BIOMEDICAL ENGINEERING FORMULAS

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A. ENGINEERING MATHEMATICS

1. Convergence and Divergence of a Series

- $\sum_{n=1}^{\infty} u_n$ is convergent ⇔ The sum S_n is up bounded
- $0 < u_n \leq v_n \forall n \geq N$:

  If $\sum_{n=1}^{\infty} u_n$ is divergent, then $\sum_{n=1}^{\infty} v_n$ is divergent

  If $\sum_{n=1}^{\infty} v_n$ is convergent, then $\sum_{n=1}^{\infty} u_n$ is convergent
- D'Alembert: $\lim\limits_{n \to \infty} \frac{u_{n+1}}{u_n} < 1$: convergent

  > 1: divergent

  $= 1$: unidentified
- Cauchy: $\lim\limits_{n \to \infty} \sqrt[n]{u_n} < 1$: convergent

  > 1: divergent

  $= 1$: unidentified
- Other forms:

  $\sum_{n=1}^{\infty} \frac{1}{n}$: divergent

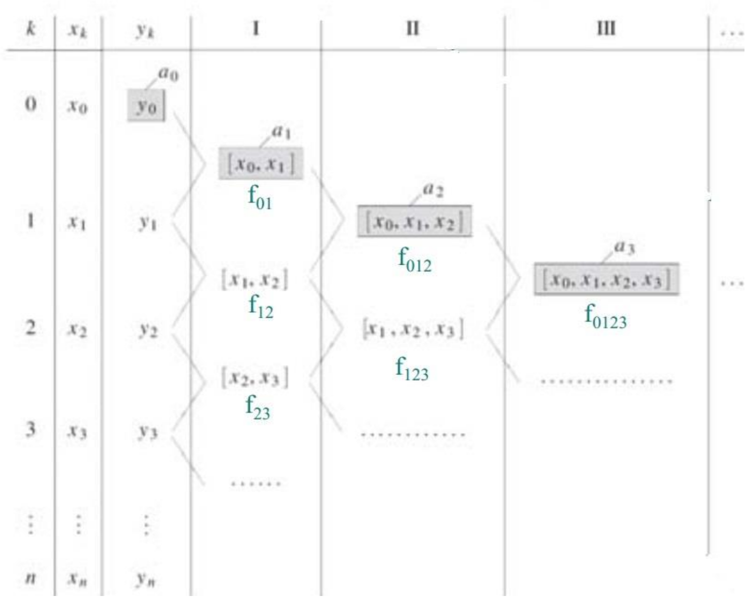
  $\sum_{n=1}^{\infty} \frac{1}{n^2}$: convergent

  $\sum_{n=1}^{\infty} (-1)^n . \frac{1}{n}$: convergent

  $\sum_{n=0}^{\infty} \frac{x^n}{n!}$: convergent for all x

2. Interpolation

- $f_{ij} = \frac{y_j - y_i}{x_j - x_i}$
- $p(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \ldots.$



3. Lim L'Hospital

$$\lim_{x \to x_0} f(x) = \lim_{x \to x_0} g(x) = 0 \to \lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)} = \frac{f'(x)}{g'(x)} \ (g'(x_0) \neq 0)$$

Ausdruck	Typ	Umformung
$\frac{1}{f(x)} - \frac{1}{g(x)}$	$\infty - \infty$	$\frac{g(x)-f(x)}{f(x) \cdot g(x)}$
$f(x) \cdot g(x)$	$0 \cdot \infty$	$\frac{g(x)}{\frac{1}{f(x)}}$
$f(x)^{g(x)}$	1^{∞}	$e^{g(x) \cdot \ln(f(x))} = e^{\frac{\ln(f(x))}{\frac{1}{g(x)}}}$

4. Taylor Series and Maclaurin Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} . (x - x_0)^n = a_n (x - x_0)^n$$

Convergence Radius: $\mathrm{r} = \lim_{\mathrm{n} \to \infty} \left| \frac{\mathrm{a_n}}{\mathrm{a_{n+1}}} \right|$

$$f(x) = \sum_{n=0}^{m} a_n (x - x_0)^n + R_m(x) = f_m(x) + R_m(x)$$

$\mathrm{y} = \mathrm{f_m}(\mathrm{x})$ is the m-th degree equation which is the most approximate function for y = f(x)

Maclaurin Series is a special case of Taylor Series with $\mathrm{x_0} = 0$

5. Integral

5.1. Improper Integral

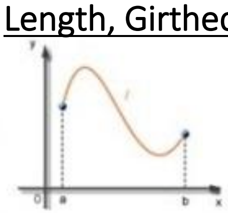
$$\int_a^{\infty} f(x)\, dx = \lim_{\lambda \to \infty} I(\lambda) = \lim_{\lambda \to \infty} \int_a^{\lambda} f(x)\, dx$$

$$\int_{-\infty}^{\infty} f(x) dx = \lim_{\lambda \to \infty} \lim_{\mu \to -\infty} \int_{\mu}^{\lambda} f(x) dx$$

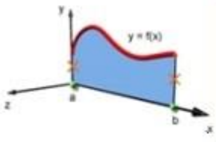
$$\int_a^{b} f(x)\, dx = \lim_{\lambda \to 0} I(\lambda) = \lim_{\lambda \to 0} \int_a^{b-\lambda} f(x)\, dx$$

(When f(x) is unidentified on [a;b])

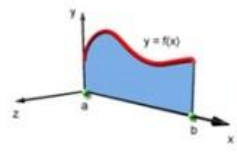
5.2. Length, Girthed Area and Volume with Integral



$$l = \boxed{L_a^b(f) = \int_a^b \sqrt{1 + (f'(x))^2} dx}$$



$$M = \int dM = 2\pi \int_a^b f(x) \sqrt{1 + f'(x)^2} \, dx$$



$$V = \pi \int_a^b f(x)^2 dx$$

5.3. Line Integral

$$\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) \, dt$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_C (F_x(x; y) \, dx + F_y(x; y) \, dy)$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_{t_1}^{t_2} (\vec{F} \cdot \dot{\vec{r}}) \, dt = V(\overrightarrow{E}) - V(\overrightarrow{A})$$

5.4. Multiple Integral

$$\int_{x_0}^{x_1} \int_{y_0}^{y_1} f(x,y) dy dx = \int\int_R f \, dA = \iint_R f(r, \varphi) \, dA = \int_{\varphi = a}^{b} \int_{r = 0}^{r(\varphi)} f(r, \varphi) \, r \, dr \, d\varphi$$

$$\int f \, \mathrm{d}(x,y,z) = \int_r \int_\varphi \int_\vartheta f(r, \varphi, \vartheta) \cdot r^2 \cdot \sin\vartheta \;\; \mathrm{d}\vartheta \, \mathrm{d}\varphi \, \mathrm{d}r$$