ASSIGNMENT - GROUP 8

Consider the problem of the linear differential equation of second order

$$\begin{cases} y''(x) + \dfrac{2x^3 + 1}{2x + 3} y'(x) - (7x + 15)\ln(2x + 5)y(x) = -\dfrac{6x^3 + 5x + 7}{3x + 2}\,\mathrm{e}^x, & 1 \leqslant x \leqslant 2 \\ y(1) = 1; \quad y(2) = 0 \end{cases}$$

1. Use the Finite Difference Method to approximate the function $y(x)$ in the closed interval $[1, 2]$ with the step size $h = \dfrac{1}{20}$ $(n = 20)$. Sketch the graph of $y(x)$.

2. Use the results in Question 1 and the Simpson Formula to approximate the integral

$$I = \int\limits_1^2 y(x)\, dx$$

with the number of subintervals n is as in Question 1.

3. Use the results in Question 1 at the nodes $x_k = 1 + kh, \ k = 0, 1, \ldots, n$ to construct a natural cubic spline $g(x) \approx y(x)$ in $[1, 2]$. Sketch the graph of $g(x)$.

4. Use the natural cubic spline in Question 3 to calculate the integrals

$$I_k = \int\limits_{x_k}^{x_{k+1}} g_k(x)\, dx, \ k = 0, 1, \ldots, n - 1$$

Then calculate the sum $J = \sum\limits_{k=0}^{n-1} I_k$ and compare with the result in Question 2. State your comment.