Via Afrika Mathematics

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Grade 11 Study Guide

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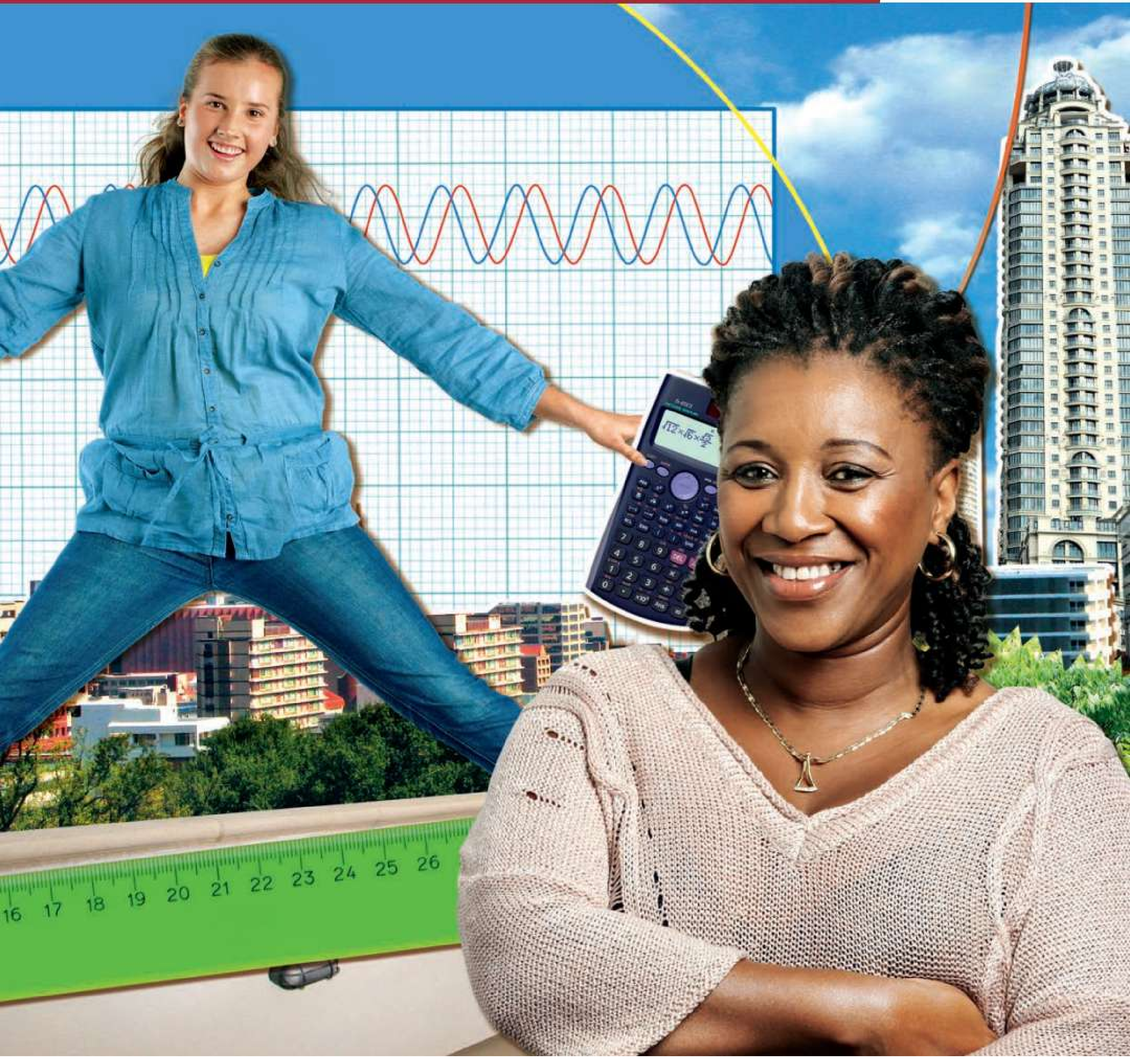
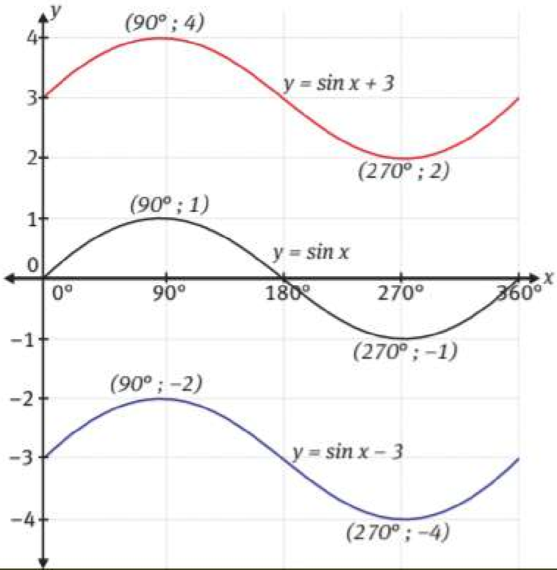
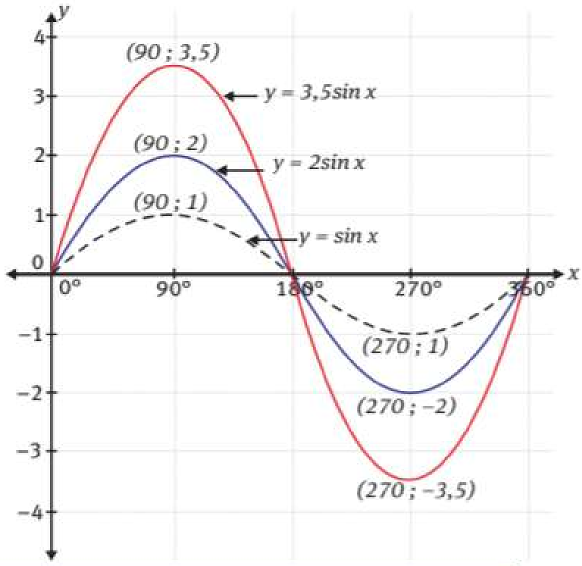


Trigonometric graphs

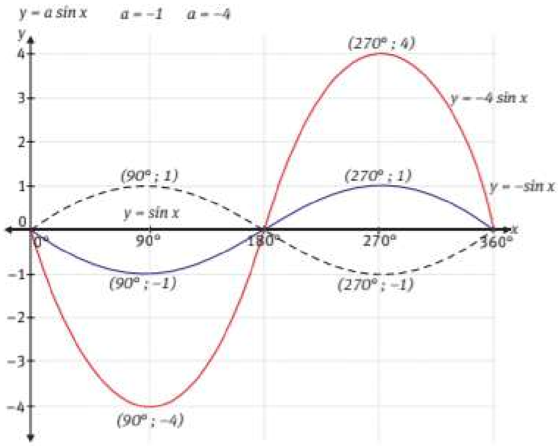
3.1 Revision of the basic graphs

The effect of a and q on the graph $y = a \sin x + q \ for \ x \in [-360°;\ 360°]$



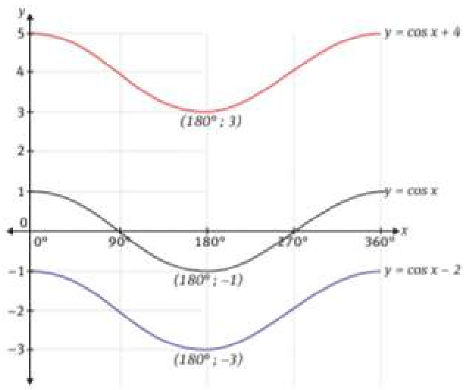


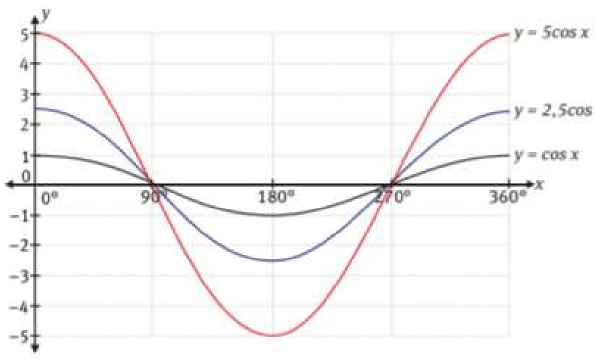
Manipulating the sin graph, a > 0	
$q > 0$: the graph shifts upwards. $q < 0$: the graph shifts downwards. The shape does not change.	$a > 0$: multiplying by a causes a vertical stretch of the basic graph. The shape of the basic graph changes.
Domain: $x \in [-360°;\ 360°]$	**Domain:** $x \in [-360°;\ 360°]$
Range: $y = \sin x + 3$: $2 \leq y \leq 4$; $[2;\ 4]$ $y = \sin x$: $-1 \leq y \leq 1$; $[-1;\ 1]$ $y = \sin x - 3$: $-4 \leq y \leq -2$; $[-4;\ -2]$	**Range:** $y = 3{,}5 \sin x$: $-3{,}5 \leq y \leq 3{,}5$; $y \in [-3{,}5;\ 3{,}5]$ $y = 2 \sin x$: $-2 \leq y \leq 2$; $y \in [-2;\ 2]$ $y = \sin x$: $-1 \leq y \leq 1$; $y \in [-1;\ 1]$
Amplitude: ((highest y −value) – (lowest y-value))/2 For all three graphs this equals 1.	**Amplitude:** The amplitude varies for the three graphs, e.g. for $y = 3{,}5 \sin x$: $(3{,}5 - (-3{,}5))/2 = 3{,}5$
Period: 360° for all three graphs	**Period:** 360° for all three graphs



Manipulating the sin graph, a < 0
$a < 0$: multiplying by negative a causes a reflection of $y = a \sin x$ in the x −axis
Domain: $x \in [-360°;\ 360°]$
Range: $y = (-1)\sin x$: $\quad -1 \leq y \leq 1;\quad y \in [-1;\ 1]$ $y = (-4)\sin x$: $\quad -4 \leq y \leq 4;\quad y \in [-4;\ 4]$
Amplitude: The amplitude varies for the three graphs, e.g. for $y = (-4)\sin x$: $\quad (4 - (-4))/2 = 4$
Period: 360° for all three graphs

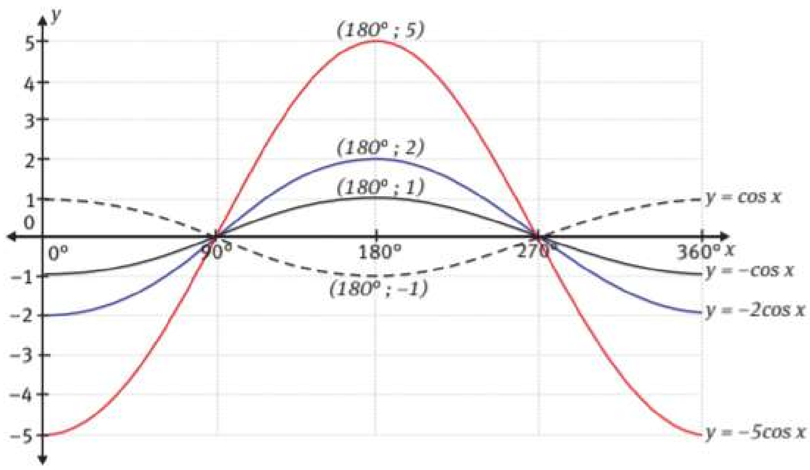
The effect of a and q on the graph $y = a \cos x + q$ for $x \in [-360°;\ 360°]$





Manipulating the cos graph, a > 0	
$q > 0$: the graph shifts upwards. $q < 0$: the graph shifts downwards. The shape does not change.	$a > 0$: multiplying by a causes a vertical stretch of the basic graph.

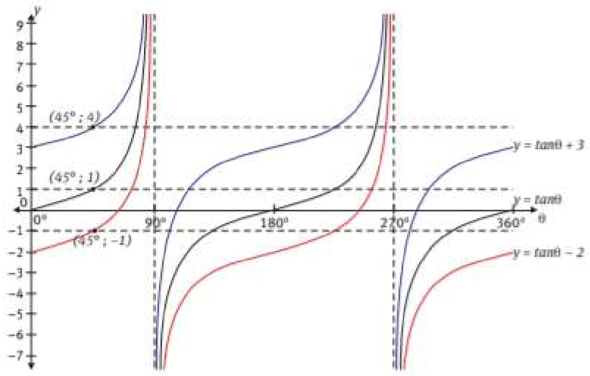
Domain: $x \in [-360°; 360°]$	**Domain:** $x \in [-360°; 360°]$
Range: $y = \cos x + 4$: $3 \leq y \leq 5$; $[3; 5]$ $y = \cos x$: $-1 \leq y \leq 1$; $[-1; 1]$ $y = \cos x - 2$: $-3 \leq y \leq -1$; $[-3; -1]$	**Range:** $y = 5 \cos x$: $-5 \leq y \leq$; $5\ y \in [-5; 5]$ $y = 2 \cos x$: $-2{,}5 \leq y \leq 2{,}5$; $y \in [-2{,}5; 2{,}5]$ $y = \cos x$: $-1 \leq y \leq 1$; $y \in [-1; 1]$
Amplitude: ((highest y –value) – (lowest y-value))/2 For all three graphs this equals 1.	**Amplitude:** The amplitude varies for the three graphs, e.g. for $y = 5\ cos\ x$: $(5 - (-5))/2 = 5$
Period: 360° for all three graphs	**Period:** 360° for all three graphs.

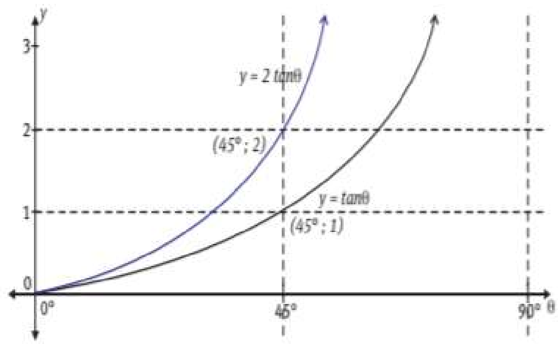


Manipulating the cos graph, a < 0
$a < 0$: multiplying by negative a causes a reflection of $y = a \cos x$ in the x-axis.
Domain: $x \in [-360°; 360°]$
Range: $y = (-5)\cos x$: $-5 \leq y \leq 5$; $y \in [-5; 5]$ $y = -\cos x$: $-1 \leq y \leq 1$; $y \in [-1; 1]$
Amplitude: The amplitude varies for the three graphs, e.g. for $y = (-5) \cos x$: $(5 - (-5))/2 = 5$
Period: 360° for all three graphs.

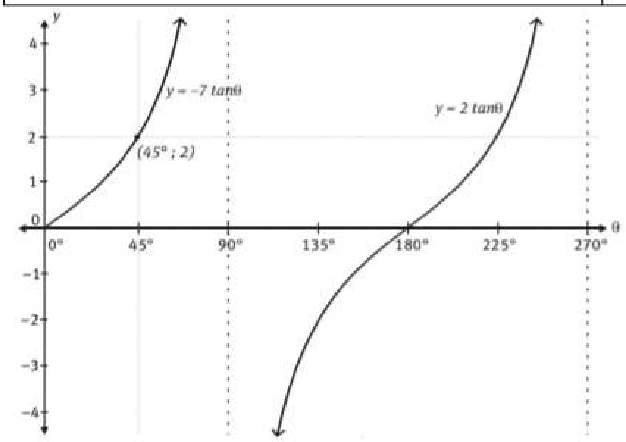
The effect of a and q on the graph $y = a\ tan\ \theta + q$ for $x \in [-360°; 360°]$

Note: The horizontal axis is no longer the x –axis, but is now the θ –axis!





Manipulating the tan graph	
Domain: $x \in [-360°;\ 360°]$	**Domain:** $x \in [-90°;\ 90°]$
Range: Undefined	**Range:** Undefined
Period: 180°	**Period:** 180°
Amplitude: Undefined	**Amplitude:** Undefined
Asymptotes: -270°, -90°, 90° and 270°	**Asymptotes:** -90° and 90°



Manipulating the tan graph
$y = \tan\theta$
Domain: $x \in [-90°;\ 270°]$
Range: Undefined
Period: 180°

Amplitude: Undefined
Asymptotes: -90°, 90° and 270°

3.2 Investigating the effect of the parameter *p* on the graph of $y = \sin(x + p)$

- The parameter p causes a horizontal shift of the sine graph:
  - If $p > 0°$, the graph shifts p degrees to the left.
  - If $p < 0°$, the graph shifts p degrees to the right.
  - The period, amplitude and shape stay the same.
- The parameter p causes a horizontal shift of the cosine graph:
  - If $p > 0°$, the graph shifts p degrees to the left.
  - If $p < 0°$, the graph shifts p degrees to the right.
  - The period, amplitude and shape stay the same.
- The parameter p causes a horizontal shift of the tan graph:
  - If $p > 0°$, the graph shifts p degrees to the left.
  - If $p < 0°$, the graph shifts p degrees to the right.
  - The asymptotes shift to the left or right by $p°$.
  - The period stays the same, and the amplitude is undefined.

3.3 Sketch graphs of trigonometric functions containing at most two of the parameters *a*, *p* and *q*

- Now that we know what the parameters mean and what influence they have on the trigonometric graphs, it is easy to sketch the graphs of these functions.
- Remember to sketch the basic function of the graph you are trying to sketch in light pencil to guide you.
- The basic graphs that we have studied up to now are:

Basic trigonometric graphs			
Basic functions	$y = \sin x$	$y = \cos x$	$y = \tan x$
q causes a vertical shift	$y = \sin x + q$	$y = \cos x + q$	$y = \tan x + q$
a causes a vertical shift	$y = a \sin x$	$y = a \cos x$	$y = a \tan x$

k influences the period	$y = \sin kx$	$y = \cos kx$	$y = \tan kx$
p causes a horizontal shift of $p°$ to the left ($p > 0$) or to the right ($p < 0$)	$y = \sin (x + p)$	$y = \cos (x + p)$	$y = \tan (x + p)$

- When you are asked to sketch a graph with more than one parameter influencing it, don't fret. Take it one step at a time:
  - Start on the LHS – is there a value for a? What does this mean?
  - Move on – is there a value for p? What does this mean?
  - Move on – is there a value for q? What does this mean?

  Once you have answers to all these questions you will be able to sketch the graph.

3.4 Interpreting graphs

- Remember: Every ordered number pair on the graph of a trigonometric function indicates the size of the angle and the value of the trigonometric ratio for that specific angle.
- Each y-value indicates the distance from that point to the x –axis. Distance is never negative!

Question 16

Draw neat sketch graphs of the groups of functions given below on the same system of axes for the domain given. Indicate the coordinates of the turning points (where applicable) as well as the intercepts with the axes. Remember to indicate the asymptotes where applicable.

16.1 $f(\theta) = 3\sin\theta$; $g(x) = -3\sin\theta$; $h(x) = -3\sin\theta - 3$; $0^\circ \leq \theta \leq 360^\circ$

16.2 $f(x) = 5\cos x$; $g(x) = -5\cos x$; $h(x) = -5\sin x + 2$; $-180^\circ \leq \theta \leq 180^\circ$

16.3 $f(x) = \tan x$; $g(x) = \tan x - 3$; $h(x) = \tan x + 5$; $0^\circ \leq \theta \leq 360^\circ$

Question 17

Copy and complete the table. The domain for all the graphs is $0^\circ \leq \theta \leq 360^\circ$.

	Highest y-value	**Lowest y-value**	**Amplitude**	**Range**	**Perio d**	**Ordered number pair at 45°**
$y = 3\sin x + 5$						
$y = \cos x - 5$						
$y = -\tan x - 3$						
$y = -4\cos x + 7$						
$y = \tan x + 5$						
$y = -5\sin x + 2$						

Question 18

Draw the graphs of the given functions for the indicated domains:

18.1 $y = 3\sin x$, $x \in [-180°; 180°]$

18.2 $y = -3\cos x + 4$, $x \in [0°; 180°]$

18.3 $y = 3\tan x - 2$, $x \in [-360°; 360°]$

Question 19

Sketch the graphs of the functions given below, for the given domain, on separate axes. Clearly indicate the coordinates of the intercepts with the axes, as well as those of the turning points.

Write down the period, amplitude and range of each graph.

19.1 $g(x) = \sin\left(\frac{1}{3}\right)x$, $x \in [-360°; 360°]$

19.2 $f(x) = \cos 3x$, $x \in [-180°; 180°]$

19.3 $k(x) = -\tan 5x$, $x \in [0°; 360°]$

Question 20

Draw the graphs of the functions indicated on separate sets of axes. The domain for each graph is given.

Where applicable, indicate the:

- intercepts with the axes
- coordinates of the turning points
- position of the asymptote(s) with a dotted line.

20.1 $y = \sin(x + 30°)$, $x \in [-30°; 330°]$

20.2 $y = \cos(x - 45°)$, $x \in [-45°; 315°]$

20.3 $y = \tan(x + 60°)$, $x \in [-60°; 300°]$

20.4 $y = \sin(x + 20°)$, $x \in [-160°; 160°]$

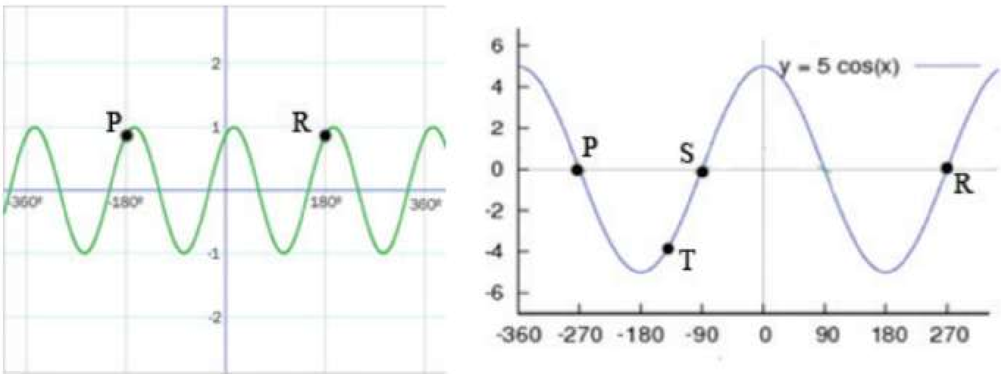
Question 21

Match the function in column A to the correct description in column B.

	Column A		Column B
A	$y = 2\sin 3x$	**E**	The graph of $y = \sin x$ is shifted upwards by three units.
B	$y = \sin x + 3$	**F**	The graph has a period of 180° and is shifted 30° to the right.
C	$y = \sin 2(x - 30°)$	**G**	The amplitude is 2 and the period 120°.

Question 22

The graphs of $g(x) = \sin 2(x - 30°)$ and $p(x) = 5\cos x$ for $x \in [-180°;\ 90°]$ are sketched below.



22.1 Write down the range of p.

22.2 Write down the coordinates of R if the coordinates of P are $(-180°;\ 0{,}866)$ and $(-270°;\ 0)$ respectively.

22.3 Write down the period of $g(2x)$.

22.4 Determine the length of ST correct to four decimal places.

22.5 For which values of x is $g(x) = p(x)$?

22.6 For which values of x is $g(x) \leq p(x)$?

22.7 Write down the equation of h if g is shifted 45° to the right and 2,5 units downwards.