3.2/39. Show that if f and g are real-valued functions such that $f(x)$ is $O(g(x))$, then for every positive integer n, $f^n(x)$ is $O(g^n(x))$.

Proof: Since f is $O(g)$

Then, there are c and k such that $|f(x)| \leq c|g(x)|$, $x > k$

It follows that $|f^n(x)| \leq c^n |g^n(x)|$ for $x > k$

The witnesses for "$f^n(x)$ is $O(g^n(x))$" are c^n and k.

3.2/45. If $f_1(x)$ and $f_2(x)$ are function from the set of positive integers to the set of positive real numbers and $f_1(x)$ and $f_2(x)$ are both $\Theta(g(x))$, is $(f_1 - f_2)(x)$ also $\Theta(g(x))$? Either prove that it is or give a counterexample

Proof: Take $f_1(x) = f_2(x) = g(x) = x$

Then, $f_1 - f_2 = 0$, which is not in $\Theta(g)$

Therefore, the answer is negative

3.2/68. Suppose that $f(x)$ is $O(g(x))$. Does it follow that $\log|f(x)|$ is $O(\log|g(x)|)$

Proof: Let $f(x) = 2^x$ and $g(x) = 3^x$. Then $f(x)$ is $O\, g(x)$

But, $\log f(x) = x \log 2$ and $\log g(x) = x \log 3$

$$\lim_{x \to 0} \frac{\log(f(x))}{\log(g(x))} = \frac{x \log 2}{x \log 3} = \log_3 2 \neq 0$$

Therefore, the answer is no.

1.1/16 Let p, q, and r be the propositions

p: You get an A on the final exam

q: You do every exercise in this book

r: You get an A in this class

a. You get an A in this class, but you do not do every exercise in this book.

$r \land \neg q$

b. You get an A on the final, you do every exercise in this book, and you get an A in this class.

$p \land q \land r$

c. To get an A in this class, it is necessary for you to get an A on the final.

$r \to p$

d. You get an A on the final, but you don't do every exercise in this book; neverthelesss, you get an A in this class.

$(p \land \neg q) \land r$

e. Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class

$(p \land q) \to r$

f. You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final. $r \leftrightarrow (p \lor q)$

1.1/24 Write each of these statements in the form "if p, the q"

a) It is necessary to wash the boss's car to get promoted.

If you want to get promoted, then he wash the boss car

b) Winds from the south imply a spring thaw

If the wind come from the south, then there will be the spring thaw

c) A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.

- If you bough the computer less than a year ago, then the warranty is good.

d) Willy gets caught whenever he cheats.

- If Willy cheats, then he gets caught.

e) You can access the website only if you pay a subcription fee.

- If you access the website, then you pay a subcription fee.

f) Getting elected follows from knowing the right people.

- If you know the right people, then you will get elected.

g) Carol gets seasick whenever she is on a boat

- If Carol is on a boat, the she gets seasick

1.1/29 State the converse, contrapositive, and inverse

a) If it snows today, I will ski tomorrow. $p \to q$

$q \to p$: I will ski tomorrow only if it snows today.

$\neg p \to \neg q$: If it doesn't snows today, then I won't ski tomorrow.

$\neg q \to \neg p$: If I do not ski tomorrow, then it will not have snowed today.

b) I come to class whenever there is going to be a quiz. $p \to q$

$q \to p$: If I come to class, then there will be a quiz.

$\neg q \to \neg p$: If I donot come to class, then there won't be a quiz.

$\neg p \to \neg q$: If there is not going to be a quiz, then I don't come to class

c) A positive integer is a prime only if it has no divisors other than 1 and itself. $p \to q$

$q \to p$: If A positive integer has no divisors other than 1 and itself, then It is a prime.

$\neg q \to \neg p$: If a positive integer has divisors other than 1 and itself, then it is not a prime.

$\neg p \to \neg q$: If a positive integer is not a prime, then it has divisors other than 1 and itself.

1.1/33

a) $p \land \neg p$

p	$\neg p$	$p \land \neg p$
T	F	F
F	T	F

b) $p \lor \neg p$

p	$\neg p$	$p \lor \neg p$
T	F	T
F	T	T

c) $(p \lor \neg q) \rightarrow q$

p	q	$\neg q$	$p \lor \neg q$	$(p \lor \neg q) \Rightarrow q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

d) $(p \lor q) \rightarrow (p \land q)$

p	q	$p \lor q$	$p \land q$	$(p \lor q) \rightarrow (p \land q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

e) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg q \rightarrow \neg p$	$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

f) $(p \to q) \to (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

1.1/52 Is the assertion "This statement is false" a proposition?

No, it is not a proposition because it can not be assigned a definite truth value without leading to a contradiction.

1,2/40

	Alice	Carlos	John	Diana
Alice did it	F	F	T	T
Carlos did it	T	F	T	T
John did it	F	F	F	T
Diana did it	F	T	T	F

a) John

b) Carlos

10) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$\neg p$	$\neg q$	$(p \wedge q)$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T