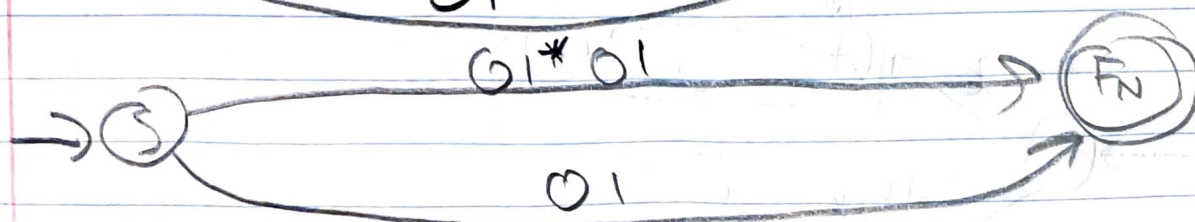
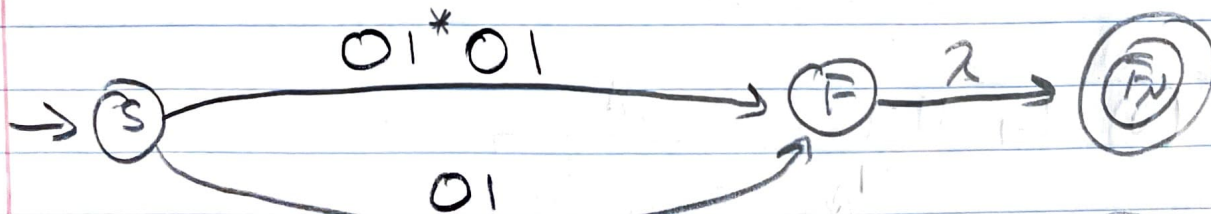
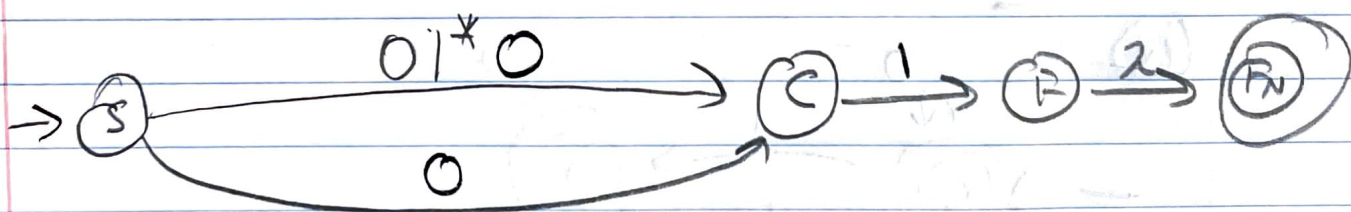
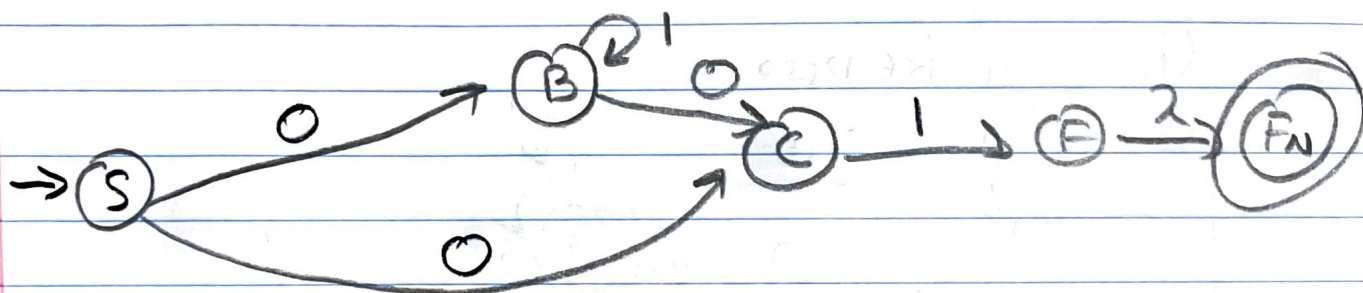
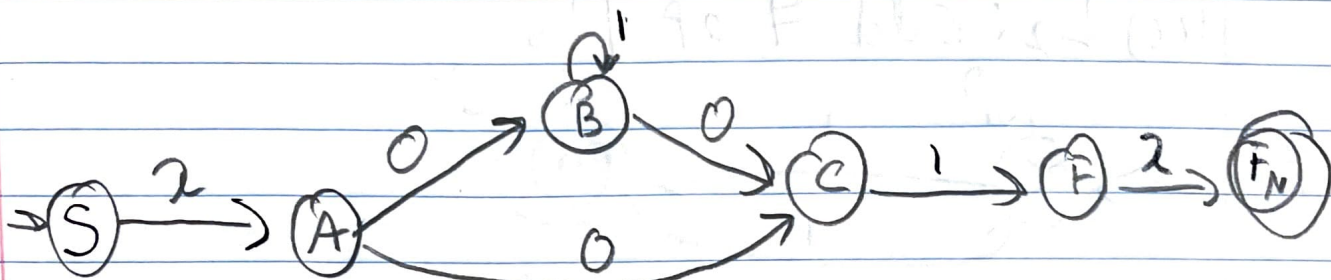
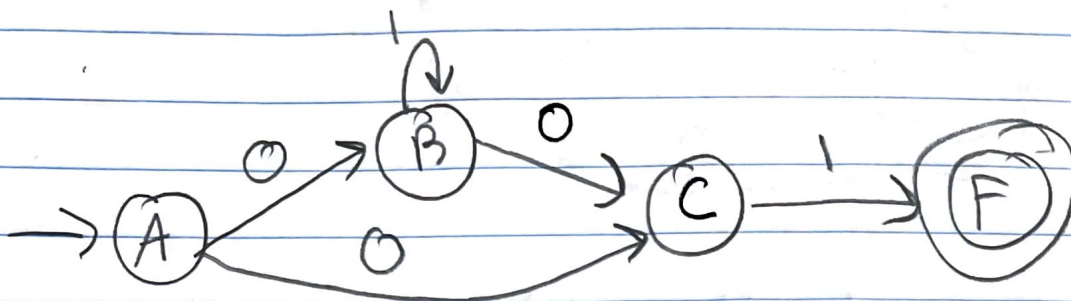


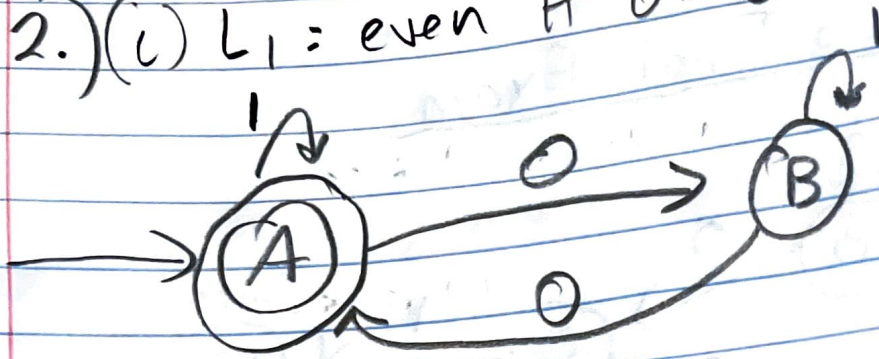
CS 3186 Final Exam

1.)

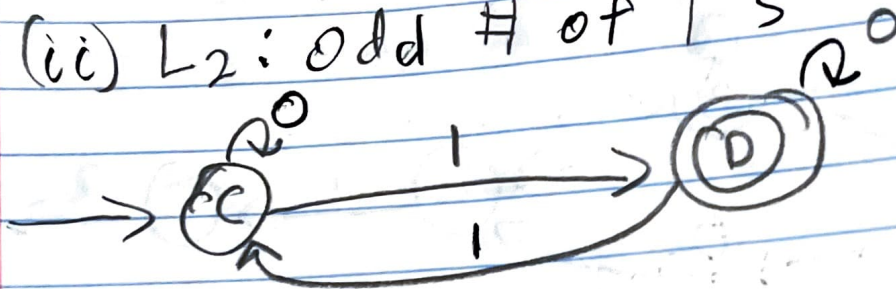


Regular expression:  $(01^*0 + 0)1$

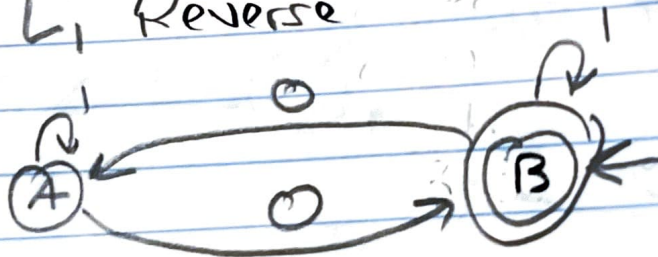
2.) (i)  $L_1$ : even # of 0's



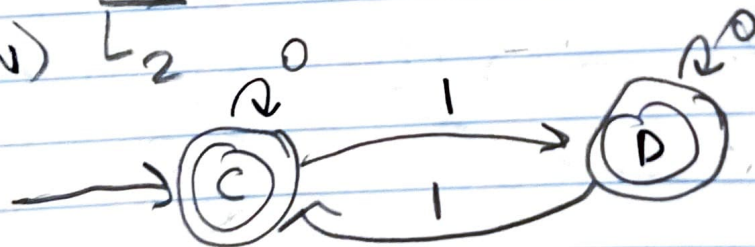
(ii)  $L_2$ : odd # of 1's



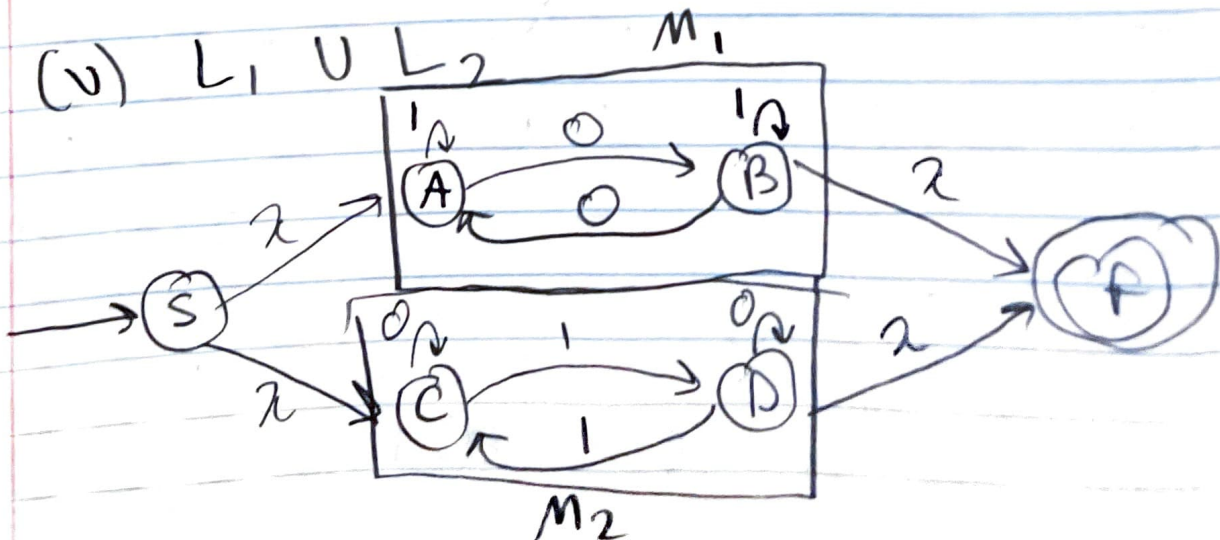
(iii)  $L_1$  Reverse



(iv)  $\overline{L_2}$



(v)  $L_1 \cup L_2$





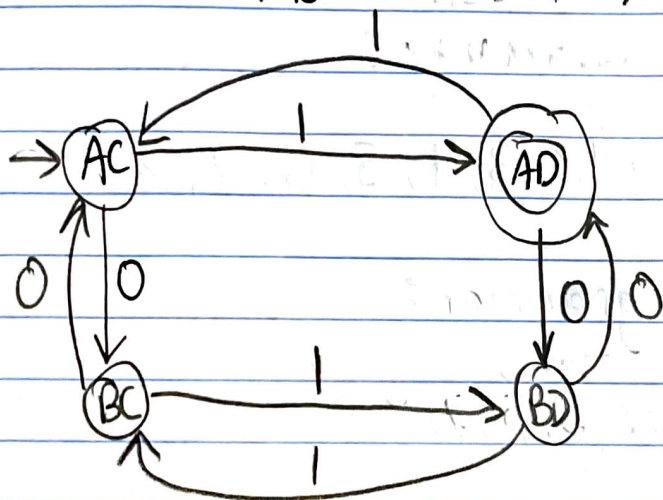
(vi)  $L_1 \cap L_2$

$$\delta(AC, 0) = BC \quad \delta(AC, 1) = AD$$

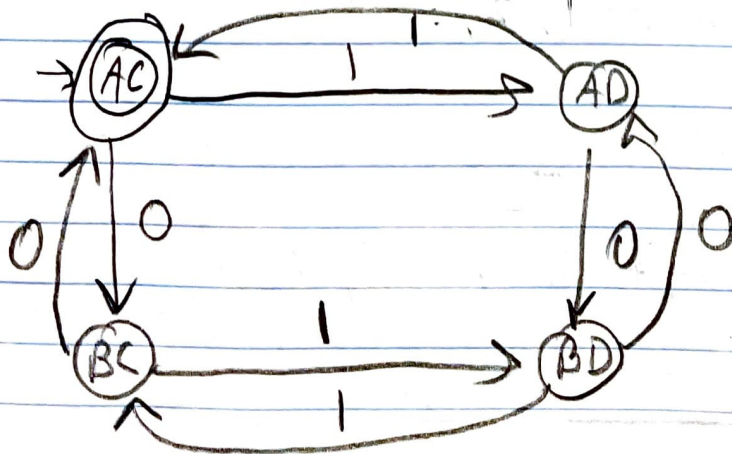
$$\delta(AD, 0) = BD \quad \delta(AD, 1) = AC$$

$$\delta(BC, 0) = AC \quad \delta(BC, 1) = BD$$

$$\delta(BD, 0) = AD \quad \delta(BD, 1) = BC$$



(vii)  $L_1 - L_2 = L_1 \cap \overline{L_2}$



- 3.)
1.  $S \rightarrow OA$
  2.  $S \rightarrow IA$
  3.  $S \rightarrow A\emptyset$
  4.  $S \rightarrow AB$
  5.  $S \rightarrow \emptyset$
  6.  $S \rightarrow OA1$
  7.  $A \rightarrow 1$
  8.  $S \rightarrow \lambda$

(i) Context-free grammar:

Production rules: 1, 2, 3, 4, 5, 6, 7, 8

(ii) Left-linear grammar:

Production rules: 3, 5, 7, 8

(iii) Right Linear regular grammar:

Production rules: 1, 2, 5, 7, 8

(iv) Linear grammar:

Production rules: 1, 2, 3, 5, 7, 8



4.) Use pumping lemma to show  $L = \{a^m b^m c^{m+1}\}$

Given  $L$  is an infinite language, assume  $L$  is context free

There exists a PDA with " $n$ " of production rules & # of variables

Choose  $w = a^n b^n c^{n+1}$

$$|w| = 3n+1 \geq n \text{ (as desired)}$$

$$w = a^n b^n c^{n+1} = uvxy z$$

Since  $|vxy| \leq n$ , leads to many cases

Case 1:  $vxy$  is within  $a$ 's

For  $i=0$ ; you have less # of  $a$ 's than  $b$ 's  
(Since  $|vy| \geq 1$ )

Case 2:  $vxy$  is within  $b$ 's

For  $i=0$ ; you have less # of  $b$ 's than  $a$ 's  
(Since  $|vy| \geq 1$ )

Case 3:  $vxy$  is within  $c$ 's

For  $i=0$ ; you have same or less # of  $c$ 's than  $a$ 's and  $b$ 's (Since  $|vy| \geq 1$ )

Case 4:  $vxy$  spans a's and b's

For  $i \geq 0$ , you have at least 1 less than 2 a's or b's than c's. (Since  $|vy| \geq 1$ ).

Case 5:  $vxy$  spans b's and c's

For  $i \geq 0$ , you have less # of b's or same # of c's than a's (Since  $|vy| \geq 1$ ).

Example I used  $i \geq 0$ ,  $uxz \in L$

All the cases lead to a contradiction

Hence, our assumption that  $L$  is context free is not true.  $L$  is not a CFL.

$$5.) L = \{a^{2n}b^n \mid n \geq 1\} \cup \{b^{n+1}a^n \mid n \geq 1\}$$

(i) Algorithm:

If the first letter is a, then:

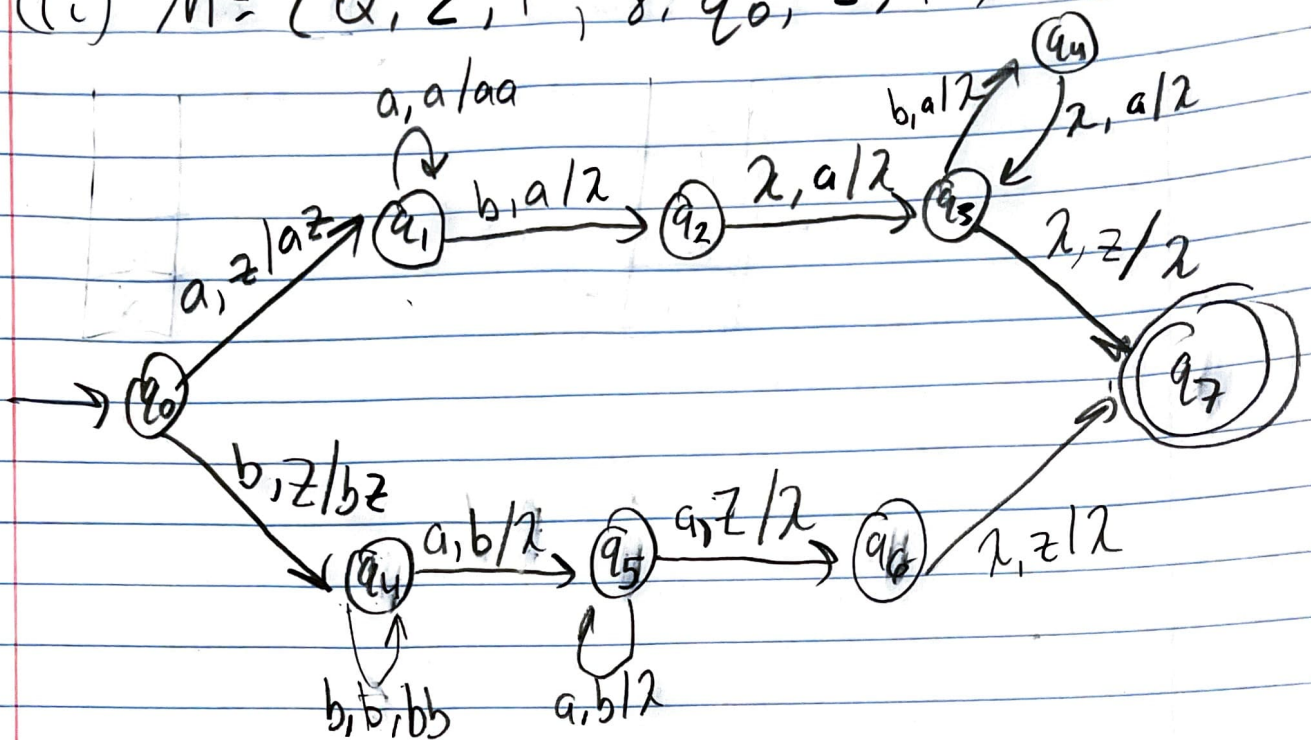
When a, if top is  $\epsilon$  or a, then push a  
When b, if top is a, then pop two a's

If the first letter is b, then:

When b, if top is  $\epsilon$  or b, push a b  
When a, if top is b, pop, pop  $(n+1)$  b's  
When a, if top is  $\epsilon$ , do nothing



(i)  $M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$



Complete 7-tuple:  $(\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}, \{a, b\}, \{z, a, b\}, \delta, q_0, z, \{q_7\})$

(iii.) Show  $bbba$  leads to rejection

$\delta(q_0, bbb a, z) \vdash \delta(q_1, bbb a, bz) \vdash \delta(q_4, ba, bbz)$   
 $\vdash \delta(q_4, a, bbbz) \vdash \delta(q_5, \lambda, bbbz)$

↓  
 No transition  
 defined leads to  
 rejection.