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Study Guide

Mathematics

Grade 10

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Chapter 3

Number patterns

Overview

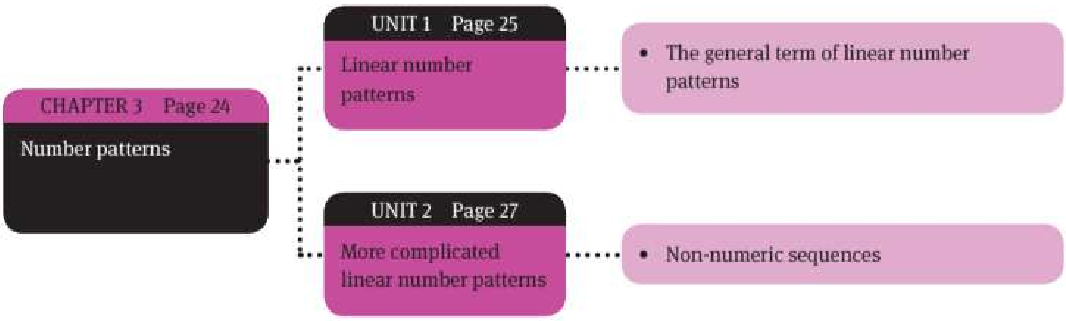
A number pattern is a list of numbers. The numbers are in a definite order, and follow a fixed rule, or pattern. Examples of number patterns are:

- Even numbers 2; 4; 6; 8; 10; 12; ...
- Odd numbers 1; 3; 5; 7; 9; 11; ...
- Square numbers 1; 4; 9; 16; 25; 36; 49; ...
- Cube numbers 1; 8; 27; 64; 125; 216; ...
- Fibonacci numbers 1; 1; 2; 3; 5; 8; 13; 21; 34; ...

In a number pattern:

- the numbers in the sequence are called terms
- each term has a position in the sequence.

A linear number pattern is a special kind of number pattern, in which the difference between the terms is always a fixed value.



Unit 1

Linear number patterns

1.1 The general term of linear number patterns

A linear number pattern has a constant difference between consecutive terms.

For example, consider the pattern: 2; 5; 8; 11; ... We obtain each term of the pattern by adding 3 to the previous term. Therefore, the difference between the second and first term is 3. The difference between the third and the second term is also 3, and so on.

We represent the first term in the pattern by the letter a. We represent the constant difference between terms as d. Therefore, in our previous example:

$a = 2$ (first term)
$d = 3$ (constant difference)

We define the first term (a) as T_1, the second term as T_2, the third term as T_3, and so on. In our example, It is relatively easy to find the sixth term in the pattern (17), but it is not as easy to find, for example the 1 000th term. To do so, we need to develop a rule, or formula.

The general formula to calculate the value of a term in a linear number pattern is:

$$T_n = a + (n - 1) \times d$$

Therefore, the 1 000th term is: $T_{1\,000} = 2 + (999) \times 3 = 2\,999$

Example

The general term (T_n) of a sequence is given by $T_n = 6n + 2$. Find the first four terms.	$T_1 = 6(1) + 2 = 8$ Replace n with "1" for Term 1 $T_2 = 6(2) + 2 = 14$ Replace n with "2" for Term 2 $T_3 = 6(3) + 2 = 20$ $T_4 = 6(4) + 2 = 26$ $\therefore$ sequence is 8; 14; 20; 26; ...
Given the sequence 3; 7; 11; 15 ... Determine the formula for T_n.	3 ; 7 ; 11 ; 15 ... +4 +4 +4 Constant difference $T_n = an + d$ a = constant difference Therefore: $T_n = 4n + d$ Substitute $n = 1$: $T_1 = 4(1) + d = 3$ $\therefore b = -1$ Therefore, $T_n = 4n - 1$

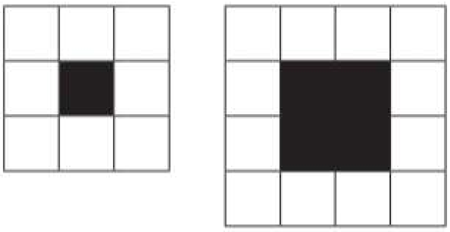
Given the sequence 10; 2; −6; −14; … Find T_n.	$10 \; ; \; 2 \; ; \; -6 \; ; \; -14$ $-8 \quad -8 \quad -8$ Constant difference $T_n = an + d$ a = constant difference Therefore: $T_n = -8n + d$ Substitute $n = 1$: $T_1 = -8(1) + d = 10$ $b = 18$ Therefore, $T_n = -8n + 18$

Unit 2

More complicated linear number patterns

2.1 Non-numeric sequences

Matches are used to build **squares** around a **central area (in black)**, as shown below. Complete the table that shows how wide the squares are, and how many matches are used in each square.



Number of matches wide	3	4	5	7	156	n
Number of matches used	24	36	48	?	?	?

There is a constant difference of 12 between matches used. Therefore:

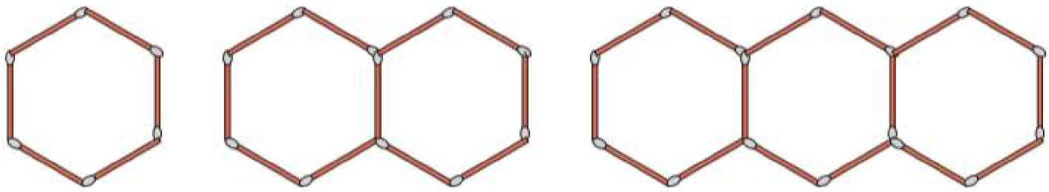
$24 = 12(1) + d$

$d = 12$

$T_n = 12n + 12 = 12(n + 1)$

Term 7 = 12(7) + 12 = 96 matches (if the square is 27 matches wide)
Term 156 = 12 (157) = 1 884 matches (if the square is 156 matches wide)

Rapunzel created a chain by forming hexagons with matches. She predicts that the number of matchsticks needed to form a chain of 20 hexagons will be 121.



To test this prediction, we need a rule:
H1 = 6 matches
H2 = 6 + 5 = 11 matches
H3 = 6 + 5 + 5 = 16 matches

Note that 6 is the starting value and then we 5 each time, one less than the number of hexagons. Therefore:

$$H_n = 6 + (n - 1) \times 5 = 5n + 1$$

Apply the rule for 20 hexagons: $5(20) + 1 = 101$
Therefore, the prediction is false.

Questions

1 Complete the following patterns by writing down the next two terms.

a 12 ; 19 ; 26 ; ...

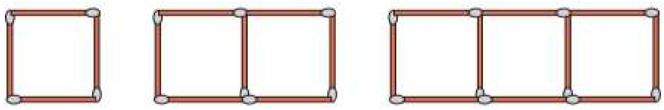
b $\frac{2}{3}; \frac{3}{5}; \frac{4}{7}; ...$

2 Determine the nth term and 16th term of the following patterns:

a 25; 22; 19; 16;...

b 1; 2; 4; 8; 16;...

3 Chains of squares can be built with matchsticks as shown below.



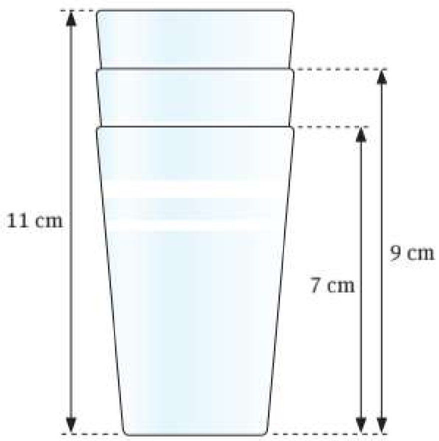
a How many matches are used to create a chain of 4 squares?

b How many matches are used to create a chain of 5 squares?

c What is the general formula for calculating how many matches there will be in a chain of squares?

d Now determine how many matches will be needed to build a chain of 100 squares.

4 You are stacking polystyrene cups, which fit into each other as in the diagram below. The first cup is 7 cm high. Two cups stacked together are 9 cm high, and three cups stacked together are 11 cm high.



a How high would a stack of four cups be?

b Write down the sequence of numbers in stacks with 1, 2, 3, 4, 5 and 6 cups.

c Find a general formula for the height of a stack of n cups.

d How many cups will stack on a shelf if the height between shelves is 22 cm?

e Can you build a stack of cups that has a height of 81 cm? If so, how many cups do you need?