CSCE 222—Spring 2024
Homework #1
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1. Which of these sentences are propositions? What is the truth value of those that are propositions?

   1. Boston is the capital of Massachusetts.
      This sentence is a proposition. The truth value of the proposition is True.

   2. Miami is the capital of Florida.
      This sentence is a proposition. The truth value of the proposition is False.

   3. 2+3=5.
      This sentence is a proposition. The truth value of the proposition is True.

   4. 5+7=10.
      This sentence is a proposition. The truth value of the proposition is False.

   5. x+2=11.
      This sentence is not a proposition.

   6. Answer this question.
      This sentence is not a proposition.

2. Let p and q be the following propositions.

p :	I bought a lottery ticket this week.
q :	I won the million dollar jackpot on Friday.

Express each of these propositions as an English sentence.

1. ¬p : I did not buy a lottery ticket this week.

2. p ∨ q : I either bought a lottery ticket this week or I won the million dollar jackpot on Friday (or both).

3. p → q : If I bought a lottery ticket this week, then I won the million dollar jackpot on Friday.

4. p ∧ q : I bought a lottery ticket this week, and I won the million dollar jackpot on Friday.

5. p ⟷ q : I bought a lottery ticket this week if and only if I won the million dollar jackpot on Friday.

6. ¬p → ¬q : If I did not buy a lottery ticket this week, then I did not win the million dollar jackpot on Friday.

7. ¬p ∧ ¬q : I did not buy a lottery ticket this week, and I did not win the million dollar jackpot on Friday.

8. $\neg p \vee (p \wedge q)$: I did not buy a lottery ticket this week or I bought a lottery ticket this week and won the million dollar jackpot on Friday (or both).

3. Let S stand for the statement "Steve is happy" and G for "George is happy." What English sentences are represented by the following formulas? (How To Prove It, §1.1 Ex. 7)

    1. $(S \lor G) \land (\neg S \lor \neg G)$

       Either Steve or George is happy, but one of them is not.

    2. $(S \lor (G \land \neg S)) \lor \neg G$
       $\equiv ((S \lor G) \land (S \lor \neg S)) \lor \neg G$
       $\equiv S \lor (G \lor \neg G)$

       Steve is happy, or George is either happy or not happy.

4. Let p and q be the following propositions.

$p:$	It is below freezing.
$q:$	It is snowing.

Write these propositions using p, q and logical connectives.

1. It is below freezing and snowing.
   $p \wedge q$
2. It is below freezing but not snowing.
   $p \wedge \neg q$
3. It is not below freezing and it is not snowing.
   $\neg p \wedge \neg q$
4. It is either snowing or below freezing (or both)
   $p \vee q$
5. If it is below freezing, it is also snowing.
   $p \rightarrow q$
6. It is either below freezing or it is snowing, but it is not snowing if it is below freezing.
   $(p \vee q) \wedge (p \rightarrow \neg q)$
7. That it is below freezing is necessary and sufficient for it to be snowing.
   $p \longleftrightarrow q$

5. Write each of these statements in the form "**if** p, **then** q" in English.

   1. It is necessary to wash the boss's car to get promoted.

      - If you want to get promoted, then you need to wash the boss's car.

   2. Winds from the south imply a spring thaw.

      -If the wind come from the south, then there will be the spring thaw.

   3. A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.

      -If you bought the computer less than a year ago, then the warranty is good

   4. Willy gets caught whenever he cheats.

      -If Willy cheats, then he gets caught.

   5. You can access the website only if you pay a subscription fee.

      - If you access the website, then you pay a subscription fee..

   6. Getting elected follows from knowing the right people.

      -If you know the right people, then you will get elected.

   7. Carol gets seasick whenever she is on a boat.

      -If Carol is on a boat, then she gets seasick.

6. There are exactly two truth environments (assignments) for the variables M, N, P, Q, R, S that satisfy the following formula:

$$\underbrace{(\bar{P} \vee Q)}_{\text{clause (1)}} \wedge \underbrace{(\bar{Q} \vee R)}_{\text{clause (2)}} \wedge \underbrace{(\bar{R} \vee S)}_{\text{clause (3)}} \wedge \underbrace{(\bar{S} \vee P)}_{\text{clause (4)}} \wedge M \wedge \bar{N}$$

1. This claim could be proved by truth-table. How many rows would the truth table have?

   - There are 64 rows for truth table.(2^6)

2. Instead of a truth-table, prove this claim with an argument by cases according to the truth value of P. Hint: The formula is in CNF, so for the formula to be T, each clause must be true. Can you figure out the assignments to M, N, Q, R, S when P is set to T,and when P is set to F?

   - Case 1: When P is T.
   Clause 1 is True, then Q must be T.
   Clause 2 is True, then R must be T (because Q is T).
   Clause 3 is True, then S must be T (because R is T).
   Clause 4 is True, then P must be T (because S is T).
   M is T , and N is F (negation)
   Therefore, M = P = Q = R = S = P = True , N = False .

   - Case 2: When P is F.
   Clause 4 is True, then $\overline{S}$ is T, thus S is F.
   Clause 3 is True, then $\overline{R}$ is T, thus R is F.
   Clause 2 is True, then $\overline{Q}$ is T, thus Q is F.
   Therefore, M = True, P = Q = R = S = N = False.

7. The five-variable propositional formula

$$P ::= (A \wedge B \wedge \bar{C} \wedge D \wedge \bar{E}) \vee (\bar{A} \wedge B \wedge \bar{C} \wedge \bar{E})$$

is in Disjunctive Normal Form with two "AND-of-literal" clauses.

1. Find a full Disjunctive Normal Form that is equivalent to P , and explain your reasoning.

   Proof. -The formula consists of two main clauses connected by a disjunction ($\vee$).
   -The first clause, $(A \wedge B \wedge \bar{C} \wedge D \wedge \bar{E})$ is a conjunction of five literals.
   -The second clause, $(\bar{A} \wedge B \wedge \bar{C} \wedge \bar{E})$ is a conjunction of 4 literals.
   -Both clauses share the term $(B \wedge \bar{C} \wedge \bar{E})$, so we will consider the remaining clauses: $(A \wedge D) \vee (\bar{A})$
   -Full disjunctive normal form if each of its variables appears exactly once in every conjunction and each conjunction appears at most once. Thus, we will have 2 cases for $\bar{A}$: $(\bar{A} \wedge D)$; $(\bar{A} \wedge \bar{D})$
   - Therefore, full Disjunctive Normal Form that is equivalent to P is :

   $(A \wedge B \wedge \bar{C} \wedge D \wedge \bar{E}) \vee (\bar{A} \wedge B \wedge \bar{C} \wedge D \wedge \bar{E}) \vee (\bar{A} \wedge B \wedge \bar{C} \wedge \bar{D} \wedge \bar{E})$ □

2. Let C be a full Conjunctive Normal Form that is equivalent to P. Assume that C has been simplified so that none of its "OR-of-literals" clauses are equivalent to each other. How many clauses are there in C?

   Proof. - We have 5 literals, so we have $(2^5) = 32$ cases able to happen.
   - We have 3 cases in full Disjunctive Normal Form giving True results. This means there will be 29 cases with False results.
   - Converting DNF to CNF involves De Morgan's Law: $\neg(A \vee D) = \bar{A} \wedge \bar{D}$.
   - Flip all "OR"s and "AND"s and negate all literals.
   - Then, we have 29 clause in C that satisfy a full Conjunctive Normal Form.

   □

8. For which values of p, q, and r is the following logical expression true?

   (a) $(\neg p \vee q) \wedge (q \rightarrow r) \wedge (\neg r \vee p)$

   For a logical expression to be true, each clause must be true.
   Case 1: Assume p is true, then q must be true. Hence, r is also true.
   Case 2: Assume p is false, then r must be false. Hence, q is false.
   Therefore, p, q, and are must either be all true or all false.

   (b) Show that the following two expressions aren't logically equivalent:
   $(p \rightarrow q) \wedge r$ and $p \rightarrow (q \wedge r)$

   Consider first expression:
   r must be true
   If $p \rightarrow q$ is true, then there are 3 possible cases for the clause to be true: (p = T, q = T), (p = F, q = T), (p = F, q = F).

   Consider second expression:
   Let r is false, p is false, and q is true. Hence, the second expression is true.
   But, this leads to a value conflict with the first expression (first expression is false with r = F, p = F, and q = T). Because expressions A and B are logically equivalent if they evaluate to the same value in "all possible worlds".
   Therefore, two expression aren't logically equivalent.

9. Simplify the following propositions as much as possible.

(a) $(\neg p \to q) \wedge (q \wedge p \to \neg p)$
$\equiv (\neg(\neg p) \vee q) \wedge (\neg(p \wedge q) \vee \neg p)$ by the truth table for implies
$\equiv (p \vee q) \wedge (\neg p \vee \neg q \vee \neg p)$ by De Morgan law
$\equiv (p \vee q) \wedge (\neg p \vee \neg q)$ by idempotent laws
$\equiv (p \vee q) \wedge \neg(p \wedge q)$ by De Morgan law
$\equiv p \oplus q$ by the truth table.

(b) $(p \to \neg p) \to ((q \to (p \to p)) \to p)$
$\equiv (\neg p \vee \neg p) \to ((q \to T) \to p)$ by truth table
$\equiv \neg p \to (T \to p)$ by idempotent laws
$\equiv \neg p \to p$ by truth table
$\equiv \neg(\neg p) \vee p$ by truth table for implies
$\equiv p$ double negation laws.

(c) $(p \to p) \to (\neg p \to \neg p) \wedge q$
$\equiv T \to (T \wedge q)$ by the truth table for implies
$\equiv T \to q$ by the identity laws
$\equiv q$ by the truth table for implies

(d) "Every proposition over the single variable p is either logically equivalent to p or it is logically equivalent to $\neg p$."
The following claim is correct. Because every proposition only accepts 1 of 2 values: True or False

10. What is X in the compound proposition below? Explain your reasoning. No points will be given without correct reasoning.

$$(\neg p \wedge (\neg q \to p)) \to X$$

Proof. X in the compound proposition $(\neg p \wedge (\neg q \to p)) \to X$ is logically equivalent to q.

p	q	$\neg q$	$\neg p$	$\neg q \to p$	$\neg p \wedge (\neg q \to p)$	$\neg p \wedge (\neg q \to p) \to q$
T	T	F	F	T	F	T
T	F	T	F	T	F	T
F	T	F	T	T	T	T
F	F	T	T	F	F	T

To make the entire proposition true, X should logically follow from $(\neg p \wedge (\neg q \to p))$. The only way this can be true is if X is equivalent to q, because q and combining it with the previous conditions ensures the truth of the entire proposition. $\square$

11. Use a truth table to determine for which truth values of p, q, and r

$$(\neg(p \wedge (q \vee r))) \longleftrightarrow ((\neg p \vee \neg q) \wedge (\neg p \vee \neg r))$$

is true.

p	q	r	$p \vee (q \vee r)$	$\neg p$	$\neg q$	$\neg r$	$p \wedge (q \vee r)$	$\neg(p \wedge (q \vee r))$	$\neg p \vee \neg q$	$\neg p \vee \neg r$	$(\neg p \vee \neg q) \wedge (\neg p \vee \neg r)$	$\neg(p \wedge (q \vee r)) \leftrightarrow ((\neg p \vee \neg q) \wedge (\neg p \vee \neg r))$
True	True	True	True	False	False	False	True	False	False	False	False	True
True	True	False	True	False	False	True	True	False	False	True	False	True
True	False	True	True	False	True	False	True	False	True	False	False	True
True	False	False	False	False	True	True	False	True	True	True	True	True
False	True	True	True	True	False	False	False	True	True	True	True	True
False	True	False	True	True	False	True	False	True	True	True	True	True
False	False	True	True	True	True	False	False	True	True	True	True	True
False	False	False	False	True	True	True	False	True	True	True	True	True

12. Show that the conclusion

$$(p \to (q \to r)) \to (p \to r)$$

follows from the premise $p \to q$.

Proof. We'll use the hint provided to express the conclusion in terms of implications.

1. Start with the premise: $p \to q$
This means that " if p is true, then q is true ".
2. Use the hint to express $(p \to q) \to (p \to r)$:
This means that "if $(p \to q)$ is true, then $(p \to r)$ is true."
- According to the hint, this expression is equivalent to $p \to (q \to r)$
3. Now, we want to show $(p \to (q \to r)) \to (p \to r)$:
This expression states that " if $(p \to (q \to r))$ is true, then $(p \to r)$ is true."
From the previous proof shows us the value of $(p \to (q \to r))$ and $(p \to r)$ are true. Hence, this expression is true.
Therefore, we showed that the conclusion

$$(p \to (q \to r)) \to (p \to r)$$

follows from the premise $p \to q$.

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \to ((p \to (q \to r)) \to (p \to r))$
True	True	True	True	True	True	True
True	True	False	True	False	False	True
True	False	True	False	True	True	True
True	False	False	False	True	False	True
False	True	True	True	True	True	True
False	True	False	True	False	True	True
False	False	True	True	True	True	True
False	False	False	True	True	True	True

□

13. Using a truth table show that

$$((p \to q) \wedge (q \to r)) \to (p \to r)$$

is a tautology.

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \wedge (q \to r)$	$((p \to q) \wedge (q \to r)) \to (p \to r)$
True	True	True	True	True	True	True	True
True	True	False	True	False	False	False	True
True	False	True	False	True	True	False	True
True	False	False	False	True	False	False	True
False	True	True	True	True	True	True	True
False	True	False	True	False	True	False	True
False	False	True	True	True	True	True	True
False	False	False	True	True	True	True	True

14. **Errors in reasoning**. Show using a counter example that the following arguments are invalid.

   (a) a) (2 points) Converse Error.

      (a) If $x \geq 2$, then $x \geq 0$.
      (b) Therefore, $x \geq 0 \Rightarrow x \geq 2$.

      Proof. If $x \geq 0$, it does not necessarily mean that $x \geq 2$.
      Counterexample: Let $x = 1$. Hence, $1 \geq 0$, but $1 < 2$.
      Therefore, the argument is invalid. $\square$

   (b) (2 points) Inverse Error.

      (a) If $x \geq 2$, then $x \geq 0$.
      (b) Therefore, $x \ngeq 2 \Rightarrow x \ngeq 0$.

      Proof. If $x \ngeq 2$, it does not necessarily mean that $x \ngeq 0$.
      Counterexample: Let $x = 1$. Hence, $1 < 2$, but $1 > 0$.
      Therefore, the argument is invalid. $\square$