1. Consider the language $Y = (w + c)(oul + oo)d$. List all words for language Y.

   Solution: $Y = \{\lambda, d, wd, cd, ould, ood, would, could, wood, cood\}$

2. Let $X = \{a, b\}$ and $Y = \{\lambda, ba, ab\}$

   (a) List the strings in the set XY.

   Solution: $XY = \{a, aba, aab, b, bba, bab\}$

   (b) List the strings in the set YX.

   Solution: $YX = \{a, b, baa, bab, aba, abb\}$

   (c) How many strings of length 4 are there in Y^*?

   Solution: 4

   (d) List the strings in the set Y^* of length 3 or less.

   Solution: $\{\lambda, ba, ab\}$

   (e) List the strings in the set X^*Y^* of length four or less.

   Solution: $\{\lambda, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb, aaaa, aaab,$
   $aaba, aabb, abaa, abab, abba, abbb, baaa, baab, baba, babb, bbaa, bbab, bbba, bbbb\}$

3. Consider the language S^*, where $S = \{aa, b\}$

   (a) How many words does this language have of length 2?

   Solution: 2

   (b) How many words does this language have of length 3?

   Solution: 3

4. For each of the following, write **two strings** IN the language, **two strings** that **NOT IN** the language and give a **short description** of the language using your own words.

   Solution:

Languages	2 valid strings (IN)	2 invalid strings (NOT IN)	Description
$ab(a+b)^*$	ab, aba	$babb, aab$	All strings in this language starts with ab, followed by a's and b's in any order.
$ab(a+b)^*ba$	$abba, abaaba$	$abaab, baba$	All strings in this language starts with ab and ends with ba.
$(a+b)b(a+b)^*$	$aba, bbaa$	$aaa, aaaa$	All strings in this language must contain at least one b.
$(a(a+b)^*b) +$ $(b(a+b)^*a)$	$abb, baba$	$abaa, bbb$	All strings in this language either starts with a and ends in b, or starts in b and ends in a.

5. Consider the regular expression $(a+b)^*a(a+b)$.

   (a) What string is NOT in this language?

   Solution: Strings that do not have an a in the second to last position is not in this language.

   (b) Write out all of the words in this language with 4 or fewer letters.

   Solution: $aa, ab, aaa, aab, baa, bab, aaaa, aaab, abaa, abab, baaa, baab, bbaa, bbab$

6. Generate all possible strings for each of the following regular expression (at least for 3 values of Kleene star * i.e: 0, 1, 2):

   (a) $a(a+b)^*$

   Solution: $\{a, aa, ab, aaa, aab, aba, abb, aaaa, aaab, aaba, aabb, abaa, abab, abba, abbb, \dots\}$

   (b) a^*b^*

   Solution: $\{\lambda, a, b, aa, ab, bb, aaa, aab, abb, aaaa, aabb, abbb, aaab, \dots\}$

   (c) $(ab)^*$

   Solution: $\{\lambda, ab, abab, ababab, abababab, \dots\}$

7. For the alphabet $\Sigma = \{a, b\}$, give a regular expression for the following languages:

(a) L_1 = All strings

Solution: $(a + b)^*$

(b) L_2 = All strings except empty string

Solution: $a(a + b)^* + b(a + b)^*$

(c) L_3 = All strings starting with ab.

Solution: $ab(a + b)^*$

(d) L_4 = All strings ending with ab.

Solution: $(a + b)^*ab$

(e) L_5 = All strings that begin AND end with ab.

Solution: $ab(a + b)^*ab$

(f) L_6 = All strings that begin OR end with ab

Solution: $ab(a + b)^* + (a + b)^*ab$

(g) L_7 = All strings that contain the substring ab.

Solution: $(a + b)^*ab(a + b)^*$

(h) L_8 = All strings that contain the substring ba

Solution: $(a + b)^*ba(a + b)^*$

(i) L_9 = All strings that contain the substring ab or ba

Solution: $(a + b)^*ab(a + b)^* + (a + b)^*ba(a + b)^*$

(j) L_{10} = All strings that contain the substring ab and ba

Solution: $(a + b)^*ab(a + b)^*ba(a + b)^*$

(k) L_{11} = All strings containing exactly two a's.

Solution: $(a + b)^*aa(a + b)^*$

(l) L_{12} = All strings containing at least two a's.

Solution: $(a + b)^*(aa)^*(a + b)^*$