1. Let M be the Turing machine defined by Table 1 where the start and final states are q_0 and q_2, respectively.

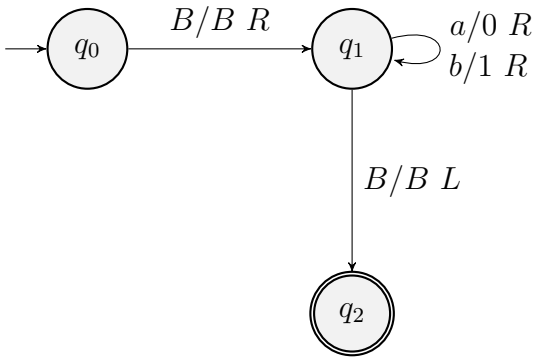
Table 1

δ	B	a	b
q_0	q_1, B, R	-	-
q_1	q_2, B, R	$q_1, 0, R$	$q_1, 1, R$
q_2	-	-	-

Note: B – blank symbol

(a) Give the state diagram of M.

Solution:



(b) Trace the computation for the input string *aabba*.

Solution:

$$\begin{aligned} q_0BaabbaB &\vdash Bq_1aabbaB \\ &\vdash B0q_1abbaB \\ &\vdash B00q_1bbaB \\ &\vdash B001q_1baB \\ &\vdash B0011q_1aB \\ &\vdash B00110q_1B \\ &\vdash B0011q_20B \end{aligned}$$

(c) Trace the computation for the input string *baba*.

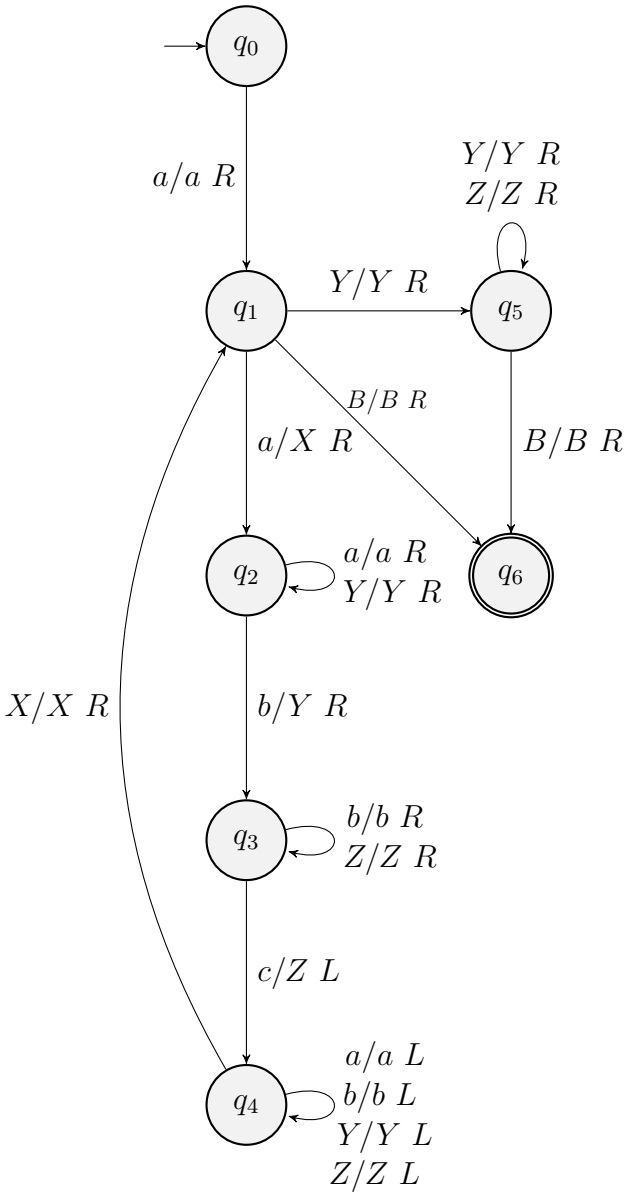
Solution:

$$\begin{aligned} q_0BbabaB &\vdash Bq_1babaB \\ &\vdash B1q_1abaB \\ &\vdash B10q_1baB \\ &\vdash B101q_1aB \\ &\vdash B1010q_1B \\ &\vdash B101q_20B \end{aligned}$$

(d) Describe the result of a computation in M.

Solution: The computation in M maps every a in the input string to 0, and every b to 1.

2. Let M be the Turing Machine diagram as follows:



(a) Give the transition table of M.

Solution:

δ	B	a	b	c	X	Y	Z
q_0	-	q_1, a, R	-	-	-	-	-
q_1	q_6, B, R	q_2, X, R	-	-	-	q_5, Y, R	-
q_2	-	q_2, a, R	q_3, Y, R	-	-	q_2, Y, R	-
q_3	-	-	q_3, b, R	q_4, Z, L	-	-	q_3, Z, R
q_4	-	q_4, a, L	q_4, b, L	-	q_1, X, R	q_4, Y, L	q_4, Z, L
q_5	q_6, B, R	-	-	-	-	q_5, Y, R	q_5, Z, R
q_6	-	-	-	-	-	-	-

(b) Trace the computations of M on input strings abc and $aabc$.

Solution: abc :

$$q_0abc \vdash aq_1bc$$

No transitions, machine halts...

$aabc$:

$$\begin{aligned} q_0aabc &\vdash aq_1abc \\ &\vdash aXq_2bc \\ &\vdash aXYq_3c \\ &\vdash aXq_4YZ \\ &\vdash aq_4XYZ \\ &\vdash aXq_1YZ \\ &\vdash aXYq_5Z \\ &\vdash aXYZq_5B \\ &\vdash aXYZBq_6B \end{aligned}$$

(c) Give a set-theoretic definition to define the language.

Solution: $L(M) = \{a^{n+1}b^nc^n \mid n \geq 0\}$

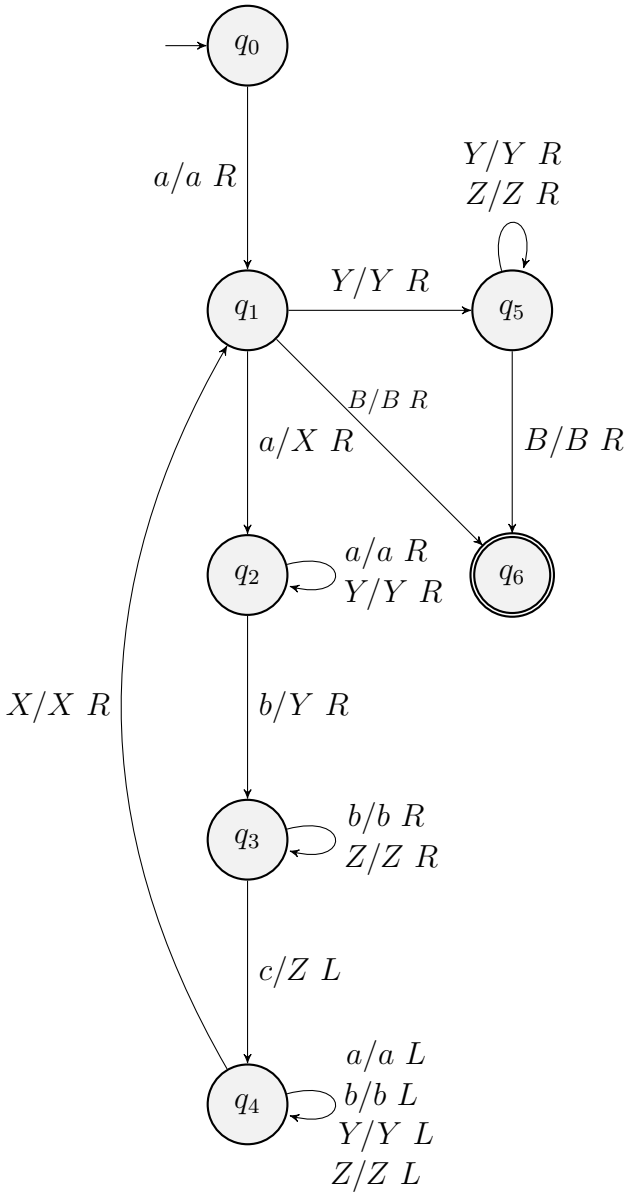
1. Given the languages, L as follows in Table 1. For each languages answer questions (a) - (b):

Table 1

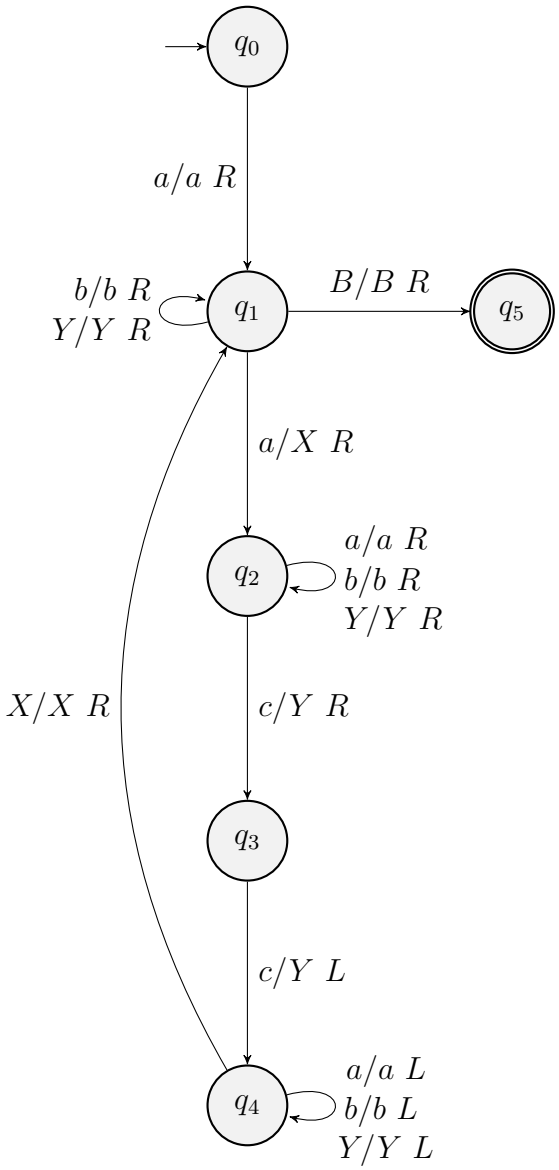
Languages
i. $L = \{a^{m+1}b^m c^n \mid m = n \text{ and } m, n \geq 0\}$
ii. $L = \{a^{m+1}b^n c^{2m} \mid m, n \geq 0\}$

(a) Design the Turing Machine that accept the language $L(M)$.

Solution:



$L = \{a^{m+1}b^m c^n \mid m = n \text{ and } m, n \geq 0\}$



$$L = \{a^{m+1}b^n c^{2m} \mid m, n \geq 0\}$$

(b) What is the shortest string in the language accepted by the Turing Machine.

Solution:
Shortest string accepted by Language 1: a

Shortest string accepted by Language 2: a