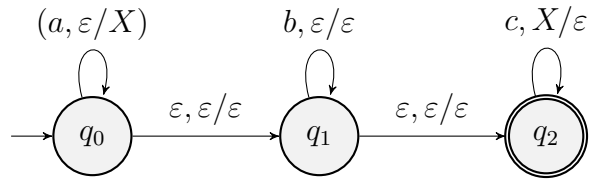


1. Let  $M$  be the PDA as follows.



- (a) Give the transition table of  $M$ .

*Solution:*

$\delta$	$a$	$b$	$c$	$\varepsilon$
$q_0$	$q_0, \varepsilon/X$	-	-	$q_1, \varepsilon/\varepsilon$
$q_1$	-	$q_1, \varepsilon/\varepsilon$	-	$q_2, \varepsilon/\varepsilon$
$q_2$	-	-	$q_2, X/\varepsilon$	-

- (b) Trace all computations of strings  $ac, abbc, aabcc$  in  $M$ .

*Solution:*

$$\begin{aligned}
 (q_0, ac, \$) &\vdash (q_0, c, X\$) \\
 &\vdash (q_1, c, X\$) \\
 &\vdash (q_2, c, X\$) \\
 &\vdash (q_2, \lambda, \$)
 \end{aligned}$$

$$\begin{aligned}
 (q_0, abbc, \$) &\vdash (q_0, bbc, X\$) \\
 &\vdash (q_1, bbc, X\$) \\
 &\vdash (q_1, bc, X\$) \\
 &\vdash (q_1, c, X\$) \\
 &\vdash (q_2, c, X\$) \\
 &\vdash (q_2, \lambda, \$)
 \end{aligned}$$

$$\begin{aligned}
 (q_0, aabcc, \$) &\vdash (q_0, abcc, X\$) \\
 &\vdash (q_0, bcc, XX\$) \\
 &\vdash (q_1, bcc, XX\$) \\
 &\vdash (q_1, cc, XX\$) \\
 &\vdash (q_2, cc, XX\$) \\
 &\vdash (q_2, c, X\$) \\
 &\vdash (q_2, \lambda, \$)
 \end{aligned}$$

- (c) Give a set-theoretic definition of the language accepted by  $M$ .

$$\text{Solution: } M = \{a^m b^n c^m \mid m, n \geq 0\}$$

- (d) Write a grammar for the corresponding PDA.

*Solution:*

$$\begin{aligned}
 S &\rightarrow aSc \mid B \mid \lambda \\
 B &\rightarrow bB \mid \lambda
 \end{aligned}$$

- (e) Write the shortest string that is IN and NOT IN the language  $L(M)$ .

*Solution:*

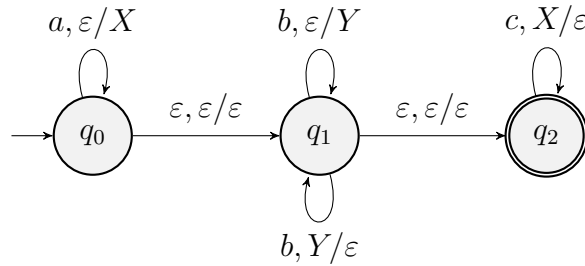
Shortest string IN  $L(M)$ :  $\lambda$

Shortest string NOT IN  $L(M)$ :  $a$

2. Given the language  $L(M) = \{a^m b^{2n} c^m \mid m, n \geq 0\}$

- (a) Design a pushdown automata (PDA) that accept the language  $L(M)$ .

*Solution:*



- (b) Trace the computation of the PDA that process the string  $abbc$  and  $abc$ .

*Solution:*

$(q_0, abbc, \$) \vdash (q_0, bbc, X\$)$   
 $\vdash (q_1, bbc, X\$)$   
 $\vdash (q_1, bc, YX\$)$   
 $\vdash (q_1, c, X\$)$   
 $\vdash (q_2, c, X\$)$   
 $\vdash (q_2, \lambda, \$)$

$(q_0, abc, \$) \vdash (q_0, bc, X\$)$   
 $\vdash (q_1, bc, X\$)$   
 $\vdash (q_1, c, YX\$)$   
 $\vdash (q_2, c, YX\$)$   
*Machine hangs...*

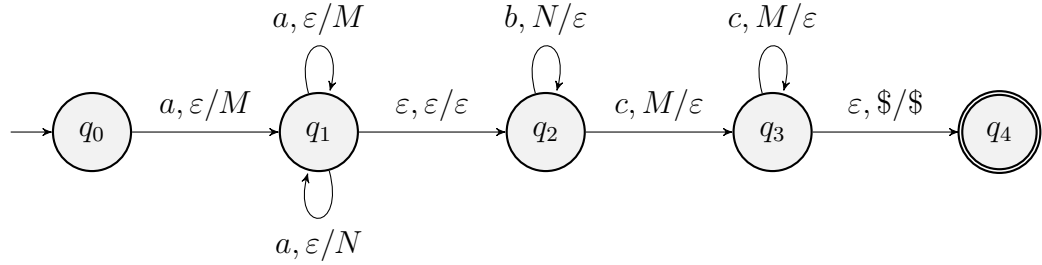
- (c) What is the shortest string in the language accepted by the PDA.

*Solution:*  $\lambda$

1. Design a PDA for the following languages

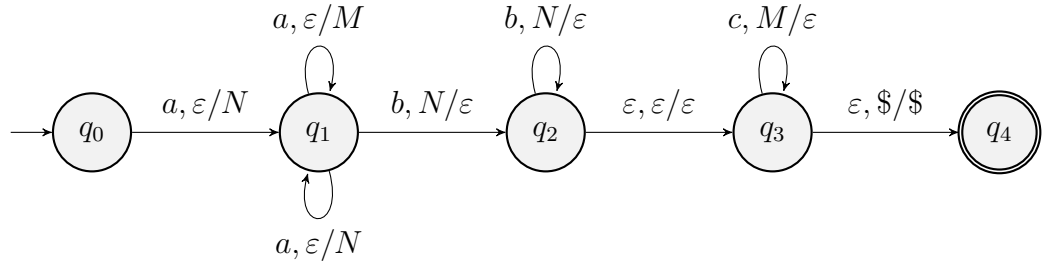
(a)  $\{a^{m+n}b^n c^m \mid m > 0, n \geq 0\}$

*Solution:*



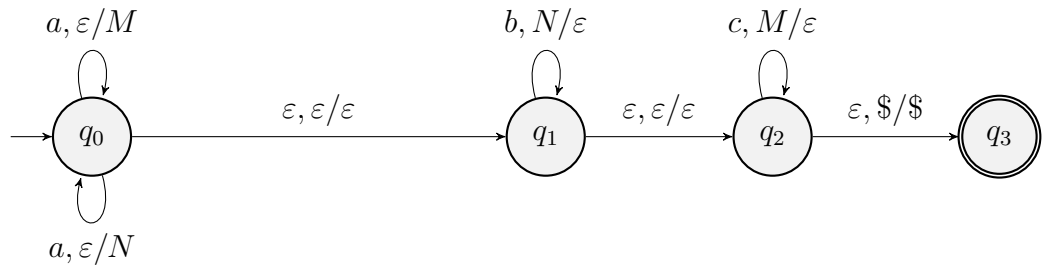
(b)  $\{a^{m+n}b^n c^m \mid m \geq 0, n > 0\}$

*Solution:*



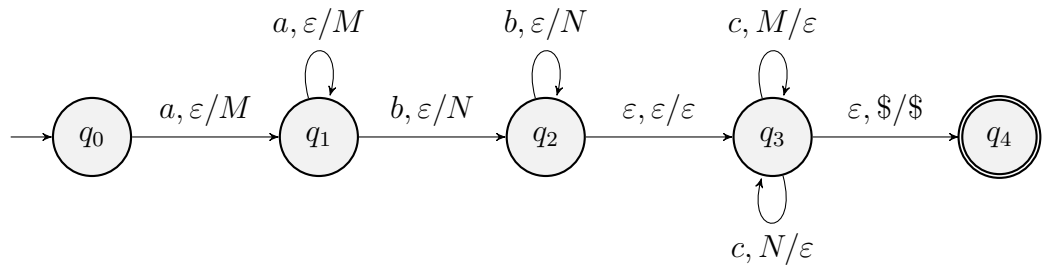
(c)  $\{a^{m+n}b^n c^m \mid m, n \geq 0\}$

*Solution:*



(d)  $\{a^m b^n c^{m+n} \mid m, n > 0\}$

*Solution:*



(e)  $\{a^m b^n c^{m+n} \mid m, n \geq 0\}$

*Solution:*

