1.3/18 $(\neg p \wedge (p \to q)) \to \neg q$ is a tautology ?

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg p \wedge (p \to q)$	$(\neg p \wedge (p \to q)) \to \neg q$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	T

$\Rightarrow$ not a tautology

1.3/21. Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$	$p \wedge q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
T	T	F	F	T	T	F	T
T	F	F	T	F	F	F	F
F	T	T	F	F	F	F	F
F	F	T	T	T	F	T	T

$\Rightarrow \quad p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

1.3/26

$$
\begin{aligned}
(p \to q) \wedge (p \to r) &\equiv (\neg p \vee q) \wedge (\neg p \vee r) && (\text{Thm } 1 \to) \\
&\equiv \neg p \vee (q \wedge r) && (\text{Distributes laws}) \\
&\equiv p \to (q \wedge r) && (\text{Thm } 1 \to)
\end{aligned}
$$

1.4/19 $P(x)$: "x can speak Russian"

$Q(x)$: " x knows the computer language C++ "

Domain x is all students at school

a) There is a student at your school who can speak Russian and who knows C++.

$\exists x\,(P(x) \land Q(x))$

b) There is a student at your school who can speak Russian but who doesn't know C++

$\exists x\,(P(x) \land \neg Q(x))$

c) Every student at your school either can speak Russian or knows C++

$\forall x\,(P(x) \lor Q(x))$

d) No student at your school can speak Russian or knows C++

$\forall x\, \neg(P(x) \lor Q(x))$

1.4/12 : $Q(x)$: " $x+1 > 2x$ ". Domain: all integers

a) $Q(0)$: T

b) $Q(-1)$: T

c) $Q(1)$: F

d) $\exists x\, Q(x)$: T

e) $\forall x\, Q(x)$: F

f) $\exists x\, \neg Q(x)$: T

g) $\forall x\, \neg Q(x)$: F

1.4/25 Domain: all people , $P(x)$: x is perfect

$Q(x)$: x is your friend

a) No one is perfect

$\forall x \neg P(x)$

b) Not everyone is perfect

$\neg \forall x P(x)$

c) All your friend are perfect

$\forall x (Q(x) \to P(x))$

d) At least one of your friends is perfect

$\exists x (Q(x) \land P(x))$

e) Everyone is your friend and is perfect

$\forall x \, (P(x) \land Q(x))$

f) Not everybody is your friend or someone is not perfect.

$(\neg \forall x \, Qx) \lor (\exists x \neg P(x))$

1.4/46 Whether $\forall x (P(x) \to Q(x))$ and $(\forall x P(x) \to \forall x Q(x))$ are $\equiv$?

Counterexample: $P(x)$: $x \geq 0$ Domain: $\mathbb{R}$

$Q(x)$: $x \geq 1$

$\forall x P(x) \to \forall x Q(x)$ is true

$\forall x (P(x) \to Q(x))$ is false

$\Rightarrow \forall x (P(x) \to Q(x)) \not\equiv (\forall x P(x) \to \forall x Q(x))$

1.5/24 Domain: all real number

a) $\exists x \forall y (x+y=y)$

There is a real number x such that for every real number y, the sum x and y equals y.

b) $\forall x \forall y (((x \geq 0) \wedge (y<0)) \rightarrow (x-y>0))$

For all real number x and y, If x is greater than 0 and y is less than 0, then the difference between x and y is positive

c) $\exists x \exists y (((x \leq 0) \wedge (y \leq 0)) \wedge (x-y)>0))$

There is exist real numbes x and y such that both x and y are less than or equal to 0, and the difference between x and y is positive

d) $\forall x \forall y (((x \neq 0) \wedge (y \neq 0)) \leftrightarrow (x.y \neq 0))$

For all real number x and y, x and y are both non-zero if and only if the product of x and y is non-zero.

1.5/30

a) $\forall x \forall y \neg P(x,y)$

b) $\exists x \forall y \; P(x,y)$

c) $\forall y (\neg Q(y) \vee \exists y \; R(x,y))$

d) $\forall y (\forall y \neg R(x,y) \wedge \exists x \neg S(x,y))$

e) $\forall y(\exists x \forall z \neg T(x,y,z) \land \forall y \exists z \neg U(x,y,z))$

1.5/48 Show that $\forall x P(x) \lor \forall x Q(x) \equiv \forall x \forall y (P(x) \lor Q(y))$

+ $\forall x P(x) \lor \forall x Q(x) \Rightarrow \forall x \forall y (P(x) \lor Q(y))$

Assume $\forall x P(x) \lor \forall x Q(x)$

If $\forall x P(x)$, then for every x $P(x)$ is true

- Now consider any y in the domain. Since $P(x)$ is true for every x, $P(y)$ must also be true.
- Therefore, $P(x) \lor Q(y)$ is true for every x & y, which means $\forall x \forall y (P(x) \lor Q(y))$ holds.

If $\forall x Q(x)$, then for every x, $Q(x)$ is true

- Similarly, considering any y in the domain, $Q(y)$ must also be true.

Hence, $P(x) \lor Q(y)$ is true for every x and y, implying $\forall x \forall y (P(x) \lor Q(y))$

+ $\forall x \forall y (P(x) \lor Q(y)) \Rightarrow \forall x P(x) \lor \forall x Q(x)$:

Assume $\forall x \forall y (P(x) \lor Q(y))$

Consider any x in the domain. For this x, $P(x) \lor Q(y)$ holds for every y.

If we set $y = x$, then $P(x) \lor Q(x)$ must be true.

Since $P(x) \lor Q(x)$ hold for every x.

Therefore, $\forall x P(x) \lor \forall x Q(x) \equiv \forall x \forall y (P(x) \lor Q(y))$