2ab + b^2)$

d Find the products and simplify: $(2a - 5b)(4a - 3b) - (a + 3b)(5a - 12b)$

e Find the product: $(k^2 + \frac{3}{4})(k - \frac{1}{2})$

2 Factorise the following expressions:

a $25x^2 + 30x + 9$

b $5x^2 - 7x - 6$

c $x^3 - \frac{1}{4}x$

d $(2t - 5)^3 - (2t - 5)^2$

e $3x^3 + x^2 - 3x - 1$

f $m^3 - m^2 - mn^2 + n^2$

g $(y + 3)(2y - 3) + (y - 1)(3 - 2y)$

h $x^2(2x - 1) - 2x(2x - 1) - 3(2x - 1)$

3 Algebraic fractions:

a Simplify $\frac{x^2 - x - 6}{x^2 - 4x + 3}$

b Simplify to the lowest terms: $\frac{4x^3 - 9x^2}{4x^3 + 6x^2}$

c Simplify as far as possible: $\frac{x^2 - 7x + 12}{4 - x}$

d Simplify as far as possible: $\frac{a(a + 1) + (a + 1)}{a^2 - 2a + 1} \times \frac{a^2 - 1}{a^2} \div \frac{a^2 + a}{a^2 - a}$

e Calculate and simplify: $\frac{1}{x - 2} + \frac{-4}{(x + 2)^2} - \frac{1}{x + 2}$

f Calculate and simplify: $\frac{p - 3}{p^2 - p - 12} + \frac{2}{3 + p} - \frac{3}{8 - 2p}$