Counting and Probability

From Disrupt Tutoring

Term	Explanation
probability	The likelihood or chance of something happening. A probability answer is ALWAYS in the range: $0 \leq x \leq 1$
trial/experiment	The process of trying something out to find the chance (probability) of an event occurring. For example: Tossing a coin 100 times.
outcome	A possible result from an experiment. For example: 'tails' is one of two possible outcomes when tossing a coin.
sample space	The sample space of an experiment is the set of all possible outcomes of that experiment.
experimental probability	The result of doing an experiment to find the chances of an event occurring. For example: An experiment was conducted to see how many tails appeared when a coin was tossed 100 times. The result was $\frac{47}{100}$.
relative frequency	The outcome of an experiment. In the above example $\frac{47}{100}$ is the relative frequency.
theoretical probability	The probability of an event happening using knowledge of numbers. The theoretical calculation. $$P(A) = \frac{n(A)}{n(S)}$$
tree diagram	One method used for counting the number of possible outcomes of an event. The last column of the tree diagram shows all the possible outcomes.
contingency table	A table showing the distribution of one variable in rows and another in columns, used to study the correlation between the two variables.
Venn diagram	A useful way to represent mathematical or logical sets of information. In a Venn diagram, the position and overlapping of circles are used to indicate the relationships between different sets of information.
union	The set of all outcomes that occur in at least one of the events. Key word: or

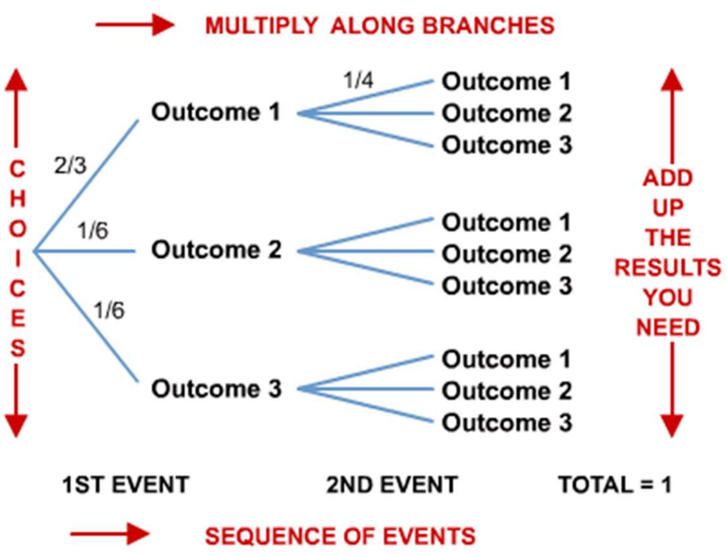
intersection	The set of outcomes that occur in all the events Key word: and
mutually exclusive events	Events with no outcomes in common (no intersection)
complementary events	Mutually exclusive events that contain all the outcomes between them.
independent events	Two events where the outcome of one event does not affect the outcome of the other.
dependent events	The outcome of one event affects the outcome of the next event.
fundamental counting principle	A way to work out the number of outcomes when different options are offered. The principle can be used in probability problems. If there are *m* ways of doing one event, *n* ways of doing a 2^{nd} event and *p* ways of doing a 3^{rd} event, then there will be $m \times n \times p$ total possible arrangements.
factorial notation	The result of multiplying a sequence of descending natural numbers down to 1 (such as 4 × 3 × 2 × 1) The symbol is '!'. $4! = 4 \times 3 \times 2 \times 1 = 24$
permutation	A permutation is an arrangement or ordering of several distinct objects where order matters.

Grade 10 and 11

Tree Diagrams

In a tree diagram (once it is complete), to find the probability of an outcome we multiply across the branches. If more than one outcome matches the results required, then addition of the answers gained from the multiplication is used.

Remember that each 'clump' of branches should add up to a probability of 1.



Independent events and dependent events

Independent events: the probability of one event is not affected by another event. For example, tossing a coin more than once – the second toss is not affected by what was tossed previously.

Dependent events: the probability of one event is affected by another event. For example, if there is a bag of two different coloured balls and one is drawn out but not replaced, this affects the probability of drawing a certain colour in a following draw.

'Not replaced'

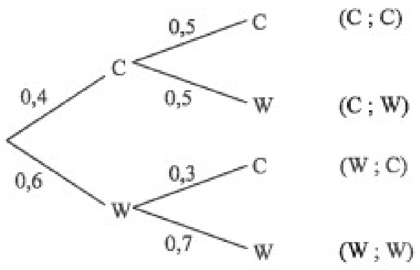
If an item IS replaced, then the following event is independent of what was drawn.

If an item is NOT replaced, then the following event is dependent on what was drawn.

Example
The probability that the first answer in a maths quiz competition will be correct is 0,4. If the first answer is correct, the probability of getting the next answer correct rises to 0,5. However, if the answer is wrong, the probability of getting the next answer correct is only 0,3. a) Represent the information on a tree diagram. Show the probabilities associated with each branch as well as the possible outcomes. b) Calculate the probability of getting the second answer correct. NSC NOV 2016
Notes This may seem like a difficult question when it is first read. The instruction to draw a tree diagram is key because this will make it much clearer. (a) Consider: How many options are given? (2) How many questions will be asked/extensions needed? (2) What are the options? (correct or wrong) (b) Look carefully at the outcomes – choose those that represent the second answer being correct. Multiply along the branches leading to these outcomes and add.

Solution:

a)



b)

P(2nd answer correct)

$= P(C\ and\ C) + P(W\ and\ C)$

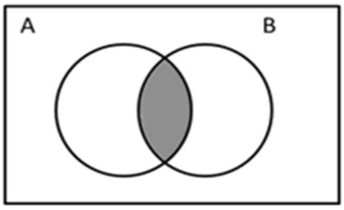
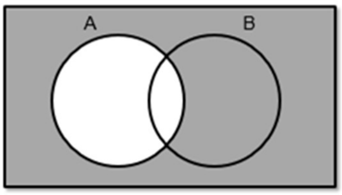
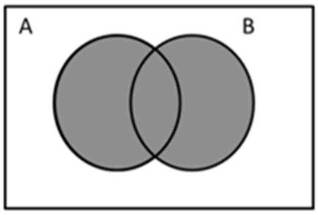
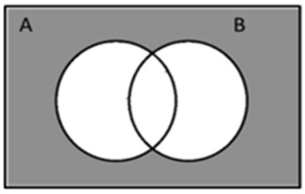
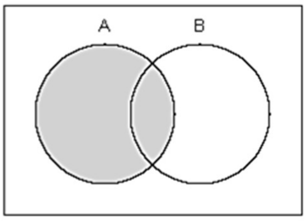
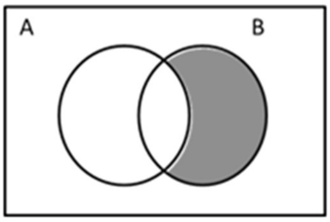
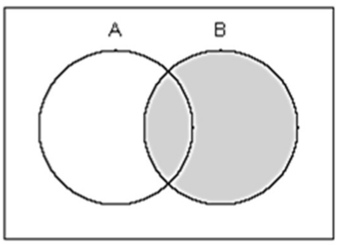
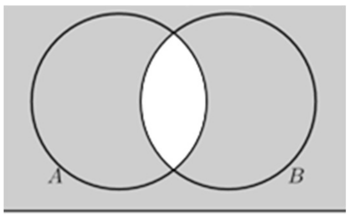
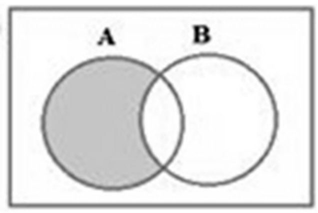
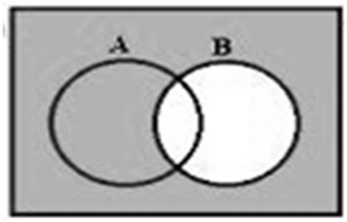
$= (0{,}4)(0{,}5) + (0{,}6)(0{,}3)$

$= 0{,}38$

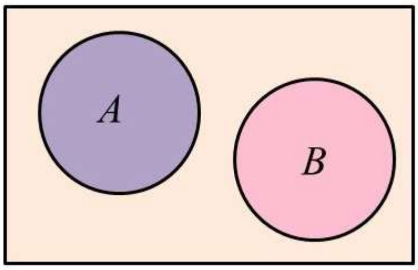
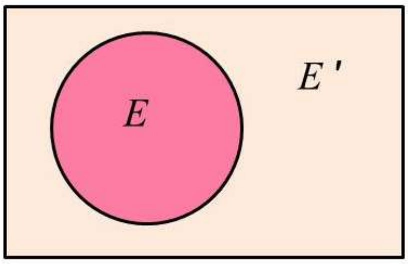
Venn diagrams

Ensure you have a clear understanding of the following phrases and how they relate to Venn diagrams.

A and B	not A	A or B
not (A or B)	A	B only
B	not (A and B)	A only
not B		

A and B 	not A 
A or B 	not (A or B) 
A 	B only 
B 	not (A and B) 
A only 	not B 

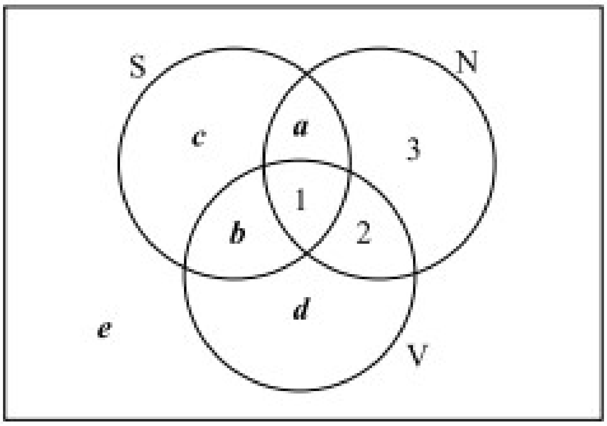
Mutually exclusive events and complementary events

	Mutually exclusive events	Complementary events
Explanation/ Example	Events with no intersection. Grade 10 learners and Grade 11 learners.	Events that contain all the possible outcomes between them. A learner either draws a pencil from the bag or they do not. Only one can happen at once but one must happen.
Venn diagram		
Notation	$P(A \; and \; B) = 0$	$P(E) = 1 - P(E')$

Example

A survey was conducted amongst 100 learners at a school to establish their involvement in three codes of sport, soccer, netball, and volleyball. The results are shown below:

- 55 learners play soccer (S)
- 21 learners play netball (N)
- 7 learners play volleyball (V)
- 3 learners play netball only
- 2 learners play soccer and volleyball
- 1 learner plays all three sports



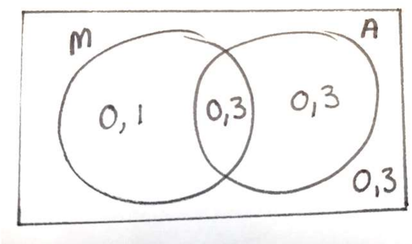
The Venn diagram below shows the information above.

a) Determine the values of a, b, c, d and e.

b) What is the probability that one of the learners chosen at random from this group plays netball or volleyball?

NSC NOV 2016

Notes This question is like needing to draw the Venn diagram from the information given. You will still need to work out what numbers go in each section. Consider: where should we always start when completing a Venn diagram? (the intersection) So, we need to work our way outwards from there until the last areas filled in are those that represent ONLY a certain set. Note that the 1 has been filled in. How will we find the value of ‘a’? (It is the only missing value, so knowing that 21 learners play netball will be used and the other numbers subtracted). How will we find the value of ‘b’? (It forms part of the intersection of soccer and volleyball and we know that is 2 but 1 has already been counted). c and d can now both be found using the totals for each sport and e will be found in a similar way – we know the total surveyed. (b) Remember that ‘or’ means ‘more’ - there is m<u>or</u>e chance when the option of ‘<u>or</u>’ is given. (only c and e are excluded)
Solution: a) $a = 15\,; b = 1\,; c = 38\,; d = 3\,; e = 37$ b) $P(N\ or\ V) = \frac{25}{100} = \frac{1}{4}$

Example
William writes a Mathematics examination and an Accounting examination. He estimates that he has a 40% chance of passing the Mathematics examination. He estimates that he has a 60% chance of passing the Accounting examination. He estimates that he has a 30% chance of passing both. Determine the probability that William will fail Mathematics and Accounting. EC 2015
Note that this question does not give a suggestion as to what method could be used to assist in calculating the answer. The key word is that there is a 'both' - in other words an intersection which in turn implies that a Venn diagram may be useful. Consider: *how many sets are represented?* (2 – mathematics and accounting) *Are they inclusive?* (yes – the chance of passing both) Draw the frame and the two overlapping circles. *Which part of the Venn diagram must always be completed first?* (the intersection) Find the information that describes this (30% chance of passing both) and fill it in. *How much chance is still left after the intersection has been completed of passing each exam?* (10% for mathematics and 30% for accounting). *Fill this in then finally check if it totals to 100% - if not there is something outside the sets (in other words it is not exhaustive).* Failing both will be the probability on the outside of the sets.
Solution:  P(fail both)$=0{,}3$

Contingency tables

A contingency table is a table showing the distribution of one variable in rows and another in columns. It is used to study the correlation between the two variables.
The following example will be used to explain further:

Medicine Taken

Cold Length

	yes	no	Total
1 -3 days	86	19	105
4 - 7 days	16	79	95
Total	102	98	200

You should be able to answer each of the following questions.
The first two are read from the rows (1-3 days and 4-7 days).

- How many people had a cold for 1 – 3 days? (105)
- How many people had a cold for 4 – 7 days? (95)

The next two are read from the columns (yes and no)

- How many people took medicine for their cold? (102)
- How many people did not take medicine for their cold? (98)

These two need you to look at two headings. First the number of days then the yes or no column.

- How many people had a cold for 1 – 3 days AND took medicine? (86)
- How many people had a cold for 4 – 7 days and did NOT take medicine? (79)

Contingency tables can be read from to calculate probabilities. The same table will be used now to answer probability questions:

	Notes	
Find the probability that a person chosen at random:		
had a cold for 1 – 3 days	Note that each of these questions has a focus on one of the totals of the various headings (other than the grand total which will be the denominator).	$\frac{105}{200}$
had a cold for 4 - 7 days		$\frac{95}{200}$
took medicine for their cold		$\frac{102}{200}$
did not take medicine for their cold		$\frac{98}{200}$
had a cold for 1 – 3 days and took medicine	Note that each of these questions has a focus on an item from a column and an item from a row – in other words, two of the headings (other than the grand total which will be the denominator).	$\frac{86}{200}$
took medicine and had a cold for 4 – 7 days		$\frac{16}{200}$
had a cold for 4 – 7 days and did not take medicine		$\frac{79}{200}$
did not take medicine and had a cold for 1 – 3 days		$\frac{19}{200}$
took medicine, given that they had a cold for 1 – 3 days	Note that each of these questions has a focus on a different area for the total since the question states, given that. This means that a particular group is already chosen. For example, given that they took medicine means that the total number of people who took medicine is the new sample space.	$\frac{86}{105}$
had a cold for 4 – 7 days given that they took medicine		$\frac{16}{102}$
did not take medicine, given that they had a cold for 4 - 7 days.		$\frac{79}{95}$
had a cold for 1 - 3 days given that they took medicine		$\frac{86}{102}$

Remember that independent events were discussed earlier when tree diagrams were covered.

For events to be independent, the probability of one event multiplied by the probability of the other event is equal to the probability of the intersection of the 2 events.

$$P(A) \times P(B) = P(A \text{ and } B)$$

Contingency tables can be used in a question relating to whether two events are independent or not.

For example, are the events, taking medicine and having a cold for 4 – 7 days independent of each other?

Solution:	Notes
P(taking medicine) × P(4-7 day cold) $= \frac{102}{200} \times \frac{95}{200}$ $= \frac{969}{4000} = 0{,}24225$	Find the probability of each of the events on their own and multiply them.
P(taking medicine AND 4 – 7 day cold) $= \frac{86}{200} = 0{,}43$	Find the probability of the event of the intersection of the two events.
$0{,}24225 \neq 0{,}43$ ∴ the events are not independent	If the answers are equal, the events are independent.

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=gtCFMsxp6ag

(Teaching Grade 11 probability)

https://www.youtube.com/watch?v=mdYilUV7GwQ

The fundamental counting principle and factorial notation

Consider the following scenario:

You and your friend decide to make key holders to make some extra money. The choices you offer people are as follows:

Wood: Pine or oak Size: Small, medium or large

Work out how many possible options there are that could be made.

You should have found 6 choices:

- Small pine
- Medium pine
- Large pine
- Small oak
- Medium oak
- Large oak

Note that there were TWO wood choices and THREE size choices.

To find ALL possible options (permutations), multiply the number of options to equal the total possible.

This is the fundamental counting principle.

Definition:

If one event can occur in *m* ways and another event can occur in *n* ways, then there are $m \times n$ ways of doing BOTH.

This rule is not limited to two events. For example, if a person is buying a new car and is given the following option:

- Automatic or manual
- White, red, blue or black
- Hatchback or sedan

There will be $2 \times 4 \times 2 = 16$ possible different cars for the buyer to choose.

The following example clearly requires a knowledge of the rule, rather than thinking through all the possibilities.

When a crime has been committed and a witness saw one of the criminals getting away, the police may ask the witness to look through photographs of various features to try and put together an identikit of the criminal. The police have the following available to be chosen from: • 40 hairlines • 48 eyes and eyebrows • 56 noses • 35 mouths • 65 chins and cheeks

How many different faces can be put together using this database?

$(40 \times 48 \times 56 \times 35 \times 65 = 244\,608\,000)$

If the witness was positive about the hairline, eyes and eyebrows and nose, there will only be one choice for those features. How many different faces could still be made?

$(1 \times 1 \times 1 \times 35 \times 65 = 2\,275)$

Note why '1' was used for those choices. It is an important idea throughout this section.

There was only ONE of those aspects of the face that he was certain about.

This idea will now be extended further.

Consider number plates.

Number plates in South Africa are made up in various ways according to the province. Below are three types.

	Description	Example
Type 1	The province (or city) abbreviation and 5 digits	CA 56819
Type 2	3 letters of the alphabet, 3 digits and the province abbreviation	DST 551 MP
Type 3	2 letters of the alphabet, 2 digits, 2 letters of the alphabet and the province abbreviation	DG 71 SZ GP
Type 2 and 3: No vowels may be used and the digit zero may not be used. All 3 types: Repeats are allowed.		

When answering this type of question, always make a dash for each possible item that needs to be considered.

For example, ___ ___ ___ ___ ___ if 5 items need considering.

We are going to calculate how many of each of the types mentioned are possible to make.

For each type, make the dashes to represent where different symbols (letters or numbers) could go. In other words, ignore the province or town represented as there are no options for that. However, you could use '1' option because that is already set.

NB: There are some basics that you need to know for this probability section. Ensure you know: There are 26 letters in the alphabet. 5 of those are vowels and 21 are consonants. There are 10 digits (zero and 1 – 9) There are 52 cards in a standard pack of cards. Those 52 are made up of 4 suits each – 13 hearts and 13 diamonds make up 26 red cards and 13 spades and 13 clubs make up the 26 black cards. Each suit has the Ace and 2 to 10 as well as 3 picture cards – Jack, Queen and King.

	Notes
Type 1	There are 5 place holders for the different options: _____ _____ _____ _____ _____ Consider: how many digits are there that could go in position 1? (9 – there are 10 digits in total but zero may not be used) Place 9 on the dash. How many digits are there that could go in position 2? (9 as repeats are allowed). Place 9 on the 2^{nd} dash. Repeat the question for each place holder. Once all the place holders have a number, multiply.
Total different number plates possible	$9 \times 9 \times 9 \times 9 \times 9$ $= 59\ 049$

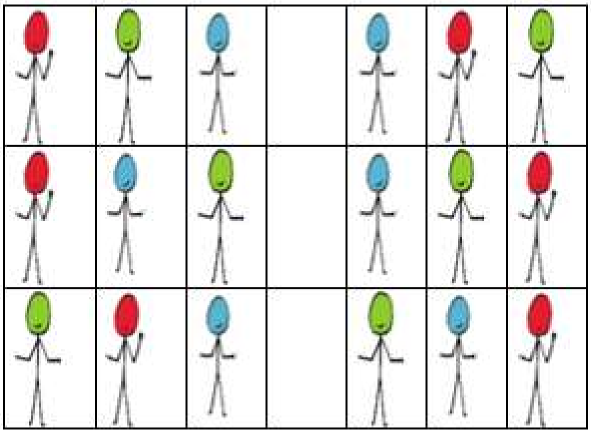
<table>
<tr><td>Type 2</td><td>There are 6 place holders for the different options:
____ ____ ____   ____ ____ ____
Note the bigger space where letters end, and digits begin. This can make the process easier.
How many letters are there that could go in position 1?
(21 – as vowels may not be used) Place 21 on the 1st dash.
How many letters are there that could go in position 2?
(21 as repeats are allowed) Place 21 on the 2nd dash.
How many letters are there that could go in position 3?
(21 as repeats are allowed) Place 21 on the 3rd dash.
How many digits are there that could go in position 4?
(9 as repeats are allowed) Place 9 on the 4th dash.
How many digits are there that could go in position 5?
(9 as repeats are allowed) Place 9 on the 5th dash.
How many digits are there that could go in position 6?
(9 as repeats are allowed) Place 9 on the 6th dash.
Once all the place holders have a number, multiply.</td></tr>
<tr><td colspan="2">Total different number plates possible $21 \times 21 \times 21 \times 9 \times 9 \times 9$
$= 6\ 751\ 269$</td></tr>
<tr><td>Type 3</td><td>There are 6 place holders for the different options:
____ ____   ____ ____   ____ ____
Note the bigger spaces again.
How many letters are there that could go in position 1?
(21 – as vowels may not be used) Place 21 on the dash.
How many letters are there that could go in position 2?
(21 as repeats are allowed)
How many digits are there that could go in position 3?
(9 as repeats are allowed)
How many digits are there that could go in position 4?
(9 as repeats are allowed)
How many letters are there that could go in position 5?
(21 as repeats are allowed)
How many letters are there that could go in position 6?
(21 as repeats are allowed)</td></tr>
<tr><td colspan="2">Total different number plates possible $21 \times 21 \times 9 \times 9 \times 21 \times 21$
$= 15\ 752\ 961$</td></tr>
</table>

When letters or numbers CAN be repeated, it is in fact the same number that is being repeated; therefore, we can simplify the calculation using exponents.
For example, we have the digits 1; 3 and 5 and 7 to make a 4-digit code. How many different codes can be made if digits can be repeated.

___ ___ ___ ___

Consider: How many digits are there that could go in position 1? (4) and position 2? (4) position 3? (4) position 4? (4)
Therefore, the number of codes that can be made is: $4 \times 4 \times 4 \times 4 = 256$
But there is a shorter way of writing this: 4^4

Examples where repeating is not allowed:
Think of three people and in how many ways they can stand in a line.
There should be 6.



Consider the word CAT.
How many different words (they don't have to be real) can be formed with these letters?
Using the fundamental counting principle:
First make dashes to represent the positions ____ ____ ____
How many letters are there that could go in position 1? (3 – the C or A or T)
How many letters are there that could go in position 2? (2, because one has already been used in position 1)
How many letters are there that could go in position 3? (1, because two of them have already been used in position 1 and 2)
Multiply. $3 \times 2 \times 1 = 6$. There are 6 different 'words' that can be made with the letters C, A and T.
Use this idea on the three people and see how it is also $3 \times 2 \times 1 = 6$

An example with a longer word: SAVOURY

As there are SEVEN different letters, there are $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5\,040$.

There are 5 040 different 'words' that can be made from the 7 letters in the word, savoury.

How many ways could we order the letters of the alphabet?

$$26 \times 25 \times 24 \times 23 \times 22 \times 21 \times 20 \ldots \ldots \ldots \times 2 \times 1$$

Important to notice in these examples - they all have something in common even though we didn't start with the same number.

Each one of them starts with the number of objects available and each successive number being multiplied goes down one until we get to 1.

There is a function on the calculator that can assist us in doing these calculations quickly and there is a special name for multiplying a list of numbers in this manner. It is called factorial notation.

Definition: The product of a sequence of descending natural numbers.

For example, $4 \times 3 \times 2 \times 1$ and this is written as $4!$

On the calculator, look for $x!$

Notice that it is a second function key.

All these different ways of ordering objects also has a special name – they are permutations.

Definition: An ordering of n objects is a permutation. In general, the number of permutations of n distinct objects is $n! = n \times (n-1) \times (n-2) \times (n-3) \ldots \ldots \times 3 \times 2 \times 1$.

Further reading, listening or viewing activities related to this topic are available on the following web links:

http://www.classzone.com/eservices/home/pdf/student/LA212AAD.pdf

http://learn.mindset.co.za/sites/default/files/resourcelib/emshare-topic-overview-asset/Maths%2012-3%20A%20Guide%20to%20Counting%20and%20Probability.pdf

http://www.amesa.org.za/AMESA2014/Proceedings/papers/1%20hour%20workshops/8.%20Desiree%20Timmet%20-%20The%20Counting%20principle.pdf

http://virtualnerd.com/algebra-2/probability-statistics/permutations-combinations/counting-outcomes/fundamental-counting-principle-definition

https://www.youtube.com/watch?v=TZj5nrtgol0

https://www.youtube.com/watch?v=bsGmzMpIpD0
(using the calculator for factorial calculations)

Solving problems using the fundamental counting principle

How many permutations are possible using the word PASTOR

$(6! = 720)$

But what if the instruction changed to - how many different words can we make if the word must start with ‘R’?

Draw the dashes and fill in the R on the first one as that must happen – there is no choice.

R ____ ____ ____ ____ ____

Now ask yourself how many letters are there that could go in the 1st position requiring a letter?

5 – because the R has already been used

and position 2? (4) position 3? (3) position 4? (2) position 5? (1)

R __5__ __4__ __3__ __2__ __1__

∴ number of permutations: $5! = 120$

Further examples using the same word:

How many different words can we make if the word must start with ‘R’ and end with ‘P’?

Draw your dashes and fill in the R on the first one and the P in the last position as that must happen – there is no choice.

R ____ ____ ____ ____ P

How many letters are there that could go in the 1st position requiring a letter?

(4 – the R and P have already been used) and position 2? (3) position 3? (2) position 4? (1)

R __4__ __3__ __2__ __1__ P

∴ number of permutations: $4! = 24$

Let’s look at a more unusual question – how many different words can we make if the word must start with a consonant and end with a vowel?

Draw your dashes

____ ____ ____ ____ ____ ____

The first and last letter have specific requirements and must therefore be considered first.

How many letters are there that could go in the 1st position requiring a consonant?

(4 – P, S, T or R)

How many letters are there that could go in the last position requiring a vowel? (2 – A or O)

__4__ ____ ____ ____ ____ __2__

Note that no matter which letters end up in those positions, TWO letters will now have been used – one in position 1 and one in position 6.

How many letters are there that could go in the 1st position still requiring a letter (position 2)? (4 as two are already accounted for)

How many letters are there that could go in the 2nd position requiring a letter (position 3)?(3)

How many letters are there that could go in the 3rd position requiring a letter (position 4)? (2)

How many letters are there that could go in the 4th position requiring a letter (position 5)? (1)

__4__ __4__ __3__ __2__ __1__ __2__

∴ number of permutations: $4 \times 2 \times 4! = 192$ $(4 \times 4 \times 3 \times 2 \times 1 \times 2)$

Note where the $4!$ Comes from – the middle four spaces. 4×2 was written first to cover the specific requirements that were stated.

The next idea we are going to look at is when certain objects need to be grouped together. For instance, to continue using this word, let's use the idea of the vowels needing to be together.

The question would therefore read as: how many different arrangements are possible if the vowels must be next to each other?

A good way to imagine (and ensure) that the vowels stay together is to think about them being tied together with an elastic band. The list of letters would then look as follows:

P S T R A O

It is like the vowels are one object, they must move together into a position.

How many objects do I have to move around? (5 because the vowels need to combine into 1)

However, there is an added complication – do the vowels have to be in the same position we see them in now? No – they could change positions and still be next to each other!

So, inside the group of vowels there is $2!$ ways that they could be arranged.

It is a good idea to write this inside the group.

Make the dashes (only 5 as there are 5 objects).

____ ____ ____ ____ ____

How many letters are there that could go in the 1st position requiring a letter?

(5 – any one of the objects is acceptable) and position 2? (4) position 3? (3) position 4? (2) position 5? (1)

Remember that one of those objects has some arrangements of its own so that needs to be included in the calculation.

∴ total arrangements possible: $5! \times 2! = 240$

Let's try the idea again but wanting all the consonants to be together.

P S T R A O

How many objects do I have to move around? (3 because the consonants need to combine into 1)

But remember the added complication – the consonants could change positions and still be next to each other!

In how many ways could they be arranged inside their own little group? (4!)

Remember to write it inside the group.

Make the dashes (only 3 as there are 3 objects).

_____ _____ _____

How many letters are there that could go in the 1st position requiring a letter?

(3 – any one of the objects is acceptable) and position 2? (2) position 3? (1)

Remember that one of those objects has some arrangements of its own so that needs to be included in the calculation.

∴ total arrangements possible: $3! \times 4! = 144$

Example

Consider the word DAUGHTER.

a) How many arrangements can be made with this word?
b) How many arrangements can be made that start with a vowel?
c) What is the probability that the new word will start with a vowel?

Notes:

a) This should be straightforward. Make use of the dashes if you don't find it easy. Keep asking yourself, how many letters can possibly go into position 1? Then how many will be left for position 2 and so on.
b) Make use of the dashes. First deal with the specific requirement of needing a vowel in the front so the number of letters available for position 1 is now limited.
c) Note that in (a) you found the TOTAL possible ways with no restrictions and in (b) you find the total ways with the vowel restriction. To find the probability you will need to use these two totals.

a) $8! = 40\,320$ b) $3 \times 7! = 15\,120$ c) $\frac{15120}{40320} = 0{,}375$

Look at the final answer and check it looks reasonable. 3 of the 8 words are vowels so yes, 37,5% looks about right.

Fully worked examples from past papers

Example 1
Seven cars, of different manufacturers, of which 3 are silver, are to be parked in a straight line. a) In how many different ways can ALL the cars be parked? b) If the 3 silver cars must be parked next to each other, determine in how many different ways the cars can be parked? NOV 2014
Notes (a) Use the dashes and consider how many options there are for each place (b) The 3 silver cars must be together. Imagine them tied up and made into one. Think of the new total number of objects but don't forget that the 3 silver cars could also be arranged in different ways.
Solution: a) $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5\,040$ b) Car 1 Car 2 Car 3 Car 4 [Silver Car 1 Silver Car 2 Silver Car 3] [3!] $3! \times 5! = 3 \times 2 \times 1 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Example 2
A group of 3 South Africans, 2 Australians and 2 Englishmen are staying at the same hotel while on holiday. Each person has his/her own room and the rooms are next to each other in a straight corridor. If the rooms are allocated at random, determine the probability that the 2 Australians will have adjacent rooms and the Englishmen will also have adjacent rooms. MAR 2017
Notice immediately that it is a probability question (underline it in an exam situation). Therefore, you will need: • Total possible arrangements with no restrictions • Total possible arrangements with the restriction given This will give the values used to find probability. Continue using the dashes and remember to group together and think of the arrangements inside the groups.
Solution: Working: *n(S):* $7! = 5\ 040$ __7__ __6__ __5__ __4__ __3__ __2__ __1__ *n(E):* $5! \times 2! \times 2!$ $= 480$ SA SA SA AUS AUS ENG ENG 2! 2! __5__ __4__ __3__ __2__ __1__ $\therefore P(E) = \frac{5! \times 2! \times 2!}{7!} = \frac{2}{21} = 0{,}095$

Example 3

A Banana Airways aeroplane has 6 seats in each row.

a) How many possible arrangements are there for 6 people to sit in a row of 6 seats?

b) Xoliswa, Anees and 4 other passengers sit in a certain row on a Banana Airways flight. In how many different ways can these 6 passengers be seated if Xoliswa and Anees must be seated next to each other?

c) Mary and 5 other passengers are to be seated in a certain row. If seats are allocated at random, what is the probability that Mary will sit at the end of the row? MAR 2016

Notes

(a) A straightforward question

(b) Use the dashes and group Xoliswa and Anees together as well as remember that the two of them can also be arranged in more than one way.

(c) Notice that it is a probability question (underline it in an exam situation).

Therefore, you will need:

- Total possible arrangements with no restrictions (already found in (a))
- Total possible arrangements with the restriction given

This will give the values used to find probability.

This question, however, has a different element to it – Mary could be on the left of the row or the right of the row and still be at the end.

Consider: what do we do when there is more than one possibility? Think about tree diagrams. (add the possibilities). Continue using the dashes

Solution:

a) $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

b) X A 2! P1 P2 P3 P4

__5__ __4__ __3__ __2__ __1__

$5! \times 2! = 240$

c) Number of ways Mary could be on the end:

__1__ __5__ __4__ __3__ __2__ __1__

(Mary)

But she could also be at the other end

__5__ __4__ __3__ __2__ __1__ __1__

(Mary)

$\therefore 1 \times 5! + 5! \times 1 = 240$

$\therefore$ P(Mary will be on the end) $= \frac{240}{720} = \frac{1}{3}$

Example
Nametso may choose DVD's from 3 categories as listed below:

Drama	Romance	Comedy
• *Last Hero*	• *One Heart*	• *Laughing Dragon*
• *Midnight*	• *You and Me*	• *Falling Down*
• *Stranger Calls*	• *Love Song*	• *Sitting on the Stairs*
• *Missing in Action*	• *Bird's First Nest*	
• *Only 40 Seconds Left*		

a) Nametso must choose one DVD from the Drama category. What is the probability that she will choose Midnight?

b) How many different selections are possible if her selection must include ONE drama, ONE romance and ONE comedy?

c) Calculate the probability that she will have 'Last Hero' and 'Laughing Dragon' as part of her selection in (b)?

MAR 2015

Notes

(a) Basic probability

(b) Basic use of the fundamental counting principle BUT you will not use factorial notation as there are only 3 options.

Use the dashes.

(c) This could be calculated using basic probability principles but to practice the fundamental counting principle, that method should be used.

Notice that it is a probability question.

Therefore, you will need:

- Total possible arrangements with no restrictions (already found in (b))
- Total possible arrangements with the restriction given

Use the dashes.

Solution:

a) $\frac{1}{5}$

b) __5__ __4__ __3__

$5 \times 4 \times 3 = 60$

c) __1__ __4__ __1__

(LH) (LD)

$\therefore$ P(LH and LD) $= \frac{1\times4\times1}{60} = \frac{1}{15}$

Example
The digits 1 – 7 are used to create a 4-digit code to enter a locked room. How many different codes are possible if the digits may not be repeated and the code must be an even number bigger than 5 000? NOV 2016
Notes This scenario creates more than one possibility, so addition will be required. (There is more than one method of doing this, so you may want to explore another route yourself). How many digits can be in the 1st position? (3 – 5, 6 and 7 will make a number greater than 5 000) How many digits can be in the last position? (3 – 2, 4 and 6 will make it an even number). But, note that 6 is on both lists. This can be addressed by considering 3 possibilities: ending in 2, ending in 4 and ending in 6. The case of the number ending in 6 will affect how many digits are available to start the number. Use of dashes.
Solution: Ending in 2: __3__ __5__ __4__ __1__ Ending in 4: __3__ __5__ __4__ __1__ Ending in 6: __2__ __5__ __4__ __1__ Start with the 1 possibility in position 4, then consider the 3 possibilities in position 1 before filling in the number of digits that would be left over for the middle two positions (after 2 have been used there are 5 then 4 leftover). When 6 is in the last place, there are only 2 digits that can be in position 1 but there are still 5 and 4 digits left for the middle positions. Number of different codes: $(3 \times 5 \times 4 \times 1) + (3 \times 5 \times 4 \times 1) + (2 \times 5 \times 4 \times 1)$ $= 60 + 60 + 40 = 160$

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=guvaQtcVlM4

https://www.youtube.com/watch?v=fPdYKStm7Nw

https://www.youtube.com/watch?v=Yw-Oaumh2Ww

https://www.youtube.com/watch?v=aegBgqk5jeg