

MORE ON
LINEAR REGRESSION

Recap: Linear Regression

Hypothesis: $h_{\theta}(x) = \theta_0 + \theta_1 x$

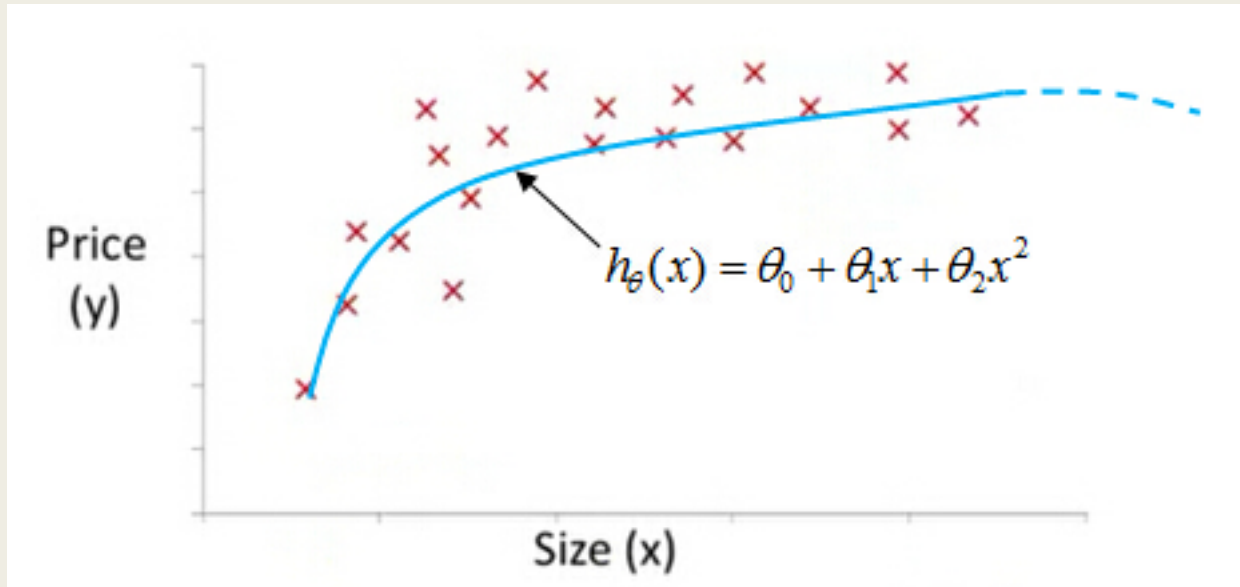
Parameters: θ_0, θ_1

Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$

Goal: minimize $J(\theta_0, \theta_1)$
 θ_0, θ_1

Polynomial Regression

- A polynomial function may fit the data better than a linear function



Set

- $x_1 = x$
- $x_2 = x^2$

Model representation: $h(x) = \theta_0 + \theta_1 x_1 + \theta_1 x_2$

Polynomial Regression

Model representation: $h(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2$

Cost Function

$$J(\theta_0, \theta_1, \theta_2) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Gradient Descent (Parameters Learning)

repeat until convergence: {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_0^{(i)}$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_1^{(i)}$$

$$\theta_2 := \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_2^{(i)}$$

...

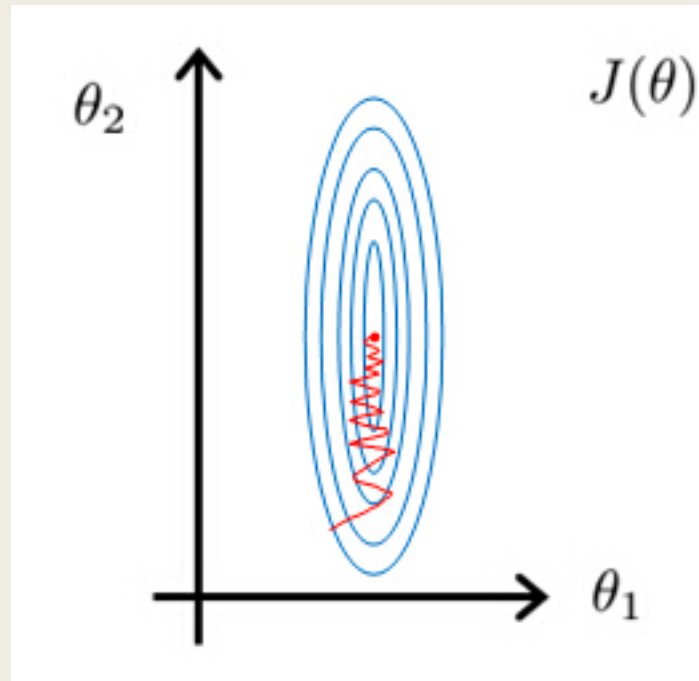
}

Feature Scaling

Idea: make sure features are on a similar scale

x_1 = size (0-2000 feet²)

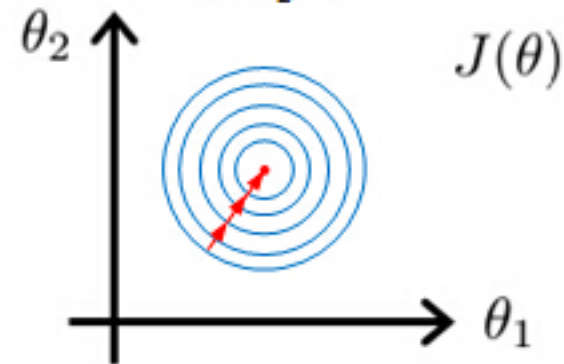
x_2 = number of bedrooms (1-5)



$$x_1 = \frac{\text{size(feet}^2\text{)}}{2000}$$
$$x_2 = \frac{\text{number of bedrooms}}{5}$$

$$0 \leq x_1 \leq 1$$

$$0 \leq x_2 \leq 1$$



Feature Scaling

Mean Normalization

$$x_i \leftarrow \frac{x_i - mean}{std}$$

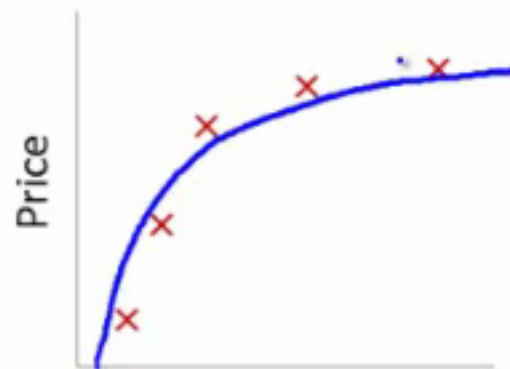
So all features would have an average of zero and similar range

The problem of overfitting



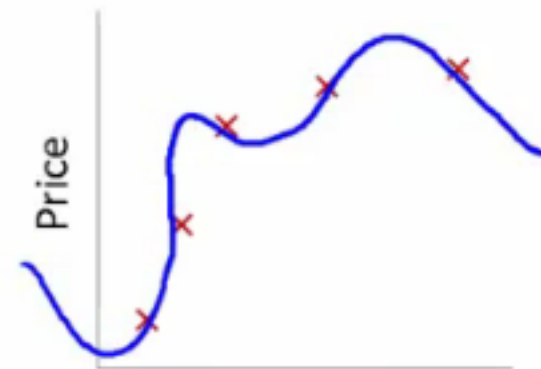
$$\theta_0 + \theta_1 x$$

High bias
(underfit)



$$\theta_0 + \theta_1 x + \theta_2 x^2$$

"Just right"



$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

High variance
(overfit)

Addressing overfitting

1) Reduce number of features

- Manually select which features to keep
- Model selection algorithms are discussed later (good for reducing number of features)
- But, in reducing the number of features we lose some information

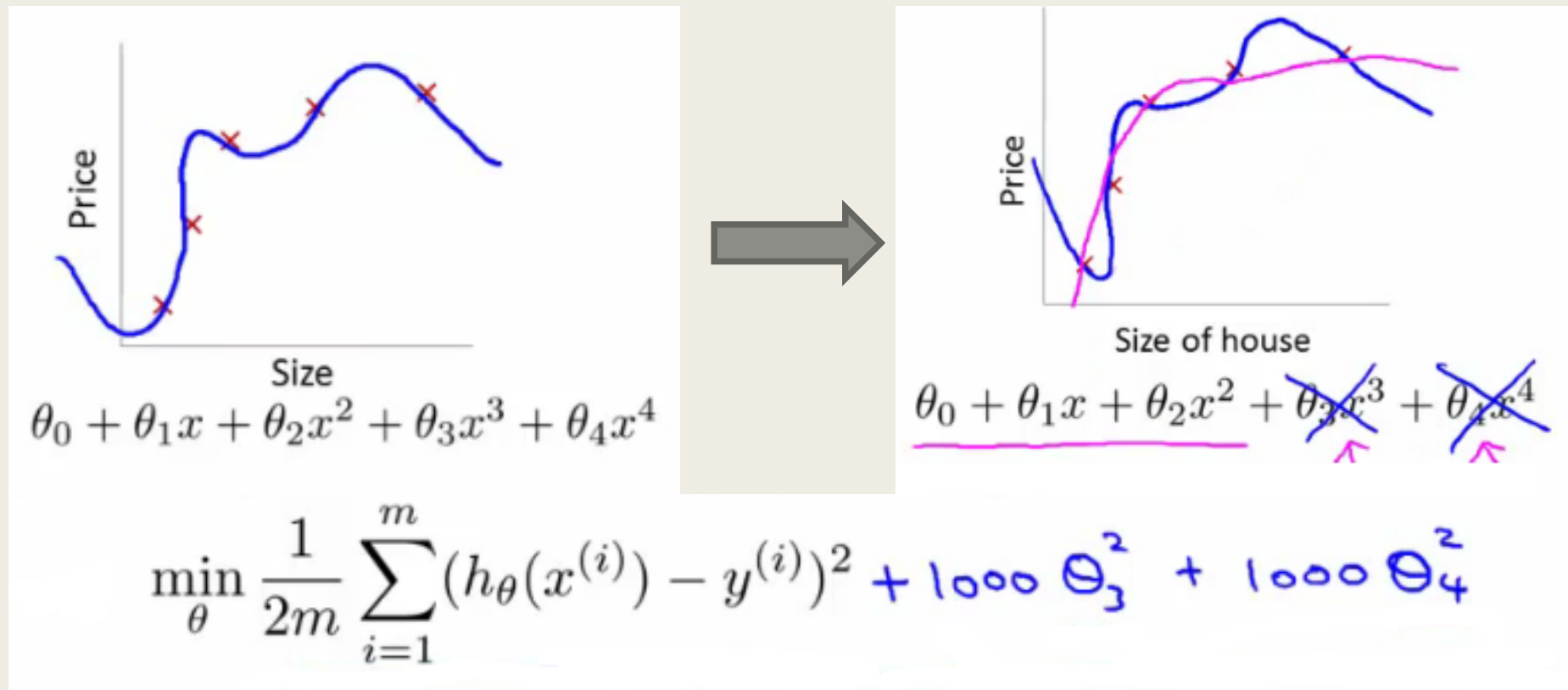
2) Regularization

- Keep all features, but reduce magnitude of parameters θ
- Works well when we have a lot of features, each of which contributes a bit to predicting y

Regularization

Intuition: making theta so small that is almost equivalent to zero

Penalize and make some of the θ parameters really small.g. here θ_3 and θ_4



Ridge regression (L2 Regularization)

$$J(\boldsymbol{\theta}) = \frac{1}{2m} \sum_{i=1}^m (h_{\boldsymbol{\theta}}(x^i) - y^i)^2 + \lambda \sum_{j=1}^n \theta_j^2$$

- Small values for parameters corresponds to a simpler hypothesis (you effectively get rid of some of the terms)
- A simpler hypothesis is less prone to overfitting
- λ is the **regularization parameter**
Controls a trade off between our two goals
 1. Want to fit the training set well
 2. Want to keep parameters small

This is what we call "**Hyper-parameters**"

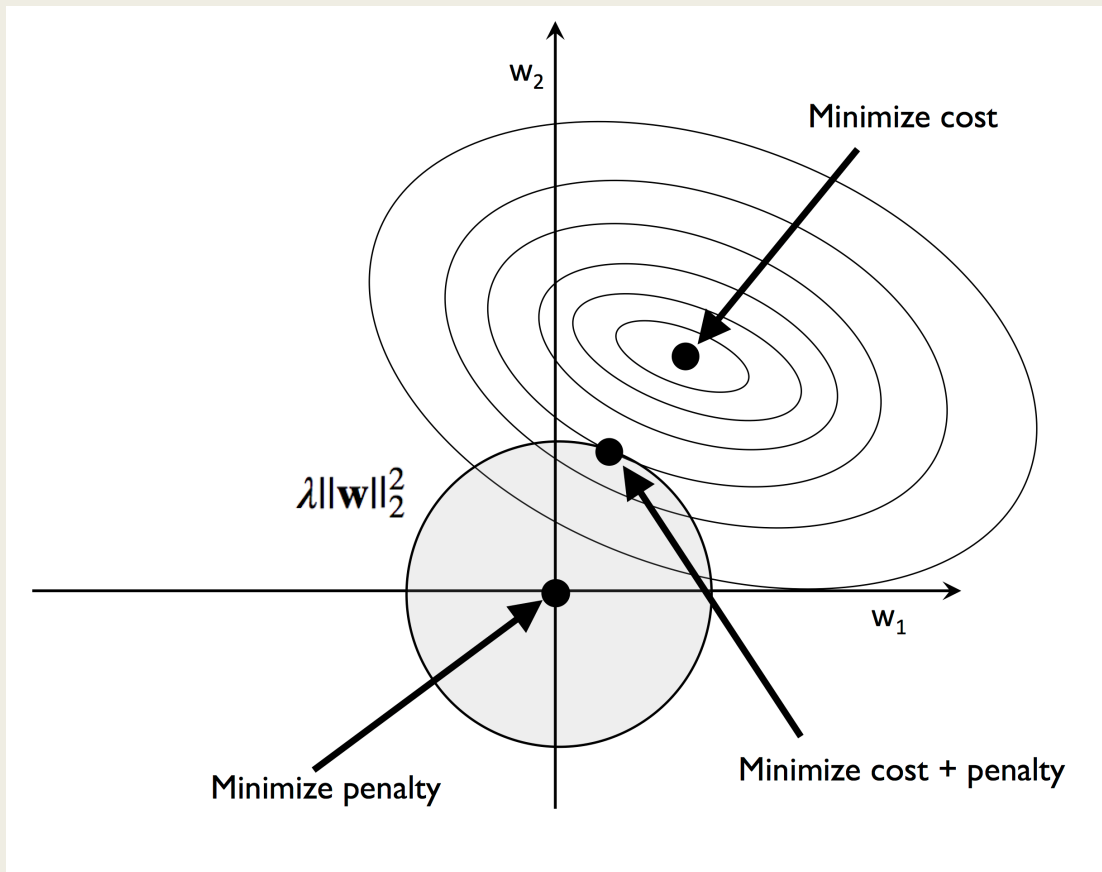
Lasso regression (L1 Regularization)

$$J(\boldsymbol{\theta}) = \frac{1}{2m} \sum_{i=1}^m (h_{\boldsymbol{\theta}}(x^i) - y^i)^2 + \lambda \sum_{j=1}^n |\theta_j|$$

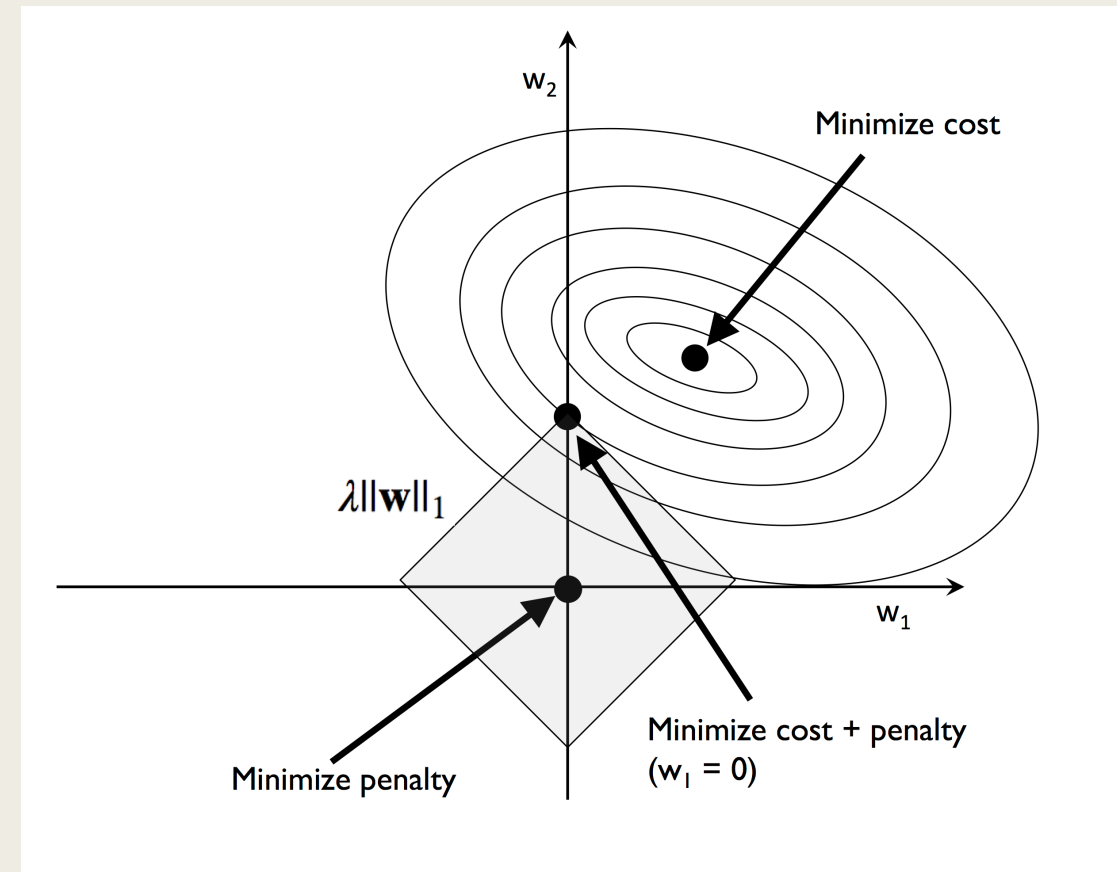
- Very similar to L2 regularization.
- However, instead of a quadratic penalty term as in L2, we penalize the model by the absolute weight coefficients
- This is a subtle, but important change. Some of the coefficients may be shrunk exactly to zero.
- Perform both “Feature selection” and “Regularization”

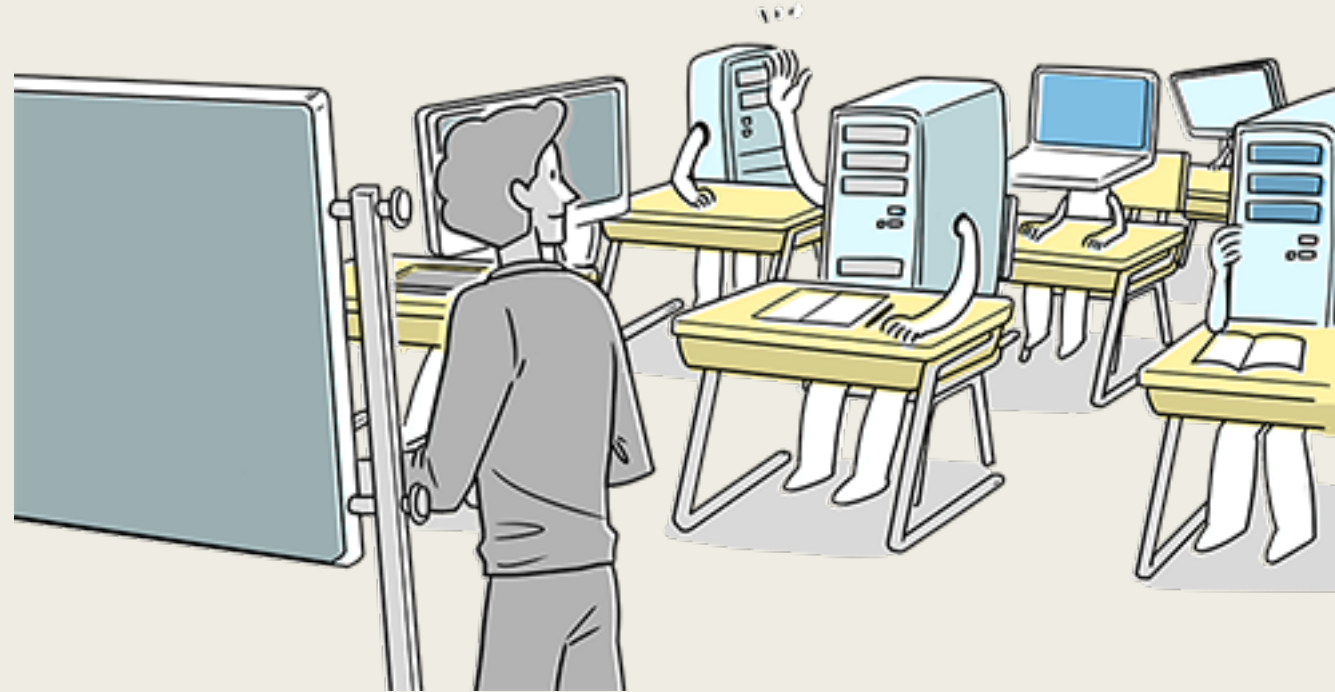
Ridge (L2) vs Lasso (L1)

Ridge



Lasso





CLASSIFICATION

Classification

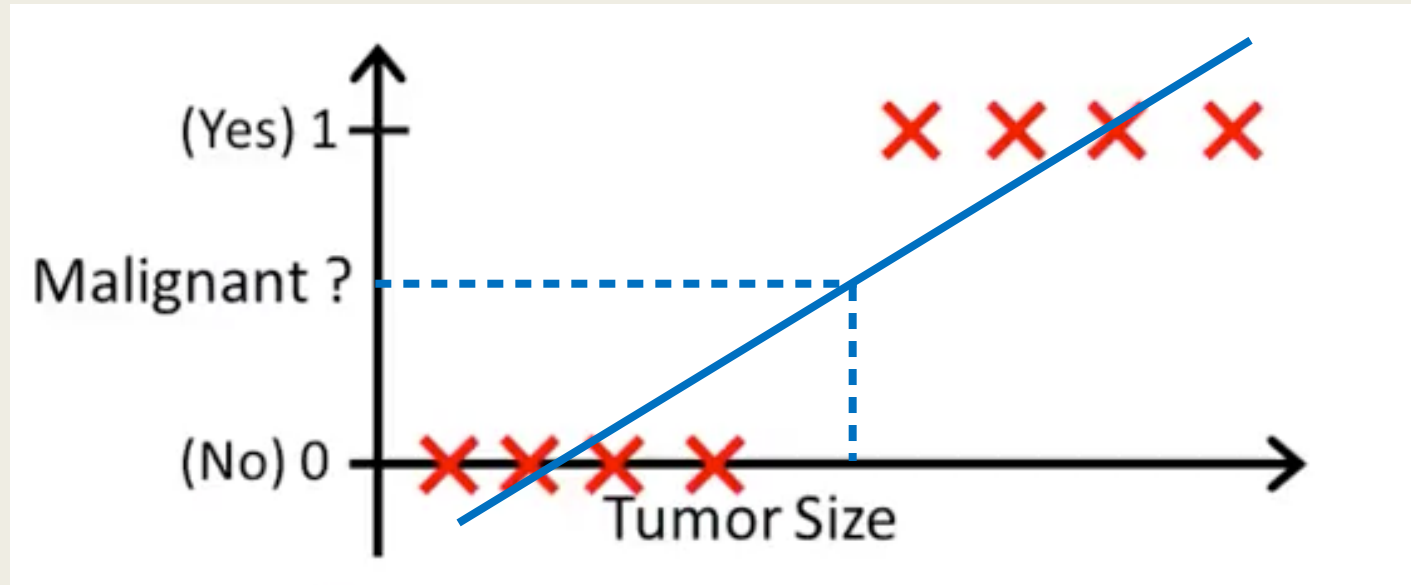
- Email: Spam/ Not Spam ?
- Online Transactions: Fraudulent (Yes/No) ?
- Tumor: Malignant/ Benign?

$$y \in \{0, 1\}$$

0: “Negative Class” (e.g., benign tumor)

1: “Positive Class” (e.g., malignant tumor)

How to develop a classification algorithm?

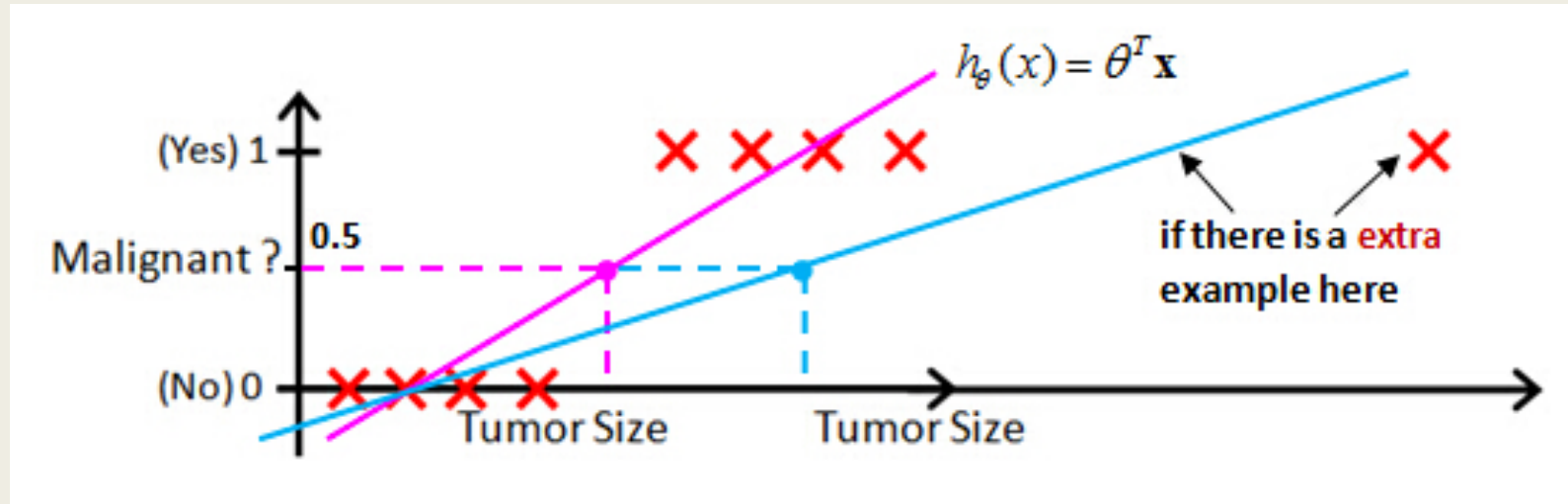


Can we use linear regression ? $h(x) = \theta_0 + \theta_1 x_1$

If $h(x) \geq 0.5$, predict “y = 1”

If $h(x) < 0.5$, predict “y = 0”

How to develop a classification algorithm?



Linear regression to a classification problem often isn't a great idea

Classification: $y = 1$ or 0

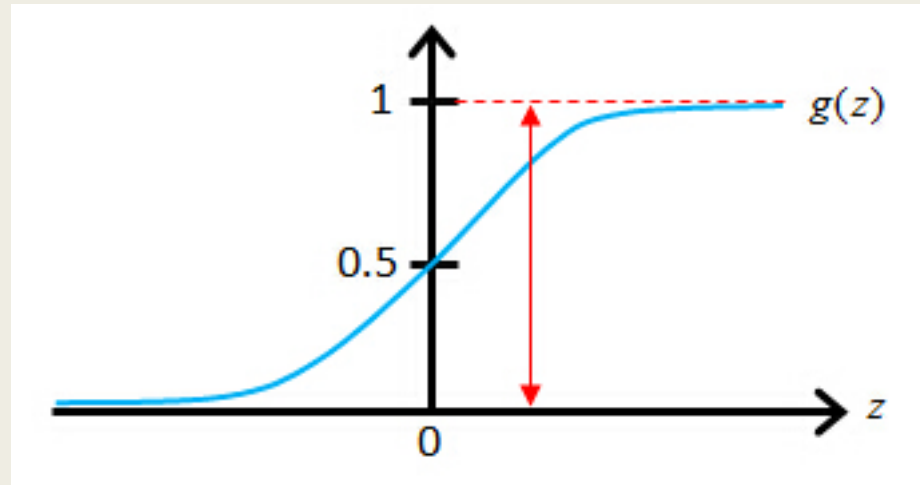
But for linear regression: $h(x)$ can be > 1 or < 0

We want $0 \leq h(x) \leq 1$

Logistic Regression Model

We want: $0 \leq h(x) \leq 1$

$$h(x) = g(\theta_0 + \theta_1 x_1) \quad \Rightarrow \quad h(x) = \frac{1}{1 + e^{-(\theta_0 + \theta_1 x_1)}}$$
$$g(z) = \frac{1}{1 + e^{-z}}$$



The sigmoid/logistic function $g(z)$

Interpretation of Hypothesis Output

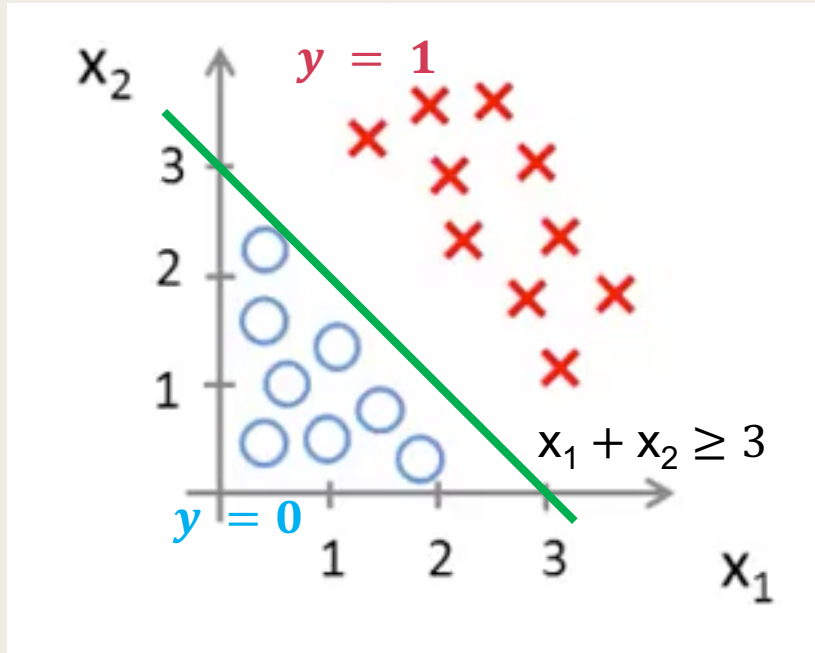
$$\mathbf{h}(\mathbf{x}) = \frac{1}{1 + e^{-(\theta_0 + \theta_1 \mathbf{x}_1)}}$$

$h(x)$ = estimated probability that $y = 1$ on input x

$$h(x) = P(y = 1 | x)$$

e.g. $P(\text{email is spam} \mid \text{number of word "discount" in the email})$

Decision Boundary



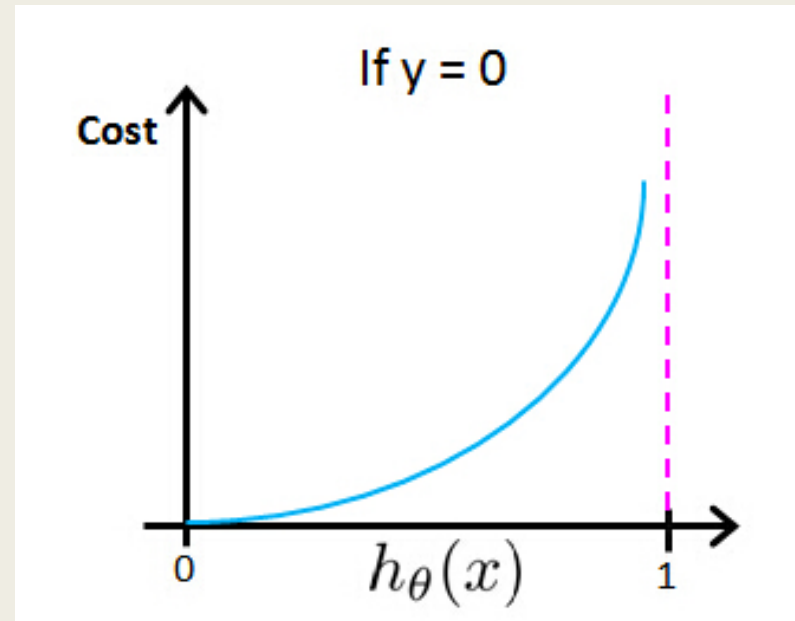
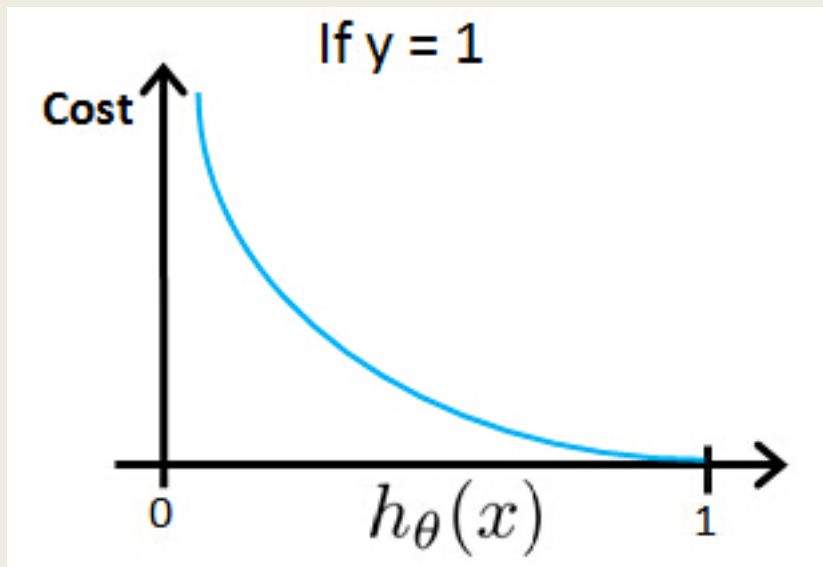
$$h(x) = \frac{1}{1 + e^{-(-3 + 1x_1 + 1x_2)}}$$

Predict $y = 1$ if $h(x) \geq 0.5$

equivalent to $-3 + x_1 + x_2 \geq 0$

Logistic Regression Cost function

$$J(\theta) = \text{Cost}(h_{\theta}(x), y) = \begin{cases} -\log(h_{\theta}(x)) & \text{if } y = 1 \\ -\log(1 - h_{\theta}(x)) & \text{if } y = 0 \end{cases}$$



Parameter Learning with Gradient Descent

$$J(\theta) = -\frac{1}{m} \left[\sum_{i=1}^m y^{(i)} \log h_{\theta}(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_{\theta}(x^{(i)})) \right]$$

Want $\min_{\theta} J(\theta)$:

Repeat {

$$\theta_j := \theta_j - \alpha \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

}

Algorithm looks identical to linear regression!!

(simultaneously update all θ_j)