(1) Define a PDA as a 7-tuple and describe each of the components.

$PDA = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$

Q : set of states

Σ : input alphabet

Γ : stack alphabet

δ : transition function

q_0 : initial state

z : stack start symbol

F : set of final states

(2) Create a PDA that recognizes the following context free language with terminals {a,b}

L = {w | number of a's = twice the number of b's; String w can only have a's followed by b's or b's followed by a's}

i.e., it should accept aab, aaaabb, baa, bbaaaa,.. and so on.

(i) Describe your algorithm

(ii) Give the description as a complete 7-tuple with a transition diagram

(iii) Show configuration sequences on **aaaabb** leading to acceptance.

(Note that this is an easier problem than simply saying that
L = {w | number of a's = twice the number of b's}
Then we need to account for strings like aba, abbaaa,.. which complicates the logic.

i) Algorithm:

If first letter is a, then:

when a:

If top is z or a, then push a

when b:

If top is a, then pop two a's

If first letter is b, then:

when b:

If top is z or a then push two b's

when a:

If top is b then pop b

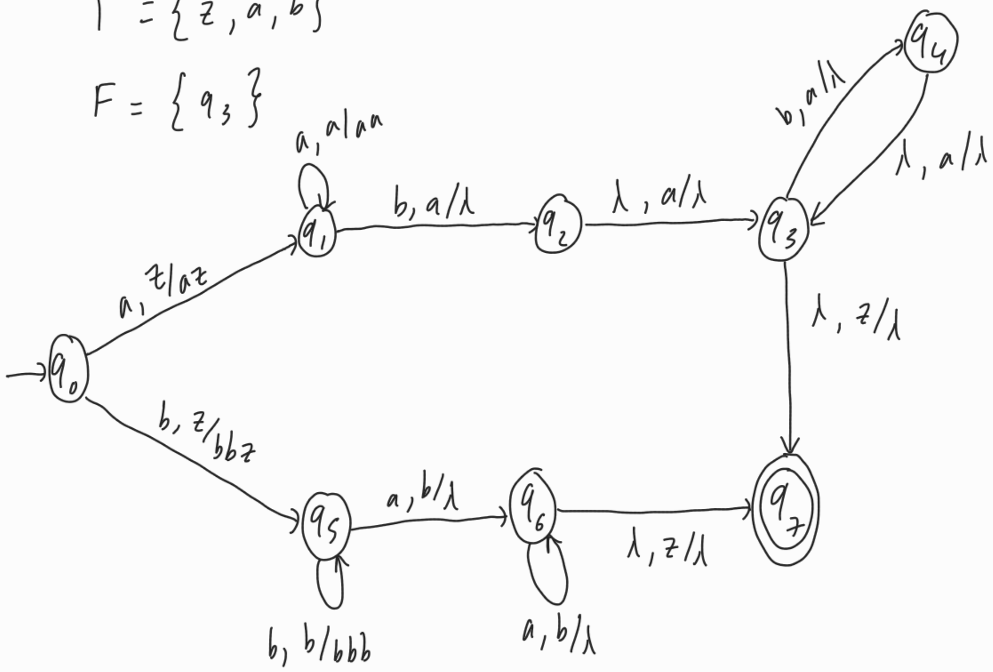
ii) $NPDA = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$

$\Gamma = \{z, a, b\}$

$F = \{q_3\}$



$PDA = \{Q, \Sigma, \Gamma, \delta, q_0, z, F\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$

$\Sigma = \{a, b\}$

$\Gamma = \{z, a, b\}$

$F = \{q_7\}$

iii)

$\delta(q_0, aabbbb, z)$

$\delta(q_1, abbbb, az)$

$\delta(q_1, bbbb, aaz)$

$\delta(q_2, bbb, az)$

$\delta(q_3, bbb, z)$

$\delta(q_7, bbb, z)$

No action defined $\rightarrow$ reject

(3) Create a PDA that recognizes the following context free language with terminals {a,b,c}
$L = \{wc^k \mid w \in \{a, b\}^* \text{ and } k = |w|\}$
(Hint: It is only asking for the # of c's = total number of a's + b's)
(i) Describe your algorithm
(ii) Give the description as a complete 7-tuple with a transition diagram
(iii) Show configuration sequences on babbcccc leading to acceptance.

i) Algorithm:

When a:
- if top is z, then push a
- if top is a, then push a
- if top is b, then push a

When b:
- if top is z, then push b
- if top is a, then push b
- if top is b, then push b

When c:
- if top is a, then pop a
- if top is b, then pop b

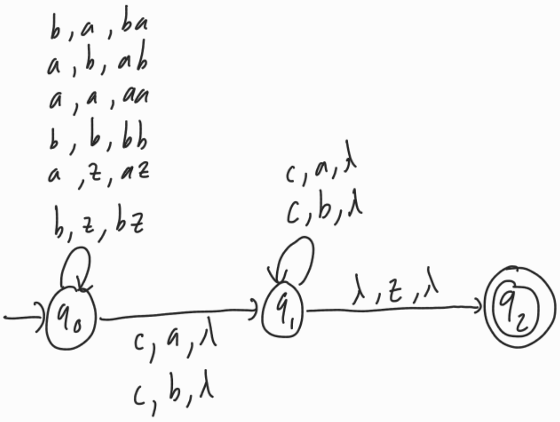
ii) NPDA = $\{Q, \Sigma, \Gamma, \delta, q_0, z, F\}$

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b, c\}$

$\Gamma = \{z, a, b, c\}$

$F = \{q_2\}$



iii) $\delta(q_0, babbcccc, z)$

$\delta(q_0, abbcccc, bz)$

$\delta(q_0, bbcccc, abz)$

$\delta(q_0, bcccc, babz)$

$\delta(q_0, cccc, bbabz)$

$\delta(q_1, ccc, babz)$

$\delta(q_1, cc, abz)$

$\delta(q_1, c, bz)$

$\delta(q_1, \lambda, z)$

$\delta(q_2, \lambda, \lambda)$

(4) Example 7.5 is considered in the notes. Give all possible configuration sequences to account for all the choices on string babbba (similar to slide #45)