2 Work, energy and power

The work done on an object by a constant force F is F $\Delta x \cos\Theta$, where F is the magnitude of the force, Δx the magnitude of the displacement and Θ the angle between the force and the displacement. W = F$\Delta x \cos\theta$

$$W = F\,\Delta x\,\cos\theta$$

Work
in Joule
scalar

MAGNITUDE
of the force
in Newton

MAGNITUDE
of the displacement
in meter

angle between the
force and displacement

W scalar (no direction): A **negative** W is energy **removed** from object.

Net work: $W_{net} = W_g + W_T + W_{fric} + W_N$ (use the whole force)
$W_{net} = F_{net}\,\Delta\,x\cos\theta$ (use components and calculate F_{net})

Work-energy theorem:
The net/total work done on an object is equal to the change in the object's kinetic energy.
In symbols:

$$W_{net} = \Delta\,E_K$$
$$W_{net} = \tfrac{1}{2}m({v_f}^2 - {v_i}^2)$$

Conservative force: The work done by the force in moving an object between 2 points is independent of the path taken ex. gravitational, electrostatic and elastic forces.
Non-conservative force: The work done by the force in moving an object between 2 points depends the path taken ex. frictional force, air resistance, tension in a chord.

Work done by non-conservative forces: $W_{nc} = \Delta\,E_K + \Delta\,E_P$ since $W_g = -\Delta\,E_P$

All the W except W_g

useful when 'no corner' is given for an inclined plane

Mechanical energy: $E_{mech} = E_k + E_p$
Kinetic energy energy due to movement: $E_k = \frac{1}{2}mv^2$
Gravitational potential energy: energy due to position: $E_p = mgh$

The principle of conservation of mechanical energy:
The total mechanical energy (sum of gravitational potential energy and kinetic energy) in an isolated system remains constant.

$$E_{mech(i)} = E_{mech(f)} \quad \text{(Only } F_g\text{)}$$
$$E_{pi} + E_{ki} = E_{pf} + E_{kf}$$
$$mgh + \tfrac{1}{2}mv^2 = mgh + \tfrac{1}{2}mv^2$$

(g and v no + or $-$)

Power: rate at which work is done/energy is expended. $P = \frac{W}{\Delta t}$ or $P_{ave} = Fv_{ave}$

(For v constant)

(A power of 200 W means 200 J energy is used/work is done per second.)

Work

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Work
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of the force
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angle between the
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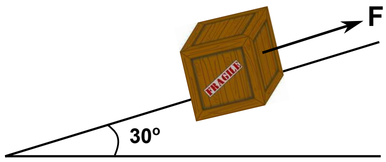
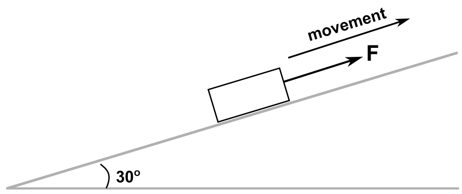
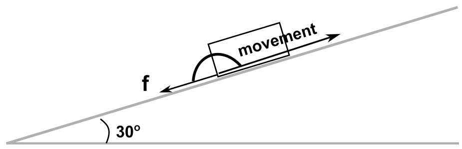
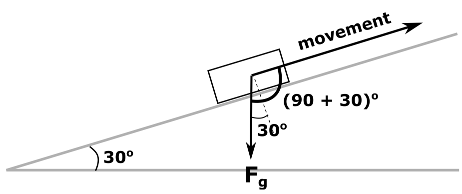
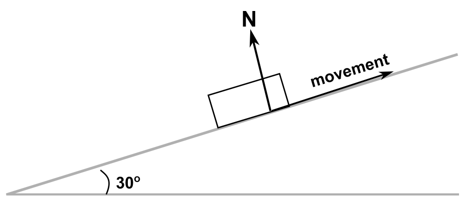
Example

A crate, with mass 10 kg, is pulled 4 m up an inclined plane that makes an angle of 30° with the ground. The crate is pulled with a force of 180 N and experiences a frictional force of 10N. Calculate the work done by each of the forces working on the crate.



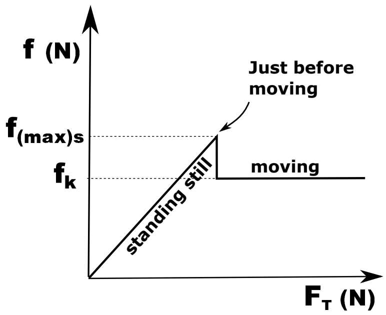
Applied force: $W_F = F\Delta x\cos\theta$ $= 180(4)\cos 0^\circ$ $= 720$ J	
Friction: $W_f = f\Delta x\cos\theta$ $= 10(4)\cos 180^\circ$ $= -40$ J	
Gravity: $W_g = F_g\Delta x\cos\theta$ $= (10\times 9{,}8)(4)\cos 120^\circ$ $= -196$ J	
Normal force: $W_N = N\Delta x\cos\theta$ $= N\Delta x\cos 90^\circ$ $= 0$ J	

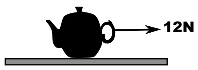
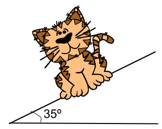
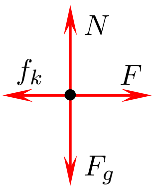
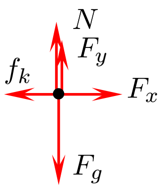
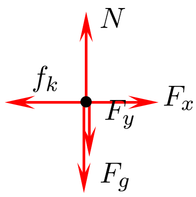
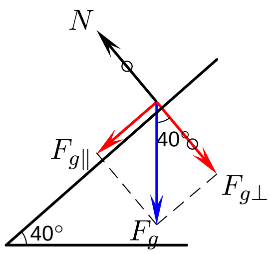
$F_x = F\cos\theta$ and $F_y = F\sin\theta$

$F_{g\perp} = F_g\cos\theta$ and $F_{g||} = F_g\sin\theta$

θ relative to horisontal

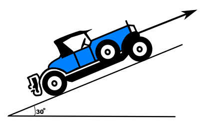
$f_{s(max)} = \mu_s N$ $f_{s(max)} = \mu_s N$

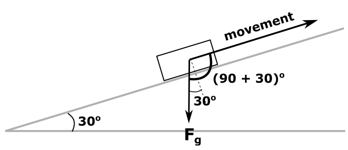
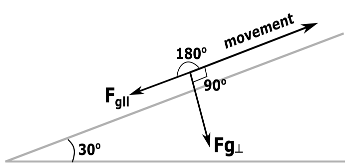
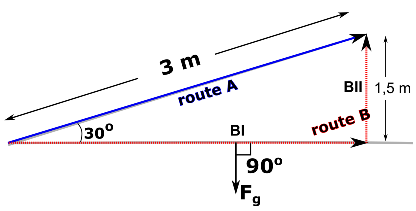


			
			
Vertical: $F_{net} = 0$ $N + (-F_g) = 0$ $N = F_g$	Vertical: $F_{net} = 0$ $N + F_y + (-F_g) = 0$ $N = F_g - F_y$	Vertical: $F_{net} = 0$ $N + (-F_g) + (-F_y) = 0$ $N = F_g + F_y$	$N = F_{g\perp}$

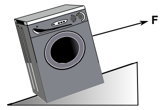
Different methods to calculate W$_g$

A 10 kg toy car is pulled 3 m up an inclined plane. The plane is at a 30° angle to the ground and the height is 1,5 m. Calculate the work done by gravity.



Method 1 Accoding to definition	Method 2 F$_g$ components	Method 3 F$_g$ conservative force
		
$W_g = F_g\Delta x\cos\theta$ $= 98(3)\cos(90° + 30°)$ $= 98(3)\cos(120°)$ $= -147{,}00J$	$W_g = W_{g\|\|} + W_{g\perp}$ $= F_{g\|\|}\Delta x\cos\theta + 0$ $= (98\sin 30°)(3)\cos 180°$ $= 49(3)\cos 180°$ $= -147{,}00J$	$W_{g\ routeA} = W_{g\ routeB}$ $W_g = W_{g(BI)} + W_{g(BII)}$ $= 0 + F_g(h)\cos 180°$ $= 98(1{,}5)\cos 180°$ $= -147{,}00J$

A man pulls a 50 kg-washing machine 3 m up an inclined plane by exerting a force of 2000 N parallel to the plane. The plane makes an angle of 40° with the horizon. The washing machine experiences 20 N frictional force.



a. Draw a free body-diagram of all the forces acting on the machine. (No components)

b. Calculate the work done by every force.

c. Use the previous answers to calculate the net work.

a. Draw a free body-diagram of all the forces on the machine. Use components of F_g.

b. Calculate the net force on the machine.

c. Use the F_{net} to calculate the net work.

The washing machine starts from rest. Use the work-energy principle to prove that after 3 m the magnitude of the velocity is 14,14 m·s^{-1}.

Calculate the average power of the man with $p = \frac{W}{\Delta t}$

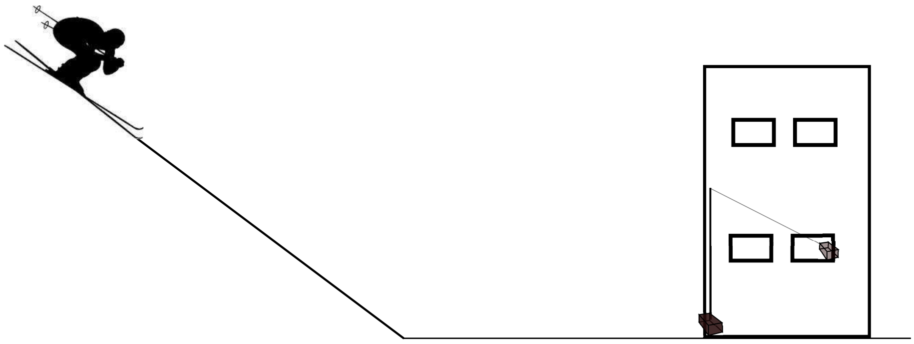
Calculate the average power of the man with $p_{ave} = Fv_{ave}$

(Most teachers prefer $P_{ave} = Fv_{ave}$ only for constant v.)

Closed system **No friction or applied force**	**Any system** **With or without friction**
Conservation of mechanical energy	**Work-energy principle**
$E_{mech(i)} = E_{mech(f)}$ $E_{pi} + E_{ki} = E_{pf} + E_{kf}$ $mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$ $gh_i + \frac{1}{2}v_i^2 = gh_f + \frac{1}{2}v_f^2$ Pendulums & free fall Inclined planes & curved planes v and g only magnitude (no sign)	Δx given $W_{net} = \Delta E_K$ $\underbrace{W_T + W_f + W_N + W_g}_{\substack{Every\ W = F\Delta x \cos\theta \\ No\ components}} = \frac{1}{2}m({v_f}^2 - {v_i}^2)$ $or \quad W_{net} = \Delta E_K$ $\underbrace{F_{net}\Delta x \cos\theta}_{Use\ components} = \frac{1}{2}m({v_f}^2 - {v_i}^2)$ v only magnitude (no sign)
Conservation of momentum	**Impulse-momentum principle**
Collisions and explosions NB: Directions!!! $\Sigma p_i = \Sigma p_f$ $p_{1i} + p_{2i} = p_{1f} + p_{2f}$ $m_1v_{i1} +_2 mv_{i2} = m_1v_f + m_2v_f$	Δt given NB: Directions!!! $F_{net}\Delta t = \Delta p$ $F_{net}\Delta t = p_f - p_i$ $F_{net}\Delta t = m(v_f - v_i)$
Sometimes Elastic collisions **(Conservation of kinetic energy)**	**Work-energy principle** **for non-conservative forces**
$\Sigma E_{k(i)} = \Sigma E_{k(f)}$ $E_{k1i} + E_{k2i} = E_{k1f} + E_{k2f}$ $\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$ v only magnitude (no sign) If collision is elastic: $\Sigma E_{k(i)} = \Sigma E_{k(f)}$ Is the collision elastic? Calculate $\Sigma E_{k(i)}$ and $\Sigma E_{k(f)}$ and compare	$W_{net} = \Delta E_K$ $W_{nc} = \Delta E_K + \Delta E_P$ $\underbrace{W_T + W_f + W_N}_{All\ W\ except\ W_g} = \frac{1}{2}m({v_f}^2 - {v_i}^2) + mg(h_f - h_i)$ v and g only magnitude (no sign) Inclined planes with no angle

The script of a new James Bond movie includes the following scenario:

James Bond (80 kg) starts from rest and skis down a 25 m slope with a villain at his heels. The slope makes an 38° angle with the ground and James experiences a frictional force of
10 N. At the bottom of the slope he covers a horizontal plane for 15 s and experiences a 15 N frictional force. It brings him to a parcel (1 kg) fixed to an inelastic rope. He grabs the parcel and swings up to the window on the second floor 5,2 m above the ground. He releases the parcel, breaks the window and escapes through the building. 5,4 $\times$ 10^5 J is required to break the window.



You are the technical advisor to the producer and must determine if the scenario is possible.