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Study Guide

Mathematics

Grade 10

Via Afrika Publishers

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Chapter 8

Analytical geometry

Overview

When answering questions on analytical geometry:

- ALWAYS make a sketch. It can be a rough sketch, but at least have the points in the correct quadrants of the Cartesian plane.
- Read carefully and put all the information in the sketch.
- Always ask yourself: Is this answer possible if I look at my drawing?
- When using a calculator, always round off at the final answer only.

Unit 1

The distance formula

The distance between the two points is given by the formula:

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ (Remember, the answer is a length)

1.2 Using the distance formula

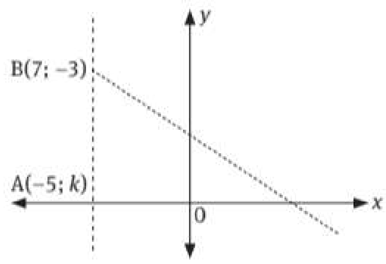
Example

1 Determine the distance between P(2; 5) and Q(−4; 1) on the Cartesian coordinate system.

$PQ^2 = (x_p - x_q)^2 + (y_p - y_q)^2$
$= (2 - (-4))^2 + (5 - 1)^2$ (Use the given values)
$= (6)^2 + (4)^2$
$= 36 + 16$
$= 52$
$\therefore PQ = \sqrt{52} = 7{,}21$ units (Usually rounded off to two decimal places)

2 The distance between A(−5; *k*) and B(7; −3) is 13 units. Determine the value(s) of *k*.

Here, point A can be anywhere along the vertical dashed line in the graph, as long as the distance between A and B is 8 units.



$AB^2 = (x_a - x_b)^2 + (y_a - y_b)^2$
Substitute in the values given:
$132 = (-5 - 7)^2 + (k - (-3))^2$
$169 = 144 + (k + 3)^2$
$= 144 + k^2 + 6k + 9$
$k^2 + 6k - 16 = 0$
$(k + 8)(k - 2) = 0$
$\therefore k = -8$ or 2

Unit 2

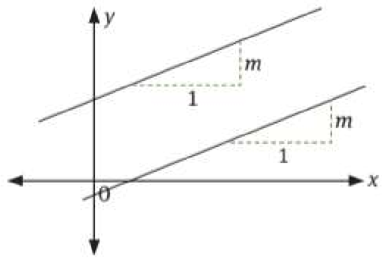
The gradient formula

The gradient of the line joining the points is given by the following formula:

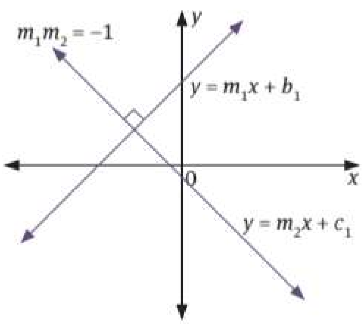
$m = \frac{y_1 - y_2}{x_1 - x_2}$

2.2 Parallel and perpendicular lines

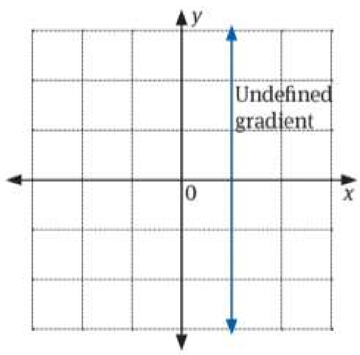
- When two lines are parallel, their gradients are the same.



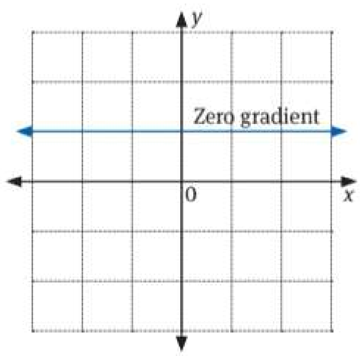
- When two lines are perpendicular then the **product** of their gradients equals –1.



- A vertical line has an **undefined** gradient.

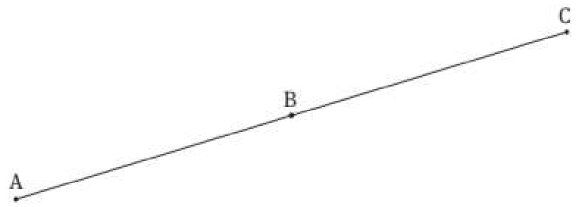


- A horizontal line has a gradient of 0.



2.3 Collinear points

Points A, B and C are collinear if AB + BC = AC or if $m_{AB} = m_{BC}$ AND point B is a common point.



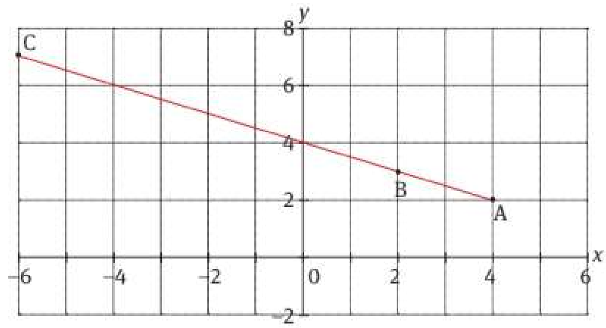
Example

1 Show that ABC is a straight line (in other words, show that A(4; –2), B(2; 3) and C(–6; 7) are collinear).

We need to calculate the gradients between AB and BC. If the gradients are the same, the points are collinear.

$$m_{AB} = \frac{y_A - y_B}{x_A - x_B} = \frac{2-3}{4-2} = -\frac{1}{2}$$

$$m_{BC} = \frac{y_C - y_B}{x_C - x_B} = \frac{7-3}{-6-2} = -\frac{1}{2}$$



Therefore, ABC is a straight line.

2 Determine the value(s) of k if P(8; –4); Q(k; 1) and R(–1; 3) are collinear.

If the points are collinear, then m_{PQ} must equal m_{PQ}. Therefore:

$$= \frac{y_p - y_q}{x_p - x_q} = \frac{y_r - y_q}{x_r - x_q}$$

$$= \frac{-4-1}{8-k} = \frac{3-1}{-1-k}$$ (Replace with coordinate values)

$$-3(-1-k) = 2(8-k)$$ (solve for k)

$$3 + k = 16 - 2k$$

$$3k = 13$$

$$\therefore k = 4$$

Unit 3

The midpoint formula

The coordinates of the midpoint of the line joining the points is given by:

$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$ (Remember, the answer is a set of coordinates)

Example

1 Determine the midpoint, M, of CD, if the points are C(–4: 5) and D(2; –3).

$$M\left(\frac{x_c + x_d}{2}; \frac{y_c + y_d}{2}\right) = M\left(\frac{-4 + 2}{2}; \frac{5 + (-3)}{2}\right)$$

$$= M(-1; 1)$$

2 Determine the coordinates of P if Q is (–1; 5) and point R(3; 7) is the midpoint of PQ. Suppose the coordinates for P are $(x_p; y_p)$. Use the midpoint formula to solve two equations, one for the x-value of P and one for the y-value for P.

$$(3; 7) = \left(\frac{x_p + x_q}{2}; \frac{y_p + y_q}{2}\right)$$

x-coordinate of the midpoint:

$$x_R = \frac{x_p + x_q}{2}$$

$$3 = \frac{x_p + (-1)}{2}$$

$$6 = x_p - 1$$

$$\therefore x_p = 7$$

y-coordinate of the midpoint:

$$y_R = \frac{y_p + y_q}{2}$$

$$7 = \frac{y_p + 5}{2}$$

$$14 = y_p + 5$$

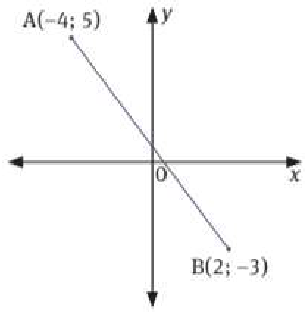
$$\therefore y_p = 9$$

$\therefore$ P is the point (7; 9)

Questions

1 Given: P(–4; –1); Q(6; 3); R(6; *b*) and S(–4; –3):
   a Determine the gradient of PQ.
   b If PQ is parallel to SR, determine the value of *b*.
   c Show that PQ = SR.
   d Is quadrilateral PQRS a parallelogram? Give a reason for your answer.
   e Calculate the midpoint of:
      i PR   ii SQ
   f What can you subsequently deduce regarding the diagonals of a parallelogram?
   g A rhombus is a parallelogram with two consecutive sides that are equal in length. Show that PQRS is not a rhombus.

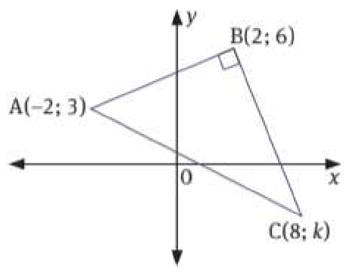
2 Using the sketch alongside (not drawn to scale), calculate:
   a AB
   b the midpoint, K, of AB
   c the gradient of A



3 BD and AC are the diagonals of a parallelogram. If B = (2; 3), D = (6; 0), C = (7; 5) and A = (*x*; *y*), find the values of *x* and *y*.

4 Given points A(3; 7), B(5; 11) and C(6; 3), find:
   a the length of AB, leaving your answer as a surd.
   b the length of BC, correct to two decimal places
   c the midpoint of AC
   d the gradient of AB and BC

5 Triangle ABC (alongside) is made up of the points A(–2; 3), B(2; 6) and C (8; *k*). Find the value of *k*, given that ΔABC = 90°.



6 G(3; 7), H(–5; 1), K(1; –3) and D(*x*; *y*) are the vertices of a parallelogram.
   a Calculate the length of the line HK.
   b Calculate the length of KD.
   c Find the coordinates of D.
   d Find the coordinates of the midpoint of HD.

7 The points M(–4 0), N(3; –7) and P(7; 4) are shown in the diagram alongside. Calculate the following:

a Calculate the midpoint, Q, of MN.

b Calculate the gradients of MN and PQ

c Calculate the product of the gradients of MN and PQ.

d What can you say about MN and PQ?

e Calculate the lengths of PM and PN.

f Draw a conclusion about ΔPMN.