FPT UNIVERSITY

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Chapter 2: Basic Structures: Sets, Functions, Sequences, and Sum

DISCRETE MATHEMATIS

Lecture: Dr. Nguyen Kieu Linh

Hanoi, 2020

Content

Content

2.1 Sets

- An unordered collection of objects
- The objects in a set are called the elements, or members. A set is said to contain its elements.
- Some important sets in discrete mathematics

$$\begin{aligned}
\mathbb{N} &= \{0, 1, 2, 3, \dots\} \\
\mathbb{Z} &= \{\dots, -2, -1, 0, 1, 2, \dots\} \\
\mathbb{Q} &= \left\{ r = \frac{p}{q} : p \in \mathbb{Z}, 0 \neq \in \mathbb{Z} \right\} \\
\mathbb{R} &= \text{the set of real numbers} \\
V &= \{o, e, u, a, i\}
\end{aligned}$$

2.1 Sets

Definition

- **Finite set**: Set has n element, n is a nonnegative interger.
- A set is an **infinite** set if is not finite
- **Cardinality** of a set $|S|$: Number of elements of S.
- $\emptyset$: **empty set (null set)**, the set with no element.
- Two sets are **equal** if and only if they have the same elements
  $A = B$ if and only if $\forall x(x \in A \leftrightarrow x \in B)$
- $A \subseteq B$: The set A is a **subset** of the set B
  $A \subseteq B$ if and only if $\forall x(x \in A \rightarrow x \in B)$
- $A \subset B$: The set A is a **proper subset** of the set B
  $A \subset B$ if and only if $(A \subseteq B) \wedge (A \neq B)$.

Power set

- Given a set S, **power set** $P(S)$ of S is a set of all subsets of the set S.

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- Exemple: $S = \{1, 2, 3\}$
- Then $P(S) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}\{2, 3\}, \{1, 2, 3\}\}$.

Cartesian Products

- The ordered $n-$tuple $(a_1, a_2, \ldots, a_n)$ is the ordered collection that has a_1 as its first element, a_2 as its second element, $\ldots$, and a_n as its n^{th} element.
- Let A and B be sets. The Cartesian product of A and B, denoted by $A \times B$,
$$A \times B = \{(a, b) | a \in A, b \in B\}$$
- Exemple 1: $A = \{a, b\}; B = \{1, 2, 3\}$

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- Then $A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$.
- The Cartesian product of $A_1, A_2, \ldots, A_n$, denoted $A_1 \times A_2 \times \ldots A_n$, is the set of ordered $n-$tuples$(a_1, a_2, \ldots, a_n)$,
$$A_1 \times A_2 \times \ldots \times A_n = \{(a_1, a_2, \ldots, a_n) | a_i \in A_i, i = 1, 2, \ldots, n\}$$
- Exemple 2: $A = \{a, b\}; B = \{1, 2, 3\}, C = \{@, *\}$

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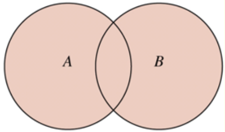
- Exemple 2: $A = \{a, b\}; B = \{1, 2, 3\}, C = \{@, *\}$
- Then $A \times B \times C = \{(a, 1, @), (a, 1, *), (a, 2, @), (a, 2, *), (a, 3, @), (a, 3, *), (b, 1, @), (b, 1, *), (b, 2, @), (b, 2, *), (b, 3, @), (b, 3, *)\}$.

Content

2.2 Set operations

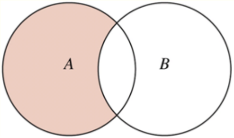
- The Union of sets A and B, denoted by $A \cup B$

$$A \cup B = \{x | x \in A \vee x \in B\}$$



- The difference of A and B, denoted by $A - B$

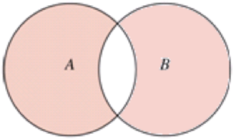
$$A - B = \{x | x \in A \wedge x \notin B\}$$



2.2 Set operations

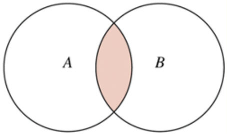
- The symmetric difference of A and B, denoted by $A \oplus B$

$$A \oplus B = A \cup B - A \cap B = \{x | (x \in A \vee x \in B) \wedge (x \notin A \cap B)\}$$



- The intersection of A and B

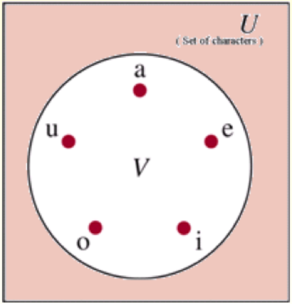
$$A \cap B = \{x | x \in A \wedge x \in B\}$$



2.2 Set operations

- U is the universal set, complement of A is denoted by $\bar{A}$

$$\bar{A} = U - A = \{x | x \notin A\}$$



Set Identities

Identity – See proofs : pages 125, 126	Name
$A \cup \emptyset = A$ $\quad$ $A \cap \mathbf{U} = \mathbf{A}$	Identity laws - Luật đồng nhất
$A \cup \mathbf{U} = \mathbf{U}$ $\quad$ $A \cap \emptyset = \emptyset$	Domination laws - Luật thống trị
$A \cup A = A$ $\quad$ $A \cap A = A$	Idempotent laws – Luật bất biến
$\overline{\overline{A}} = A$	Complementation law – Luật bù đôi
$A \cup B = B \cup A$ $\quad$ $A \cap B = B \cap A$	Commutative laws – Luật giao hoán
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws – Luật kết hợp
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive laws Luật phân phối
$\overline{A \cup B} = \overline{A} \cap \overline{B}$ $\quad$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$	De Morgan laws
$A \cup (A \cap B) = A$ $\quad$ $A \cap (A \cup B) = A$	Absorption – Luật hấp phụ
$A \cup \overline{A} = U$ $\quad$ $A \cap \overline{A} = \emptyset$	Complement laws – Luật bù

Computer Representation of Sets

- Use bit string $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- $A = \{1, 3, 5, 7, 9\}$

Computer Representation of Sets

- Use bit string $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- $A = \{1, 3, 5, 7, 9\} \to A =$ "$1010\ 1010\ 10''$
- $B = \{1, 8, 9\}$

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- $A \cup B$

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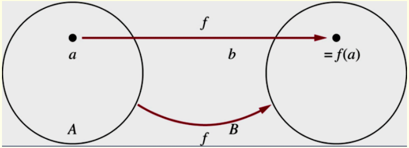
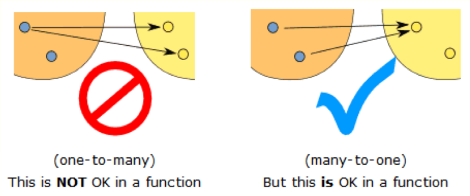
- $A \cap B = 1010\ 1010\ 10 \wedge 1000\ 0001\ 10 = 1000\ 0000\ 10$
  $\to A \cap B = \{1, 9\}$.

Content

2.3 Functions, Mapping, Transformation - Ánh Xạ

- $f : A \to B$ function f from A to B (or function f maps A to B)
- A: domain of f
- B: codomain of f





2.3 Functions, Mapping, Transformation

What are functions?

- $f : \mathbb{Z} \to \mathbb{R} : f(x) = x^2 + 2$
- $f : \mathbb{Z} \to \mathbb{R} : f(x) = \dfrac{1}{(x-1)^2} + 5x$
- $f : \mathbb{R} \to \mathbb{R} : f(x) = \dfrac{(2x+5)}{7}$
- $f : \mathbb{Z} \to \mathbb{R} : f(x) = \dfrac{(2x+5)^2}{7-2x}$

Some Important Functions

- Floor function: $f : \mathbb{R} \to \mathbb{Z}$ such that $f(x) = \lfloor x \rfloor$ = largest integer that less than or equal to x.

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- Example: $\lceil 4,3 \rceil = 5, \lceil -4,3 \rceil = -4$

One-to-One/ Injective functions (đơn ánh)

- Function f is one-to-one (or injective) if and only if $a \neq b \rightarrow f(a) \neq f(b)$ for all a, b in the domain f.
- Example: $f : \mathbb{Z} \rightarrow \mathbb{Z} : f(x) = x^2$

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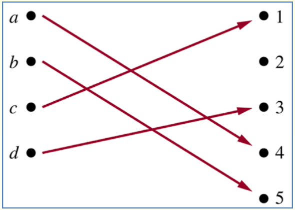
- Example: $f : \mathbb{Z} \rightarrow \mathbb{Z} : f(x) = x^2$ is not one-to-one.



Onto Functions - Ánh xạ trên (toàn ánh)

- A function f from A to B is called onto, or surjective, iff for every element b in B there is an element a in A with $f(a) = b$.
- Example: $f : \mathbb{Z} \to \mathbb{Z} : f(m) = m - 1$

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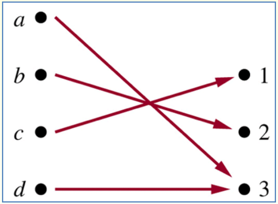
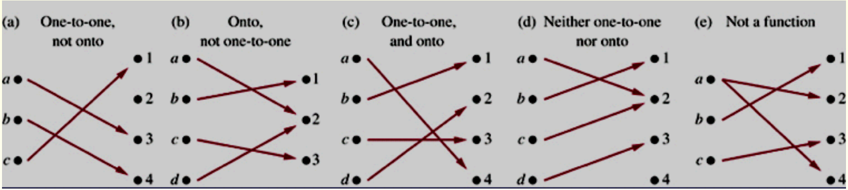
- Example: $f : \mathbb{Z} \to \mathbb{Z} : f(m) = m - 1$ is onto because $\forall y \in \mathbb{Z}, y = f(m) = m - 1$, where $m = y + 1$.

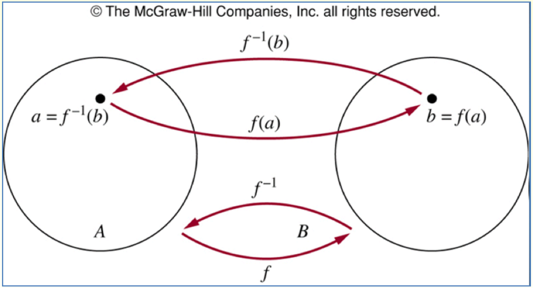


One-to-one correspodent/Bijective function (song ánh)

- Function f is a one-to-one corespondence or a bijection if it is both one-to-one and onto.



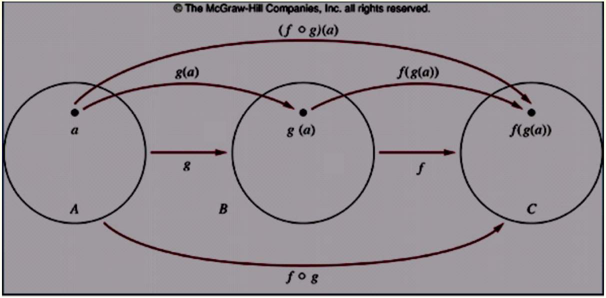
- Let f is a bijection from A to B. The inverse function, denoted by f^{-1}, of f is the function that assigns to an element b belonging to B the unique element a in A such that $f(a) = b$. Hence $f^{-1}(b) = a$ when $f(a) = b$.



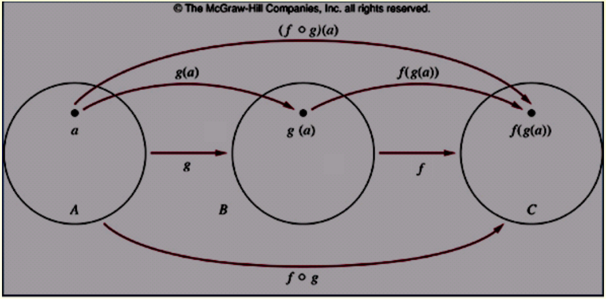
Composition of Functions – Ánh xạ hợp

- Let $g : A \to B; f : B \to C$.
  The composition of f and g, denoted by $f \circ g$, is defined by:

$$(f \circ g)(x) = f(g(x))$$



Composition of Functions – Ánh xạ hợp



- Let $f : \mathbb{Z} \to \mathbb{Z}, f(x) = x + 1$
- Let $g : \mathbb{Z} \to \mathbb{Z}, g(x) = x^2$
- $(f \circ g)(x) = f(g(x)) = f(x^2) = x^2 + 1.$
- $(g \circ f)(x) = g(f(x)) = g(x + 1) = (x + 1)^2.$

Exercises

Exercise 1

Let A, B be sets. The statement

$$A \cup (B \cap \bar{A}) = A \cup B$$

is True of False?

Select one:

A. True

B. False

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Select one:

A. **True**

B. False

Exercises

Exercise 2

Find the cardinality of the set $\{a, \{a\}, \{a, \{a\}\}\}$.

A. 3

B. 4

C. 5

D. 6

Exercises

Exercise 2

Find the cardinality of the set $\{a, \{a\}, \{a, \{a\}\}\}$.

A. **3**

B. 4

C. 5

D. 6

Exercise 3

Let $A = \{1, 2, 4, 6, 7, 9, 8\}, B = \{3, 1, 5, 7, 6\}$. Which set has the maximum cardinality?

Select one:

A. $B - A$

B. $A \cup B$

C. $A - B$

D. $A \cap B$

Exercises

Exercise 4

Let $A = \{0, a\}, B = \{0, b\}$. Determine $B \times A$. Select one:

A. $\{(0,0),(a,b)\}$

B. $\{(0,0),(0,b),(a,0),(a,b)\}$

C. $\{(0,0),(0,b),(a,0),(a,b),(b,a),(0,a),(b,0)\}$

D. $\{(0,0),(b,a),(0,a),(b,0)\}$

Exercise 5

Let $f(X) = 5X + 4, g(X) = 4X + 3$. Suppose that $g \circ f(X) = aX + b$. Find b.

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Exercise 7

Compute

$$\lfloor \frac{3}{2} - \lceil 3 + \frac{5}{4} \rceil \rfloor \text{ and } \lfloor \left(\frac{3}{2}\right)^2 \rfloor - \left(\lfloor \frac{7}{2} \rfloor\right)^2$$

Exercises

Exercise 8

Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}, f(m, n) = n + 1$. Choose correct answer:

A. $f(x)$ is neither one-to-one nor onto

B. f is onto but not one-to-one

C. f is a bijection

D. f is one-to-one but not onto.

Content

Definition

A sequence is a function from a subset of the set of integers to a set S.We use the notation a_n to denote the image of the integer n. We call a_n a *term* of the sequence.

- We use the notation $\{a_n\}$ to describe the sequence.
- a_n is called the general term of $\{a_n\}$.

Example

Consider the sequence $\{a_n\}$, where $a_n = \dfrac{1}{n}$. The list of the terms of this sequence, beginning with a_1, namely,

$$a_1 = 1; a_2 = \frac{1}{2}; a_3 = \frac{1}{3}, \dots$$

Definition

A *geometric progression* is a sequence of the form $a, ar, ar^2, ..., ar^n, ...$ where the *initial term* a and the *common ratio* r are real numbers..

Example

Consider the sequence $\{b_n\}$, where $b_n = 2\left(\frac{1}{3}\right)^n$.

2.4.1. Sequences

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Example

Consider the sequence $\{b_n\}$, where $b_n = 2\left(\frac{1}{3}\right)^n$.

$\{b_n\}$ is a geometric progression, beginning with $b_0 = 2$ and the common ratio is $r = \frac{1}{3}$.

Definition

An *arithmetic progression* is a sequence of the form $a, a+d, a+2d, ..., a+nd, ...$ where the *initial term* a and the *common difference* d are real numbers.

Example

Consider the sequence $\{c_n\}$, where $c_n = -1 + 4n$.

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$\{c_n\}$ is a arithmetic progression, beginning with $c_0 = -1$ and the common ratio is $d = 4$.

2.4.2. Recurrence Relations (quan hệ truy hồi)

Definition

A *recurrence relation* for the sequence $\{a_n\}$ is an equation that expresses a_n in terms of one or more of the previous terms of the sequence, namely, $a_0, a_1, ..., a_{n-1}$, for all integers n with $n \geq n_0$, where n_0 is a nonnegative integer.

Example

- Consider the sequence $\{a_n\}$, where $a_n = a_{n-1} + 3$ with $n = 1, 2, ...$ and $a_0 = 2$.

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2.4.3. Hints for deducing a possible formula for the terms of a sequence

Example

- Are there runs of same value?

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- Are terms obtained by combining previous terms in a certain way?
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- Are they cycles among terms?

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- 1 7 25 79 241 727 .... $\rightarrow a_n = 3^n - 2, n = 0, 1, 2, 3, ...$
- 3 4 6 9 13 18 24 .... $\rightarrow a_n = \dfrac{n^2 - n + 6}{2}, n = 1, 2, 3, ...$

2.4.4. Summations

Summation notation

- $a_m + a_{m+1} + a_{m+2} + ... + a_n = \sum_{j=m}^{n} a_j = \sum_{j=m}^{n} a_j = \sum_{m \le j \le n} a_j$,
  where a : Sequence; j : Index of summation; m: Lower limit; n : Upper limit.

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Example:

$$\sum_{1 \le j \le 5} j^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55.$$

2.4.4. Summations

Sum	Closed Form
$\sum_{k=0}^{n} ar^k \ (r \neq 0)$	$\frac{ar^{n+1} - a}{r - 1}, r \neq 1$
$\sum_{k=1}^{n} k$	$\frac{n(n+1)}{2}$
$\sum_{k=1}^{n} k^2$	$\frac{n(n+1)(2n+1)}{6}$
$\sum_{k=1}^{n} k^3$	$\frac{n^2(n+1)^2}{4}$
$\sum_{k=0}^{\infty} x^k, \lvert x \rvert < 1$	$\frac{1}{1-x}$
$\sum_{k=1}^{\infty}, kx^{k-1}, \lvert x \rvert < 1$	$\frac{1}{(1-x)^2}$

2.4.5. Cardinality – Lực Lượng

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- For example, $|\mathbb{N}| = \aleph_0$ because $\mathbb{N}$ is countable and infinite but $\mathbb{R}$ is uncountable and infinite, and we say $|\mathbb{R}| = 2^{\aleph_0}$.

2.4.5. Cardinality – Lực Lượng

sets	countable	uncountable	cardinality
{a, b, …, z}, $\{x \mid x^5 - 3x^2 - 11 = 0\}$, …	✓	✗	$< \aleph_0$
{0, 2, 4, …, }	✓	✗	$\aleph_0$
N, Z^+, Z, Q, …	✓	✗	$\aleph_0$
$\{x \mid 0 < x < 1\}$, R,…	✗	✓	$2^{\aleph_0}$