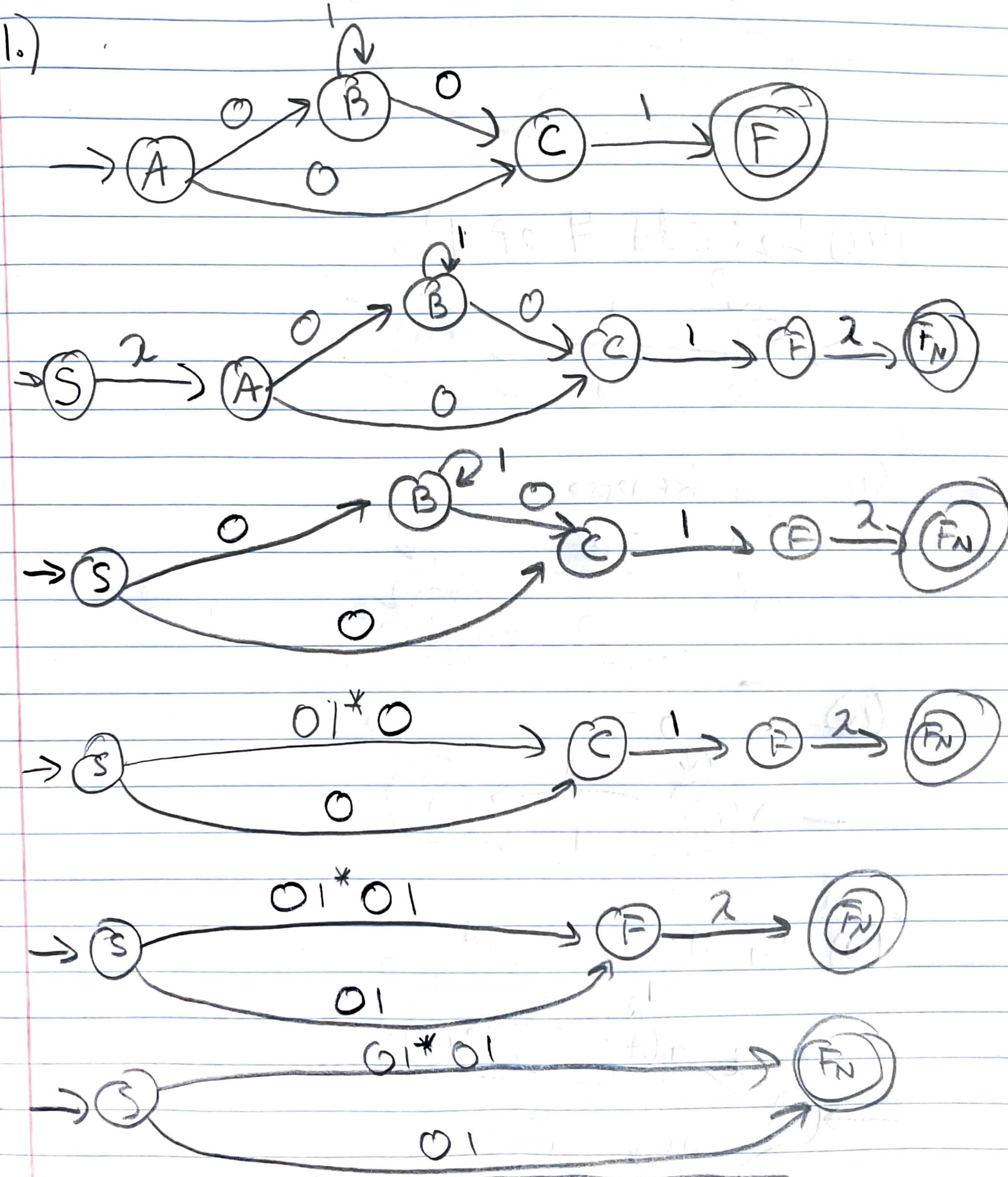
Nshan Kazaryan 5/19/22

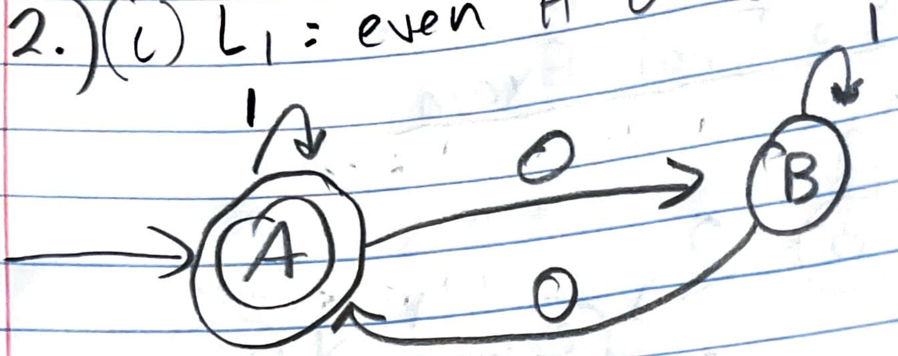
CS 3186 Final Exam

1.)

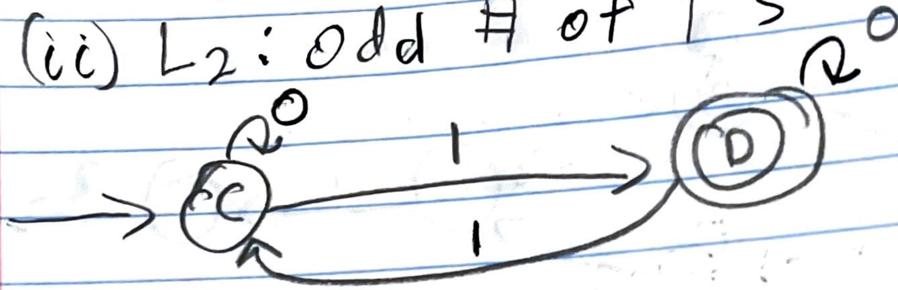


Regular expression: $(01^*0+0)1$

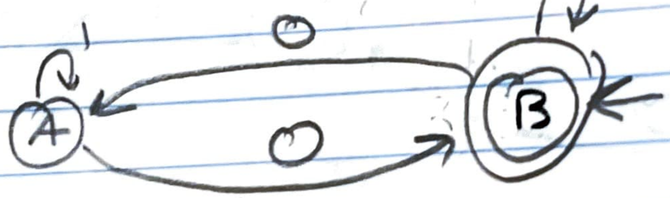
2.) (i) L_1 : even # of 0's



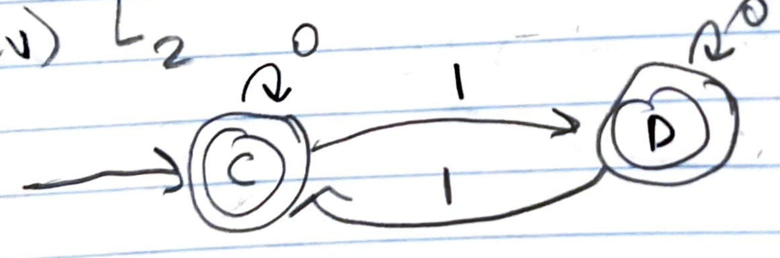
(ii) L_2 : Odd # of 1's



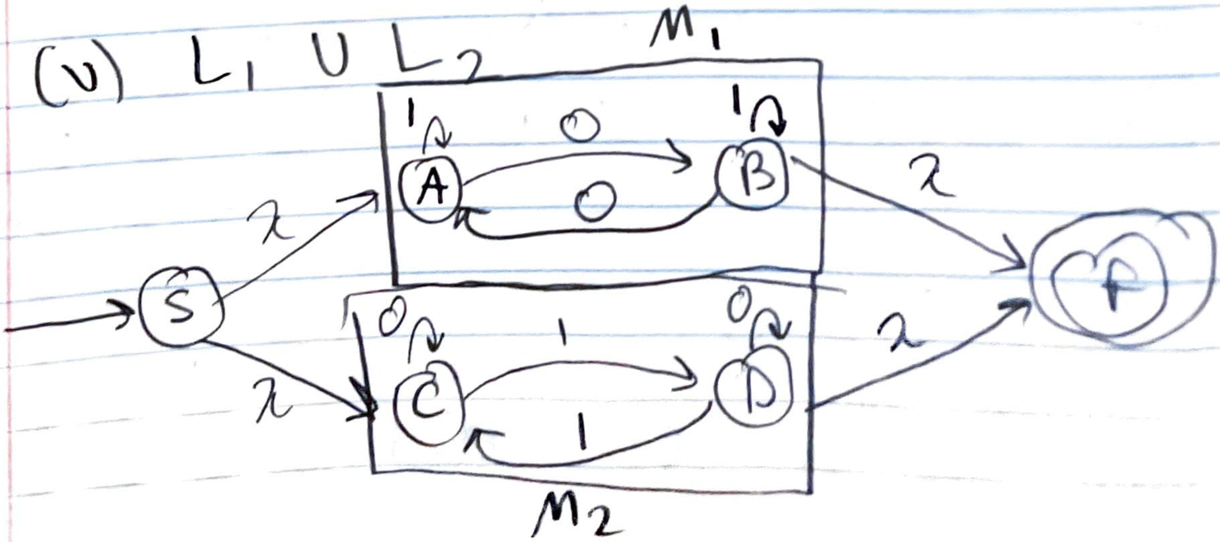
(iii.) L_1 Reverse



(iv) $\overline{L_2}$



(v) $L_1 \cup L_2$



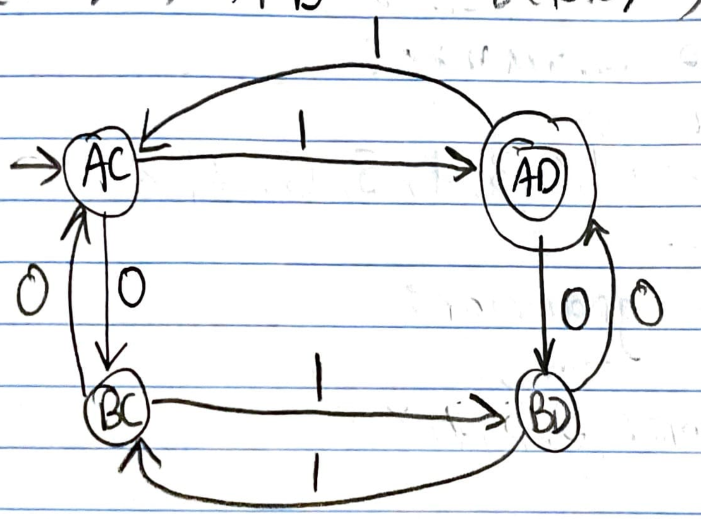
(vi) $L_1 \cap L_2$

$\delta(AC, 0) = BC$ $\delta(AC, 1) = AD$

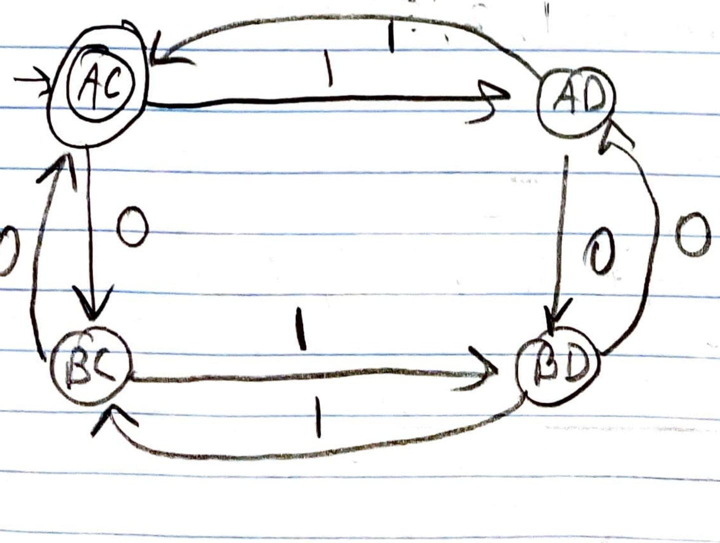
$\delta(AD, 0) = BD$ $\delta(AD, 1) = AC$

$\delta(BC, 0) = AC$ $\delta(BC, 1) = BD$

$\delta(BD, 0) = AD$ $\delta(BD, 1) = BC$



(vii) $L_1 - L_2 = L_1 \cap \overline{L_2}$



3.) 1. $S \to 0A$
2. $S \to 1A$
3. $S \to A0$
4. $S \to AB$
5. $S \to 0$
6. $S \to 0A1$
7. $A \to 1$
8. $S \to \lambda$

(i) Context-free grammar:

Production rules: 1, 2, 3, 4, 5, 6, 7, 8

(ii) Left-linear grammar:

Production rules: 3, 5, 7, 8

(iii) Right Linear regular grammar:

Production rules: 1, 2, 5, 7, 8

(iv) Linear grammar:

Production rules: 1, 2, 3, 5, 7, 8.

4.) Use pumping lemma to show $L = \{a^m b^m c^{m+1}\}$

Given L is an infinite language, assume L is context free

There exists a PDA with "n" of production rules * # of variables

Choose $w = a^n b^n c^{n+1}$

$|w| = 3n+1 \geq n$ (as desired)

$w = a^n b^n c^{n+1} = uvxyz$

Since $|vxy| \leq n$, leads to many cases

Case 1: vxy is within a's

For $i=0$; you have less # of a's than b's (Since $|vy| \geq 1$)

Case 2: vxy is within b's

For $i=0$; you have less # of b's than a's (Since $|vy| \geq 1$)

Case 3: vxy is within c's

For $i=0$; you have same or less # of c's than a's and b's (Since $|vy| \geq 1$)

Case 4: vxy spans a's and b's

For $i=0$; you have at least less than 2 a's or b's than c's. (Since $|vy| \geq 1$).

Case 5: vxy spans b's and c's

For $i=0$: you have less # of b's or same # of c's than a's (Since $|vy| \geq 1$)

Example I used $i=0$; $uxz \in L$

All the cases lead to a contradiction

Hence, our assumption that L is context free is not true. L is not a CFL.

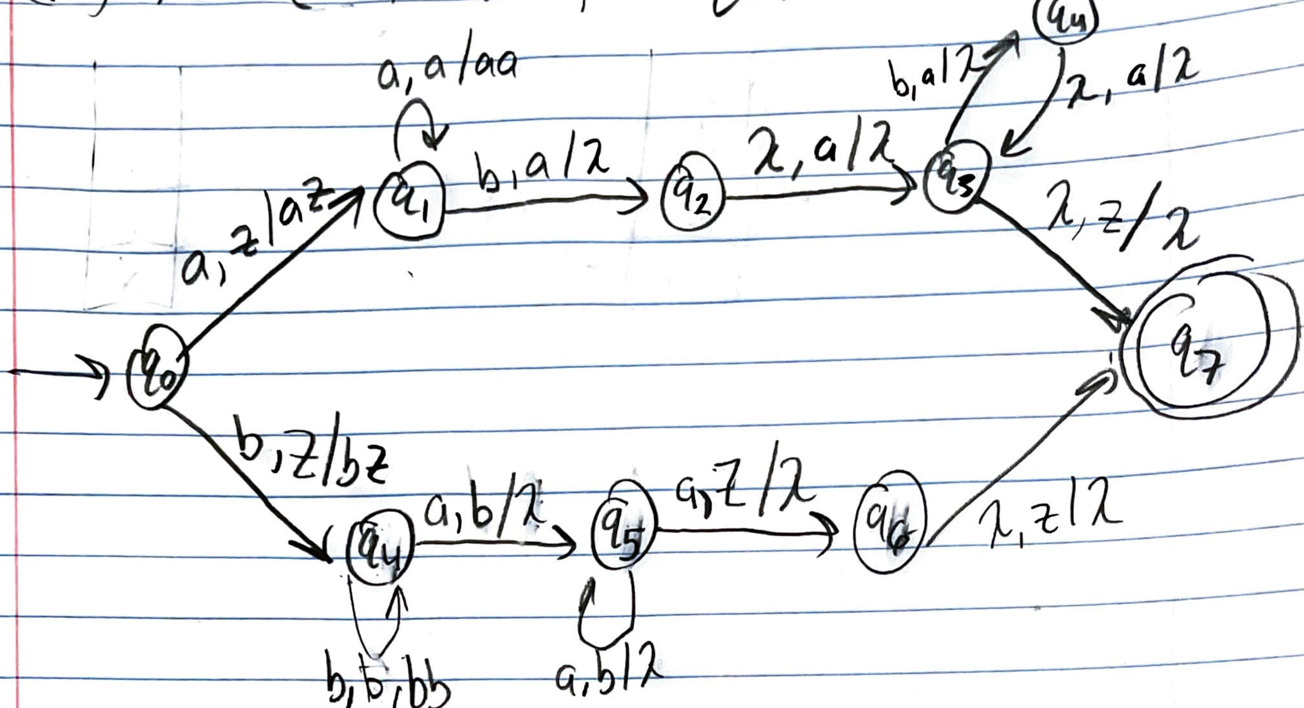
5.) $L = \{a^{2n} b^n \mid n \geq 1\} \cup \{b^{n+1} a^n \mid n \geq 1\}$

(i) Algorithm:

If the first letter is a, then:
When a, if top is Z or a, then push a
when b, if top is a, then pop two a's

If the first letter is b, then:
When b, if top is Z or b, push a b
When a, if top is b, pop, pop $(n+1)$ bs
When a, if top is Z, do nothing

(ii) $M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$



Complete 7-tuple: $(\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}, \{a, b\}, \{z, a, b\}, \delta, q_0, z, \{q_7\})$

(iii.) Show bbba leads to rejection

$\delta(q_0, bbba, z) \vdash \delta(q_4, bba, bz) \vdash \delta(q_4, ba, bbz)$

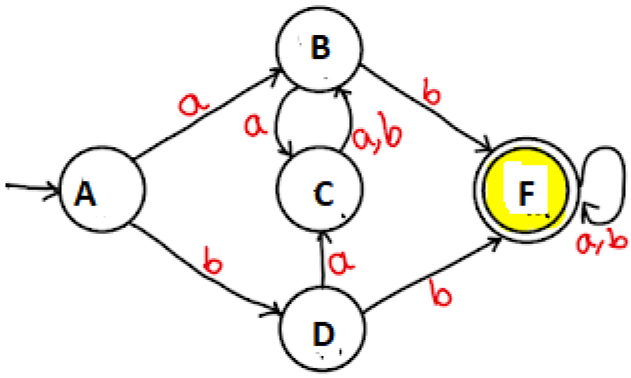
$\vdash \delta(q_4, a, bbbz) \vdash \delta(q_5, \lambda, bbz)$

No transistion defined leads to rejection

Final

Name ____________________

1. (15 points) Minimize following DFA:



2. (30 points)

(i) Give a DFA, M_1, that accepts a Language L_1 that contains even number of 0's. (Hint: only 2 states)
(ii) Give a DFA, M_2, that accepts a Language L_2 that contains even number of 1's.
(iii) Give acceptor for Reverse of L_1
(iv) Give acceptor for complement of L_2
(v) Give acceptor for L_1 union L_2
(vi) Give acceptor for L_1 intersection L_2
(vii) Give acceptor for L_1 - L_2

3. (15 points) Given the following grammar with production rules numbered from 1 to 8,

$$\{ \ 1. S \longrightarrow 0A,$$
$$2. S \longrightarrow 1A,$$
$$3. S \longrightarrow A0,$$
$$4. S \longrightarrow AB,$$
$$5. S \longrightarrow 0,$$
$$6. S \longrightarrow 0A1,$$
$$7. A \longrightarrow 1,$$
$$8. S \rightarrow \lambda \ \}$$

(i) Indicate which of the rules satisfy the conditions for a context-free grammar (Sample answer: Production rules 1,5,8)

(ii) Indicate which of the rules satisfy the conditions for a left linear grammar
(iii) Indicate which of the rules satisfy the conditions for a right linear regular grammar
(iv) Indicate which of the rules satisfy the conditions for a linear grammar

4. (20 points) Use pumping lemma to show that the language L = $\{a^{m+1}b^m c^m\}$ is not context-free.

5. (20 points) Create a PDA that recognizes the following context free language with terminals $\{a, b\}$: L = $\{a^{n+1}b^n | n \geq 1\} \cup \{b^n a^{n+2} | n \geq 1\}$.
(i) Describe your algorithm
(ii) Give the description as a complete 7-tuple with a transition diagram
(iii) Show configuration sequences on **aaabb** leading to acceptance.