**FPT UNIVERSITY**

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# THE FOUNDATIONS: LOGIC AND PROOFS

## DISCRETE MATHEMATIS

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# Content

# Content

# 1.1.1 Propositional Logic

## Definition

A proposition (proposition logic) is a declarative sentence that is either true or false, but not both.

## Example

All the following declarative sentences are propositions.

- Hanoi is the capital of Vietnam.
- Ho Chi Minh city is a big center of Laos.
- $1 + 1 = 2$.
- $2 + 3 = 4$.

# 1.1.1 Propositional Logic

**Example**

All the following sentences are not propositions.

- What time is it?
- Read this carefully.
- $x + 1 = 2$.
- $x + y = z$.

# 1.1.1 Propositional Logic

## Note

- We use letters $p, q, r, s, \ldots$ to denote propositionals.
- The truth value of a proposition is true, denoted by T, if it is a true proposition, and the truth value of a proposition is false, denoted by F, if it is a false proposition.

## Truth table

- $p$: "I am a Math teacher"

| p |
|---|
| True/ T / 1 |
| False / F / 0 |

# 1.1.2 Logical operators

## Definition

Let $p$ be a proposition. **The negation** of $p$, denoted by $\neg p$ (also denoted by $\bar{p}$), is the statement "It is not the case that $p$." The proposition $\neg p$ is read "not $p$." The truth value of the negation of $p$, $\neg p$, is the opposite of the truth value of $p$.

| p | $\bar{p}$ |
|---|---|
| 1 | 0 |
| 0 | 1 |

# 1.1.2 Logical operators

## Example

- $p_1$ : Michael's PC runs Linux.
  $\neg p_1$ : Michael's PC does not run Linux.
  or
  $\neg p_1$ : It is not the case that Michael's PC runs Linux.
- $p_2 : 2 > 1$
  $\neg p_2 : 2 \leq 1$

# 1.1.2 Logical operators

## Definition

Let $p$ and $q$ be propositions. **The conjunction** of $p$ and $q$, denoted by $p \wedge q$, is the proposition "p and q." The conjunction $p \wedge q$ is true when both $p$ and $q$ are true and is false otherwise.

| p | q | P ∧ q |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

## Example

- $p$ : The summer is hot. $q$ : The winter is cold.

# 1.1.2 Logical operators

## Definition

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| p | q | P ∧ q |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

## Example

- $p$ : The summer is hot. $q$ : The winter is cold.
- $p \wedge q$ : The summer is hot, and the winter is cold
  or
  $p \wedge q$ : The summer is hot, but the winter is cold.

## Definition

Let $p$ and $q$ be propositions. **The disjunction** of $p$ and $q$, denoted by $p \vee q$, is the proposition "$p$ or $q$." The disjunction $p \vee q$ is false when both $p$ and $q$ are false and is true otherwise.

| p | q | P ∨ q |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

## Example

- $p$ : The summer is hot. $q$ : The winter is cold.

# 1.1.2 Logical operators

## Definition

Let $p$ and $q$ be propositions. **The disjunction** of $p$ and $q$, denoted by $p \vee q$, is the proposition "$p$ or $q$." The disjunction $p \vee q$ is false when both $p$ and $q$ are false and is true otherwise.

| p | q | P ∨ q |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

## Example

- $p$ : The summer is hot. $q$ : The winter is cold.
- $p \vee q$ : The summer is hot, or the winter is cold

**Definition**

Let $p$ and $q$ be propositions. **The exclusive or (xor)** of $p$ and $q$, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

| p | q | $p \oplus q$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

# 1.1.2 Logical operators

## Definition

- Let $p$ and $q$ be propositions. **The conditional statement (implication)** $p \to q$ is the proposition "if $p$, then $q$." The conditional statement $p \to q$ is false when $p$ is true and $q$ is false, and true otherwise.
- In the conditional statement $p \to q$, $p$ is called the hypothesis (or antecedent or premise) and $q$ is called the conclusion (or consequence).

| p | q | p → q |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

- "If $1 + 1 = 3$, then dogs can fly".

# 1.1.2 Logical operators

Because conditional statements play such an essential role in mathematical reasoning, a variety of terminology is used to express $P \to q$. You will encounter most if not all of the following ways to express this conditional statement:

- "if p, then q" or "if p, q" or "p only if q."
- "p is sufficient for q" or "a necessary condition for p is q"
- "a sufficient condition for q is p" or "q whenever p"
- "q if p" or "q when p"
- "a necessary condition for p is q" or "q unless $\neg$p"
- "p implies q" or "p only if q"
- "q is necessary for p" or "q follows from p."

# 1.1.2 Logical operators

## Example

$p$ : Maria learns discrete mathematics, $q$ : she will find a good job.

# 1.1.2 Logical operators

**Example**

$p$ : Maria learns discrete mathematics, $q$ : she will find a good job.

- $p \to q$ : If Maria learns discrete mathematics, then she will find a good job.
- $p \to q$ : Maria will find a good job when she learns discrete mathematics.
- $p \to q$ : For Maria to get a good job, it is sufficient for her to learn discrete mathematics.
- $p \to q$ : Maria will find a good job unless she does not learn discrete mathematics.

# 1.1.2 Logical operators

## Definition

Let $p$ and $q$ be propositions. The **biconditional statement** $p \leftrightarrow q$ is the proposition "$p$ if and only if $q$." The biconditional statement $p \leftrightarrow q$ is true when $p$ and $q$ have the same truth values, and is false otherwise. Biconditional statements are also called bi-implications.

| p | q | p→q | q→p | (p→q)∧(q→p) | p ↔ q |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | **1** | **1** |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | **1** | **1** |

## Example

$p$ : You can take the flight, $q$ : You buy a ticket.

# 1.1.2 Logical operators

## Definition

Let $p$ and $q$ be propositions. The **biconditional statement** $p \leftrightarrow q$ is the proposition "$p$ if and only if $q$." The biconditional statement $p \leftrightarrow q$ is true when $p$ and $q$ have the same truth values, and is false otherwise. Biconditional statements are also called bi-implications.

| p | q | p→q | q→p | (p→q)∧(q→p) | p ↔ q |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |

## Example

$p$ : You can take the flight, $q$ : You buy a ticket.

- $p \leftrightarrow q$ : You can take the flight if and only if you buy a ticket.

**Example**

Construct the truth table of the compound proposition

1. $(\neg p \wedge q) \rightarrow (p \vee q)$
2. $(p \wedge \neg q) \rightarrow (p \oplus q)$
3. $(p \oplus \neg q) \leftrightarrow (p \rightarrow q)$
4. $(p \leftrightarrow q) \oplus (p \vee q)$

## 1.1.4 Precedence of Logical Operators

The following able displays the precedence levels of the logical operators, $\neg, \wedge, \vee, \rightarrow$and $\leftrightarrow$.

| Operator | Precedence |
|---|---|
| $\neg$ | 1 |
| $\wedge$ | 2 |
| $\vee$ | 3 |
| $\rightarrow$ | 4 |
| $\leftrightarrow$ | 5 |

**Example**

i. $\neg p \wedge q$ is the same as $(\neg p) \wedge q$

ii. $p \vee q \rightarrow r$ is the same as $(p \vee q) \rightarrow r$

# Content

**Definition**

A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a **tautology** (hằng đúng). A compound proposition that is always false is called a **contradiction** (mâu thuẫn).
When $p \leftrightarrow q$ is tautology, we say "$p$ and $q$ are called logically equivalence". Notation: $p \equiv q$.

**Example**

Consider the truth tables of $p \vee \neg p$ and $p \wedge \neg p$?

# 1.2.2 Logical Equivalences

## Definition

The compound propositions $p$ and $q$ are called **logically equivalent** if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes that $p$ and $q$ are logically equivalent.

## Example

Show that the following compound propositions are logically equivalents (De Morgan laws).

- $\neg(p \vee q)$ and $\neg p \wedge \neg q$.
- $\neg(p \wedge q)$ and $\neg p \vee \neg q$.

## Example

Show that $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent.

# 1.2.2 Logical Equivalences

| Equivalence | Name |
|---|---|
| $p \wedge T \equiv p$ $\quad$ $p \vee F \equiv p$ | Identity laws- Luật đồng nhất |
| $p \vee T \equiv T$ $\quad$ $p \wedge F \equiv F$ | Domination Laws – Luật chi phối |
| $p \vee p \equiv p$ $\quad$ $p \wedge p \equiv p$ | Idempotent Laws – Luật bất biến |
| $\neg(\neg p) \equiv p$ | Double Negation Laws – Luật đảo kép |
| $p \vee q \equiv q \vee p$ $\quad$ $p \wedge q \equiv q \wedge p$ | Commutative Laws – Luật giao hoán |
| $(p \vee q) \vee r \equiv p \vee (q \vee r)$<br>$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ | Associative Laws – Luật kết hợp |
| $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$<br>$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ | Distributive Laws – Luật phân phối |
| $\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\quad$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$ | De Morgan Laws |
| $p \vee (p \wedge q) \equiv p$ $\quad$ $p \wedge (p \vee q) \equiv p$ | Absorption Laws – Luật hấp thụ |
| $p \vee \neg p \equiv T$ $\quad$ $p \wedge \neg p \equiv F$ | Negation Laws - Luật nghịch đảo |

# 1.2.2 Logical Equivalences

| Equivalences | Equivalences |
|---|---|
| $p \rightarrow q \equiv \neg p \lor q$ | $p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p)$ |
| $p \rightarrow q \equiv \neg q \rightarrow \neg p$ | $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$ |
| $p \lor q \equiv \neg p \rightarrow q$ | $p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$ |
| $p \land q \equiv \neg (p \rightarrow \neg q)$ | $\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$ |
| $\neg(p \rightarrow q) \equiv p \land \neg q$ | |
| $(p \rightarrow q) \land (p \rightarrow r) \equiv p \rightarrow (q \land r)$ | |
| $(p \rightarrow r) \land (q \rightarrow r) \equiv (p \lor q) \rightarrow r$ | |
| $(p \rightarrow q) \lor (p \rightarrow r) \equiv p \rightarrow (q \lor r)$ | |
| $(p \rightarrow r) \lor (q \rightarrow r) \equiv (p \land q) \rightarrow r$ | |

# Content

**Definition**

- **Predicate logic (vị từ)** is used to express the meaning of a wide range of statements in mathematics and computer science in ways that permit us to reason and explore relationships between objects.
- There is an important way, called **quantification (lượng từ)**, to create a proposition from a propositional function.

**Example**

1. $P(x) =$ "$x$ is a prime number", called **propositional function** at $x$.
   - $P(2) =$ "2 is a prime number" $\rightarrow$ True
   - $P(4) =$ "4 is a prime number" $\rightarrow$ False

### Example

2. $Q(x, y) =$ "$x = y + 3$" also called an **2-place predicate or a 2-ary predicate**
   - $Q(1, 2) =$ "$1 = 2 + 3$": False
   - $Q(5, 2) =$ "$5 = 2 + 3$": True
3. $R(x, y, z) =$ "$x^2 = y^2 + z^2$" also called an **3-place predicate or a 3-ary predicate**
   - $R(2, 3, 5) =$ "$2^2 = 3^2 + 5^2$": False
   - $R(10, 8, 6) =$ "$10^2 = 8^2 + 6^2$": True

## Quantifiers (Lượng từ)

- The words in natural language: all, some, many, none, few, . . . .are used in quantifications.
- The universal quantification (lượng từ phổ dụng) of $P(x)$ is the statement "$P(x)$ for all values of $x$ in the domain". Notation : $\forall x P(x)$.
- The existential quantification (lượng từ tồn tại) of $P(x)$ is the statement "There exists an element $x$ in the domain such that $P(x)$". Notation : $\exists x P(x)$.
- Uniqueness quantifier (Lượng từ tồn tại duy nhất): $\exists! x P(x)$ or $\exists_1 x P(x)$.

| Statement | When True? | When False? |
| --- | --- | --- |
| $\forall x P(x)$ | $P(x)$ is true for every $x$ | There is an $x$ for which $P(x)$ is false |
| $\exists x P(x)$ | There is an $x$ for which $P(x)$ is true | $P(x)$ is false for every $x$ |

- *Example*: Let $P(x)$ be the statement "$x + 1 > x$". What is the truth value of the quantification $\forall x P(x)$, where the domain consists of all real numbers?
- *Solution*: Because $P(x)$ is true for all real numbers $x$, the quantification $\forall x P(x)$ is true.

# 1.3 Predicates and Quantifiers

- *Example*: Let $Q(x)$ be the statement "$x < 2$." What is the truth value of the quantification $\forall x Q(x)$, where the domain consists of all real numbers?
- *Solution*: $Q(x)$ is not true for every real number $x$, because, for instance, $Q(3)$ is false. That is, $x = 3$ is a counterexample for the statement $\forall x Q(x)$. Thus $\forall x Q(x)$ is false.

- *Example*: Let $P(x)$ denote the statement "$x > 3$." What is the truth value of the quantification $\exists x P(x)$, where the domain consists of all real numbers?
- *Solution*: Because "$x > 3$" is sometimes true for instance, when $x = 4$ the existential quantification of $P(x)$, which is $\exists x P(x)$, is true.

**Note**

- The quantifiers $\forall$ and $\exists$ have higher precedence than all logical operators from propositional calculus.
- *Example*: $\forall x P(x) \vee Q(x)$ is the disjunction of $\forall x P(x)$ and $Q(x)$. In other words, it means $(\forall x P(x)) \vee Q(x)$ rather than $\forall x(P(x) \vee Q(x))$.
- *Example*: $\forall x < 0(x^2 > 0)$: states that for every real number $x$ with $x < 0, x^2 > 0$. This statement is the same as $\forall x(x < 0 \rightarrow x^2 > 0)$.
- *Example* $\exists z > 0(z^2 = 2)$: states that there exists a real number $z$ with $z > 0$ such that $z^2 = 2$. This statement is equivalent to $\exists z(z > 0 \wedge z^2 = 2)$.

# Logical Equivalences Involving Quantifiers

## Note

- Statements involving predicates and quantifiers are **logically equivalent if and only if they have the same truth value.**
- *Example*: $\forall x(P(x) \wedge Q(x)) \equiv \forall x P(x) \wedge \forall x Q(x)$.

| Expression | Equivalence | Expression | Negation |
|---|---|---|---|
| $\neg\exists x P(x)$ | $\forall x \neg P(x)$ | $\exists x P(x)$ | $\forall x \neg P(x)$ |
| $\neg \forall x P(x)$ | $\exists x \neg P(x)$ | $\forall x P(x)$ | $\exists x \neg P(x)$ |

# Exercises

1. Which compound proposition is True when $p = r =$ True and $q =$ False, and is False otherwise?
Select 1:

a. $p \wedge \neg q \wedge \neg r$
b. $p \vee q \wedge \neg r$
c. $\neg p \wedge \neg q \wedge \neg r$
d. $p \wedge q \wedge \neg r$
e. $p \wedge \neg q \wedge r$

2. Given two propositions:

p = "I went to Paris." q = "I visit Eiffel Tower"

- I visit Eiffel Tower only if I go to Paris.
- I cannot visit Eiffel Tower if I do not go to Paris.
- Whenever I go to Paris, I visit Eiffel Tower.
- Going to Paris is a sufficient condition for visiting Eiffel Tower.

# Exercises

3. Let $p,\ q$ be two propositions. Which propositions are logically equivalent to $p \to q$?

- $\bar{p} \vee q$
- $\bar{p} \vee \bar{q}$
- $p \vee q$
- $\bar{p} \wedge q$

4. Which propositions are contradiction?

- $(p \to q) \land (q \to p) \land (p \oplus q)$
- $[(p \to q) \lor (q \to p)] \land (p \oplus q)$
- $(p \to q) \lor (q \to p) \lor (p \oplus q)$

# Content

# 1.4 Nested Quantifiers

- $\forall x \exists y(x+y=0)$ is is the same thing as $\forall x Q(x)$, where $Q(x)$ is $\exists y P(x,y)$, where $P(x,y)$ is $x+y=0$.
- $\forall x \forall y(x+y=y+x)$: $x+y=y+x$ for all real numbers $x$ and $y$".
- $\forall x \exists y(x+y=0)$: "for every real number $x$ there is a real number $y$ such that $x+y=0$."

| *Statement* | *When True?* | *When False?* |
|---|---|---|
| $\forall x \forall y\, P(x,y)$<br>$\forall y \forall x P(x,y)$ | P(x, y) is true for every pair x, y. | There is a pair x, y for which P(x, y) is false. |
| $\forall x \exists y P(x,y)$ | For every *x* there is a *y* for which P(x, y) is true. | There is an *x* such that P(x, y) is false for every y. |
| $\exists x \forall y P(x,y)$ | There is an *x* for which P(x,y) is true for every y. | For every *x* there is a *y* for which P (x, y) is false. |
| $\exists x \exists y P(x,y)$<br>$\exists y \exists x P(x,y)$ | There is a pair x, y for which P(x, y) is true. | P(x, y) is false for every pair x, y. |

# 1.4 Nested Quantifiers

- Translate the statement $\forall x(C(x) \vee \exists y(C(y) \wedge F(x, y)))$ into English, where $C(x)$ is "$x$ has a computer," $F(x, y)$ is "$x$ and $y$ are friends," and the domain for both $x$ and $y$ consists of all students in your school.
- For every student $x$ in your school, $x$ has a computer or there is a student $y$ such that $y$ has a computer and $x$ and $y$ are friends.
- Or: every student in your school has a computer or has a friend who has a computer.

- Translate the logical expression into sentence, domain is all real numbers

$$\forall x \forall y(((x < 0) \wedge (y < 0)) \rightarrow (xy > 0))$$

Select one:

a. The product of two negative numbers is negative.
b. For each negative number $x$ there is a negative number $y$ such that $xy$ is positive.
c. There is a negative number $x$ and there is a negative number $y$ such that $xy$ is positve.
d. The product of two negative numbers is positive

Let
$P(x) =$ "$x$ goes to class regularly"
$Q(x) =$ "$x$ reads books"
$R(x) =$ "$x$ passed the exam."
Tranlate the sentence into logical expression, domain is the set of all students in class.
"Some student who goes to class regularly and reads books has failed the exam."

# 1.4 Nested Quantifiers

| TABLE 2 De Morgan's Laws for Quantifiers. | | | |
|---|---|---|---|
| ***Negation*** | ***Equivalent Statement*** | ***When Is Negation True?*** | ***When False?*** |
| $\neg\exists x P(x)$ | $\forall x \neg P(x)$ | For every $x$, $P(x)$ is false. | There is an $x$ for which $P(x)$ is true. |
| $\neg\forall x P(x)$ | $\exists x \neg P(x)$ | There is an $x$ for which $P(x)$ is false. | $P(x)$ is true for every $x$. |

$$\begin{aligned}\neg\forall x\exists y(xy=1) &\equiv \exists x\neg\exists y(xy=1)(\text{De Morgan laws})\\ &\equiv (\exists x)(\forall y)\neg(xy=1)\\ &\equiv (\exists x)(\forall y)(xy\neq 1)\end{aligned}$$

# Content

# 1.5 Rules of Inference

- "If you have a current password, then you can log onto the network."
- "You have a current password."
- Therefore,
  "You can log onto the network"

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

# 1.5 Rules of Inference

- An **argument** in propositional logic is a sequence of propositions. All but the final proposition in the argument are called **premises** and the final proposition is called the **conclusion**. An argument is valid if the truth of all its premises implies that the conclusion is true.
- An **argument form** in propositional logic is a sequence of compound propositions involving propositional variables. An argument form is **valid** no matter which particular propositions are substituted for the propositional variables in its premises, the conclusion is true if the premises are all true.
- The argument form with premises $p_1, p_2, \ldots, p_n$ and conclusion $q$ is valid, when $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \to q$ is a tautology.

# 1.5 Rules of Inference

- Proposition 1 (Hypothesis - giả thiết)
- Proposition 2
- Proposition 3
- Proposition 4
- ......
- Proposition $n$ (Conclusion)

# 1.5 Rules of Inference

| Rule | Tautology | Name |
|---|---|---|
| $p$<br>$p \to q$<br>$\therefore q$ | $[p \wedge (p \to q)] \to q$<br>You work hard<br>If you work hard then you will pass the examination<br>∴ you will pass the examination | Modus ponen (law of detachment-luật tách) |
| $\neg q$<br>$p \to q$<br>$\therefore \neg p$ | $[\neg q \wedge (p \to q)] \to \neg p$<br>She did not get a prize<br>If she is good at learning she will get a prize<br>∴ She is not good at learning | Modus tollen |
| $p \to q$<br>$q \to r$<br>$\therefore p \to r$ | $[(p \to q) \wedge (q \to r)] \to (p \to r)$<br>If the prime interest rate goes up then the stock prices go down.<br>If the stock prices go down then most people are unhappy.<br>If the prime interest rate goes up then most people are unhappy. | Hypothetical syllogism – Tam đoạn luận giả thiết, Quy tắc bắc cầu |

# 1.5 Rules of Inference

| Rule | Tautology | Name |
| --- | --- | --- |
| $p \vee q$<br>$\underline{\neg p}$<br>$\therefore q$ | $[(p \vee q) \wedge \neg p] \rightarrow q$<br>Power puts off or the lamp is malfunctional<br>Power doesn't put off<br>the lamp is malfunctional | Disjunctive syllogism (Tam đoạn luận) |
| $\underline{p}$<br>$\therefore p \vee q$ | $p \rightarrow (p \vee q)$<br>It is below freezing now<br>It is below freezing now or raining now | Addition |
| $\underline{p \wedge q}$<br>$\therefore p$ | $(p \wedge q) \rightarrow p$<br>It is below freezing now and raining now<br>It is below freezing now | Simplication |
| $p$<br>$\underline{q}$<br>$\therefore p \wedge q$ | $[(p) \wedge (q)] \rightarrow (p \wedge q)$<br>She studied hard<br>She got good grades<br>She studied hard and got good grades | Conjunction |
| $p \vee q$<br>$\underline{\neg p \vee r}$<br>$\therefore q \vee r$ | $[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$<br>Jasmin is skiing OR it is not snowing<br>It is snowing OR Bart is playing hockey<br>Jasmin is skiing OR Bart is playing hockey | Resolution |

# 1.5 Rules of Inference

| Rule | Name |
|---|---|
| $\forall x P(x)$<br>$\therefore P(c)$ | Universal Instantiation<br>Cụ thể hóa lượng từ phổ dụng |
| $P(c)$ for arbitrary c<br>$\therefore \forall x P(x)$ | Universal generalization<br>Tổng quát hóa bằng lượng từ phổ dụng |
| $\exists x P(x)$<br>$\therefore P(c)$ for some element c | Existential instantiation<br>Chuyên biệt hóa |
| $P(c)$ for some element c<br>$\therefore \exists x P(x)$ | Existential generalization<br>Khái quát hóa bằng lượng từ tồn tại |

# Fallacies – ngụy biện – sai logic

- If you do every problem in this book then you will learn discrete mathematic.
- you learned mathematic
  Therefore, you did every problem in this book.
- $(p \to q) \wedge q \to p$ (false)
- $(p \to q) \wedge q \to q$ (absorption law - luật hấp thụ)
  $\therefore$ no infnormation for $p$
- $p$ can true or false, so you may learn discrete mathematic but you might do some problems only.

# Exercises

1. Given an argument: ' If Jack is a soccer player then Jack is rich. Jack only plays pingpong. Therefore Jack is not rich." Choose correct statement:

a. This valid argument is based on modus tollens

b. This valid argument is based on disjunctive syllogism

c. This valid argument is based on hypothetical syllogism

d. This valid argument is based on modus ponens

e. This argument is a fallacy.

# Exercises

1. Given an argument: ‘ If Jack is a soccer player then Jack is rich. Jack only plays pingpong. Therefore Jack is not rich.” Choose correct statement:

a. This valid argument is based on modus tollens
b. This valid argument is based on disjunctive syllogism
c. This valid argument is based on hypothetical syllogism
d. This valid argument is based on modus ponens
e. **This argument is a fallacy.**

# Exercises

2. Find the negation of

$$\exists y(Q(x,y) \land \forall x \neg R(x,y))$$

Seclect one:

a. $\forall y(\neg Q(x,y) \land \exists x R(x,y))$

b. $\forall y(\neg Q(x,y) \lor \exists x \neg R(x,y))$

c. $\forall y(\neg Q(x,y) \lor \forall x R(x,y))$

d. $\forall y(\neg Q(x,y) \lor \exists x R(x,y))$

# Exercises

2. Find the negation of

$$\exists y(Q(x,y) \wedge \forall x \neg R(x,y))$$

Seclect one:

a. $\forall y(\neg Q(x,y) \wedge \exists x R(x,y))$

b. $\forall y(\neg Q(x,y) \vee \exists x \neg R(x,y))$

c. $\forall y(\neg Q(x,y) \vee \forall x R(x,y))$

d. $\forall y(\neg Q(x,y) \vee \exists x R(x,y))$

# Exercises

3. Which statements are correct?

Seclect one:

a. $\forall x(P(x) \to Q(x))$ and $\forall x P(x) \to \forall x Q(x)$ have the same truth values

b. $\forall x(P(x) \vee Q(x))$ and $\forall x P(x) \vee \forall x Q(x)$ have the same truth values

c. $\forall x(P(x) \wedge Q(x))$ and $\forall x P(x) \wedge \forall x Q(x)$ have the same truth values.

# Exercises

3. Which statements are correct?

Seclect one:

a. $\forall x(P(x) \to Q(x))$ and $\forall x P(x) \to \forall x Q(x)$ have the same truth values

b. $\forall x(P(x) \vee Q(x))$ and $\forall x P(x) \vee \forall x Q(x)$ have the same truth values

c. $\forall x(P(x) \wedge Q(x))$ **and** $\forall x P(x) \wedge \forall x Q(x)$ **have the same truth values.**