CS 3186 --- Assignment #14

(I) State the Pumping Lemma for regular languages and the Pumping lemma for context-free languages.

The pumping lemma proves the language is not regular using:
w = xyz
|xy| ≤ m
|y| ≥ 1

The pumping lemma proves the language is not context free using:
w = uvxyz
|vxy| ≤ m
|vy| ≥ 1

(II) Given a CFL L_1 described by grammar
G_1: $S_1 \rightarrow aS_1b \mid \lambda$
CFL L_2 described by
G_2: $S_2 \rightarrow cS_2d \mid \lambda$

(i) Show that $L_1 \cup L_2$ is context free by constructing a complete grammar.

$G_1 \Rightarrow L_1 \Rightarrow \{a^x b^x\}$ [same amount of a's and b's]
$G_2 \Rightarrow L_2 \Rightarrow \{c^y d^y\}$ [same amount of c's and d's]
$L_1 \cup L_2 = \{a^x b^x\} \cup \{c^y d^y\} \Rightarrow S_1 \mid S_2$

(ii) Using this grammar derive a string that belongs to L_1

$L_1 \Rightarrow S_1 \Rightarrow aS_1b \Rightarrow aaS_1bb \Rightarrow aa\lambda bb \Rightarrow aabb$

(iii) Using this grammar derive a string that belongs to L_2

$L_2 => S_2 => cS_2d => ccS_2dd => cc\lambda dd => ccdd$

(III) Consider the grammars G_1 and G_2 above. Show that $L_1 L_2$ is context free by constructing a complete grammar.

$L_1L_2 = a^xb^xc^yd^y => S_1S_2$

(i) Derive any string w1 that belongs to L_1 and any string w_2 that belongs to L_2.

$w_1 => S_1 => aS_1b => a\lambda b => ab$

$w_2 => S_2 => cS_2d => c\lambda d => cd$

(ii) Show that $w_1 w_2$ that belongs to $L_1 L_2$

$w_1w_2 => S_1S_2 => aS_1bS_2 => a\lambda bS_2 => abcS_2d => abc\lambda d => abcd => L_1L_2$

(IV) Consider the grammars G_1 above. Show that L_1^* is context free by constructing a complete grammar.

$L_1^* => S_1 -> SS_1 \mid \lambda$

(V) Name two closure properties that are true for regular languages that are not necessarily true for CFL

- Intersection
- Complementation
- Set Difference