- 1. (a) M1: $\{q_0\}$ M2: $\{q_0\}$
 - (b) M1: $\{q_1\}$ M2: $\{q_0, q_3\}$

(c) M1:
$$[q_0, aabb] \vdash [q_1, abb]$$

 $\vdash [q_2, bb]$
 $\vdash [q_0, b]$
 $\vdash [q_0, \lambda]$

Sequence of states: $q_0 \rightarrow q_1 \rightarrow q_2 \rightarrow q_0 \rightarrow q_0$

M2:
$$[q_0, aabb] \vdash [q_0, abb]$$

 $\vdash [q_0, bb]$
 $\vdash [q_1, b]$
 $\vdash [q_3, \lambda]$

Sequence of states: $q_0 \rightarrow q_0 \rightarrow q_0 \rightarrow q_1 \rightarrow q_3$

- (d) M1 does not accept the string aabb but M2 does.
- (e) M1 does not accept ε but M2 does.
- (f) M1 = $(\{q_0, q_1, q_2\}, \{a, b\}, \delta, q_0, q_1)$ where δ is described as:

$$\begin{array}{c|ccc} & a & b \\ \hline q_0 & q_1 & q_0 \\ q_1 & q_2 & q_2 \\ q_2 & q_1 & q_0 \\ \end{array}$$

 $M2 = (\{q_0, q_1, q_2, q_3\}), \{a, b\}, \delta, q_0, \{q_0, q_3\})$ where δ is described as:

$$\begin{array}{c|cccc} & a & b \\ \hline q_0 & q_0 & q_1 \\ q_1 & q_2 & q_3 \\ q_2 & q_1 & q_0 \\ q_3 & q_2 & q_3 \end{array}$$

 $2. \quad (a)$

$$\begin{array}{c}
a, b \\
\hline
q_0 \\
a, b
\end{array}$$

(b)
$$abbb$$
: $[q_0, abbb] \vdash [q_1, bbb]$
 $\vdash [q_0, bb]$
 $\vdash [q_1, b]$
 $\vdash [q_0, \lambda]$

All alphabets are traced and the machine stops at q_0 , which is an accepting state. Thus, this machine accepts the string abbb.

bbb:
$$[q_0, bbb] \vdash [q_1, bb]$$

 $\vdash [q_0, b]$
 $\vdash [q_1, \lambda]$

All alphabets are traced and the machine stops at q_1 , which is not an accepting state. Thus, this machine rejects the string bbb.

baa:
$$[q_0, baa] \vdash [q_1, aa]$$

 $\vdash [q_0, a]$
 $\vdash [q_1, \lambda]$

All alphabets are traced and the machine stops at q_1 , which is not an accepting state. Thus, this machine rejects the string baa.

$$\begin{aligned} \pmb{baab} \colon & [q_0, baab] \vdash [q_1, aab] \\ & \vdash [q_0, ab] \\ & \vdash [q_1, b] \\ & \vdash [q_0, \lambda] \end{aligned}$$

All alphabets are traced and the machine stops at q_0 , which is an accepting state. Thus, this machine accepts the string baab.

- (c) The strings abbb and baab are accepted by the machine M.
- (d) $((a+b)(a+b))^*$

1. (a)
$$\begin{array}{c|cccc}
 & a & b \\
\hline
q_0 & q_1 & q_0 \\
q_1 & q_2 & q_0 \\
q_2 & q_3 & q_0 \\
q_3 & q_3 & q_3
\end{array}$$

M is a deterministic FA.

(b)
$$\boldsymbol{aaa}$$
: $[q_0, aaa] \vdash [q_1, aa] \vdash [q_2, a] \vdash [q_3, \lambda]$

All alphabets are traced and the machine stops at q_3 , which is an accepting state. Thus, this machine accepts the string aaa.

$$aab$$
: $[q_0, aab] \vdash [q_1, ab]$
 $\vdash [q_2, b]$
 $\vdash [q_0, \lambda]$

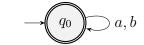
All alphabets are traced and the machine stops at q_0 , which is not an accepting state. Thus, this machine rejects the string aab.

baaab:
$$[q_0, baaab] \vdash [q_0, aaab]$$

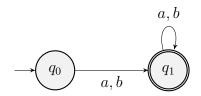
 $\vdash [q_1, aab]$
 $\vdash [q_2, ab]$
 $\vdash [q_3, b]$
 $\vdash [q_3, \lambda]$

All alphabets are traced and the machine stops at q_3 , which is an accepting state. Thus, this machine rejects the string baaab.

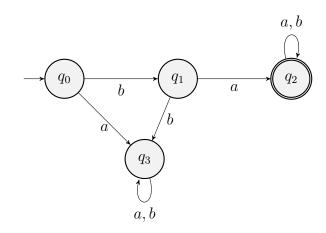
- (c) The strings aaa and baaab are accepted by M.
- (d) $(a+b)^*aaa(a+b)^*$
- 2. (a) $L_1 = (a+b)^*$



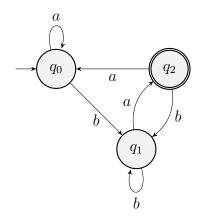
(b) $L_2 = (a+b)(a+b)^*$



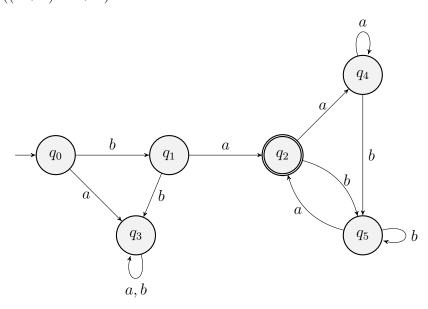
(c) $L_3 = ba(a+b)^*$



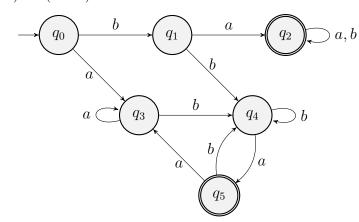
(d) $L_4 = (a+b)^*ba$



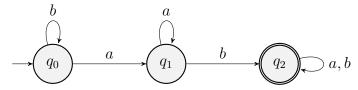
(e) $L_5 = ba((a+b)^*ba + \lambda)$



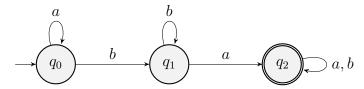
(f) $L_6 = ba(a+b)^* + (a+b)^*ba$



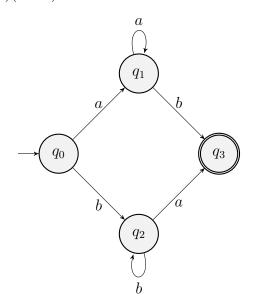
(g) $L_7 = (a+b)^*ab(a+b)^*$



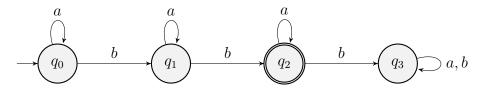
(h) $L_8 = (a+b)^*ba(a+b)^*$



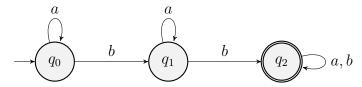
(i) $L_9 = (a+b)^*(ab+ba)(a+b)^*$



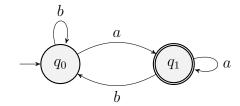
(j) $L_{10} = a^*ba^*ba^*$



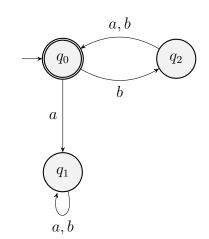
(k)
$$L_{11} = (a+b)^*b(a+b)^*b(a+b)^*$$



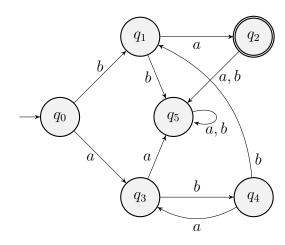
3. (a) $L = (a+b)^*a$



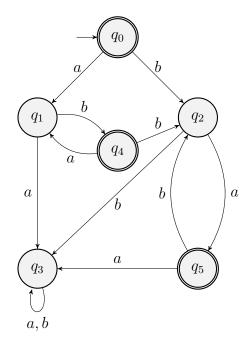
(b) $L = (bb + ba)^*$



(c) $L = (ab)^*ba$



(d) $L = (ab)^*(ba)^*$



(e) $L = (ab^*a)^*$

