HO CHI MINH CITY UNIVERSITY OF TECHNOLOGY

OFFICE FOR INTERNATIONAL STUDY PROGRAMS

REPORT ASSIGNMENT

NUMERICAL METHODS

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Assignment – Group 8		

Consider the problem of the linear differential equation of second order

$$\begin{cases} y''(x) + \dfrac{2x^3+1}{2x+3}y'(x) - (7x+15)\ln(2x+5)y(x) = -\dfrac{6x^3+5x+7}{3x+2}e^x, & 1 \leqslant x \leqslant 2 \\ y(1) = 1; \quad y(2) = 0 \end{cases}$$

1. Use the Finite Difference Method to approximate the function $y(x)$ in the closed interval $[1, 2]$ with the step size $h = \dfrac{1}{20}$ $(n = 20)$. Sketch the graph of $y(x)$.

2. Use the results in Question 1 and the Simpson Formula to approximate the integral

$$I = \int_1^2 y(x)\,dx$$

with the number of subintervals n is as in Question 1.

3. Use the results in Question 1 at the nodes $x_k = 1 + kh$, $k = 0, 1, \ldots, n$ to construct a natural cubic spline $g(x) \approx y(x)$ in $[1, 2]$. Sketch the graph of $g(x)$.

4. Use the natural cubic spline in Question 3 to calculate the integrals

$$I_k = \int_{x_k}^{x_{k+1}} g_k(x)\,dx,\ k = 0, 1, \ldots, n-1$$

Then calculate the sum $J = \sum\limits_{k=0}^{n-1} I_k$ and compare with the result in Question 2. State your comment.

Name	Workload
Nguyễn Duy Thành	Report + Formatting
Võ Duy Thành	Matlab Code
Lê Ngọc Thành	Report + Formatting
Nguyễn Đô Trưởng	Formula + Report

\+ In order to solve functions and sketch graphs, we use Matlab (version may vary depends on each individual personal computer). The following code entries will show the detailed functions of each assignments that are executed in Matlab (complete program will be put in the end of report). The solving method involved many formulas and methods, which includes:

1) Finite difference method
2) Simpson formula
3) Natural cubic spline construction

- **Question 1**: Finite Difference Method

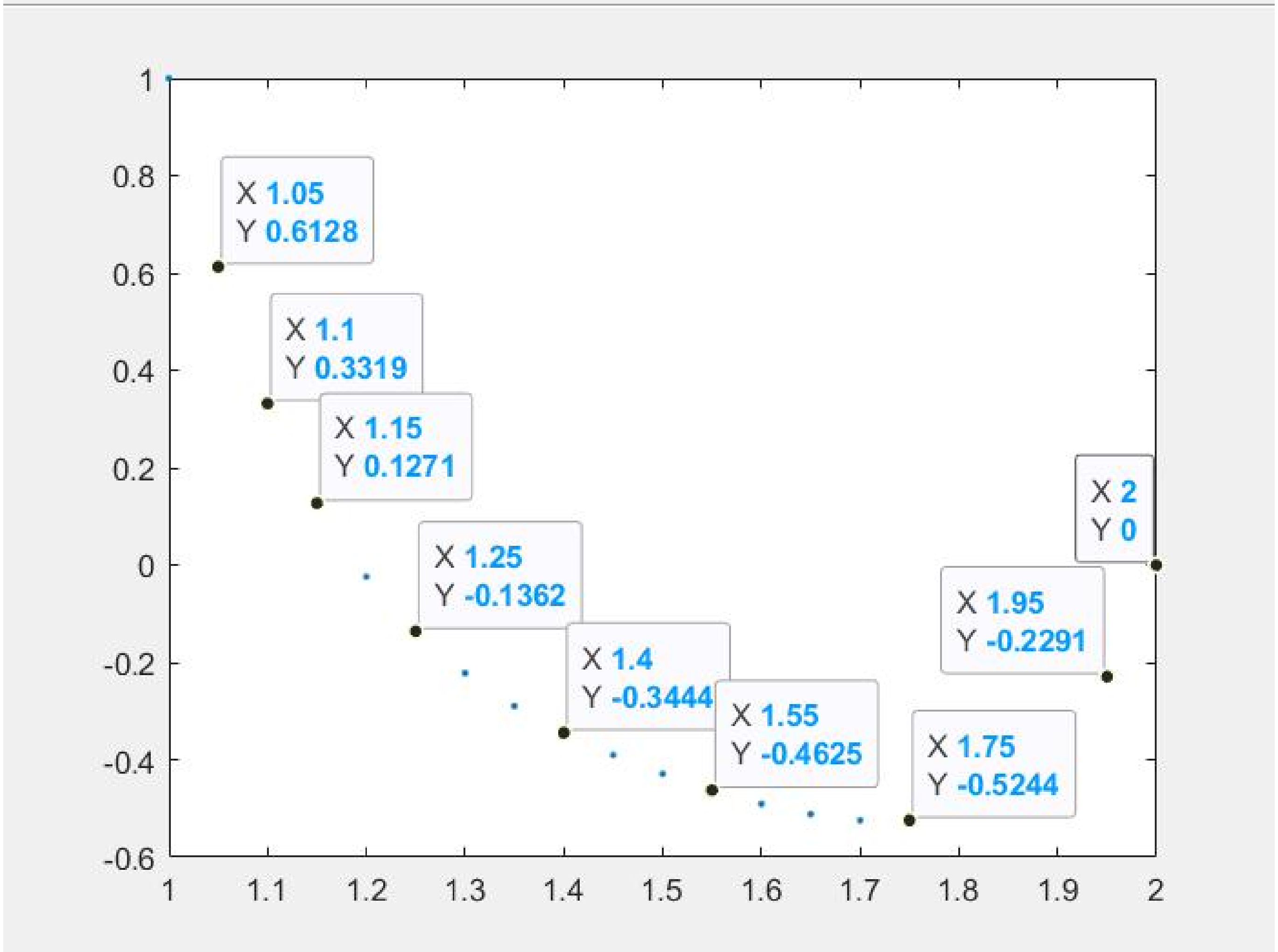
```
%test 1
x = 1:1/20:2;
%Functions settings and work environment
%Rearrange functions and equations

X = x;
B = (6*x.^3 + 5*x + 7)./(3*x + 2).*exp(x);
U = (2*x.^3 + 1)./(2*x + 3);
L = -(7*x + 15).*log(2*x + 5);
h = 1/20; %dentax
A = zeros(length(x));

A(1,1)        = 1;
A(end,end)    = 1;
B(1)          = 1;
B(end)        = 0;

 for i = 2:length(x)-1
     A(i,i)    = -2/h^2 + U(i)/h +L(i);
     A(i,i-1)  = 1/h^2 - U(i)/h;
     A(i,i+1)  = 1/h^2;
 end
 Y = inv(A)*B';
```

```
plot(x,Y,'.')
disp('TEST 1')
Y
```



Output:

Test 1:

N	1	2	3	4	5	6	7
X	1	1.05	1.1	1.15	1.2	1.25	1.3
Y	1	0.6128	0.3319	0.1271	-0.0236	-0.1362	-0.2222

N	8	9	10	11	12	13	14
X	1.35	1.4	1.45	1.5	1.55	1.6	1.65
Y	-0.2897	-0.3444	-0.3900	-0.4290	-0.4625	-0.4905	-0.5119

N	15	16	17	18	19	20	21
X	1.7	1.75	1.8	1.85	1.9	1.95	2
Y	-0.5245	-0.5244	-0.5057	-0.4596	-0.3735	-0.2291	0

- **<u>Question 2</u>**: Simpson Formula

```
%Apply Simpson formula for problem 2%
 disp('Test 2')
I =
   (h/3)*(  ( Y(1)  + Y(end))
     + 4*( Y(2)  + Y(4)  + Y(6)  + Y(8)  + Y(12)  +
   Y(14)  + Y(16)  + Y(18)  + Y(20)  )
     + 2*(Y(3)  + Y(5)  + Y(7)  + Y(9)  + Y(11)  +
   Y(13)  + Y(15)  + Y(17)  + Y(19)))
```

Test 2:

```
 I = - 0.1943
```

- **<u>Question 3</u>**: Natural Cubic Spline Construction

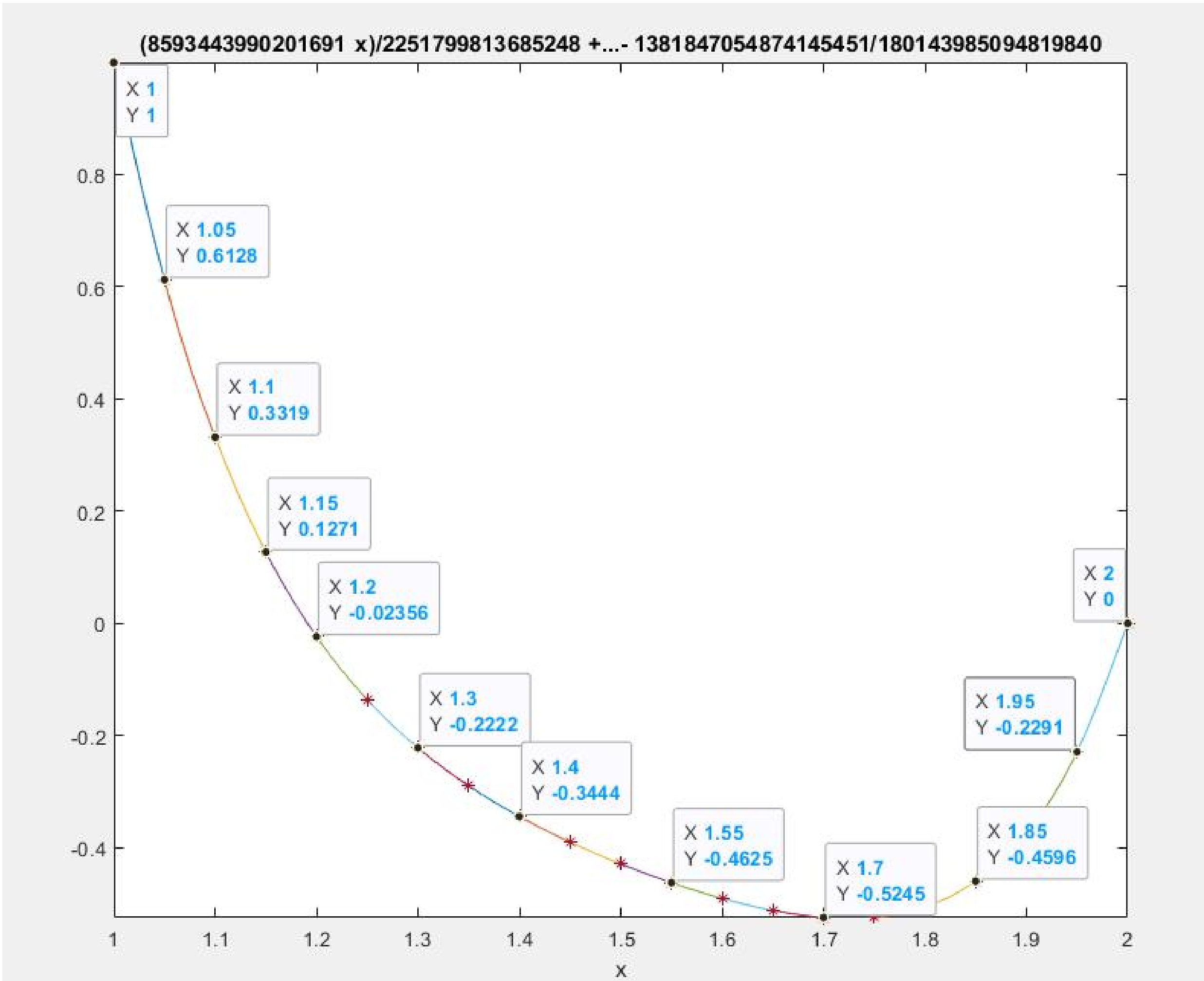
```
%Construct a natural cubic spline
 disp('Test 3')
 f = Spline_bac_3_tu_nhien(x,Y)
%%
 %Graph sketch for y(x) in problem 1
 function f = Spline_bac_3_tu_nhien(x,y)
plot(x,y,'.')
hold on
% x = input('nhap x:')
% y = input('nhap y:');
 h = diff(x); % h (j) = x(j+1) -x(j)
 a = y; % a(j) = y(j);
% d = diff(c)/(3*h); % d(j) = (c(j+1)-
c(j))/(3h(j))
% b = diff(a)/h - h(c+2*c)/3

%Natural Cubic Spline graph sketch for g(x)
based on calculated y(x)nodes
n       = length(x);
A       =  zeros(n);
A(1,1) = 1;
A(n,n) = 1;
j = 1;
for i = 2 : n-1
    A(i,i)    = 2*(h(j) + h(j+1));
    A(i,i-1) = h(j);
    A(i,i+1) = h(j+1);
    j = j+1;
end
% B
v = diff(a);
B = zeros(n,1);
for i = 2 : length(B) - 1
```

```
    B(i) = 3/h(i) * v(i) - 3/h(i-1) * v(i-1);
end
c = inv(A) * B;
m = diff(c);
for i =1 : length(c)-1
d(i) = m(i) / (3*h(i));
end
for j = 1 : length(a)-1
    b(j) =
(a(j+1)-a(j))/h(j)-h(j) *( c(j+1) + 2*c(j))/3;
end

X = x;
syms x
a = double(a);
b = double(b);
c = double(c);
d = double(d);
for i = 1 : length(d)
    f(i)= a(i) + b(i)*(x - X(i)) + c(i) *(x -
X(i))^2 + d(i)*(x - X(i) )^3;
    ezplot(f(i), [X(i) X(i+1)])
end
plot(X,y,'*')
axis([min(X) max(X) min(y) max(y)])
f = f.';
```

Test 3:



y(x) $\approx$ g(x) =

- $k = 0$;[1, 1.05]

$$g(x) = \frac{3363757302804821 * \left(c - \frac{21}{20}\right)^3}{17592186044416} - \frac{4628621916743865 * x}{562949953421312} + \frac{5191571870165177}{562949953421312}$$

- $k = 1$;[1.05, 1.1]

$$g(x)=\frac{4036508763365789*\left(c-\frac{21}{20}\right)^2}{140737488355328}-\frac{1910660082035353*x}{281474976710656}-\frac{7418221893101037*\left(x-\frac{21}{20}\right)^3}{70368744177664}+\frac{34858804725565529}{45035996273704960}$$

- $k=2$;[1.1, 1.15]

$$g(x)=\frac{452760548858869*\left(c-\frac{11}{10}\right)^2}{35184372088832}-\frac{662952493077613*x}{140737488355328}-\frac{6367276398399323*\left(x-\frac{11}{10}\right)^3}{562949953421312}+\frac{62076914277476929}{11258999068426240}$$

- $k=3$;[1.15, 1.2]

$$g(x)=\frac{6289077321982009*\left(c-\frac{23}{20}\right)^2}{562949953421312}-\frac{7900590668497035*x}{2251799813685248}-\frac{903001352379943*\left(x-\frac{23}{20}\right)^3}{35184372088832}+\frac{74975954410056941}{18014398509481984}$$

- $k=4$;[1.2, 1.25]

$$g(x)=\frac{64404282441721*\left(c-\frac{6}{5}\right)^2}{8796093022208}-\frac{2909200194423301*x}{1125899906842624}-\frac{969406063922435*\left(x-\frac{6}{5}\right)^3}{70368744177664}+\frac{4434580945969522131}{1441151880758558720}$$

- $k = 5$;[1.25, 1.3]

$$g(x) = \frac{2958586799563221 * \left(c - \frac{5}{4}\right)^2}{562949953421312} - \frac{550288526709991 * x}{281474976710656} - \frac{6198518062068221 * \left(x - \frac{5}{4}\right)^3}{562949953421312} + \frac{41570069005008379}{18014398509481984}$$

- $k = 6$;[1.3, 1.35]

$$g(x) = \frac{2028809090252987 * \left(c - \frac{13}{10}\right)^2}{562949953421312} - \frac{3404829035716685 * x}{2251799813685248} - \frac{8255244024345293 * \left(x - \frac{13}{10}\right)^3}{1125899906842624} + \frac{62816356851814871}{36028797018963968}$$

- $k = 7$;[1.35, 1.4]

$$g(x) = \frac{2819331576854179 * \left(c - \frac{27}{20}\right)^2}{1125899906842624} - \frac{5434268119961339 * x}{4503599627370496} - \frac{1406151638568075 * \left(x - \frac{27}{20}\right)^3}{281474976710656} + \frac{6031716731782503 9}{45035996273704960}$$

- $k = 8$;[1.4, 1.45]

$$g(x) = \frac{1975640593713337 * \left(c - \frac{7}{5}\right)^2}{1125899906842624} - \frac{8950547371695677 * x}{9007199254740992} - \frac{6881005407523147 * \left(x - \frac{7}{5}\right)^3}{2251799813685248} + \frac{94290110730300193}{90071992547409920}$$

- $k = 9$;[1.45, 1.5]

$$g(x) = \frac{2919130376298201 * \left(c - \frac{29}{20}\right)^2}{2251799813685248} - \frac{7576465058950701 * x}{9007199254740992} - \frac{3355973497816367 * \left(x - \frac{29}{20}\right)^3}{2251799813685248} + \frac{149453280574406179}{180143985094819840}$$

- $k = 10$;[1.5, 1.55]

$$g(x) = \frac{4831468703251491 * \left(c - \frac{3}{2}\right)^2}{4503599627370496} - \frac{6509492113365911 * x}{9007199254740992} - \frac{2399763083349491 * \left(x - \frac{3}{2}\right)^3}{36028797018963968} + \frac{11799435854714395}{18014398509481984}$$

- $k = 11$;[1.55, 1.6]

$$g(x) = \frac{149577285794959 * \left(c - \frac{31}{20}\right)^2}{140737488355328} - \frac{1386924482124223 * x}{2251799813685248} - \frac{6130795431567375 * \left(x - \frac{31}{20}\right)^3}{4503599627370496} + \frac{22165096458650603}{45035996273704960}$$

- $k = 12$;[1.6, 1.65]

$$g(x) = \frac{5706092460173795 * \left(c - \frac{8}{5}\right)^2}{4503599627370496} - \frac{8996882735871285 * x}{18014398509481984} + \frac{3395640622128107 * \left(x - \frac{8}{5}\right)^3}{1125899906842624} + \frac{5559347718834413}{18014398509481984}$$

- $k = 13$;[1.65, 1.7]

$$g(x) = \frac{1935869208362663 * \left(c - \frac{33}{20}\right)^2}{1125899906842624} - \frac{6306968877146407 * x}{18014398509481984} + \frac{5573433713062965 * \left(x - \frac{33}{20}\right)^3}{1125899906842624} + \frac{23696841155687191}{36028797018963968 0}$$

- $k = 14$;[1.7, 1.75]

$$g(x) = \frac{5543768530644217 * \left(c - \frac{17}{10}\right)^2}{2251799813685248} - \frac{5081532196397171 * x}{36028797018963968} + \frac{4499971422083701 * \left(x - \frac{17}{10}\right)^3}{562949953421312} + \frac{102582420613326093}{360287970189639680}$$

- $k = 15$;[1.75, 1.8]

$$g(x) = \frac{1030468922986805 * \left(c - \frac{7}{4}\right)^2}{281474976710656} + \frac{5948483735233765 * x}{36028797018963968} + \frac{5835585718105145 * \left(x - \frac{7}{4}\right)^3}{562949953421312} - \frac{117212077716337235}{144115188075855872}$$

- $k = 16$;[1.8, 1.85]

$$g(x) = \frac{5872551407378765 * \left(c - \frac{9}{5}\right)^2}{1125899906842624} + \frac{685611471692355 * x}{1125899906842624} + \frac{5762831159655167 * \left(x - \frac{9}{5}\right)^3}{281474976710656} - \frac{901727105219415}{562949953421312}$$

- $k = 17$;[1.85, 1.9]

$$g(x) = \frac{2332562525792967 * \left(c - \frac{37}{20}\right)^2}{281474976710656} + \frac{2891503094439775 * x}{2251799813685248} + \frac{5280994985121585 * \left(x - \frac{37}{20}\right)^3}{562949953421312} - \frac{51074408897063849}{18014398509481984}$$

- $k = 18$;[1.9, 1.95]

$$g(x) = \frac{5457274299354169 * \left(c - \frac{19}{10}\right)^2}{562949953421312} + \frac{1228995741156947 * x}{562949953421312} + \frac{6227297107634299 * \left(x - \frac{19}{10}\right)^3}{70368744177664} - \frac{407260844699271463}{90071992547409920}$$

- $k = 19$;[1.95, 2]

$$g(x) = \frac{6465015414257667 * \left(c - \frac{39}{20}\right)^2}{281474976710656} + \frac{8593443990201691 * x}{2251799813685248} - \frac{2693756422607359 * \left(x - \frac{39}{20}\right)^3}{17592186044416} - \frac{1381847054874145451}{180143985094819840}$$

- **<u>Question 4</u>**: Comparison

```
%Calculate the intergral and sum J (noted by S
in this project)
 disp('Test 4')
 syms x
 S = 0;
 for i = 1: length(f)
     S = S + int(f(i), x, X(i), X(i+1));
 end

%Comparision format between S and I in problem
2
 S = double(S)
 if S > I
     disp('S>I')
 elseif S == I
     disp('S==I')
 else
     disp('S<I')
 end
```

Test 4:

S = - 0.2200

S<I at the difference of: -0.1943 - (-0.2200) = 0.0257

- **<u>Source code</u>**

```
clc
clear all
%test 1
x = 1:1/20:2;
%Functions settings and work environment
%Rearrange functions and equations
X =x;
```

```
B = (6*x.^3 +5*x+7)./(3*x+2).*exp(x);
U = (2*x.^3+1)./(2*x+3);
L = -(7*x+15).*log(2*x+5);
h=1/20; %dentax
A =zeros(length(x));
A(1,1) =1;
A(end,end) = 1;
B(1) = 1;
B(end) = 0;
 for i =2:length(x)-1
     A(i,i) = -2/h^2 +U(i)/h +L(i);
     A(i,i-1) = 1/h^2 -U(i)/h;
     A(i,i+1) = 1/h^2;
 end
 Y = inv(A)*B';
 plot(x,Y,'.')
 disp('TEST 1')
 Y

%Apply Simpson formula for problem 2%
 disp('Test 2')
I=(h/3)*((Y(1)+Y(end))+4*(Y(2)+Y(4)+Y(6)+Y(8)+Y(12)+Y(1
4)+Y(16)+Y(18)+Y(20))+2*(Y(3)+Y(5)+Y(7)+Y(9)+Y(11)+Y(13)
+Y(15)+Y(17)+Y(19)))

%Construct a natural cubic spline
 disp('Test 3')
 f = Spline_bac_3_tu_nhien(x,Y)

%Calculate the intergral and sum J (noted by S in this
project)
 disp('Test 4')
 syms x
 S =0;
 for i = 1: length(f)
     S = S + int(f(i),x,X(i),X(i+1));
 end

%Comparision format between S and I in problem 2
 S = double(S)
```

```
 if S > I
     disp('S>I')
 elseif S == I
     disp('S==I')
 else
     disp('S<I')
 end
 %%
 %Graph sketch for y(x) in problem 1
 function f =Spline_bac_3_tu_nhien(x,y)
plot(x,y,'.')
hold on
% x = input('nhap x:')
% y = input('nhap y:');
h = diff(x) ; % h (j) = x(j+1) -x(j)
 a = y; % a(j) = y(j);
% d = diff(c)/(3*h); % d(j) = (c(j+1)-c(j))/(3h(j))
% b = diff(a)/h - h(c+2*c)/3

%Natural Cubic Spline graph sketch for g(x) based on
calculated y(x)nodes
n = length(x);
A  =  zeros(n);
A(1,1) = 1;
A(n,n) = 1;
j = 1;
for i =2 : n-1
    A(i,i) = 2*(h(j) +h(j+1));
    A(i,i-1) = h(j);
    A(i,i+1) = h(j+1);
    j =j+1;
end
% B
v =diff(a);
B = zeros(n,1);
for i =2 : length(B)-1
    B(i) = 3/h(i)*v(i) -3/h(i-1)*v(i-1);
end
c = inv(A)*B;
m = diff(c);
```

```
for i =1 : length(c)-1
d(i)= m(i)/(3*h(i));
end
for j = 1 : length(a)-1
    b(j) = (a(j+1) -a(j))/h(j) -h(j) *(c(j+1)+2*c(j))/3;
end

X = x;
syms x
a = double(a);
b = double(b);
c = double(c);
d = double(d);
for i =1 : length(d)
    f(i)= a(i) + b(i)*(x-X(i)) + c(i) *(x-
X(i))^2+d(i)*(x-X(i))^3;
    ezplot(f(i), [X(i) X(i+1)])
end
plot(X,y,'*')
axis([min(X) max(X) min(y) max(y)])
f =f.';

end
```