1) $T(n) = 3T(\frac{n}{2}) + n^2$

$a = 3,\ b = 2,\ f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

$f(n) = n^2$ $\Rightarrow f(n) > n^{\log_b a}$ Case 3

Regularity Condition: $af(\frac{n}{b}) \leq cf(n)\ ,\ c < 1$

$\Leftrightarrow 3f(\frac{n}{2}) \leq cf(n)$

$\Leftrightarrow 3(\frac{n}{2})^2 \leq cn^2 \Rightarrow \frac{3n^2}{4} \leq cn^2$

This is true for $c = \frac{3}{4}$

$T(n) = \Theta(f(n)) = \Theta(n^2)$

2) $T(n) = 4T(\frac{n}{2}) + n^2$

$a = 4,\ b = 2,\ f(n) = n^2$

$n^{\log_b a} = n^{\log_2 4} = n^2$ $\Rightarrow f(n) = n^{\log_b a}$ Case 2

$f(n) = n^2$

$T(n) = \Theta(n^{\log_b a} \log n) = \Theta(n^2 \log n)$

3) $T(n) = T(\frac{n}{2}) + 2^n$

$a = 1,\ b = 2,\ f(n) = 2^n$

$n^{\log_b a} = n^{\log_2 1} = 1$ $\Rightarrow f(n) > n^{\log_b a}$ Case 3

$f(n) = 2^n$

Regularity Condition: $af(\frac{n}{b}) \leq cf(n)$

$f(\frac{n}{2}) \leq cf(n) \Rightarrow 2^{\frac{n}{2}} \leq c(2^n) \Rightarrow \sqrt{2^n} \leq c(2^n)\ ,\ c < 1$

$T(n) = \Theta(f(n)) = \Theta(2^n)$

4) $T(n) = 2^n T\left(\frac{n}{2}\right) + n^n$

$a = 2^n$, $b = 2$, $f(n) = n^n$

$n^{\log_b a} = n^{\log_2 2^n} = n^n \Rightarrow f(n) = n^{\log_b a}$ Case 2

$f(n) = n$

$T(n) = \Theta(n^{\log_b a} \log n) = \Theta(n^n \log n)$

5) $T(n) = 16T\left(\frac{n}{4}\right) + n$

$a = 16$, $b = 4$, $f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2 \Rightarrow f(n) = n^{\log_b a - \varepsilon}$ $(\varepsilon = 1)$ Case 1

$f(n) = n$

$T(n) = \Theta(n^{\log_b a}) = \Theta(n^2)$

6) $T(n) = 2T\left(\frac{n}{2}\right) + n\log n$

$a = 2$, $b = 2$, $f(n) = n\log n$

$n^{\log_b a} = n^{\log_2 2} = n$

$f(n) = n\log n \Rightarrow f(n) > n^{\log_b a}$ Case 3

Regularity Condition: $a f\left(\frac{n}{b}\right) \leq c f(n)$

$2 f\left(\frac{n}{2}\right) \leq c f(n)$

$2\left(\frac{n}{2}\log\left(\frac{n}{2}\right)\right) \leq c\, n\log n \Rightarrow n\log n - n\log 2 \leq c\, n\log n$

$(1-c)\, n\log n \leq n\log 2$ True for $c < 1$

$T(n) = \Theta(f(n)) = \Theta(n\log n)$

7) $T(n) = 2T(n/2) + \frac{n}{\log n}$

$a = 2$, $b = 2$, $f(n) = \frac{n}{\log n}$

$n^{\log_b a} = n^{\log_2 2} = n$

$f(n) = \frac{n}{\log n} \Rightarrow f(n) = n^{\log_b a - \varepsilon}$ Case 1

$T(n) = \Theta(n^{\log_b a}) = \Theta(n)$

8) $T(n) = 2T(\frac{n}{4}) + n^{0.51}$

$a = 2,\ b = 4,\ f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5} \Rightarrow f(n) > n^{\log_b a}$ Case 3

$f(n) = n^{0.51}$

Regularity Condition: $af(\frac{n}{b}) \leq cf(n)$

$2f(\frac{n}{4}) \leq cf(n) \Rightarrow 2(\frac{n}{4})^{0.51} \leq c(n^{0.51})$

$\approx 0.986 \leq c \leq 1$

$T(n) = \Theta(f(n)) = \Theta(n^{0.51})$

9) $T(n) = 0.5T(\frac{n}{2}) + \frac{1}{n}$

$a = 0.5,\ b = 2,\ f(n) = \frac{1}{n}$

$n^{\log_b a} = n^{\log_2 0.5} = n^{-1} \Rightarrow f(n) = n^{\log_b a}$ Case 2

$f(n) = \frac{1}{n}$

$T(n) = \Theta(n^{\log_b a} \log n) = \Theta(n^{-1} \log n)$

10) $T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4,\ f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2 \Rightarrow f(n) > n^{\log_b a}\ (n > 3)$ Case 3

$f(n) = n!$

$n!$ is superpolynomial $\Rightarrow$ We can skip regularity condition

$\Rightarrow T(n) = \Theta(f(n)) = \Theta(n!)$