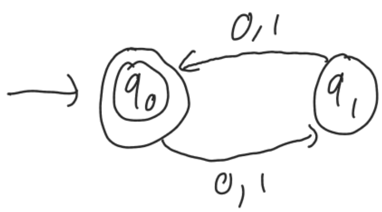
(I) Give a formal definition with any notations for the following:

- **DFA as a 5 tuple**
    - Deterministic Finite Accepter
    - M = (Q, Σ, δ, q_0 F) where
        - Q = Set of states
        - Σ = input alphabet
        - Δ = transition function
        - q_0 = initial state
        - F = set of finite states F ⊆ Q
- **Language accepted by automaton**
    - Let M be an Automaton
    - L(M) is accepted by M if it contains all input strings accepted by M
    - L(M) = { w ∈ Σ*: δ(q_0, w) ∈ F } for a DFA M = (Q, Σ, δ, q_0 F)
- **Regular language**
    - A language is regular if there is a DFA M such that L = L(M)

(II) Given the alphabet as {0, 1}, write a DFA for the following three regular languages. (Give the complete description of the DFA, and also as a transition graph)

(i) L = {w | w is a string of even length}

$\Sigma = \{0, 1\}$

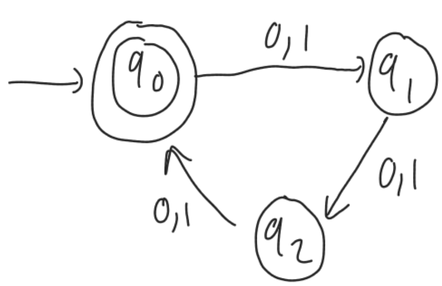


δ	0	1
q_0	q_1	q_1
q_1	q_0	q_0

w	|w|	δ
λ	0	q_0
0	1	q_1
1	1	q_1
01	2	q_0
10	2	q_0
010	3	q_1

(ii) L = {w | |w| mod 3 = 0}

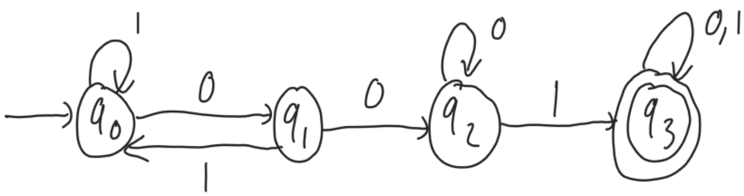
$\Sigma = \{0, 1\}$



δ	0	1
q_0	q_1	q_1
q_1	q_2	q_2
q_2	q_0	q_0

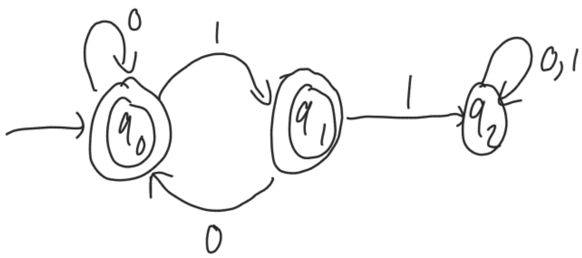
w	|w|	δ
λ	0	q_0
0	1	q_1
10	2	q_2
010	3	q_0
1010	4	q_1

(iii) L = {w | w contains the string 001 as a substring}



δ	0	1
q_0	q_1	q_0
q_1	q_2	q_0
q_2	q_2	q_3
q_3	q_3	q_3

(iv) L = {w | w does not contain two consecutive 1's}



δ	0	1
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_2	q_2

(III)Describe the extended transition function on a string recursively using transitions one symbol at a time for the automaton of following problems.
(i) A string (of length >=4) that belongs to the language

i) $L = \{w \mid |w| \text{ is even}\}$

0000

$\delta^*(q_0, 0000)$

$\delta(\delta^*(q_0, 000), 0)$

$\delta(\delta(\delta^*(q_0, 00), 0), 0)$

$\delta(\delta(\delta(\delta^*(q_0, 0), 0), 0), 0)$

$\delta(\delta(\delta(\delta(q_0, \lambda), 0), 0), 0), 0)$

$\delta(\delta(\delta(q_1, 0), 0), 0)$

$\delta(\delta(q_0, 0), 0)$

$\delta(q_1, 0) = q_0 \in F$

ii) $L = \{ w \mid |w| \% 2 = 0 \}$

001100

$$\delta^*(q_0, 001100)$$

$$\delta(\delta^*(q_0, 00110), 0)$$

$$\delta(\delta(\delta^*(q_0, 0011), 0), 0)$$

$$\delta(\delta(\delta(\delta^*(q_0, 001), 1), 0), 0)$$

$$\delta(\delta(\delta(\delta(\delta^*(q_0, 00), 1), 1), 0), 0)$$

$$\delta(\delta(\delta(\delta(\delta(\delta^*(q_0, 0), 0), 1), 1), 0), 0)$$

$$\delta(\delta(\delta(\delta(\delta(\delta(\delta(q_0, \lambda), 0), 0), 1), 1), 0), 0)$$

$$\delta(\delta(\delta(\delta(\delta(q_1, 0), 1), 1), 0), 0)$$

$$\delta(\delta(\delta(\delta(q_2, 1), 1), 0), 0)$$

$$\delta(\delta(\delta(q_0, 1), 0), 0)$$

$$\delta(\delta(q_1, 0), 0)$$

$$\delta(q_2, 0) = q_0 \in F$$

iii) w contains 001

0001

$\delta^*(q_0, 0001)$

$\delta(\delta^*(q_0, 000), 1)$

$\delta(\delta(\delta^*(q_0, 00), 0), 1)$

$\delta(\delta(\delta(\delta^*(q_0, 0), 0), 0), 1)$

$\delta(\delta(\delta(\delta(q_0, \lambda), 0), 0), 0), 1)$

$\delta(\delta(\delta(q_1, 0), 0), 1)$

$\delta(\delta(q_2, 0), 1)$

$\delta(q_2, 1) = q_3 \in F$

iv) w does not contain 2 consecutive 1's

1010

$\delta^*(q_0, 1010)$

$\delta(\delta^*(q_0, 101), 0)$

$\delta(\delta(\delta^*(q_0, 10), 1), 0)$

$\delta(\delta(\delta(\delta^*(q_0, 1), 0), 1), 0)$

$\delta(\delta(\delta(\delta(q_0, \lambda), 1), 0), 1), 0)$

$\delta(\delta(\delta(q_1, 0), 1), 0)$

$\delta(\delta(q_0, 1), 0)$

$\delta(q_1, 0) = q_0 \in F$

(II) A string (of length >=4) that does not belong to the language

i) 00000

$\delta^*(q_0, 00000)$

$\delta(\delta^*(q_0, 0000), 0)$

$\delta(\delta(\delta^*(q_0, 000), 0), 0)$

$\delta(\delta(\delta(\delta^*(q_0, 00), 0), 0), 0)$

$\delta(\delta(\delta(\delta(\delta^*(q_0, 0), 0), 0), 0), 0)$

$\delta(\delta(\delta(\delta(\delta(\delta(q_0, \lambda), 0), 0), 0), 0), 0)$

$\delta(\delta(\delta(\delta(q_1, 0), 0), 0), 0)$

$\delta(\delta(\delta(q_2, 0), 0), 0)$

$\delta(\delta(q_0, 0), 0)$

$\delta(q_1, 0) = q_1 \notin F$

ii) 00110

$\delta^*(q_0, 00110)$

$\delta(\delta^*(q_0, 0011), 0)$

$\delta(\delta(\delta^*(q_0, 001), 1), 0)$

$\delta(\delta(\delta(\delta^*(q_0, 00), 1), 1), 0)$

$\delta(\delta(\delta(\delta(\delta^*(q_0, 0), 0), 1), 1), 0)$

$\delta(\delta(\delta(\delta(\delta(\delta(q_0, \lambda), 0), 0), 1), 1), 0)$

$\delta(\delta(\delta(\delta(q_1, 0), 1), 1), 0)$

$\delta(\delta(\delta(q_2, 1), 1), 0)$

$\delta(\delta(q_0, 1), 0)$

$\delta(q_1, 0) = q_2 \notin F$

iii) 0101

$\delta^*(q_0, 0101)$

$\delta(\delta^*(q_0, 010), 1)$

$\delta(\delta(\delta^*(q_0, 01), 0), 1)$

$\delta(\delta(\delta(\delta^*(q_0, 0), 1), 0), 1)$

$\delta(\delta(\delta(\delta(\delta(q_0, \lambda), 0), 1), 0), 1)$

$\delta(\delta(\delta(q_1, 1), 0), 1)$

$\delta(\delta(q_0, 0), 1)$

$\delta(q_1, 1) = q_0 \notin F$

iv) 0110

$\delta^*(q_0, 0110)$

$\delta(\delta^*(q_0, 011), 0)$

$\delta(\delta(\delta^*(q_0, 01), 1), 0)$

$\delta(\delta(\delta(\delta^*(q_0, 0), 1), 1), 0)$

$\delta(\delta(\delta(\delta(\delta(q_0, \lambda), 0), 1), 1), 0)$

$\delta(\delta(\delta(q_0, 1), 1), 0)$

$\delta(\delta(q_1, 1), 0)$

$\delta(q_2, 0) = q_2 \notin F$