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Study Guide

Mathematics

Grade 10

Via Afrika Publishers

Our Teachers. Our Future.

Chapter 11

Measurement

Overview

In this chapter, we focus on three-dimensional (3D) solids. The surface area of a 3D solid refers to the area of the outside surface of the solid. The volume refers to the amount of space inside the 3D solid.

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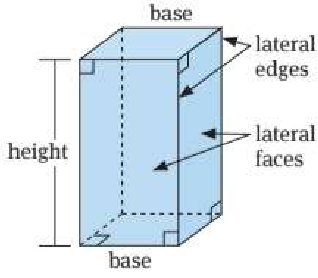
Measurement

- **UNIT 1 and 2 Page 111**
  Right prisms and cylinders
- **UNIT 3 Page 113**
  The volume and surface area of complex-shaped solids
  - A Sphere
  - A right circular cone
  - A pyramid

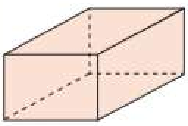
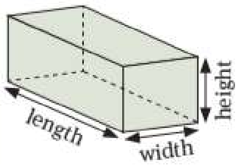
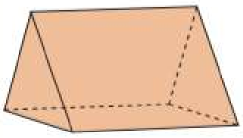
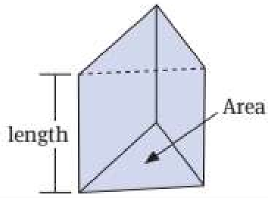
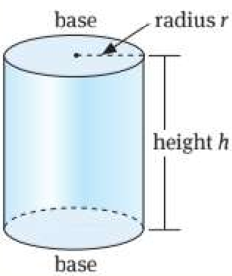
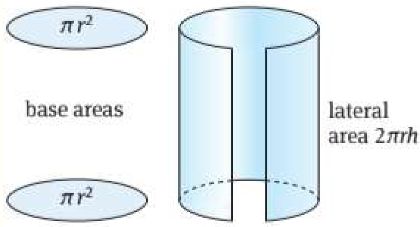
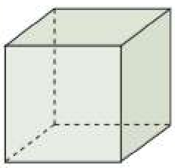
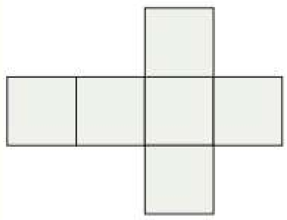
Unit 1 and 2

Right prisms and cylinders

- A prism is a polyhedron (solid) with two congruent faces, called bases, that lie in parallel planes.
- The other faces, called lateral faces, are parallelograms formed by connecting the corresponding vertices of the bases.
- The segments connecting these vertices are lateral edges.
- A prism can be cut into slices, which are all the same shape.

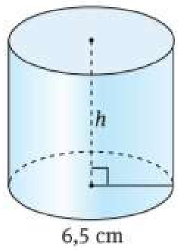


Right rectangular prism

Prism	Volume	Surface area
Rectangular prism 	length × breadth × height	$2lw + 2lh + 2wh$ 
Triangular prism 	$\frac{\text{base} \times \text{height}}{2} \times$ height of prism	$b.h + (S_1 + S_2 + S_3) \times h$ 
Cylinder 	$V = \pi r^2 \times h$	$2\pi rh + 2\pi r^2$ 
Cube 	$V = s^3$ s = side length of cube	Area of six cubes: Surface area = $6s^2$ 

Example

Find the height of a cylinder which has a radius of 6,5 cm and a surface area of 592,19 cm^2.



1 Make h the subject of the formula.
2 Substitute values into the formula.
3 Do not leave answer in terms of π, unless specifically asked to do so.
4 Do not forget the units.

$$\text{Surface area} = 2\pi rh + 2\pi r^2$$
$$2\pi rh = \text{SA} - 2\pi r^2$$
$$h = \frac{\text{SA} - 2\pi r^2}{2\pi r}$$
$$h = 592{,}19 - 2\pi(6{,}5)^2$$
$$h \approx 8 \text{ cm}$$

Example

A right circular prism (cylinder) has a volume of 50 units^3, a radius r and height h. If the radius is tripled and the height halved, what is the new volume of the cylinder?

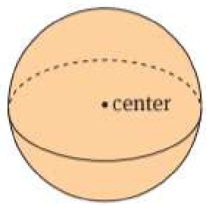
$$V_{\text{new}} = \pi(3r)^2\left(\frac{h}{2}\right)$$
$$= 9r^2\pi\left(\frac{h}{2}\right)$$
$$= \left(\frac{9}{2}\right)\pi r^2 h \qquad (\pi r^2 h = 50)$$
$$= \left(\frac{9}{2}\right) \times 50$$
$$= 225 \text{ units}^3$$

Unit 3

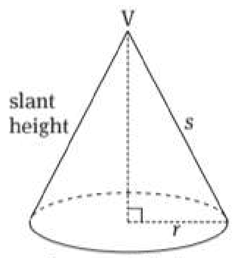
The volume and surface area of complex-shaped solids

3.1 A sphere

- Surface area = $4\pi r^2$
- $V = \frac{4}{3}\pi r^3$

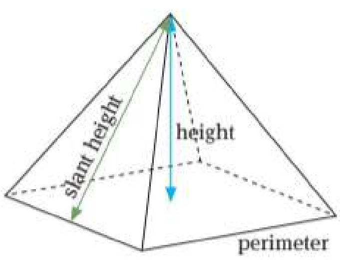


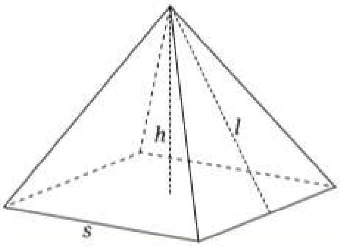
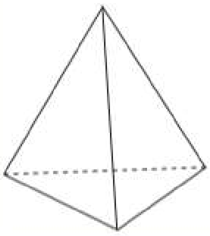
3.2 A right circular cone

A cone is simply a pyramid with a circular base.  Circumference of the base = $2\pi r$	$V = \frac{1}{3}\pi r^2 h$	$\pi rs + \pi rs^2$ • Area of the cone section + area of circle • Curved surface area (without the base) = πrs • s = slant height

3.3 A pyramid

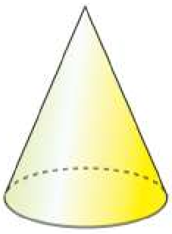
A pyramid is made by connecting a base to an apex.

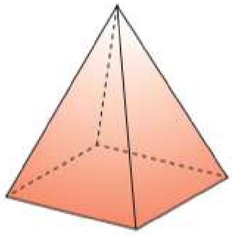


Rectangular base pyramid 	$V = \frac{1}{3}Ah$ A = Area of the base	Area of the base + $\frac{1}{2}$ perimeter of base × slant height
Triangular base pyramid (also called a tetrahedron) 	$V = \frac{1}{3}Ah$ A = Area of the base $V = \frac{1}{3}Ah$ $V = \frac{1}{3} \times \frac{1}{2} \times b \times h_{base} \times H_{pyramid}$ $V = \frac{1}{6}bh \times H$	4 × Area triangle: Area of regular triangle $= \frac{\sqrt{3}}{4} \times \text{side}^2$ $SA = 4 \times \frac{\sqrt{3}}{4} \times \text{side}^2$ $\therefore SA = \sqrt{3} \times \text{side}^2$

Example

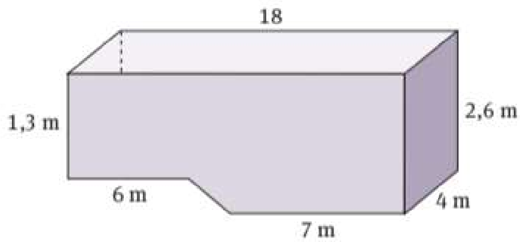
1 Consider two types of containers, each 15 cm deep. One is a rectangle pyramid, with a base of 4 cm by 7 cm, and the other is a cone with radius 3 cm. Determine which container holds more water when full, and by how much?





Volume of pyramid $= \frac{1}{3} \times 4 \times 7 \times 15 = 140 \text{ cm}^3$
Volume of cone $= \frac{1}{3} \times 3 \times 3 \times 15 \times \pi = 45\pi \text{ cm}^3$
Difference $= 45\pi \text{ cm}^3 - 140 \text{ cm}^3 = 1{,}37 \text{ cm}^3$
Therefore, the cone holds more water.

2 The sketch alongside shows the cross-section of a swimming pool. Determine the surface area and the volume of the pool. (Remember: the top is open!).



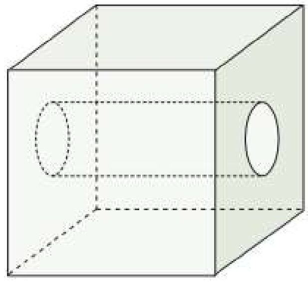
$$\text{Surface area} = (2{,}6 \times 4) + (1{,}3 \times 4) + (6 \times 4) + (7 \times 4) + (4 \times \sqrt{26{,}69}) + 2(1{,}3 \times 18) + 2 \times \frac{1{,}3}{2}(12 + 7)$$
$$= 159{,}76 \text{ m}^2$$
$$\text{Volume} = [(1{,}3 \times 18) + \frac{1{,}3}{2}(12 + 7)] \times 4$$
$$= 143 \text{ m}^3$$

Note: If you have a cone with no base, don't add the base area. There are different ways to calculate the surface area of a cone. Remember the basic concept: you add the cone's slanted area to the cone's base area.

Questions

1 A cube with a side length of 12 cm is filled to the top with water. The water is carefully poured into a rectangular prism with a length of 18 cm and a width of 8 cm. Calculate the height of the water in the rectangular prism.

2 A cylindrical hole has been drilled through the centre of a 10 cm solid cube (see figure alongside). The diameter of the cylindrical hole is 3 cm and its height is perpendicular to the two opposite faces of the cube. What is the total surface area of the cube (correct to two decimal places)?



3 A hollow sphere (e.g. a tennis ball) has an interior radius of 15 mm and an exterior radius of 20 mm. Calculate the volume of the material forming the sphere in cubic centimetres.

4 A metal top in the shape of a cone has a perpendicular height of 70 mm. If it displaces 4 224 cm^3 of water when fully immersed, calculate the total surface area.