WORK, ENERGY & POWER

IMPORTANT TERMS & DEFINITIONS

The work done on an object by a constant force F $W = F \Delta x \cos \theta$	The work done on an object by a constant force F, where FΔxcos θ, F is the magnitude of the force, Δx the magnitude of the displacement and θ the angle between the force and the displacement
Work-energy Theorem $W_{net} = \Delta K$ or $W_{net} = \Delta E_k$	The net/total work done on an object is equal to the change in the object's kinetic energy OR the work done on an object by a resultant/net force is equal to the change in the object's kinetic energy.
Conservative force	A force for which the work done in moving an object between two points is independent of the path taken.
Non-conservative force $W_{nc} = \Delta K + \Delta U$ or $W_{nc} = \Delta E_k + \Delta E_p$	A force for which the work done in moving an object between two points depends on the path taken.
The principle of conservation of mechanical energy $(E_k + E_p)_{top/A} = (E_k + E_p)_{bottom/B}$	The total mechanical energy (sum of gravitational potential energy and kinetic energy) in an isolated system remains constant.
Power $P = \frac{W}{\Delta t}$	The rate at which work is done or energy is expended.

- Work is a form of energy. Work is done by a force F on mass m when the force and the displacement Δx are parallel. In general: $W = F \Delta x \cos \theta$, and work is a SCALAR.
- Isolated system is a system on which no external forces acting on an object (i.e friction)

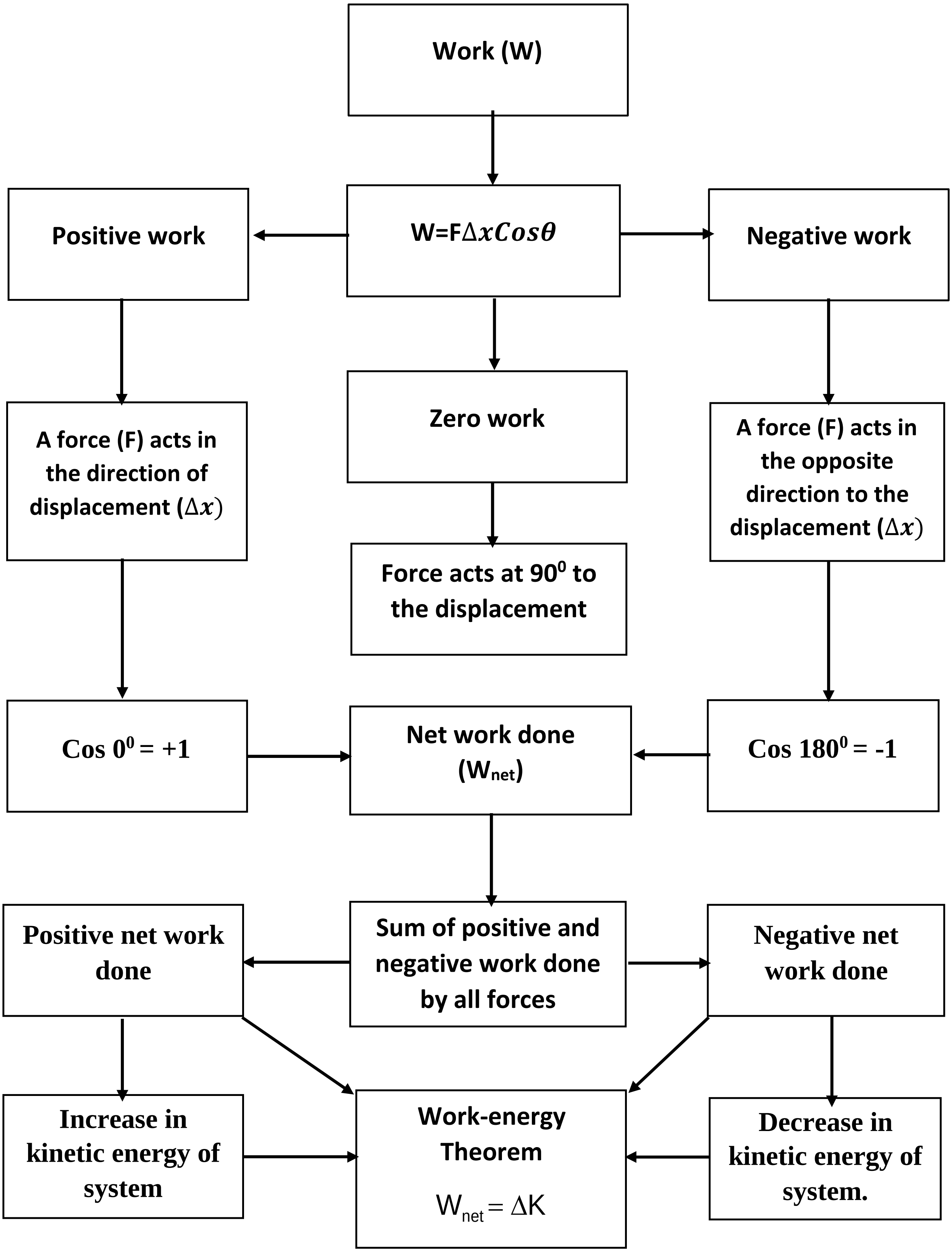
FORMULAE TABLES:

FORCE

$F_{net} = ma$	$p = mv$
$f_s^{max} = \mu_s N$	$f_k = \mu_k N$
$F_{net}\Delta t = \Delta p$ $\Delta p = mv_f - mv_i$	$w = mg$
$F = \frac{Gm_1 m_2}{d^2}$	$g = G\frac{M}{d^2}$

WORK, ENERGY AND POWER

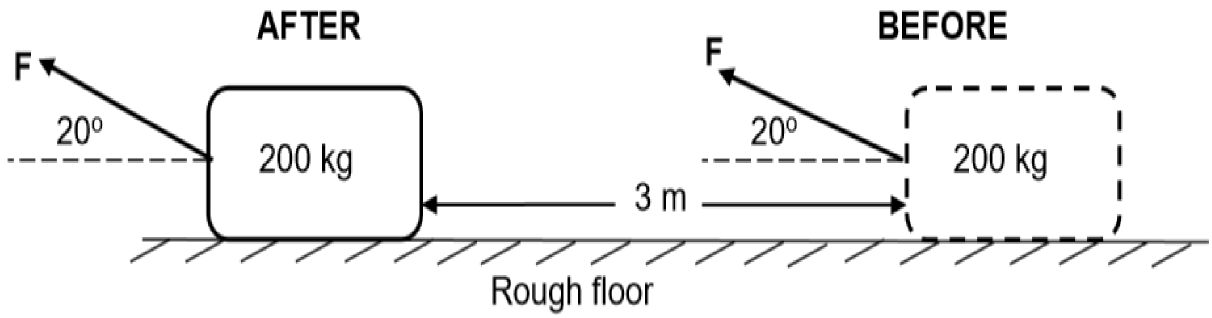
$W = F\Delta x \cos\theta$	$U = mgh$ or/*of* $E_P = mgh$
$K = \frac{1}{2}mv^2$ or/*of* $E_k = \frac{1}{2}mv^2$	$W_{net} = \Delta K$ or/*of* $W_{net} = \Delta E_k$ $\Delta K = K_f - K_i$ or/*of* $\Delta E_k = E_{kf} - E_{ki}$
$W_{nc} = \Delta K + \Delta U$ or/*of* $W_{nc} = \Delta E_k + \Delta E_p$	$P = \frac{W}{\Delta t}$
$P_{ave} = Fv_{ave}$	


Work (W)
W=FΔxCosθ
Positive work
Negative work
Zero work
A force (F) acts in the direction of displacement (Δx)
Force acts at 90⁰ to the displacement
A force (F) acts in the opposite direction to the displacement (Δx)
Cos 0⁰ = +1
Net work done (W_net)
Cos 180⁰ = -1
Positive net work done
Sum of positive and negative work done by all forces
Negative net work done
Increase in kinetic energy of system
Work-energy Theorem
W_net = ΔK
Decrease in kinetic energy of system.

LEARNER WORKED ACTIVITIES & EXAMPLES
Work (W) or net work (W_{net}) done can be calculated or applied by considering an object that moves: ***1. horizontally*** ***2. vertically*** ***3. at an incline,*** under the influence of one or more forces
Key Concepts: **1.** Work. **2.** Free-body diagrams. **3.** Work - Energy theorem. **4.** Conservative and non-conservative forces. **5.** Principle of conservation of mechanical energy **6.** Power

WORKED EXAMPLE 1

A constant force **F**, applied at an angle of 20° above the horizontal, pulls a 200 kg block, over a distance of 3 m, on a rough, horizontal floor as shown in the diagram below.



The coefficient of kinetic friction, μ_k, between the floor surface and the block is 0,2.

1.1 Give a reason why the coefficient of kinetic friction has no units. (1)

1.2 State the work-energy theorem in words. (2)

1.3 Draw a free-body diagram indicating ALL the forces acting on the block while it is being pulled. (4)

1.4 Show that the work done by the kinetic frictional force (W_{fk}) on the block can be written as W_{fk} = (-1 176 + 0,205 **F**) J. (4)

1.5 Calculate the magnitude of the force **F** that has to be applied so that the net work done by all forces on the block is zero. (4)

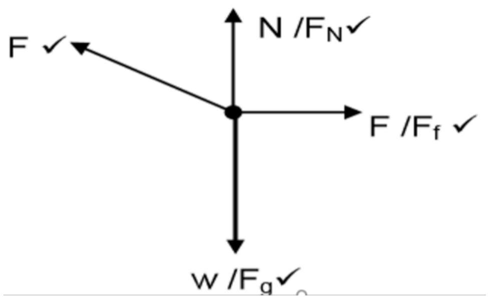
[15]

SOLUTION

1.1 It is a ratio of two forces ✓ (hence units cancel out). (1)

1.2 The net work done on an object is equal ✓to the change in kinetic energy of the object ✓ (2)

1.3



1.4 $F\sin 20^o + N = mg$✓
$N = mg - F\sin 20^o$ (4)

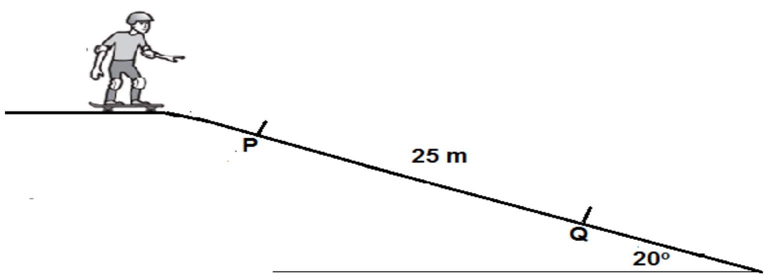
$W_{fk} = f_k\Delta x \cos\theta = \mu_k N \Delta x \cos\theta$✓
$= \mu_k(mg - F\sin 20)(3)\cos\theta$
$= \underline{(0,2)[200(9,8) - F\sin 20](3)\cos 180^o}$✓
$= (-1176 + 0,205\ F)\ J$✓ (4)

1.5 $W_{net} = [W_g] + W_f + W_F$ ✓
0 ✓ $= \underline{[0]} + \underline{[(-1176 + 0,205\ F)]} + \underline{[F(\cos 20)(3)(\cos 0)]}$✓
$F = 388,88\ N$✓ (4)
[15]

WORKED EXAMPLE 2

The diagram below shows a boy skateboarding on a ramp which is inclined at 20^o to the horizontal. A constant frictional force of 50 N acts on the skateboard as it moves from **P** to **Q**. Consider the boy and the skateboard as a single unit of mass 60 kg.

Ignore the effects of air friction.



2.1 Draw a labelled free-body diagram, showing ALL the forces acting on the boy-skateboard unit while moving down the ramp from **P** to **Q**. (3)

Points **P** and **Q** on the ramp are 25 m apart. The skateboarder passes point **P** at a speed v_i and passes point **Q** at a speed of 15 m·s^{-1}.

Ignore rotational effects due to the wheels of the skateboard.

2.2 State the work-energy theorem in words. (2)

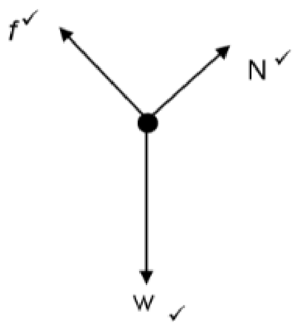
2.3 Use energy principles to calculate the speed v_i of the skateboarder at point **P**. (5)

2.4 Calculate the average power dissipated by the skateboarder to overcome friction between **P** and **Q**. (4)

[14]

SOLUTION

2.1



(3)

2.2 The net/total work done on an object equals the change in the object's kinetic energy. ✓✓

2.3 **<u>OPTION 1</u>**

$W_{net} = \Delta E_K$

$f\Delta x\cos\theta + F_g\Delta x\cos\theta = ½mv_f^2 – ½mv_i^2$ ✓Any one/

$(50)(25\cos180^o)$✓ + $(60)(9,8)(25\cos70^o)$ ✓ = $½(60)(15^2 – v_i^2)$ ✓

$-1\ 250 + 5\ 027,696 = 6\ 750 – 30v_i^2$

$v_i = 9,95(4)\ m.s^{-1}$✓

<u>OPTION 2</u>

$W_{nc} = \Delta E_K + \Delta E_P$

$f\Delta x\cos\theta = ½(mv_f^2 - mv_i^2) + (mgh_Q – mgh_P)$ ✓Any one/*Enige een*

$E_{mechP} + E_{mechQ} + W_{nc} = 0$

$(50)(25\cos180^o)$ ✓= $½(60)(15^2 – v_i^2)$ ✓ + $(60)(9,8)(-25\sin 20^o)$ ✓

$-1\ 250 = 6\ 750 – 30v_i^2 – 5\ 027,696$

$v_i = 9,95\ m.s^{-1}$✓ (5)

2.4

$P_{ave/gemid} = Fv_{ave/gemid}$ ✓

$= 50$✓ $\frac{(9,95 + 15)}{2}$ ✓

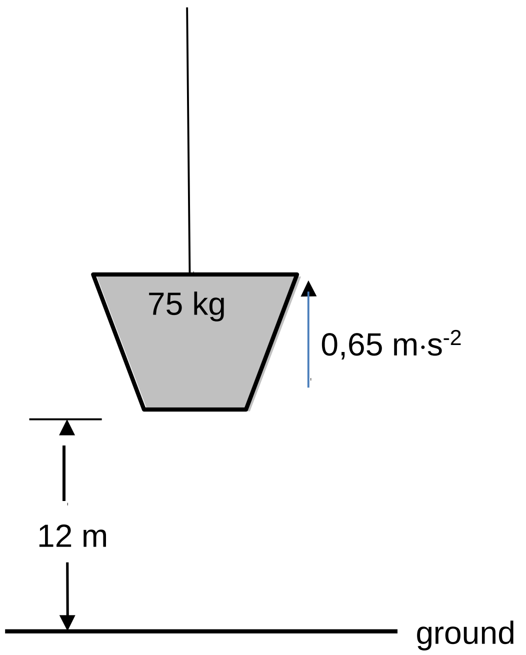
$= 623,75\ W$ ✓

(4)

[14]

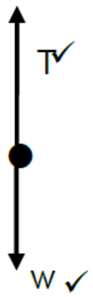
WORKED EXAMPLE 3

A load of mass 75 kg is initially at rest on the ground. It is then pulled vertically upwards at a constant acceleration of 0,65 m·s^{-2} by means of a light inextensible rope. Refer to the diagram below. Ignore air resistance, rotational effects and the mass of the rope.



3.1	Draw a labelled free-body diagram for the load while it moves upward.	(2)
3.2	Name the non-conservative force acting on the load.	(1)
3.3	Calculate the work done on the load by the gravitational force when the load has reached a height of 12 m.	(3)
3.4	State the work-energy theorem in words.	(2)
3.5	Use the work-energy theorem to calculate the speed of the load when it is at a height of 12 m.	(5)
		[13]

3.1 **SOLUTION**



(2)

3.2 Tension✓ (1)

3.3 $W = F\Delta x\cos\theta$ ✓ | 1 mark for any of these/*1 punt vir enige van hierdie*

$W_w = mg\Delta x\cos\theta$

$= \underline{75(9,8)\ (12)\cos 180^o}$✓

$= -\ 8\ 820$ J✓

OR/*OF*

$W_w = -\Delta E_p$ ✓

$= -\ (mgh - 0)$

$= -(75)(9,8)(12)$ ✓

$= -8\ 820$ J✓ (3)

3.4 The work done on an object by a <u>net</u> force is <u>equal to the change in the object's kinetic energy</u>. ✓✓

3.5

1 mark for any of these

$W_{net} = \Delta K$ ✓

$F_{net}\Delta x\cos\theta = (½\ mv_f^2 - ½\ mv_i^2)$

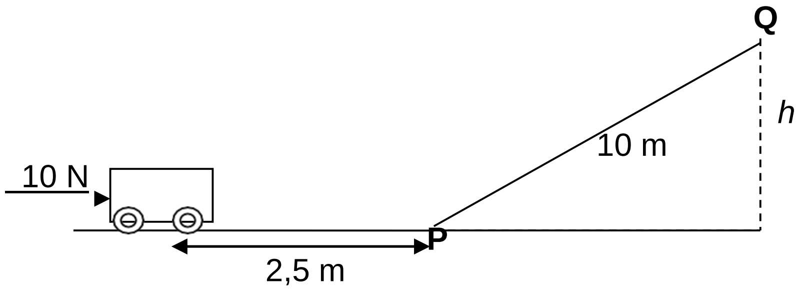
$\underline{(75)(0,65)(12)}$ ✓$\cos 0^o$ ✓ $= ½(75)(\ v_f^2 - 0)$ ✓

$v_f = 3,95\ m{\cdot}s^{-1}(3,949\ m{\cdot}s^{-1})$ ✓ (5)

[13]

WORKED EXAMPLE 4

A 3 kg trolley is at rest on a horizontal frictionless surface. A constant horizontal force of 10 N is applied to the trolley over a distance of 2,5 m.



When the force is removed at point **P**, the trolley moves a distance of 10 m up the incline until it reaches the maximum height at point **Q**. While the trolley moves up the incline, there is a constant frictional force of 2 N acting on it.

4.1 Write down the name of a non-conservative force acting on the trolley as it moves up the incline. (1)

4.2 Draw a labelled free-body diagram showing all the forces acting on the trolley as it moves along the horizontal surface. (3)

4.3 State the WORK-ENERGY THEOREM in words. (2)

4.4 Use the work-energy theorem to calculate the speed of the trolley when it reaches point **P**. (4)

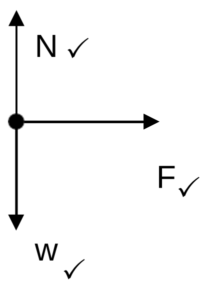
4.5 Calculate the height, *h*, that the trolley reaches at point **Q**. (5)

[15]

SOLUTION

4.1 Frictional force✓ (1)

4.2



(3)

4.3 The <u>net work done</u> ✓on an object is <u>equal to the change in kinetic energy</u> ✓of the object.

(2)

4.4 $W_{net} = \Delta E_K$ ✓

$W_F + W_w + W_{FN} = ½\, m(v_f^2 – v_i^2)$

$(10)(2,5)\cos 0^o + 0 + 0$ ✓$= ½\,(3)(v_f^2 – 0^2)$ ✓

$v_f = 4,08\ m·s^{-1}$ ✓ (4)

4.5 **<u>OPTION</u> 1**

$W_{nc} = \Delta E_p + \Delta E_k$ ✓

$f\Delta x\cos\theta = (mgh_f - mgh_i) + (\frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2)$

$(2)(10)\cos180^o$ ✓ $= (3)(9,8)h_f - 0$ ✓ $+ 0 - \frac{1}{2}(3)(4,08)^2$ ✓

$\therefore h = 0,17$ m ✓

<u>OPTION</u> 2

$W_{net} = \Delta E_k$ ✓

$mg\sin\alpha\ \Delta x\cos\theta + f\Delta x\cos\theta = \frac{1}{2} m(v_f^2 - v_i^2)$

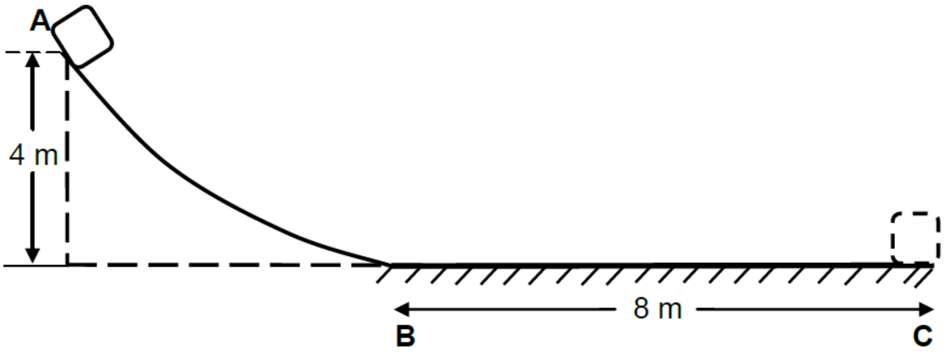
$(3)(9,8)(\frac{h}{10})(10)\cos180^o$ ✓ $+ (2)(10)\cos180^o$ ✓ $= \frac{1}{2}(3)(0^2 - 4,08^2)$ ✓

$\therefore h = 0,17$ m ✓ (5)

[15]

WORKED EXAMPLE 5

5.1 The diagram below shows a track, **ABC**. The curved section, **AB**, is frictionless. The rough horizontal section, **BC**, is 8 m long.



An object of mass 10 kg is released from point **A** which is 4 m above the ground. It slides down the track and comes to rest at point **C**.

5.1.1 State the *principle of conservation of mechanical energy* in words. (2)

5.1.2 Is mechanical energy conserved as the object slides from **A** to **C**? Write only YES or NO. (1)

5.1.3 Using ENERGY PRINCIPLES only, calculate the magnitude of the frictional force exerted on the object as it moves along **BC**. (6)

[9]

SOLUTION

5.1.1 In an isolated/closed system, ✓ the total mechanical energy is conserved / remains constant ✓ (2)

5.1.2 No ✓ (1)

5.1.3

OPTION 1	**OPTION 2**
Along **AB** $E_{\text{mechanical at A}} = E_{\text{mechanical at B}}$ $(E_p + E_k)_A = (E_p + E_k)_B$ ✓ $(mgh + ½ mv^2)_A = (mgh + ½ mv^2)_B$ $(10)(9,8)(4) + 0 = 0 + ½ (10) v_f^2$ ✓ $v_f = 8,85\ m \cdot s^{-1}$	Along **AB** $W_{net} = \Delta E_k$ ✓ $F_g \Delta h \cos\theta = ½\, m(v_f^2 - v_i^2)$ $(10)(9,8)(4)\cos 0^o = ½ (10)(v_f^2 - 0)$ ✓ $v_f = 8,85\ m \cdot s^{-1}$
Along **BC**/*Langs* **BC** $W_{net} = \Delta K$ ✓ $f \Delta x \cos\theta = \Delta K$ $\underline{f(8)\cos 180^o}$ ✓ $= \underline{½ (10)(0 - 8,85^2)}$ ✓ $f = 48,95$ N ✓	Along **BC**/*Langs* **BC** $W_{nc} = \Delta K + \Delta U$ ✓ $f\, \Delta x \cos\theta = \Delta K + \Delta U$ $\underline{f(8)\cos 180}$ ✓ $= \underline{½ (10)(0 - 8,85^2) + 0}$ ✓ $f = 48,95$ N ✓ (Accept/ *Aanvaar* 49 N)

(6)

[9]

KEY POINTS TO NOTE

✓ **Drawing free body diagrams**

Avoid doing the following:

- Drawing a force diagram instead of a free-body digram.
- Drawing a free-body diagram for an object on an incline, when the object is on a horizontal surface and vice versa
- Resolving a force (the weight of an object on an incline, and a force acting at an angle) into its components and then including the force and the components in one diagram.
- Including a frictional force when friction should be ignored or omitting the frictional force when there is friction.
- Incorrect representation of the normal force when the object is on an inclined plane.
- Incorrect labelling of forces, and drawing straight lines without arrow-heads.

✓ **Calculations/Problem solving**

- Identify the correct initial and final velocities and do not swap these two velocities
- Understand the meaning of F & F_{net} and W & W_{net}. F_{net} is the sum of all the forces acting on an object. W_{net} is the sum of the work done by all the forces. F is a single force acting on an object, while W is work done by ONE force.
- Do not leave out the subscripts in the formulae, i.e F or W instead of F_{net} or W_{net}.
- Do not include a frictional force where friction should be ignored or leave out the frictional force where there is no friction.
- Remember that friction always acts in the opposite to the motion of an object.
- Make sure you understand the concept of negative work. Note that an object can be moving whilst a force is acting in the opposite direction and this force may not necessarily be frictional force.
- When a force does negative work on an object, energy is removed from the object and converted to other forms of energy such as heat. (The object becomes warmer)