CS 3186 --- Assignment #2

***Remember to write it out instead of typing so you can get practice and muscle memory**

$\overline{L}$= L^C

I) Give a formal definition with any notations for each of the following: Alphabet, String, Language, Concatenation of strings, Reverse of a string, Substring, Length of a string, Star-Closure of an alphabet, Positive Closure of an alphabet, Sentence of a language

Alphabet - An alphabet Σ is a finite, nonempty set of symbols

String - a finite sequence of symbols from an alphabet

Language - a set of strings

Concatenation of strings - The concatenation of strings u and v means appending the symbols of v to the right end of the symbols of u, denoted as uv

Reverse of a string - The reverse of a string (denoted as w) is denoted as w^R. w^R is the string with the same symbols in reverse order

Substring - Substring is a sequence of consecutive characters taken from the original string to make a new string

Star-Closure of an alphabet - Σ^* for an alphabet Σ, is the set of all strings obtained by concatenating zero or more symbols from the alphabet

Positive-Closure of an alphabet - Σ^+, for an alphabet Σ, is the set of all strings from the alphabet Σ except (Lambda)

Sentence of a language - A string from a language is referred to as a sentence. For Language L: {a, aa, ab}, “a”, “aa”, “ab” are sentences

II) For a language L, describe the Complementation, and Star-Closure

Complementation - $L^C = \Sigma^* - L$

Star-Closure - $L^* = L^0 \cup L^1 \cup L^2 \cup ...$

III) Describe the relationship of Language, Grammar and Automata (over a given alphabet)

A language is a set of strings; a grammar is a set of rules used to define which symbols of an alphabet can be sequenced into strings of a language; and an automata is a mathematical model of a computer which can determine if a string is a part of a language.

IV) Write derivations for four strings of various lengths and describe what is the language generated, L(G) by the following grammar G.

V = {S, A, B}
T = {a, b},
P = {

S → A,
S → B,
B → bB,
A → aA,
A → λ,
B → λ

}
S is the start nonterminal

$V = \{S, A, B\}$

$T = \{a, b\}$

$S \to A \mid B$

$B \to bB \mid \lambda$

$A \to aA \mid \lambda$

1) $S \to A \to aA \to aaA \to aa\lambda \to aa$

2) $S \to A \to \lambda$

3) $S \to B \to bB \to b\lambda$

4) $S \to B \to bB \to bbB \to bbbB \to bbbbB \to bbbb\lambda$

$L(G) = \{a^n \mid b^n : n \geq 0\}$

V) Write derivations for four strings of various lengths and describe what is the language generated, L(G) by the following grammar
S → aSaa | B
B → bB | λ

$S \to aSaa \mid B$
$B \to bB \mid \lambda$
1) $S \to aSaa \to aBaa \to a\lambda aa$
2) $S \to aSaa \to aaSaaaa \to aaaSaaaaaa \to aaaBaaaaaa \to aaa\lambda aaaaaa$
3) $S \to B \to \lambda$
4) $S \to B \to bB \to bbB \to bb\lambda$

$L(G) = \{a^n b^m a^{2n} : n, m \geq 0\}$

VI) Write derivations for four strings of various lengths and describe what is the language generated, L(G) by the following grammar
S → aSaa | B
B → bB

$S \to aSaa \mid B$
$B \to bB$
1) $S \to aSaa \to aaSaaaa \to aaBaaaa \to aabBaaaa \to aabbBaaaa$ ✗ $\overset{\infty}{\text{loop}}$
2) $S \to B \to bB \to bbB \to bbbB$ ✗ $\overset{\infty}{\text{loop}}$

VII) Describe the operation of Automaton as an Acceptor

An automaton as an acceptor operates by taking an input then giving either a yes or no output.

VIII) Let Σ = {a.b} L1 = {a,ab,abb) L2 = {λ, b, bb} Describe all the following languages as a set of strings.

(i) $L_3 = L_1 \cap L_2$

$L_3 = L_1 \cap L_2 = \phi$

(ii) L_1L_3

$L_1L_3 = \phi$

$* \phi \simeq null$

$* L_1\{\lambda\} = L_1$

(iii) L_3L_1

$L_3L_1 = \phi$

(iv) L_1L_2

$L_1L_2 = \{a, ab, abb, abbb, abbbb\}$

(v) $L_1Ø$

$L_1\phi = \phi$

(vi) $|L_1|\ |L_3|$

$|L_1||L_3| = 3 \cdot 0 = 0$

(vii) $|L_1|\ |L_2|$

$$|L_1||L_2| = 3 \cdot 3 = 9$$

(viii) $|L_1L_2|$

$$|L_1L_2| = 5$$

(ix) L_1^R

$$L_1^R = \{a, ba, bba\}$$

(x) L_2^R

$$L_2^R = \{\lambda, b, b\}$$

(xi) $\sum^*$

$$\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, \ldots$$

(xii) L_2^C

$$\overline{L_2} = \{a, aa, ab, ba, aaa, aab, \ldots$$

(xiii) L_1^0

$$L_1^0 = \{\lambda\}$$

(xiv) $L_1{}^1$

$$L_1{}^1 = \{a, ab, aab\}$$

(xv) $L_1{}^2$

$$L_1{}^2 = \{a, ab, abb\}\{a, ab, abb\}$$
$$= \{aa, aab, aabb,$$
$$aba, abab, ababb,$$
$$abba, abbab, abbabb\}$$

(xvi) $L_2{}^2$

$$L_2{}^2 = \{\lambda, b, bb\}\{\lambda, b, bb\}$$
$$= \{\lambda, b, bb,$$
$$bbb,$$
$$bbbb\}$$