You **must** typeset your answers in LaTeX, generate a pdf from it, and upload the pdf to Gradescope for grading. Because you're unfamiliar with LaTeX, we'll give you a LaTeX shell of the solution to fill in so you get a better idea of how LaTeX works. We may not give you such a shell for later assignments.

Question	Points	Score
1	6	
2	8	
3	5	
4	7	
5	7	
6	10	
7	10	
8	10	
9	20	
10	4	
11	3	
12	3	
13	3	
14	4	
Total:	100	

1. (6 points) Which of these sentences are propositions? What is the truth value of those that are propositions?

    1. Boston is the capital of Massachusetts.
    2. Miami is the capital of Florida.
    3. $2 + 3 = 5$.
    4. $5 + 7 = 10$.
    5. $x + 2 = 11$.
    6. Answer this question.

2. (8 points) Let p and q be the following propositions.

$p:$	I bought a lottery ticket this week.
$q:$	I won the million dollar jackpot on Friday.

Express each of these propositions as an English sentence.

1. $\neg p$
2. $p \vee q$
3. $p \rightarrow q$
4. $p \wedge q$
5. $p \leftrightarrow q$
6. $\neg p \rightarrow \neg q$
7. $\neg p \wedge \neg q$
8. $\neg p \vee (p \wedge q)$

3. (5 points) Let S stand for the statement "Steve is happy" and G for "George is happy." What English sentences are represented by the following formulas?

1. $(S \vee G) \wedge (\neg S \vee \neg G)$.
2. $(S \vee (G \wedge \neg S)) \vee \neg G$

4. (7 points) Let p and q be the following propositions.

$p:$	It is below freezing.
$q:$	It is snowing.

Write these propositions using p, q and logical connectives.

1. It is below freezing and snowing.
2. It is below freezing but not snowing.
3. It is not below freezing and it is not snowing.
4. It is either snowing or below freezing (or both).
5. If it is below freezing, it is also snowing.
6. It is either below freezing or it is snowing, but it is not snowing if it is below freezing.
7. That it is below freezing is necessary and sufficient for it to be snowing.

5. (7 points) Write each of these statements in the form "**if** p, **then** q" in English.

1. It is necessary to wash the boss's car to get promoted.
2. Winds from the south imply a spring thaw.
3. A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.
4. Willy gets caught whenever he cheats.

5. You can access the website only if you pay a subscription fee.

6. Getting elected follows from knowing the right people.

7. Carol gets seasick whenever she is on a boat.

6. (10 points) There are exactly two truth environments (assignments) for the variables M, N, P, Q, R, S that satisfy the following formula:

$$\underbrace{(\bar{P} \vee Q)}_{\text{clause (1)}} \wedge \underbrace{(\bar{Q} \vee R)}_{\text{clause (2)}} \wedge \underbrace{(\bar{R} \vee S)}_{\text{clause (3)}} \wedge \underbrace{(\bar{S} \vee P)}_{\text{clause (4)}} \wedge M \wedge \bar{N}$$

(a) (4 points) This claim could be proved by truth-table. How many rows would the truth table have?

(b) (6 points) Instead of a truth-table, prove this claim with an argument by cases according to the truth value of P. **Hint**: The formula is in CNF, so for the formula to be **T**, each clause must be true. Can you figure out the assignments to M, N, Q, R, S when P is set to **T**, and when P is set to **F**?

7. The five-variable propositional formula

$$P ::= (A \wedge B \wedge \bar{C} \wedge D \wedge \bar{E}) \vee (\bar{A} \wedge B \wedge \bar{C} \wedge \bar{E})$$

is in Disjunctive Normal Form with two "AND-of-literal" clauses.

(a) (5 points) Find a full Disjunctive Normal Form[1] that is equivalent to P, and explain your reasoning. **Hint**: Can you narrow in on the important parts of the truth table without writing all of it? Alternatively, can you avoid the truth table altogether?

(b) (5 points) Let C be a full Conjunctive Normal Form that is equivalent to P. Assume that C has been simplified so that none of its "OR-of-literals" clauses are equivalent to each other. How many clauses are there in C? Please don't try to write out any of these clauses. Briefly explain your answer.

8. For which values of p, q, and r is the following logical expression true?

(a) (5 points) The expression

$$(\neg p \vee q) \wedge (q \rightarrow r) \wedge (\neg r \vee p)$$

Give a succinct description of which combinations of input values work, rather than the whole truth table.

(b) (5 points) Show that the following two expressions aren't logically equivalent:

$$\boxed{\begin{array}{l} (p \rightarrow q) \wedge r \\ p \rightarrow (q \wedge r) \end{array}}$$

Hint: Propositional expressions A and B are logically equivalent if they evaluate to the same value in "all possible worlds".

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[1] A DNF formula is in full disjunctive normal form if each of its variables appears exactly once in every conjunction https://en.wikipedia.org/wiki/Disjunctive_normal_form.

9. Simplify the following propositions as much as possible.

   (a) (5 points) $(\neg p \rightarrow q) \wedge (q \wedge p \rightarrow \neg p)$.

   (b) (5 points) $(p \rightarrow \neg p) \rightarrow ((q \rightarrow (p \rightarrow p)) \rightarrow p)$.

   (c) (5 points) $(p \rightarrow p) \rightarrow (\neg p \rightarrow \neg p) \wedge q$.

   (d) (5 points) Is the following claim true or false? Explain your choice.

      "Every proposition over the single variable p is either logically equivalent to p or it is logically equivalent to $\neg p$."

10. (4 points) What is X in the compound proposition below? Explain your reasoning. No points will be given without correct reasoning.

$$(\neg p \wedge (\neg q \rightarrow p)) \rightarrow X$$

11. (3 points) Use a truth table to determine for which truth values of p, q, and r

$$(\neg(p \wedge (q \vee r))) \longleftrightarrow ((\neg p \vee \neg q) \wedge (\neg p \vee \neg r))$$

is true. You can use the truth table generator at `https://schnekli-tamu.uc.r.appspot.com/logic`, or write the truth table by hand.

12. (3 points) Show that the conclusion

$$(p \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow r)$$

follows from the premise $p \rightarrow q$. **Hint**: B follows from A is another way of saying that $A \rightarrow B$.

13. (3 points) Using a truth table show that

$$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$$

is a tautology. **Hint**: When a propositional expression is a tautology, it's **T** in "all possible worlds".

14. **Errors in reasoning**. Show using a counter example that the following arguments are invalid.

   (a) (2 points) Converse Error.
      (a) If $x \geq 2$, then $x \geq 0$.
      (b) Therefore, $x \geq 0 \Rightarrow x \geq 2$.

   (b) (2 points) Inverse Error.
      (a) If $x \geq 2$, then $x \geq 0$.
      (b) Therefore $x \not\geq 2 \Rightarrow x \not\geq 0$.