

Started on	Friday, 12 November 2021, 8:10 PM
State	Finished
Completed on	Friday, 12 November 2021, 9:10 PM
Time taken	1 hour
Marks	16.00/50.00
Grade	3.20 out of 10.00 (32%)

Question 1

Correct

Mark 1.00 out of 1.00

Choose f(x) and g(x) such that

$f \circ g(x) = \sqrt{x^3 + x}$

Select one:

☒ a. $f(x) = \sqrt{x}, g(x) = x^3 + x$ ✓

☐ b. None of the other choices is correct

☐ c. $f(x) = \sqrt{x^3}, g(x) = x$

☐ d. $f(x) = \sqrt{x}, g(x) = (\sqrt{x})^3 + \sqrt{x}$

☐ e. $f(x) = x^3 + x, g(x) = \sqrt{x}$

The correct answer is: $f(x) = \sqrt{x}, g(x) = x^3 + x$

$$f \circ g(x) = \sqrt{x^3}$$
$$f \circ g(x) = \sqrt{(\sqrt{x})^3 + \sqrt{x}}$$
$$f \circ g(x) = \sqrt{x^3} + \sqrt{x}$$

✓

X

X

X

Question 2

Correct

Mark 1.00 out of 1.00

Given that $\lim_{x \rightarrow 8} f(x) = -10$ and $\lim_{x \rightarrow 8} (2f(x) + 3g(x)) = 1$, evaluate the limit $\lim_{x \rightarrow 8} g(x)$.

Select one:

☒ a. 7 ✓

☐ b. -19/3

☐ c. None of the other choices is correct

☐ d. -7

☐ e. 19/3

The correct answer is: 7

$$2 \lim f + 3 \lim g = 1$$
$$\lim g = \frac{1 + 20}{3}$$

↖

Question 3

Correct

Mark 1.00 out of 1.00

Evaluate the limit. $\lim_{x \rightarrow 16} \frac{4 - \sqrt{x}}{x - 16}$

$$\frac{0}{0} = \lim_{x \rightarrow 16} \frac{16 - x}{x - 16(4 + \sqrt{x})} = \lim_{x \rightarrow 16} \frac{-1}{4 + \sqrt{x}} = -\frac{1}{8}$$

Select one:

- ☐ a. 1/8
- ☐ b. -1/2
- ☐ c. 0
- ☐ d. 1/2
- ☐ e. Does not exist
- ☒ f. -1/8 ✓

The correct answer is: -1/8

Question 4

Correct

Mark 1.00 out of 1.00

Find the number a such that the function

$$f(x) = \begin{cases} \frac{2 - \sqrt{x}}{4 - x} & \text{if } x \neq 4 \\ a & \text{if } x = 4 \end{cases}$$

$$(-\infty, 4) \cup (4, +\infty)$$

$$\lim_{x \rightarrow 4} f(x) = f(4)$$

is continuous on its domain.

$$\Leftrightarrow \lim_{x \rightarrow 4} \frac{2 - \sqrt{x}}{4 - x} = a$$

$$\Leftrightarrow \lim_{x \rightarrow 4} \frac{4 - x}{(4 - x)(2 + \sqrt{x})} = a \quad \Leftrightarrow \lim_{x \rightarrow 4} \frac{1}{2 + \sqrt{x}} = a$$

$$a = \frac{1}{4}$$

Select one:

- ☐ a. None of the other choices is correct
- ☐ b. 1/2
- ☐ c. 1/16
- ☐ d. 1/8
- ☒ e. 1/4 ✓

The correct answer is: 1/4

Question 5

Correct

Mark 1.00 out of 1.00

If a ball is thrown into the air with a velocity of 58 ft/s, its height (in feet) after t seconds is given by H=58t-11t². Find the velocity when t = 4.

$$H(t) = 58t - 11t^2$$

$$v(t) = H'(t) = 58 - 22t$$

$$v(4) = 58 - 22 \cdot 4 = -30 \text{ ft/s}$$

↳ bang' na' vbi' vtoe' - 30 ft/s

Select one:

- ☐ a. 25ft/s
- ☐ b. 37ft/s
- ☒ c. 30ft/s ✓
- ☐ d. 27ft/s
- ☐ e. 31ft/s

The correct answer is: 30ft/s

Question 6

Correct

Mark 1.00 out of 1.00

The position of a car is given by the values in the following table. Find the average velocity for the time period beginning when $t = 2$ and lasting 2 seconds.

t (seconds)	0	1	2	3	4	5
s (feet)	0	18	35	74	110	175
			✓		✓	

Select one:

- ☐ a. 39
- ☐ b. 47.5
- ☒ c. 37.5 ✓
- ☐ d. 35.5
- ☐ e. 33.5

$$\overline{v}_{[2,4]} = \frac{s(4) - s(2)}{4 - 2} = \frac{110 - 35}{2} = \dots$$

The correct answer is: 37.5

Question 7

Correct

Mark 1.00 out of 1.00

Find the slope of the tangent line to the curve

$y^2x + x^3y = 6$

at the point (1, 2).

Implicit Def:

$$2yx \cdot y' + y^2 + 3x^2y + x^3y' = 0$$

Then $x = 1, y = 2$:

$$4y' + 4 + 6 + y' = 0$$
$$\Rightarrow y' = -2$$

Select one:

- ☒ a. -2 ✓
- ☐ b. 2
- ☐ c. -10/7
- ☐ d. 10/7
- ☐ e. None of the other choices is correct

The correct answer is: -2

Question 8

Correct

Mark 1.00 out of 1.00

Find f' in terms of g' .

$$f(x) = [g(x)]^2$$

$$f' = 2g(x) \cdot g'(x)$$

Select one:

- ☐ a. None of the other choices is correct
- ☐ b. $f'(x) = 2g(x)(g'(x) + x)$
- ☐ c. $f'(x) = 2g(x)$
- ☐ d. $f'(x) = x[g'(x)]$
- ☒ e. $f'(x) = 2g(x)g'(x)$ ✓

The correct answer is: $f'(x) = 2g(x)g'(x)$

Question 9

Correct

Mark 1.00 out of 1.00

Find $y''(2)$.

$$y = \frac{x}{3-x} \quad y' = \frac{3}{(3-x)^2} = \frac{3}{x^2 - 6x + 9} \rightarrow y'' = \frac{-3(2x-6)}{(x^2 - 6x + 9)^2}$$

(bấm máy cho nhanh)

$$y''(2) = \frac{-3 \cdot (-2)}{1} = 6$$

Select one:

- ☐ a. -1/6
- ☐ b. -6
- ☐ c. None of the other choices is correct
- ☒ d. 6 ✓
- ☐ e. 1/6

The correct answer is: 6

Question 10

Not answered

Marked out of 1.00

Find a linear approximation for $f(x) = (x^2/2) + 3x$ at $x = 2$.

Select one:

- ☐ a. $7x+6$
- ☐ b. $7x-6$
- ☐ c. $7x-20$
- ☐ d. $7x+20$
- ☐ e. None of the other choices is correct ✓

$$f(x) = x^2/2 + 3x \rightarrow f'(x) = x + 3 \rightarrow f'(2) = 5$$

$$f(2) = 8$$

$$L(x) = f'(2)(x-2) + f(2) = 5(x-2) + 8$$

The correct answer is: None of the other choices is correct

Question 11

Correct

Mark 1.00 out of 1.00

Determine where the function is increasing and where it is decreasing:

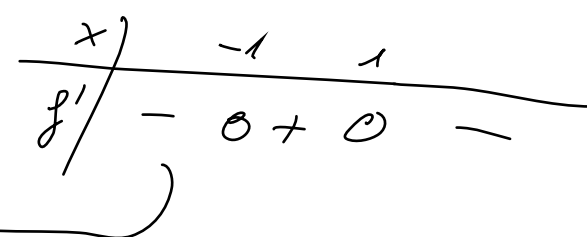
$$f(x) = 5x / (x^2 + 1)$$

$$f(x) = \frac{5x}{x^2+1} \quad f' = \frac{5(x^2+1) - 2x \cdot 5x}{(x^2+1)^2} = \frac{5-5x^2}{(x^2+1)^2}$$

Select one:

- ☐ a. None of the other choices is correct
- ☐ b. Increasing on $(-\infty, 0)$, decreasing on $(0, \infty)$
- ☐ c. Increasing on $(-\infty, -1)$ and $(1, \infty)$, decreasing on $(-1, 1)$
- ☐ d. Decreasing on $(-\infty, -1)$, increasing on $(-1, \infty)$
- ☒ e. Decreasing on $(-\infty, -1)$ and $(1, \infty)$, increasing on $(-1, 1)$ ✓

Xét dấu f' :



The correct answer is: Decreasing on $(-\infty, -1)$ and $(1, \infty)$, increasing on $(-1, 1)$

Question 12

Correct

Mark 1.00 out of 1.00

Find the horizontal asymptotes of the function.

$$y = \frac{2x^2+1}{3x-2x^2}$$

tiệm cận ngang

Select one:

- ☐ a. $y = 0$
- ☐ b. $x = 0, x = 3/2$
- ☐ c. $y = 1$
- ☐ d. None of the other choices is correct
- ☒ e. $y = -1$ ✓

$$\lim_{x \rightarrow \infty} \frac{2x^2+1}{3x-2x^2} = -1 \rightarrow y = -1$$

$$\lim_{x \rightarrow -\infty} \frac{2x^2+1}{3x-2x^2} = -1 \rightarrow y = -1$$

The correct answer is: $y = -1$

Question 13

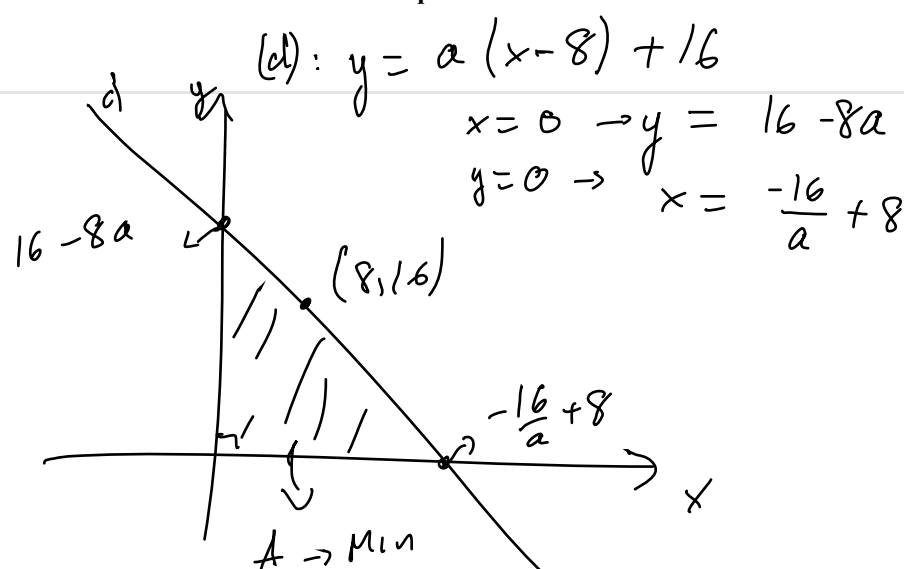
Not answered

Marked out of 1.00

Find an equation of the line through the point $(8, 16)$ that cuts off the least area from the first quadrant.

Select one:

- ☐ a. $y = x + 8$
- ☐ b. $y = -2x + 32$
- ☐ c. $y = -x + 24$
- ☐ d. $y = -4x + 48$
- ☐ e. $y = -3x + 40$



The correct answer is: $y = -2x + 32$

$$2A = (16-8a)\left(-\frac{16}{a}+8\right) = -\frac{256}{a} + 128 + 128 - 64a = f(a)$$

$$f' = \frac{256}{a^2} - 64, \quad f' = 0 \Leftrightarrow \begin{cases} a = 2 \\ a = -2 \end{cases}$$

• Đường thẳng tạo với góc phân từ thứ I tam giác

thì $a < 0 \Rightarrow$ chọn $a = -2 \Rightarrow y = -2x + 32$

Question 14

Correct

Mark 1.00 out of 1.00

Use Newton's method with the specified initial approximation x_1 to find x_3 , the third approximation to the root of the given equation.

$$x^3 - x + 3 = 0, x_1 = 2$$

(Give your answer to 4 decimal places)

Select one:

- ☒ a. 0.0944 ✓
- ☐ b. 0.1201
- ☐ c. 0.0825
- ☐ d. 0.1102
- ☐ e. None of the other choices is correct

The correct answer is: 0.0944

$$f(x) = x^3 - x + 3$$

$$f'(x) = 3x^2 - 1$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 - x_n + 3}{3x_n^2 - 1}$$

$$x_1 = 2 \rightarrow x_2 \rightarrow x_3$$

(Bấm máy)

Question 15

Correct

Mark 1.00 out of 1.00

Find the antiderivative of the function.

$$f(x) = 8x^{1/7} - 10x^{1/9}$$

ng / hām

$$= 8 \cdot \frac{7}{8} x^{8/7} - 10 \cdot \frac{9}{10} x^{10/9}$$

Select one:

- ☐ a. $(7/8)x^{8/7} - 9x^{10/9} + C$
- ☐ b. $8x^{8/7} - 9x^{10/9} + C$
- ☒ c. $7x^{8/7} - 9x^{10/9} + C$ ✓
- ☐ d. $7x^{8/7} - 9x^{10/9}$
- ☐ e. $7x^{8/7} - (10/9)x^{10/9} + C$

The correct answer is: $7x^{8/7} - 9x^{10/9} + C$

Question 16

Correct

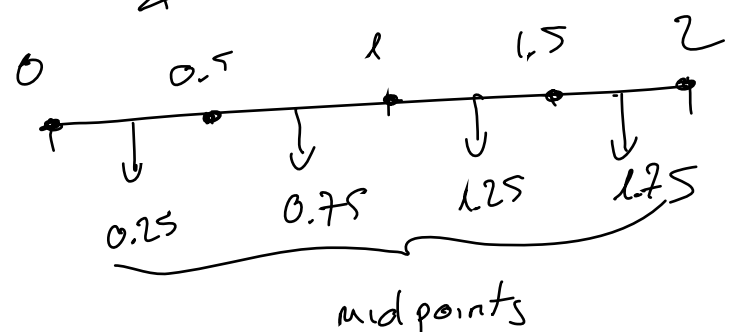
Mark 1.00 out of 1.00

Evaluate the Riemann sum for $f(r) = 2 - r^2$, $0 \leq r \leq 2$, with four subintervals, using midpoints.

Select one:

- ☒ a. 1.375 ✓
- ☐ b. None of the other choices is correct
- ☐ c. 2.75
- ☐ d. 4.5
- ☐ e. 2.25

$$\Delta x = \frac{2 - 0}{4} = 0.5$$



The correct answer is: 1.375

Riemann sum:

$$\sum f(x_i^*) \Delta x = 0.5 (f(0.25) + f(0.75) + f(1.25) + f(1.75))$$

Thay số vào hàm f.

Bấm máy:

$$x_i = x_0 + i \Delta x = 0 + 0.5i = 0.5i$$

$$\rightarrow x_{i-1} = 0.5(i-1)$$

$$\bar{x}_i = \frac{x_i + x_{i-1}}{2} = 0.5i - 0.25$$

$$\text{Bấm } 0.5 \sum_{i=1}^4 [2 - (0.5i - 0.25)^2]$$

Question 17

Correct

Mark 1.00 out of 1.00

An animal population is increasing at a rate of $13+51t^2$ per year (where t is measured in years). By how much does the animal population increase between the fourth and ninth years?

Select one:

- ☐ a. None of the other choices is correct
- ☐ b. 12300
- ☒ c. 11370 ✓
- ☐ d. 98054
- ☐ e. 15021

The correct answer is: 11370

$P(t)$: population
 $P'(t)$: rate $= 13 + 51t^2$

$$P(9) - P(4) = \int_4^9 P'(t) dt = \int_4^9 (13 + 51t^2) dt = 11370$$

Net Change Theorem.

Question 18

Correct

Mark 1.00 out of 1.00

Find the derivative of the function.

$$g(x) = \int_1^{x^2} \sqrt{2+7t} dt = F(x^2) - F(1) \quad \text{với } F \text{ là ng/hàm của } \sqrt{2+7t}$$

$$g' = F(x^2)' - 0 = \sqrt{2+7x^2} \cdot 2x \quad (\text{đh của } F \text{ là } \sqrt{2+7t} \downarrow \text{đh của } F \text{ là } \sqrt{2+7t} \text{ (đ/h hàm hợp)})$$

Select one:

☐ a. $\sqrt{2+7x^2}$

☐ b. $2\sqrt{2+7x^2}$

☒ c. $2x\sqrt{2+7x^2}$ ✓

☐ d. None of the other choices is correct

☐ e. $2x\sqrt{2+7x}$

The correct answer is: $2x\sqrt{2+7x^2}$

Question 19

Not answered

Marked out of 1.00

Evaluate the indefinite integral.

$$I = \int 6x(x^2+4)^4 dx$$

$u = x^2 + 4, du = 2x dx$

$$I = 3 \int u^4 du = \frac{3}{5} u^5 + C = \frac{3}{5} (x^2+4)^5 + C$$

Select one:

- ☐ a. $(3/5)(x^2+4)^5 + C$
- ☐ b. $(1/5)(x^2+4)^5 + C$
- ☐ c. $(4/5)(x^2+4)^5 + C$
- ☐ d. $(2/5)(x^2+4)^5 + C$

The correct answer is: $(3/5)(x^2+4)^5 + C$

Question
20

Not answered
Marked out of 1.00

Evaluate the integral.

$$I = \int (4x+3)e^{-5x} dx$$
$$u = 4x+3$$
$$dv = e^{-5x} dx$$

$$I = uv - \int v du$$
$$du = 4 dx$$
$$v = -\frac{1}{5} e^{-5x}$$
$$I = (4x+3)\left(-\frac{1}{5} e^{-5x}\right) + \frac{4}{5} \int e^{-5x} dx$$
$$= -\frac{4}{5} x e^{-5x} - \frac{3}{5} e^{-5x} - \frac{4}{25} e^{-5x} + C$$
$$= -\frac{4}{5} x e^{-5x} - \frac{19}{25} e^{-5x} + C$$

Select one:

- ☐ a. $4/5x e^{-5x} - 19/25 e^{-5x} + C$
- ☐ b. $-4/5x e^{-5x} + 19/25 e^{-5x} + C$
- ☐ c. $-4/5x e^{-5x} - 19/25 e^{-5x} + C$
- ☐ d. $4/5x e^{-5x} + 19/25 e^{-5x} + C$

The correct answer is: $-4/5x e^{-5x} - 19/25 e^{-5x} + C$

Question
21

Not answered
Marked out of 1.00

Use the Midpoint Rule with n = 3 steps to approximate the integral $\int_2^5 f(x) dx$. Round your answer to 2

decimal places.

	x_0	\overline{x}_1	x_1	\overline{x}_2	x_2	\overline{x}_3	x_3
x	2	2.5	3	3.5	4	4.5	5
f(x)	3	2	8	5	7	3	4

$$\Delta x = \frac{5-2}{3} = 1$$
$$\int_2^5 f(x) dx \approx M_3$$
$$= \Delta x (f(\overline{x}_1) + f(\overline{x}_2) + f(\overline{x}_3))$$
$$= 1(2 + 5 + 3) = 10$$

Select one:

- ☐ a. 15
- ☐ b. None of the other choices is correct
- ☐ c. 5
- ☐ d. 10
- ☐ e. 16

The correct answer is: 10

Question
22

Not answered
Marked out of 1.00

Determine whether the improper integral converges or diverges.

(i) $\int_{-1}^9 \frac{dx}{(x+1)^{1/3}}$

conv

(ii) $\int_0^\pi \frac{d\theta}{(\pi-\theta)^{5/7}}$

conv

What':

$\int_a^b \frac{dx}{(x-a)^\alpha}$

hay $\int_a^b \frac{dx}{(b-x)^\alpha}$

conv: $\alpha < 1$

div: $\alpha \geq 1$

- Select one:
- ☐ a. Only (ii) converges
 - ☐ b. Both diverges
 - ☐ c. Both converges
 - ☐ d. Only (i) converges

The correct answer is: Both converges

Question
23

Not answered
Marked out of 1.00

Determine whether the matrix is in echelon form, reduced echelon form, or neither.

1

0

-4

2

0

1

-5

-5

0

0

0

0

0

0

0

0

- Select one:
- ☒ a. Reduced echelon form ✓
 - ☐ b. Neither
 - ☐ c. Echelon but not reduced

The correct answer is: Reduced echelon form

Question 24

Not answered

Marked out of 1.00

Find all values of m so that the system

$$\begin{cases} x + y + 3z = m \\ -x + 3y + 2z = 3 \\ 3x - y + 4z = -1 \end{cases} \quad \text{3 ẩn, 3 pt}$$

has a unique solution.

hệ có n° duy nhất $\Leftrightarrow \det A \neq 0$

Select one:

- ☐ a. $m=1$
- ☐ b. all numbers but -1
- ☐ c. None of the other choices is correct
- ☐ d. $m = -1$
- ☐ e. all numbers but 1

$$\begin{vmatrix} 1 & 1 & 3 \\ -1 & 3 & 2 \\ 3 & -1 & 4 \end{vmatrix} = 0 \quad \text{nên hệ này, ko thể có duy nhất n°}$$

(Thật ra m n° ko liên quan đến việc có n° duy nhất hay ko ở bài này)

The correct answer is: None of the other choices is correct

Question 25

Not answered

Marked out of 1.00

How many solutions would a homogeneous system of linear equations of 6 equations and in 3 variables have?

$$AX = 0$$

Select one:

- ☐ a. Infinitely many solutions
- ☐ b. No solutions
- ☐ c. Only one solution
- ☐ d. 3 solutions
- ☐ e. There is not enough information

Chưa đủ thông tin:

Nên số ẩn của pt $AX = 0$ lớn hơn số pt \rightarrow lớn hơn Rank A thì khi đó chắc chắn có vô số n°

The correct answer is: There is not enough information

Question 26

Not answered

Marked out of 1.00

Find all values of a such that the system

$$\begin{cases} x + 2y + 3z = 0 \\ 4x + ay + 6z = 0 \\ 6x + 5y + 4z = 0 \end{cases}$$

has only the trivial solution.

$$\Leftrightarrow \det A \neq 0$$

$$\Leftrightarrow -14a + 70 \neq 0$$

$$a \neq 5$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & a & 6 \\ 6 & 5 & 4 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 4 & a \\ 6 & 5 \end{vmatrix}$$

$$\det A = 4a + 72 + 60$$

$$-18a - 30 - 32$$

$$= -14a + 70$$

Select one:

- ☐ a. 5
- ☐ b. All numbers but 5
- ☐ c. None of the other choices is correct
- ☐ d. All numbers but -3
- ☐ e. -3

The correct answer is: All numbers but 5

Question
27

Not answered

Marked out of 1.00

Find the entries in the first row of matrix **B** if $3A - B = 2C$, with

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 3 \\ 4 & 2 & 0 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = 3A - 2C = \begin{bmatrix} 1 & -3 & 3 \\ 0 & 0 & 0 \\ 10 & 4 & 2 \end{bmatrix}$$

Select one:

- ☐ a. -1, 3, 3
- ☐ b. 0, 2, -9
- ☐ c. 0, -2, 9
- ☐ d. None of the other choices is correct
- ☐ e. 1, -3, 3

The correct answer is: 1, -3, 3

Question
28

Not answered

Marked out of 1.00

Find the matrix product **AB** for the partitioned matrices.

$$A = \begin{bmatrix} 0 & I \\ I & F \end{bmatrix}, B = \begin{bmatrix} W & X \\ Y & Z \end{bmatrix}$$

(I is the identity matrix)

Select one:

- ☐ a. $\begin{bmatrix} 0 & Z \\ FY & FZ \end{bmatrix}$
- ☐ b. $\begin{bmatrix} Y & Z \\ W + FY & X + FZ \end{bmatrix}$
- ☐ c. $\begin{bmatrix} X & W + XF \\ Z & Y + ZF \end{bmatrix}$
- ☐ d. $\begin{bmatrix} Y & Z \\ W + YF & X + ZF \end{bmatrix}$

The correct answer is: $\begin{bmatrix} Y & Z \\ W + FY & X + FZ \end{bmatrix}$

(xem các ma trận con như
phân tử & thực hiện phép
nhân ma trận như bình
thường)

Question
29

Not answered

Marked out of 1.00

Suppose that **A** and **B** are square matrices and that $C = \begin{bmatrix} I & A \\ O & I \end{bmatrix}$ and $D = \begin{bmatrix} I & B \\ O & I \end{bmatrix}$, here **O** is zero matrix and **I** is identity matrix.

Choose the correct statements:

(i) $CD = \begin{bmatrix} I & B+A \\ O & I \end{bmatrix} \rightarrow$ các này đúng, tính như cân bên .

(ii) **C and D commute**

[nghĩa là $CD = DC$]

Select one:

- ☒ a. Both (i) and (ii) ✓
- ☐ b. (i) only
- ☐ c. (ii) only
- ☐ d. None of the other choices is correct

The correct answer is: Both (i) and (ii)

Ta chỉ cần thử xem $DC = ?$

$$DC = \begin{bmatrix} I & B \\ O & I \end{bmatrix} \begin{bmatrix} I & A \\ O & I \end{bmatrix} = \begin{bmatrix} I & A+B \\ O & I \end{bmatrix} = CD$$

Question 30

Not answered
Marked out of 1.00

Find the second entry of the second row of the matrix **A** that satisfies:

$$\left(\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} A\right)^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} * & * \\ * & 3 \end{bmatrix}$$

- Select one:
- ☐ a. None of the other choices is correct
 - ☐ b. 1
 - ☐ c. 3
 - ☐ d. 0
 - ☐ e. 2

The correct answer is: 3

Question 31

Not answered
Marked out of 1.00

Which of the following statements are true? (assume that all matrices are square matrices of the same size).

- (i) If **A** and **B** are invertible then **AB⁻¹** is also invertible and its inverse is **BA⁻¹**
- (ii) If **A** and **B** are invertible then **AB⁻¹** is also invertible and its inverse is **A⁻¹B**

$$\text{Note: } A, B: \text{invertible}$$
$$(AB)^{-1} = B^{-1}A^{-1}$$

- Select one:
- ☐ a. (i) and (ii)
 - ☐ b. (i)
 - ☐ c. None of them
 - ☐ d. (ii)

$$(AB^{-1})^{-1} = (B^{-1})^{-1}A^{-1} = BA^{-1}$$

The correct answer is: (i)

Question 32

Not answered
Marked out of 1.00

Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}$ be a linear transformation and u, v be vectors such that $T(u+v) = 1$ and $T(u-v) = 0$. Find $T(u+2v)$.

- Select one:
- ☐ a. 2/3
 - ☐ b. None of the other choices is correct
 - ☐ c. 3/2
 - ☐ d. 3
 - ☐ e. 2

$$u + 2v = a(u+v) + b(u-v)$$

$$\begin{cases} a + b = 1 \\ a - b = 2 \end{cases} \Rightarrow \begin{cases} a = 3/2 \\ b = -1/2 \end{cases}$$

$$T(u+2v) = T\left(\frac{3}{2}(u+v) - \frac{1}{2}(u-v)\right)$$

$$= \frac{3}{2}T(u+v) - \frac{1}{2}T(u-v) = \frac{3}{2} \cdot 1 - \frac{1}{2} \cdot 0$$

The correct answer is: 3/2

T : quay, sau đó đổi' x' $\longrightarrow T(x) = A(B(x)) = AB(x)$

Question 33

Not answered

Marked out of 1.00

có thể làm như:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \xrightarrow{\text{Quay}} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xrightarrow{\text{đổi } x} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \xrightarrow{\text{Quay}} \begin{bmatrix} -1 \\ 0 \end{bmatrix} \xrightarrow{\text{đổi } x} \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\rightarrow T: \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be rotation through $\frac{\pi}{2}$ followed by reflection in the line $y = x$. Find the matrix of T .

Select one:

- ☐ a. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
- ☐ b. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
- ☐ c. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- ☐ d. all of the other choices are false

The correct answer is: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

↓
B biến' $\left\{ \begin{array}{l} e_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ e_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} -1 \\ 0 \end{pmatrix} \end{array} \right\} \Rightarrow M_B:$

$$B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

A: biến' $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\rightarrow M_A: \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Vậy T có M là: $AB = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Question 34

Not answered

Marked out of 1.00

Let $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 3$. Find $\begin{vmatrix} a & p & x \\ a+2b+3c & p+2q+3r & x+2y+3z \end{vmatrix}$.

Select one:

- ☐ a. -18
- ☐ b. 6
- ☐ c. -6
- ☐ d. none of the other choices is true
- ☐ e. 18

The correct answer is: -6

$$\begin{vmatrix} a & p & x \\ c & r & z \\ 2b & 2q & 2y \end{vmatrix} = 2 \begin{vmatrix} a & p & x \\ c & r & z \\ b & q & y \end{vmatrix}$$

$$= -2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = -2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = -6$$

Question 35

Not answered

Marked out of 1.00

Let A be a 3×3 matrix. Let:

B be the matrix obtained from A by Interchanging the first and the second row.

$$|B| = -|A|$$

C be the matrix obtained from B by dividing the third row by 4.

$$|C| = \frac{1}{4} |B| = -\frac{1}{4} |A|$$

D be the matrix obtained from C by interchanging the second and the third row.

$$|D| = -|C| = \frac{1}{4} |A|$$

E be the matrix obtained from D by adding 5 times the second row to the third row.

$$|E| = |D| = \frac{1}{4} |A|$$

If $\det(E) = 2$, find $\det(A)$.

$$\rightarrow |A| = 4 |E| = 8$$

Select one:

- ☐ a. -1/2
- ☐ b. 1/2
- ☐ c. 8
- ☐ d. -40
- ☐ e. 40
- ☐ f. -8
- ☐ g. None of the other choices is correct

The correct answer is: 8

Question
36

Not answered
Marked out of 1.00

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & x \end{bmatrix}$.

Find the values of x so that A is NOT invertible. $\Leftrightarrow \det A = 0 \Leftrightarrow x - 1 = 0$

$x = 1$

- Select one:
- ☐ a. -1
 - ☐ b. None of the other choices is correct
 - ☐ c. All numbers but -1
 - ☐ d. All numbers but 1
 - ☐ e. 1

The correct answer is: 1

Question
37

Not answered
Marked out of 1.00

Let A, B be square matrices. Choose correct statements:

- (i) If $A = 0$, then $\text{adj}(A) = 0$
- (ii) If A is invertible, then $\text{adj}(A)$ is invertible

- Select one:
- ☐ a. (i) and (ii)
 - ☐ b. (ii)
 - ☐ c. None of the other choices is correct
 - ☐ d. (i)

The correct answer is: (i) and (ii)

Question
38

Not answered
Marked out of 1.00

Find the eigenvalues of the matrix

$\begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}$.

$| \lambda I - A | = 0$
 $\begin{vmatrix} \lambda - 1 & -1 \\ -4 & \lambda + 2 \end{vmatrix} = 0 \Leftrightarrow \lambda^2 + \lambda - 6 = 0$
 $\lambda = 2$
 $\lambda = -3$

- Select one:
- ☐ a. 3
 - ☐ b. -2; 3
 - ☐ c. None of the other choices is correct
 - ☐ d. 2; -3
 - ☐ e. -2

The correct answer is: 2; -3

Question 39

Not answered

Marked out of 1.00

Let $P(1,2,1)$, $Q(1,0,-1)$, $R(2,2,0)$ be the vertices of a parallelogram with adjacent sides PQ and PR . Find the other vertex S .

Select one:

- ☐ a. None of the other choices is correct
- ☐ b. $(2, 0, -2)$
- ☐ c. $(4, 0, 4)$
- ☐ d. $(0, -4, -4)$
- ☐ e. $(2, -2, 0)$

The correct answer is: $(2, 0, -2)$

$$\vec{RS} = \vec{PQ}$$

$$(x, y, z) - (2, 2, 0) = (1, 0, -1) - (1, 2, 1)$$

$$(x, y, z) = (2, 0, -2)$$

Question 40

Not answered

Marked out of 1.00

If $u = (-2, 1, 1)$ and $v = (1, 0, 1)$, then $\| \text{proj}_v(u) \|$ is:

Select one:

- ☐ a. $\frac{1}{6}$
- ☐ b. $\frac{\sqrt{6}}{6}$
- ☐ c. $\frac{\sqrt{2}}{2}$
- ☐ d. 0
- ☐ e. 1

The correct answer is: $\frac{\sqrt{2}}{2}$

$$\text{proj}_v u = \frac{u \cdot v}{v \cdot v} v$$

$$= \frac{-1}{2} (1, 0, 1)$$

$$\Rightarrow \| \text{proj}_v u \| = \frac{1}{2} \sqrt{1^2 + 0^2 + 1^2} = \frac{\sqrt{2}}{2}$$

Question 41

Not answered

Marked out of 1.00

Let the point $P(2, -1, 0)$. Find the shortest distance from the point P to the line

$$[x \ y \ z]^T = [1 \ 1 \ 0]^T + t[2 \ -1 \ -1]^T.$$

Select one:

- ☐ a. $\frac{1}{\sqrt{6}}$
- ☐ b. $\frac{1}{\sqrt{3}}$
- ☐ c. None of the other choices is correct
- ☐ d. $\frac{\sqrt{7}}{\sqrt{6}}$
- ☐ e. $\frac{\sqrt{7}}{\sqrt{3}}$

The correct answer is: $\frac{\sqrt{7}}{\sqrt{3}}$

$$\vec{HP} = \vec{P_0P} - \vec{P_0H} = \left(-\frac{1}{3}, -\frac{4}{3}, \frac{2}{3} \right)$$

$$\| \vec{HP} \| = \sqrt{\frac{1}{9} + \frac{16}{9} + \frac{4}{9}} = \sqrt{\frac{7}{3}}$$

$$P_0(1, 1, 0) \quad \vec{P_0P} = (1, -2, 0)$$

$$P(2, -1, 0)$$

$$\vec{P_0H} = \text{proj}_u \vec{P_0P} = \frac{\vec{P_0P} \cdot u}{u \cdot u} \cdot u$$

$$= \frac{4}{6} (2, -1, -1)$$

$$= \left(\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3} \right)$$

Question 42

Not answered
Marked out of 1.00

Let $u = (1, 2, 1)$; $v = (2, 3, 0)$; $w = (0, 1, 0)$. Find projection of $u \times v$ on w .

- Select one:
- ☐ a. $(0, 2, 0)$
 - ☐ b. $\frac{2}{\sqrt{15}}(-6, 4, -2)$
 - ☐ c. $(0, 10, 0)$
 - ☐ d. $(0, 0, 0)$
 - ☐ e. None of the other choices is correct

The correct answer is: $(0, 2, 0)$

$$u = (1, 2, 1)$$
$$v = (2, 3, 0)$$
$$u \times v = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$$
$$= (-3, 2, -1)$$
$$\text{proj}_w u \times v = \frac{u \times v \cdot w}{w^2} w$$
$$= \frac{2}{1} w = (0, 2, 0)$$

Question 43

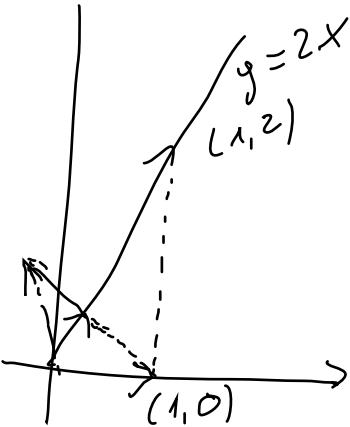
Not answered
Marked out of 1.00

Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be reflection in the x -axis followed by reflection in the line $y = 2x$. Find the sum of all entries in the first column of the matrix of T .

- Select one:
- ☐ a. $-7/5$
 - ☐ b. $-4/5$
 - ☐ c. $1/5$
 - ☐ d. $-3/5$
 - ☐ e. None of the other choices is correct

The correct answer is: $1/5$

$$T = \text{ref}_{y=2x} \circ \text{ref}_x$$
$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \xrightarrow{\text{ref}_x} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \xrightarrow{\text{ref}_{y=2x}} 2 \text{proj}_{(1,2)}(1,0) - (1,0)$$
$$\text{proj}_{(1,2)}(1,0) = \frac{1}{5}(1,2)$$
$$\left(\frac{2}{5}, \frac{4}{5} \right) - (1,0) = \left(-\frac{3}{5}, \frac{4}{5} \right)$$
$$\text{sum} = \frac{1}{5}$$



Question 44

Not answered
Marked out of 1.00

Let $u_1 = [-2, 0, 1]$, $u_2 = [3, 5, 6]$, $u_3 = [-2, 6, -4]$, $x = [-2, -32, -19]$

If express the vector x as $x = au_1 + bu_2 + cu_3$ then find c .

- Select one:
- ☐ a. 0
 - ☐ b. -3
 - ☐ c. none of the other choices is true
 - ☐ d. -1
 - ☐ e. -2

The correct answer is: -2

$$\begin{bmatrix} -2 & 3 & -2 \\ 0 & 5 & 6 \\ 1 & 6 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -2 \\ -32 \\ -19 \end{bmatrix}$$

Question 45

Not answered
Marked out of 1.00

Which of the following are subspaces of \mathbb{R}^3 ?

(i) $\{(x,y,z) \mid z = 2x+3y+2\}$ X Mặt phẳng này không qua $O(0,0,0)$

(ii) $\{(x,y,z) \mid x^2+y^2=z^2\}$ X

- Select one:
- ☐ a. (i) and (ii)
 - ☐ b. None of the other choices is correct
 - ☐ c. (i) only
 - ☐ d. (ii) only

The correct answer is: None of the other choices is correct

\mathbb{R}^3 có các kg con:

- $\{(0,0,0)\}$
- Đg thẳng qua O
- Mặt phẳng qua O
- \mathbb{R}^3

Question 46

Not answered
Marked out of 1.00

Find the dimension of the subspace $U = \text{span}\{[1, 1, 1], [2, 5, 2], [1, 2, 3]\}$.

- Select one:
- ☐ a. 1
 - ☐ b. 2
 - ☐ c. 3
 - ☐ d. none of the other choices is true

Rank $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 3 \end{bmatrix} = 3$

The correct answer is: 3

Question 47

Not answered
Marked out of 1.00

Let the set of vectors $\{u, v, w\}$ in \mathbb{R}^3 be independent. Which of the followings are true?

- (i) $\{u; v; u-v+w\}$ is also independent.
- (ii) $\{u; v+w; u+v+w\}$ is also independent.

- Select one:
- ☐ a. (ii) only
 - ☐ b. None of the other choices is correct
 - ☐ c. (i) only
 - ☐ d. (i) and (ii)

The correct answer is: (i) only

+ \rightarrow X

Question 48

Not answered
Marked out of 1.00

If we write $X = [2 \ -3 \ 2 \ 7]^T$ as a linear combination of the orthogonal basis of the subspace $U = \text{span}\{[2 \ -1 \ 0 \ 3]^T; [2 \ 1 \ -2 \ -1]^T\}$ then the sum of coefficients equals

- Select one:
- ☐ a. 1
 - ☐ b. $\frac{2}{3}$
 - ☐ c. none of the other choices is true
 - ☐ d. 0

$\begin{bmatrix} 2 \\ -3 \\ 2 \\ 7 \end{bmatrix} = a \begin{bmatrix} 2 \\ -1 \\ 0 \\ 3 \end{bmatrix} + b \begin{bmatrix} 2 \\ 1 \\ -2 \\ -1 \end{bmatrix} \rightarrow \begin{cases} a = \\ b = \end{cases} \rightarrow a+b$

Từ giải

The correct answer is: 1

Question
49

Not answered
Marked out of 1.00

Find the dimension of the subspace spanned by

[1, 1, 0, 9], [1, 1, 0, -1], [0, 0, 1, 7], [0, 0, 1, 0]

Select one:

- ☐ a. 2
- ☐ b. 4
- ☐ c. 3
- ☐ d. 1

Rank $\begin{bmatrix} 1 & 1 & 0 & 9 \\ 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 1 & 0 \end{bmatrix} = 3$

The correct answer is: 3

Question
50

Not answered
Marked out of 1.00

If A is a 7×9 matrix has rank 5, find $\dim(\text{Col}(A))$, $\dim(\text{Null}(A))$

Select one:

- ☐ a. $\dim(\text{col}(A))=4, \dim(\text{null}(A))=2$
- ☐ b. $\dim(\text{col}(A))=4, \dim(\text{null}(A))=5$
- ☐ c. $\dim(\text{col}(A))=5, \dim(\text{null}(A))=2$
- ☐ d. None of the other choices is correct
- ☐ e. $\dim(\text{col}(A))=5, \dim(\text{null}(A))=4$

The correct answer is: $\dim(\text{col}(A))=5, \dim(\text{null}(A))=4$

$A \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & \dots & \dots & \dots & \dots & \dots \\ \times & \times & \times & \times & \times & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & \dots & \dots & \dots \end{bmatrix}$

$\dim \text{Col} A = \dim \text{Row} A = \text{rank} A$
 $\dim \text{Null} A = 9 - \text{rank} A$