

ASSIGNMENT - GROUP 8

NOTES

- (i) DEADLINE: **12:00 Fri. July 23, 2021**
- (ii) The report is named as a file **ASSIGNMENT-Nxx.PDF** where xx is the group number. Do not zip this file.
- (iii) The report must include the names and duties of the team members.
- (iv) Send your assignment report to the following two email addresses (to avoid loss)

nan.thanh@lethai.vn and **thanh@lethai.edu.vn**

Consider the problem of the linear differential equation of second order

$$\begin{cases} y''(x) + \frac{2x^3 + 1}{2x + 3} y'(x) - (7x + 15) \ln(2x + 5) y(x) = -\frac{6x^3 + 5x + 7}{3x + 2} e^x, & 1 \leq x \leq 2 \\ y(1) = 1; \quad y(2) = 0 \end{cases}$$

1. Use the Finite Difference Method to approximate the function $y(x)$ in the closed interval $[1, 2]$ with the step size $h = \frac{1}{20}$ ($n = 20$). Sketch the graph of $y(x)$.
2. Use the results in Question 1 and the Simpson Formula to approximate the integral

$$I = \int_1^2 y(x) dx$$

with the number of subintervals n is as in Question 1.

3. Use the results in Question 1 at the nodes $x_k = 1 + kh$, $k = 0, 1, \dots, n$ to construct a natural cubic spline $g(x) \approx y(x)$ in $[1, 2]$. Sketch the graph of $g(x)$.
4. Use the natural cubic spline in Question 3 to calculate the integrals

$$I_k = \int_{x_k}^{x_{k+1}} g_k(x) dx, \quad k = 0, 1, \dots, n-1$$

Then calculate the sum $J = \sum_{k=0}^{n-1} I_k$ and compare with the result in Question 2. State your comment.