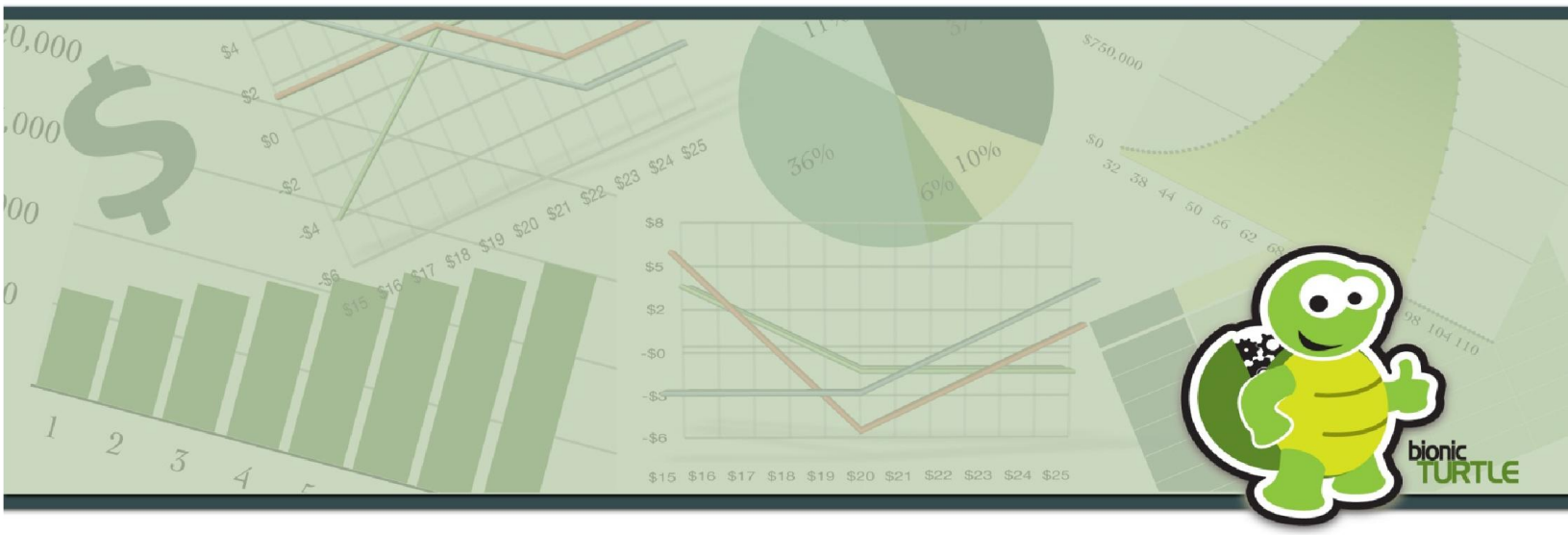


P2.T6. Credit Risk Measurement & Management

Giacomo De Laurentis, Renato Maino, and Luca Molteni, Developing, Validating and Using Internal Ratings

Bionic Turtle FRM Practice Questions

By David Harper, CFA FRM CIPM
www.bionicturtle.com

Chapter 2: Classifications and Key Concepts of Credit Risk

P2.T6.700. Credit risk classifications

Learning objectives: Describe the role of ratings in credit risk management. Describe classifications of credit risk and their correlation with other financial risks. Define default risk, recovery risk, exposure risk and calculate exposure at default.

700.1. In contrasting approaches to credit risk, De Laurentis distinguishes between *default-mode* and *value-based* valuation. A default-mode valuation (aka, loss-based valuation) is **LEAST** likely to formally incorporate which of the following risks?

a) Default risk
b) Spread risk
c) Exposure risk
d) Recovery risk

700.2. Stimway Corp has a revolving credit line with its bank. It has already drawn $1.0 million against a limit of $3.0 million. If we assume a loan equivalency factor (LEQ) of 0.70 or 70.0%, what is the exposure at default (EAD)?

a) $1.40 million
b) $2.10 million
c) $2.40 million
d) $3.70 million

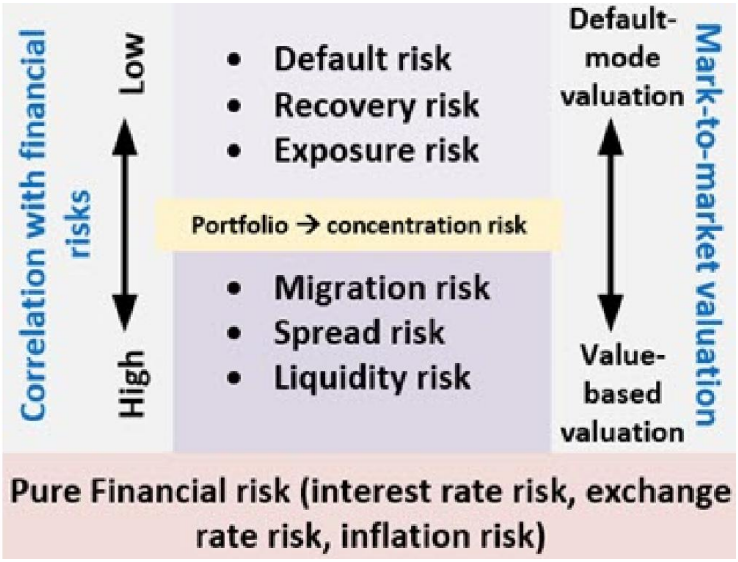
700.3. While performing a default-model valuation for a bond offering, analyst Samantha encounters a disposition of assets clause that limits the borrower's ability to sell key assets. Which measure is most likely impacted by this clause?

a) Default
b) Exposure
c) Recovery
d) None of the above (it has no direct bearing)

Answers:

700.1. B. Spread risk. See replication of De Laurentis' Table 2.1 below.

De Laurentis: "2.1.1. Default mode and value-based valuations: Credit risk can be analyzed and measured from different perspectives. Table 2.1 shows a classification of diverse credit risk concepts. Each of the listed risks depends on specific circumstances. Default risk (also called counterparty risk, borrower risk and so forth, with minor differences in meaning) is an event related to the borrower's default. Recovery risk is related to the possibility that, in the event of default, the recovered amount is lower than the full amount due. Exposure risk is linked to the possible increase in the exposure at the time of default compared to the current exposure. A default-mode valuation (sometimes also referred to as 'loss-based valuation') considers all these three risks."



700.2. C. TRUE. $2.40 million = $1.0 mm drawn + 70% * ($3.0 mm limit - $1.0 mm drawn).

Exposure at default (EAD) = Drawn + (Limit - Drawn) * LEQ; where

- **Drawn** is the amount currently used (it can be zero in case of back-up lines, letters of credit, performance bonds or similar),
- **Limit** is the maximum amount granted by the bank to the borrower for this credit facility, and
- **LEQ (Loan Equivalency Factor)** is the rate of usage of the available limit, beyond the ordinary usage, in near-to-default situations. Note: this is also known as usage given default (UGD) as found in Ong.

700.3. C. Recovery

De Laurentis (**emphasis ours**): "2.1.3 Recovery risk: The recovery rate is the complement to the loss in the event of default (typically defined as LGD, Loss Given Default, expressed as a percentage). Note that here default is given, that is to say that it has already occurred. In the event of default, the net position proceeds dependent on a series of elements. First of all, recovery procedures may be different according to the type of credit contracts involved the legal system and the court that has jurisdiction. The recovery rate also depends on the general economic conditions: results are better in periods of economic expansion. Defaulted borrowers' business sectors are important because assets values may be more or less volatile in different sectors. Also, **covenants are important; these agreements between borrower and lender raise limits to borrower's actions, in order to provide some privileges to creditors. Some covenants, such as those limiting the disposal of important assets by the borrower, should be considered in LGD estimation.** Other types of collateral may reduce the probability of default rather than the LGD; these are delicate aspects to models (Altman, Resti and Sironi, 2005; Moody's Investor Service, 2007)."

Discuss in forum here: *https://www.bionicturtle.com/forum/threads/p2-t6-700-credit-risk-classifications-de-laurentis.10068/*

P2.T6.701. Unexpected loss and return on risk-adjusted capital (RARORAC)

Learning objectives: Explain expected loss, unexpected loss, VaR, and concentration risk, and describe the differences among them. Evaluate the marginal contribution to portfolio unexpected loss. Define risk-adjusted pricing and determine risk-adjusted return on risk-adjusted capital (RARORAC).

701.1. Consider two credit positions with identical features:

- Adjusted exposure, EAD = $5.0 million each
- Probability of default, PD (aka, EDF) = 5.0%
- Loss given default, LGD = 50.0%
- Standard deviation of LGD, σ(LGD) = 40.0%
- The default correlation between the positions, ρ(position #1, position #2) = 0.20

Although unexpected loss (UL) can be calibrated according to any confidence level, we decide to keep things unrealistically simple and define UL as one standard deviation, which implicitly suggests a low confidence level. As such, each position's unexpected loss (UL) is about $705,000. Which is **nearest** to the two-asset portfolio's unexpected loss (UL)? (Bonus questions: how was the position UL derived? What is each position's risk contribution?)

a) $1.092 million
b) $1.275 million
c) $1.410 million
d) $2.033 million

701.2. Analyst Mark is evaluating a portfolio of credit-sensitive assets. He is estimating expected loss (EL), unexpected loss (UL) and credit value at risk (CVaR) under various correlation assumptions. Each of the following is true **EXCEPT** which is a false dynamic?

a) An increase in the CVaR confidence level implies an increase in either the position's or portfolio's CVaR
b) An increase in (inter-position) default correlation between credit positions in a portfolio, ρ(position X, position Y), implies an increase in the portfolio's unexpected loss (UL)
c) An increase in (inter-position) default correlation between credit positions in a portfolio, ρ(position X, position Y), implies an increase in the portfolio's expected loss (EL)
d) An increase in (intra-position) correlation between a position's own default probability (PD) and its own loss given default (LGD), ρ[PD(position X), LGD(position X)], say from its typically assumed zero to a non-zero parameter, implies an increase in the position's EL

701.3. You are analyzing a $3.0 billion retail loan portfolio and you are given the following assumptions:

- Revenue (ie, spread + fees) = $153.0 million = 5.0% of $3.0 billion portfolio assets plus (+) $3.0 million in fees
- Expected loss, EL = $60.0 million = 2.0% of $3.0 billion portfolio assets
- Cost of funds, COF or COC = $30.0 million = 1.0% of $3.0 billion liabilities (assume liabilities equal assets)
- Economic capital, EC = $300.0 million = 10.0% of portfolio assets
- Cost of operations = $23.0 million
- Tax rate = 40.0%

Which of the following is **nearest** to the after-tax risk-adjusted return on risk-adjusted capital (RARORAC)? (See answer for reconciliation with T7's RAROC)

a) Zero
b) 8.00%
c) 11.50%
d) 13.33%

Answers:

701.1. A. $1.092 million = SQRT(705.0^2 + 705.0^2 + 2*705.0*705.0*0.20). We can use portfolio UL = SQRT[UL_position_1^2 + UL_position_2^2 + 2*UL_position_1*UL_position_2*ρ(position_1, position_2)].

For calculations, see XLS @ https://www.dropbox.com/s/3medq01lgmmlmcg/T6-701-1.xlsx

- ***Position UL*** = EAD * sqrt[EDF * σ(LGD)^2 + LGD^2 * σ(EDF)^2] = $5.0 mm * sqrt[0.050 * 0.40^2 + 0.50^2*0.050*(1-0.050)], where σ(EDF)^2 = EDF*(1-EDF) = ~ $705,00
- ***Risk contribution of each position*** = $705,00 * ($705,000 + $705,000*0.20)/$1,092,181 = $546,091; i.e., one-half of the portfolio UL as we would expect!

701.2. C. FALSE. Portfolio EL is the sum of position ELs such that default correlation has no impact here.

In regard to (A), (B) and (D), each is TRUE.

- ***In regard to true (A)*****,** an increase in confidence level always increases (C)VaR because it implies a greater loss quantile
- ***In regard to true (B)*****,** portfolio UL is a dispersion measure (i.e., a multiple of standard deviation) such that it increases with correlation
- ***In regard to true (D)*****,** unlike portfolio EL which is a summation, position EL is a product: EL = PD*LGD. We normally assume independence between PD and LGD, but increasing correlation between them increases EL because covariance[PD, LGD] = E[PD*LGD] - E[PD]*E[LGD] such that EL = E[PD*LGD] = E[PD]*E[LGD] + covariance[PD, LGD], where we typically assume covariance[PD, LGD] in which case E[PD*LGD] = E[PD]*E[LGD]; note this is test of independence.

701.3. B. 8.00%. Per De Laurentis' formula (adjusted for taxes), RARORAC = [(Spread + fees - EL - COF - Ops)*(1-tax rate)]/EC = [(153.0 - 60.0 - 30.0 - 23.0)*60%]/300.0 = 24/300 = 8.00%.

Please note that De Laurentis uses RARORAC as "the most renowned" variant of RAROC where:

- RARORAC = (Spread + Fees - Expected loss - Cost of Capital - OpCosts)/(Economic capital), used in the answer.

But it's not clear to me the De Laurentis reading is rigorous with respect to RAROC/RARORAC; for example, there is no numerical illustration. For the better part of a decade, the Part 2 FRM has depended on Crouhy's rigorous RAROC (not in Crouhy's Essentials but rather the older, deeper Risk Management. See http://amzn.to/2iIsuNG). My question attempts some reconciliation with Crouhy's RAROC, which is given by:

- RAROC = (Expected Revenues +ROEC - transfers [aka, interest expense] - EL - OpCosts - taxes)/EC

Such that if we translated the same assumptions given in the question, but if we recast De Laurentis (Spread + Fees) as equal to Crouhy's (Revenue + Return on risk capital), which in this question are together equal to $153.0, then we get the same final result. That is, if we re-cast this question into Crouhy's term we have:

- Revenue = $150.0 mm = 5.0% * $3.0 billion
- Return on EC = $3.0 mm = 1.0% * $300 EC; i.e., Revenue + ROEC = $153.0 mm
- EL = $60.0 mm = 2.0% * $3.0 billion
- OpCosts = $30.0 mm
- EC = $300 mm

In summary:

- De Laurentis RARORAC numerator = (Spread + Fees - Cost of Capital - EL - OpCost)*(1-tax_rate) = (150.0 + 3.0 - 30.0 - 60.0 - 23.0)*(1-40%), which is equal to:
- Crouhy's RAROC = (Expected Revenues + ROEC - Interest exp - EL - OpCost)*(1-tax_rate) = ([150.0 + 3.0] - 30.0 - 60.0 - 23.0)*(1-40%)

Spreadsheet is here https://www.dropbox.com/s/aen6ed9f2fl0137/T6-701-2.xlsx

Discuss in forum here: *https://www.bionicturtle.com/forum/threads/p2-t6-701-unexpected-loss-and-return-on-risk-adjusted-capital-rarorac-de-laurentis.10078/*

Chapter 3: Ratings Assignment Methodologies

P2.T6.702. Credit rating assignment methodologies
P2.T6.703 Structural versus Reduced-form credit risk approaches
P2.T6.704 Linear discriminant analysis
P2.T6.705. Logistic regression and principal component analysis
P2.T6.706. Heuristic approach versus neural networks

P2.T6.702. Credit rating assignment methodologies

Learning objectives: Explain the key features of a good rating system. Describe the experts-based approaches, statistical-based models, and numerical approaches to predicting default. Describe a rating migration matrix and calculate the probability of default, cumulative probability of default, marginal probability of default, and annualized default rate.

702.1. Assume the following actual, recent but truncated (on the right) 12-month rating migration matrix from Moody's:

12-Month Rating Transition Matrices
Global Structured Finance

	Total	Aaa	Aa1	Aa2	Aa3	A1	A2	A3	B
Aaa	8,357	98.1%	0.4%	0.1%	0.1%	0.3%	0.0%	0.0%	0.1
Aa1	925	21.1%	70.9%	0.1%	0.6%	2.7%	0.2%	0.6%	0.9
Aa2	1,788	7.4%	4.1%	87.8%	0.5%	0.1%			
Aa3	870	8.5%	10.0%	10.8%	67.2%	1.4%		0.7%	0
A1	1,060	3.7%	6.1%	15.5%	8.1%	64.6%	0.8%	0.5%	
A2	1,433	3.8%	2.4%	4.5%	7.4%	7.0%	73.0%	0.8%	0
A3	1,234	1.1%	1.1%	2.0%	4.5%	10.5%	4.4%	74.6%	
Baa1	1,412	0.8%	1.1%	1.3%	2.3%	6.4%	5.6%	5.7%	72.
Baa2	1,032	0.7%	0.1%	0.9%	1.4%	2.3%	4.2%	8.2%	7.
Baa3	1,761	0.5%	0.1%	0.2%	0.5%	0.9%	1.2%	4.9%	8
Ba1	1,315	0.7%		0.4%	0.2%	0.8%	0.3%	1.8%	6.
Ba2	1,219	0.4%	0.1%	0.5%	0.2%	0.2%	0.5%	1.2%	
Ba3	1,393	0.3%		0.2%	0.1%	0.2%	0.1%	0.3%	0
B1	1,247	0.2%		0.1%		0.2%		0.2%	
B2	1,332	0.2%		0.3%	0.1%	0.2%	0.1%		
B3	1,153	0.3%			0.1%		0.1%	0.1%	0.
Caa1	1,625		0.1%		0.1%		0.1%	0.1%	
Caa2	1,808					0.1%	0.1%		0.1
Caa3	1,804			0.1%	0.1%	0.1%		0.1%	0.1

Consider an issuer who is rated "**Aa1**" today (at the beginning of the year). Which of the following is nearest to the probability that this issuer does not fall into any **LOWER** rating category at any time *over the next two years*; put another way, what is the probability the issuer never experiences a rating of "Aa2" or worse over the next two years?

a) 50.3%
b) 64.9%
c) 75.0%
d) 86.0%

702.2. Each of the following is a desirable feature of a credit rating system **EXCEPT** which is not?

a) Measurable and verifiable: Ratings should give expectations in terms of default probabilities which can be adequately and continuous back tested
b) Positive and monotonic: In comparing two ratings, the firm with a better rating will have lower financial leverage (defined as assets/equity or debt/equity)
c) Specificity: The rating system is measuring the distance from the default event without any regards to other corporate financial features not directly related to it, such as short term fluctuations in stock prices
d) Objective and homogeneity: The rating system generates judgments based only on credit risk considerations, while avoiding any influence by other considerations; and, ratings are comparable among portfolios, market segments, and customer types

702.3. According to De Laurentis among the following approaches which is is the **LEAST LIKELY** to meet the criteria of Measurability and Verifiability?

a) Internal expert-based
b) External rating agency
c) Structural approach
d) Discriminant analysis

Answers:

702.1. D. 86.0%

There are four paths which keep the issuer in the top two categories:

- Aa1 [current] > Aa1 [end of 1st year] > Aa1 [end of 2nd year] with probability 0.709^2 = 50.2681%
- Aa1 > Aa1 > Aaa with probability 0.709*0.211 = 14.95990%
- Aa1 > Aaa > Aaa with probability 0.211*0.981 = 20.69970%
- Aa1 > Aaa > Aa1 with probability 0.211*0.0040 = 0.08440%

These are mutually exclusive probabilities such that we can sum them: 50.2681% + 14.95990% + 20.69970% + 0.08440% = 86.01150%

702.2. B. FALSE. Because ratings are complex and multi-factor, the higher rating could have lower leverage. Further, "positive and monotonic" is not a key feature. Recall that ratings are ordinal rankings, not cardinal rankings; e.g., "Rating is an ordinal measure of the probability of the default event on a given time horizon, having the specific features of measurability, objectivity, and homogeneity [sic: should also include specificity], to properly confront counterparts and segments of the credit portfolio."

In regard to (A), (C) and (D) each is TRUE as a key feature, according to De Laurentis.

***De Laurentis**:* "Therefore, rating systems have three desirable features in terms of measurability and verifiability, objectivity and homogeneity, and specificity:

- ***Measurability and verifiability***: these mean that ratings have to give correct expectations in terms of default probabilities, adequately and continuously back tested.
- ***Objectivity and homogeneity***: the former means that the rating system generates judgments only based on credit risk considerations, while avoiding any influence by other considerations; the latter means that ratings are comparable among portfolios, market segments, and customer types.
- ***Specificity***: this means that the rating system is measuring the distance from the default event without any regards to other corporate financial features not directly related to it, such as short term fluctuations in stock prices. These three features help to define a measure of appropriateness of internal rating systems and are decisive in depicting their distinctive suitability for credit management. However, the ability of different methodologies and approaches to deal with these desirable profiles is a matter of specific judgment, given the tradeoffs existing among them."

702.3. A. TRUE: Among these choices, the internal expert-based is LEAST LIKELY to be measurable and verifiable

In regard to (B), (C) and (D), De Laurentis assigned either fully compliant (structural approach, discriminant analysis) or three-quarters compliant (external rating agencies)

De Laurentis: "3.2.4 Experts-based internal ratings used by banks: As previously mentioned, banks' internal classification methods have different backgrounds from agencies' ratings assignment processes. Nevertheless, sometimes their underlying processes are analogous; when banks adopt judgmental approaches to credit quality assessment, the data considered and the analytical processes are similar. Beyond any opinion on models' validity, rating agencies put forward a sound reference point to develop various internal analytical patterns. For many borrower segments, banks adopt more formalized (that is to say model based) approaches. Obviously, analytical solutions, weights, variables, components, and class granularities are different from one bank to another. But market competition and syndicated loans are strong forces leading to a higher convergence. In particular, where credit risk market prices are observable, banks tend to harmonize their valuation tools, favoring a substantial convergence of methods and results.

In principle, there is no proven inferiority or superiority of expert-based approaches versus formal ones, based on quantitative analysis such as statistical models. Certainly, judgment-based schemes need long lasting experience and repetitions, under a constant and consolidated method, to assure the convergence of judgments. It is very difficult to reach a consistency in this methodology and in its results because:

- organizational patterns are intrinsically dynamic, to adapt to changing market conditions and bank's growth, conditions that alter processes, procedures, customers' segments, organization appetite for risk and so forth;
- mergers and acquisitions that blend different credit portfolios, credit approval procedures, internal credit underwriting powers and so forth;
- over time, company culture will change, as well as experts' skills and analytical frameworks, in particular with reference to qualitative information.

Even if the predictive performances of these methods (read by appropriate accuracy measures) are good enough in a given period, it is not certain that the same performance will be reached in the future. This uncertainty could undermine the delicate and complex management systems that are based on internal rating systems in modern banking and financial organizations.

-continued on next page-

An assessment of the main features of expert-based rating systems along the three principles that have been previously introduced is proposed in Table 3.6."

Table 3.6 Summary of the main features of expert-based rating systems.

Criteria	Agencies' ratings	Internal experts-based rating systems
Measurability and verifiability		
Objectivity and homogeneity		
Specificity		

The circle is a measure of adequacy: full when completely compliant, empty if not compliant at all. Intermediate situations show different degrees of compliance.

Discuss in forum here: *https://www.bionicturtle.com/forum/threads/p2-t6-702-credit-rating-assignment-methodologies-de-laurentis.10096/*

P2.T6.703 Structural versus Reduced-form credit risk approaches

Learning outcomes: Describe rating agencies' assignment methodologies for issue and issuer ratings. Describe the relationship between borrower rating and probability of default. Compare agencies' ratings to internal experts-based rating systems. Distinguish between the structural approaches and the reduced-form approaches to predicting default.

703.1. De Laurentis explains the annualized default rate (ADR) as follows: "If it is necessary to price a credit risk exposed transaction on a five year time horizon, it is useful to reduce the five-year cumulated default rate to an annual basis for the purposes of calculation. The annualized default rate can be calculated by solving the following equation:"

$$ADR_t = 1 - \sqrt[t]{\prod_{i=1}^{t} SR_1^{forw}} = 1 - \sqrt[t]{\left(1 - PD_t^{cumulated}\right)}$$

Assume the following pattern of defaults over five years (this is similar to Table 3.5):

	Years				
	1	2	3	4	5
names $_{t=0}$	1000				
names $_{t}$	992	980	970	964	951
default $_{cumulated;\ t}$	8	20	30	36	49

What is **nearest** to the five year annualized default rate, ADR(5)?

a) 1.00%
b) 1.35%
c) 2.18%
d) 3.33%

703.2. Assume the following pattern of defaults over five years for a given rating class:

	Years				
	1	2	3	4	5
names $_{t=0}$	1000				
names $_{t}$	990	976	958	941	922
default $_{cumulated, t}$	10	24	42	59	78

Please note that what De Laurentis calls the "forward default probability" is more commonly called the *conditional* default probability; further, what he calls the "marginal default probability" is called by Hull an *unconditional* default probability by Hull. What is the relationship between the **conditional** (aka, forward) default probability and **unconditional** (aka, marginal) default probability in the third year?

a) They are equal
b) Conditional PD is higher than unconditional PD
c) Unconditional PD is higher than conditional PD
d) We need default during the third year; not enough information

703.3. Your firm's Risk Committee invites you to a meeting where they are deciding between a structural and reduced form approach to estimated credit risk for the firm. Each of the four members makes one of the statements below. Each of the statements is plausible **EXCEPT** which is probably not true?

a) We prefer stability over sensitivity to equity market movements and therefore a reduced form approach is better
b) Our experts have developed a causal (cause-and-effect) model that predicts default based on the likelihood of insolvency; this describes a structural approach
c) Our firm trades with a stock price so we do have an equity price and volatility but asset volatility is not a traded quantity for us, so the structural approach is not available to us
d) Our model is based on a correlation between explanatory variables and default but we are concerned whether we can generalize the results due to high sampling variability; this is a natural and expected risk with our reduced form approaches

Answers:

703.1. A. 1.00%. The 5-year cumulative survival, SR_cumul(5) = 951/1000 = 95.10% such that ADR(5) = 1 - 0.9510^(1/5) = 0.99979%.

703.2. B. Conditional PD is higher than unconditional PD; this will always be the case except in the first year.

Unconditional PD = (42-24)/1,000 = 1.80%
Conditional PD = (42-24)/(1,000 - 24) = (42-24)/976 = 1.844%

703.3. C. FALSE. Per the Merton model for credit risk, if the firm has a observable equity price and volatility, it can solve for asset value and volatility (of course, the firm knows its own liability structure!).

In regard to (A), (B) and (D), each is TRUE.

Recall that De Laurentis divides the credit rating systems into three approach: 1. expert-based, 2. statistical-based, and 3. heuristic and numerical.

In regard to true (A), De Laurentis writes "In comparison to agencies' ratings [ie, expert-based approaches], Merton's approaches [ie, statistical-based approaches] are:

- more sensitive to market movements and quicker and more accurate in describing the path to default;
- far more unstable (because of continuative movements in market prices, volatility, and interest rates). This aspect is not preferred by long term institutional investors that like to select investments based on counterparties' fundamentals, and dislike changing asset allocation too frequently."

In regard to true (B), De Laurentis writes "3.3.2. Structural approaches > Structural approaches are based on economic and financial theoretical assumptions describing the path to default. Model building is an estimate (similar to that of econometric models) of the formal relationships that associate the relevant variables of the theoretical model. This is opposite to the reduced form models, in which the final solution is reached using the most statistically suitable set of variables and disregarding the theoretical and conceptual causal relations among them. This distinction became very apparent after the Merton (1974) proposal: default is seen as a technical event that occurs when the company's proprietary structure is no longer worthwhile ... Merton's model is therefore a cause-and-effect approach: default prediction follows from input values. In this sense, Merton's model is a structural approach because it provides analytical insight into the default process."

-continued on next page-

In regard to true (D), De Laurentis writes "3.3.3 Reduced form approaches > Reduced form models as opposed to structural models make no ex ante assumptions about the default causal drivers. The model's relationships are estimated in order to maximize the model's prediction power: firm characteristics are associated with default, using statistical methodologies to associate them to default data ... In reduced form approaches there is a clear model risk: models intrinsically depend on the sample used to estimate them. Therefore, the possibility to generalize results requires a good degree of homogeneity between the development sample and the population to which the model will be applied. It should be clear at this point that different operational, business and organizational conditions, local market structures, fiscal and accounting regimes, contracts and applicable civil laws, may produce very different paths to default. As a consequence, this makes it clear that a model estimated in a given environment may be completely ineffective in another environment."

Discuss in forum here: *https://www.bionicturtle.com/forum/threads/p2-t6-703-structural-versus-reduced-form-credit-risk-approaches-delaurentis.10101/*

P2.T6.704 Linear discriminant analysis

Learning objectives: Apply the Merton model to calculate default probability and the distance to default and describe the limitations of using the Merton model. Describe linear discriminant analysis (LDA), define the Z-score and its usage, and apply LDA to classify a sample of firms by credit quality.

704.1. In the Merton approach to credit risk, default probability is given by this function (note the formula in De Laurentis is incorrect and should be given as follows):

$$PD = N\left(\frac{\ln(F) - \ln(V_A) - \left(\mu - {}^{1}/_{2}\,\sigma_A^2\right)T}{\sigma_A\sqrt{T}}\right)$$

Let us make the following assumptions:

- The firm's asset value, V(A), is $1.0 billion
- The expected asset return, μ, is 15.0%
- The volatility of the assets, σ(A), is 25.0%
- The face value of debt, F, is $500.0 million
- The debt matures in four years; i.e., T = 4.0 years

Which is **nearest** to the default probability (PD) estimated by Merton?

a) 0.5%
b) 1.0%
c) 2.5%
d) 5.0%

704.2. You have proposed a traditional linear discriminant analysis (LDA) for the purpose of predicting default and evaluating credit risk. Members of your firm's Risk Committee politely articulate the following objections to (i.e., arguments against) an LDA approach. Each of the following arguments is a valid drawback or shortcoming to the traditional LDA approach **EXCEPT** which is incorrect?

a) The model trains on only two outcomes, defaulting or performing, such that gradations of default are not considered
b) The model's weights in the traditional discriminant function (eg, Altman's Z) are constant, but in reality the weights are likely to vary over time
c) A traditional discriminant model will limit the number of variables and therefore is likely to ignore important factors; e.g., qualitative or macroeconomic factors
d) We cannot translate the Z-score into either a credit rating or a default probability; ie, given the Z-score is unitless neither a numerical nor analytical mapping is justified

704.3. The exhibit below (displayed in a format similar to De Laurentis' Table 3.7) shows the Altman's Z-score calculation for a hypothetical company:

Asset & Liabilities/Equities (000s)	
Fixed assets	$100.0
Inventories	90.0
Receivables	120.0
Cash	4.0
	314.0
Capital	$80.0
Accrued Cap. Reserves	40.0
Financial debts	130.0
Payables	54.0
Other net liabilities	10.0
	314.0

P&L (000s)	
Sales	$500.00
EBITDA	$35.00
Net Fin'l Expenses	$9.75
Taxes	$8.33
Profit	$16.92
Dividends	$11.34
Accrued Profits	$5.59

Equity market value	$ 49.6

Ratios	Ratio	Model Coeff.		Ratio Contribution
	(A)	(B)	(A*B)	
WC/TA	68.2%	1.210	0.825	25.8%
RE/TA	12.7%	1.400	0.178	5.6%
EBIT/TA	11.1%	3.300	0.368	11.5%
EMV/TL	38.2%	0.600	0.229	7.2%
S/TA	159.2%	0.999	1.591	49.9%
		Altman's Z-score	3.191	

Assume a stress test that downwardly shocks three of the variables as follows:

- Sales decline of 10%
- EBITDA decline of 10%
- Equity market value decline of 20%

No other accounts are affected. Which of the following is **nearest** to the outcome for the updated Altman's Z-score?

a) Unaffected
b) Z-score drops to 2.95 but the model still predicts safe ("performing"), or at least grey zone
c) Z-score drops to 2.24 and the model predicts distress ("default")
d) Z-score drops to 1.88 and the model predicts distress ("default")

Answers:

704.1. B. 1.0% = N(-2.336) = N([LN(500/1000) - (0.15 - 0.5*0.25^2)*4]/[0.25*sqrt(4)]) = N([LN(0.5) - 0.4750]/0.50] = N(-2.336) where -2.33 should be familiar as the 0.010 quantile.

Please note that due to the symmetry of the normal distribution, the PD function given in this question (which is equivalent to the Stulz version) is equivalent to the N(-d2) version in Hull which simply negates the other side of the symmetrical distribution. In addition to the negative (-), we can see that ln(F) - ln[V(A)] = ln[F/V(A)], such that we have:

$$PD = N\left(\frac{\ln(F) - \ln(V_A) - \left(\mu - {}^1\!/_2\,\sigma_A^2\right)T}{\sigma_A\sqrt{T}}\right) = N\left(-\frac{\ln(V_A/F) + \left(\mu - {}^1\!/_2\,\sigma_A^2\right)T}{\sigma_A\sqrt{T}}\right)$$

704.2. D. FALSE. Rather, we can translate the Z-score into either a credit rating or a default probability

De Laurentis: "3.3.4.3 From discriminant scores to default probabilities > The probability associated to the scoring function can be determined by adopting two main approaches: **the first being empirical, the second analytical. The empirical approach** is based on the observation of default rates associated to ascendant cumulative discrete percentiles of Z-scores in the sample. If the sample is large enough, a lot of scores are observed for defaulted and non-defaulted companies. We can then divide this distribution in discrete intervals. By calculating the default rate for each class of Z intervals, we can perceive the relationship between Z and default frequencies, which are our a priori probabilities of default. If the model is accurate and robust enough, default frequency is expected to move monotonically with Z values. Once the relationship between Z and default frequencies is set, we can infer that this relation will also hold in the future, extending these findings to new (out-of-sample) borrowers. Obviously, this correspondence has to be continuously monitored by periodic back testing to assess if the assumption is still holding. **The analytical approach** is based again on the application of Bayes' theorem. Z-scores have no inferior or superior limits whereas probabilities range between zero and one ... "

In regard to (A), (B) and (C), each is a valid argument against LDA. Because the De Laurentis reading is not strong (in my humble opinion), these choices are informed by Saunders Chapter 10:

"There are a number of problems in using the discriminant analysis model to make credit risk evaluations. The **first problem** is that these models usually discriminate only between two extreme cases of borrower behavior: no default and default. As discussed in Chapter 7, in the real world various gradations of default exist, from nonpayment or delay of interest payments (nonperforming assets) to outright default on all promised interest and principal payments. This problem suggests that a more accurate or finely calibrated sorting among borrowers may require defining more classes in the discriminant analysis model.

The **second problem** is that there is no obvious economic reason to expect that the weights in the discriminant function—or, more generally, the weights in any credit scoring model—will be constant over any but very short periods. The same concern also applies to the variables, X(j). Specifically, because of changing real and financial market conditions, other borrower-specific financial ratios may come to be increasingly relevant in explaining default risk probabilities. Moreover, the linear discriminant model assumes that the X(j) variables are independent of one another.

The **third problem** is that these models ignore important, hard-to-quantify factors that may play a crucial role in the default or no default decision. For example, reputation of the borrower and the nature of long-term borrower–lender relationship could be important borrower-specific characteristics, as could macrofactors such as the phase of the business cycle. These variables are often ignored in credit scoring models. Moreover, traditional credit scoring models rarely use publicly available information, such as the prices of outstanding public debt and equity of the borrower.

A **fourth problem** relates to default records kept by FIs. Currently, no centralized database on defaulted business loans for proprietary and other reasons exists. Some task forces set up by consortiums of commercial banks, insurance companies, and consulting firms are currently seeking to construct such databases largely in response to reforms to bank capital requirements." -- Saunders. Financial Institutions Management: A Risk Management Approach (8th edition, Page 297)

704.3. B. TRUE: Z-score drops to 2.95 but the model still predicts safe ("performing"), or at least grey zone. We only need to shock the deltas of the contributions: (-10%*0.368) + (-10%*0.2290) + (-20%*1.5910) = -0.24170 is the change (reduction) in the Z-score, from 3.191 to 2.949 = 3.191 - 0.24170.

Asset & Liabilities/Equities (000s)	
Fixed assets	$100.0
Inventories	90.0
Receivables	120.0
Cash	4.0
	314.0
Capital	$80.0
Accrued Cap. Reserves	40.0
Financial debts	130.0
Payables	54.0
Other net liabilities	10.0
	314.0
Equity market value	$ 39.7

P&L (000s)	
Sales	$450.00
EBITDA	$31.50
Net Financial E	$9.75
Taxes	$8.33
Profit	$13.42
Dividends	$11.34
Accrued Profit	$2.09

Ratios	Ratio	Model Coeff.		Ratio Contr
	(A)	(B)	(A*B)	
WC/TA	68.2%	1.210	0.825	28.0%
RE/TA	12.7%	1.400	0.178	6.0%
EBIT/TA	10.0%	3.300	0.331	11.2%
EMV/TL	30.5%	0.600	0.183	6.2%
S/TA	143.3%	0.999	1.432	48.6%
	Altman's Z-score		2.949	

Discuss here in forum: *https://www.bionicturtle.com/forum/threads/p2-t6-704-linear-discriminant-analysis-lda-according-to-de-laurentis.10112/*

P2.T6.705. Logistic regression and principal component analysis

Learning objectives: Describe the application of a logistic regression model to estimate default probability. Define and interpret cluster analysis and principal component analysis.

705.1. Logistic regression is often used to predict whether a loan will default. For example, the logit function can predict conditional default by the estimation of default probability as a function of several explanatory variables x(1), x(2) ... x(n), where X(i) for example could be income or loan-to-value. Here is the general form:

$$P(loan_{status} = 1|x_1, \dots, x_n) = \pi_1 = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + \cdots + \beta_m x_m)}}$$

In reference to this logistic regression model, each of the following is a true statement **EXCEPT** which is false?

a) The logistic regression assumes homoskedasticity and normally distributed errors terms just like the classic linear regression model (CLRM)
b) The slope coefficient β(1) can be interpreted as the change in the "odds ratio" associated with a one unit change in the explanatory (predictor) variable x(1)
c) The logistic function constrains the dependent variable (output) to a value between the [0,1] interval which is necessary to be considered a probability
d) The logistic function is a transformation of a linear regression: the link function, LN[π/(1-π)], is a linear combination of the explanatory (predictor) variables

705.2. De Laurentis says about cluster analysis that "the objective of cluster analysis is to explore if, in a dataset, groups of similar cases are observable. This classification is based on *measures of distance* of observations' characteristics. Clusters of observations can be discovered using an aggregating criterion based on a specific homogeneity definition. Therefore, groups are subsets of observations that, in the statistical domain of the (q) variables, have some similarities due to analogous variables' profiles and are distinguishable from those belonging to other groups. The usefulness of clusters depends on: (i) algorithms used to define them, and (ii) economic meanings that we can find in the extracted aggregations. Operationally, we can use two approaches: hierarchical or aggregative on the one hand, and partitioned or divisive on the other hand."

About the cluster analysis, each of the following statements is true **EXCEPT** which is false?

a) In the case of divisive clustering with (n) observations, the initial state is one cluster of size (n)
b) In the case of hierarchical clustering with (n) observations, the initial state is (n) clusters, each of size one
c) A "pre-treatment" (i.e., preliminary transformation) of variables in order to reach similar magnitude and variability
d) Both hierarchical and divisive clustering require the assumption of Euclidean distance which does not penalize greater distances

705.3. Your colleague has conducted a principal component analysis (PCA) and prepared the output below. This are De Laurentis' Tables 3.11 and 3.12. The variables are financial performance indicators, most of which are income statement and/or balance sheet ratios. Specifically, return on equity (ROE), return on investment (EBIT/invested capital), current ratio (CR; current assets/current liabilities), quick ratio, leverage (MCTI), market share (SHARE%), and intangibles (R&S%):

Principal components

Components	Eigenv-values	Explaned variance on tot variance	Cumulative variance explained
COMP1	2.762	39.5%	39.5%
COMP2	1.827	26.1%	65.6%
COMP3	1.098	15.7%	81.2%
COMP4	0.835	11.9%	93.2%
COMP5	0.226	3.2%	96.4%
COMP6	0.171	2.4%	98.8%
COMP7	0.081	1.2%	100.0%
Total	7.000	100.0%	200.0%

Correlation coeff. between original variables and components

Ratios	COMP1	COMP2	COMP3	R^2	Singularity (1-R^2)
ROE	0.367	0.875	0.053	0.902	0.098
ROI	0.486	0.798	-0.100	0.883	0.117
CR	0.874	-0.395	0.057	0.923	0.077
CQR	0.885	-0.314	-0.044	0.883	0.117
MTCI	-0.892	0.149	0.196	0.856	0.144
SHARE(%)	-0.055	0.259	-0.734	0.609	0.391
R&S(%)	-0.215	-0.286	-0.709	0.631	0.369

Each of the following statements is true about this PCA output **EXCEPT** which is inaccurate?

a) The third component (COMP3) is the current ratio (CR) variable
b) The first four components of the PCA explain over 90.0% of the total variance
c) It is possible to use the components themselves as independent (explanatory) variables in a linear regression
d) The first and second components (COMP1 and COMP2) are orthogonal to each other; i.e., uncorrelated vectors

Answers:

705.1. A. FALSE. A difference between logistic regression and the classical linear regression model (CLRM) is that logistic does not assume constant variance and the errors are not normally distributed.

De Laurentis: "To focus differences with the classical linear regression, consider that:

- In classical linear regression the dependent variable range is not limited and, therefore, may assume values outside the [0; 1] interval; when dealing with risk, this would be meaningless. Instead, a logarithmic relation has a dependent variable constrained between zero and one.
- The hypothesis of homoscedasticity of the classical linear model is meaningless in the case of a dichotomous dependent variable because, in this circumstance, variance is equal to π*(1-π)
- The hypothesis testing of regression parameters is based on the assumptions that errors in prediction of the dependent variables are distributed similarly to normal curves. But, when the dependent variable only assumes values equal to zero or one, this assumption does not hold."

In regard to (B), (C) and (D), each is TRUE.

705.2. D. FALSE. There are several different measure of distance offering various tradeoffs.

De Laurentis: "The choice of the distance measure to use is crucial in order to have meaningful final results. The measures which are most used are:

- The Euclidean distance,
- The geometric distance (also called Mahalanobis distance), which takes into account different scales of data and correlations in the variables,
- The Hamming distance, which measures the minimum number of substitutions required to change one case into another,
- Some homogeneity measures, such as the χ 2 test and the Fisher's F test.

Obviously, each criterion has its advantages and disadvantages. It is advisable to pre-treat variables in order to reach a similar magnitude and variability; indeed, many methods are highly influenced by variables' dimension and variance, and, thus, in order to avoid being unconsciously driven by some specific population feature, a preliminary transformation is highly recommended."

In regard to (A), (B), and (C), each is TRUE.

705.3. A. FALSE. Each component is not an explicit variable but rather an implicit variable informed by a combination of loadings on some combination of the explicit variables.

As De Laurentis explains about the first component "The first component is the feature that characterizes the variables set the most. In this case, we can see that it is highly characterized by the liquidity variables, either directly (for current liquidity and quick liquidity ratios) or inversely correlated (financial leverage)." [i.e., note all four of these correlations are above +0.85 or below -0.85].

In regard to (B), (C) and (D), each is TRUE.

- ***In regard to true (B)***, the first four components explain 93.2% of the total variance
- ***In regard to true (C)***, see https://en.wikipedia.org/wiki/Principal_component_regression
- ***In regard to true (D)***, "Determination of the principal components is carried out by recursive algorithms. The method is begun by extracting the first component that reaches the maximum communality; then, the second is extracted by operating on the residuals which were not explained by the previous component, under the constraint of being orthogonal, until the entire original variables set is transformed into a new principal components set."

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P2.T6.706. Heuristic approach versus neural networks

Learning objectives: Describe the use of a cash flow simulation model in assigning rating and default probability, and explain the limitations of the model. Describe the application of heuristic approaches, numeric approaches, and artificial neural networks in modeling default risk and define their strengths and weaknesses. Describe the role and management of qualitative information in assessing probability of default.

706.1. The new credit department of your firm is evaluating three different methodological approaches to the assignment of credit ratings: structural, reduced form, and cash flow simulation. In regard to these approaches, each of the following considerations is true **EXCEPT** which is false?

a) The least expensive model to build and maintain is the cash flow simulation
b) Cash flow analysis is effective in rating companies whose track records are meaningless or non-existent
c) Structural approaches are often applied to listed companies because the required input data is available
d) Structural approaches tend to be more objective and homogeneous than reduced form approaches

706.2. If we want to compare a heuristic approach to a numerical approach for the purpose of developing internal credit ratings, each of the following is a true statement according to De Laurentis **EXCEPT** which is not accurate?

a) A key risk in the application of neural networks is the model risk of over-fitting
b) Neither heuristic nor neural networks are well-suited to fuzzy logic and fuzzy environments
c) A major limit of neural networks is that we have to accept results from a so-called black box
d) From the perspective of credit rating assignment, heuristic approaches have the advantage of giving order, objectivity, and discipline to the rating process

706.3. In regard to the role and management of **QUALITATIVE** information in the development of internal credit ratings, which of the following pieces of advice does De Laurentis give?

a) Try to avoid qualitative information
b) Binary variables are better than nominal and ordinal variables
c) A good collection survey contains several open-ended questions
d) Gather only qualitative information that is not collectible in quantitative terms.

Answers:

706.1. A. False. De Laurentis says cash flow simulation is very expensive

De Laurentis: "It is very easy to understand the purposes of the method and its potential as a universal application. Nevertheless, there are a considerable number of critical points. The first is model risk. Each model is a simplification of reality; therefore, the cash flow generator module cannot be the best accurate description of possible future scenarios. But, the cash flow generator is crucial to count expected defaults. Imperfections or inaccuracies in its specification are vital in determining default probability. Hence, it is evident that we are merely transferring one problem (the direct determination of default probability through a statistical model) to another (the cash flow generator that produces the number of potential default circumstances). Moreover, future events have to be weighed by their occurrence in order to rigorously calculate default probabilities. In addition, there is the problem of defining what default is for the model. We do not know if and when a default is actually filed in real circumstances. Hence, we have to assume hypotheses about the default threshold. This threshold has to be:

- not too early, otherwise we will have many potential defaults, concluding that the transaction is very risky (but associated LGD will be low),
- not too late, otherwise we will have low default probability (showing a low risk transaction) but we could miss some pre-default or soft-default circumstances (LGD will be predicted as severe).

Finally, **the analysis costs have to be taken into consideration.** A cash flow simulation model is very often company specific or, at least, industry specific; it has to be calibrated with particular circumstances and supervised by the firm's management and a competent analyst. The model risk is amplified by the cost to build it, to verify and maintain its effectiveness over time. If we try to avoid (even partially) these costs, this could reduce the model efficiency and accuracy."

In regard to (B), (C) and (D), each is TRUE.

706.2. B. False. De Laurentis says that both heuristic and numerical approaches support, indeed are well-suited, to fuzzy logic.

De Laurentis: "Expert systems [aka, heuristic methods] may also include fuzzy logic applications. Fuzzy logic has been applied to many fields, from control theory to artificial intelligence. In default risk analysis, many rules are simply rule-of-thumb that have been derived from experts' own feelings; often, thresholds are set for ratios but, because of the complexity of real world, they can result to be both sharp and severe in many circumstances. Fuzzy logic is derived from fuzzy set theory, which is able to deal with approximate rather than precise reasoning. Fuzzy logic variables are not constrained to the two classic extremes of black and white logic (zero and one), but rather they may assume any value between the extremes. When there are several rules, the set thresholds can hide incoherencies or contradictions because of overlapping areas of uncertainty and logical mutual exclusions. Instead, adopting a more flexible approach, many clues can be integrated, reaching a solution that converges to a sounder final judgment. For example: (i) if interest coverage ratio (EBIT divided by interest paid) is less than 1.5, the company is considered as risky, (ii) if ROS (EBIT divided by revenues) is more than 20%, the company is considered to be safe. The two rules can be combined together.

Only when both are valid, that is to say ROS is lower (higher) than 20% and interest coverage is lower (higher) than 1.5, we can reach a dichotomous risky/safe solution. In all other cases, we are uncertain ....
3.4.2. Neural networks: Artificial neural networks originate from biological studies and aim to simulate the behavior of the human brain, or at least a part of the biological nervous system (Arbib, 1995; Steeb, 2008). They comprise interconnecting artificial neurons, which are software programs intended to mimic the properties of biological neurons. Artificial neurons are hierarchical nodes (or steps) connected in a network by mathematical models that are able to exploit connections by operating a mathematical transformation of information at each node, **often adopting a fuzzy logic approach** ..."

In regard to (A), (C) and (D), each is TRUE.

706.3. D. True: Gather only qualitative information that is not collectible in quantitative terms.

De Laurentis: "Qualitative variables are potentially numerous and, consequently, some ordering criterion is needed to avoid complex calculations and information overlapping. Moreover, forms to be filled in soon become very complex and difficult to be understood by analysts.

A first recommendation is to only gather qualitative information that is not collectable in quantitative terms. For instance, growth and financial structure information can be extracted from balance sheets. A second recommendation regards how to manage qualitative information in quantitative models.

A preliminary distinction is needed between different categorical types of information: (i) nominal information, such as regions of incorporation; (ii) binary information (yes/no, presence/absence of an attribute); and (iii) ordinal classification, with some graduation (linear or nonlinear) in the levels (for instance, very low/low/medium/high/very high). Binary indicators can be transformed in 0/1 ‘dummy variables’. Also, ordinal indicators can be transformed into numbers and weights can be assigned to different modalities (the choice of weights is, however, debatable). When collecting data, it is preferable to structure the information in closed form if we want to use it in quantitative models. This means forcing loan officers to select some pre-defined answers. Binary variables are difficult to managed in statistical models because of their non-normal distribution. Where possible, a multistage answer is preferable, instead of yes/no. Weights can be set using optimization techniques, like bootstrap or a preliminary test on different solutions to select the most suited one."

In regard to (A), (B) and (C), each is FALSE; i.e., De Laurentis does not advise these.

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