

0293



Total No. of Questions—**24**

Total No. of Printed Pages—**4**

Regd. No. _____

Part III

MATHEMATICS, Paper - II(B)

(English Version)

Time : 3 Hours]

[Max. Marks : 75]

Note :— This question paper consists of THREE sections A, B and C.

SECTION A

$10 \times 2 = 20$

I. Very short answer type questions :

- (i) Answer ALL questions.
- (ii) Each question carries TWO marks.

1. Write the parametric equations of the circle $2x^2 + 2y^2 = 7$.
2. Find the value of k if the points $(1, 3)$ and $(2, k)$ are conjugated with respect to the circle $x^2 + y^2 = 35$.
3. Find the equation of radical axis of the circles $x^2 + y^2 + 4x + 6y - 7 = 0$, $4(x^2 + y^2) + 8x + 12y - 9 = 0$.
4. Find the equation of the normal to the parabola $y^2 = 4x$ which is parallel to $y - 2x + 5 = 0$.
5. If the eccentricity of the hyperbola is $\frac{5}{4}$, then find the eccentricity of its conjugate hyperbola.
6. Evaluate :

$$\int \frac{1 + \cos^2 x}{1 - \cos 2x} dx, \text{ on } I \subset \mathbb{R} \setminus \{n\pi : n \in \mathbb{Z}\}.$$

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7. Evaluate :

$$\int \frac{1}{x \log x [\log (\log x)]} dx, \text{ on } (1, \infty)$$

8. Evaluate :

$$\int_0^a (\sqrt{a} - \sqrt{x})^2 dx.$$

9. Find :

$$\int_0^{\pi/2} \cos^{11} x dx.$$

10. Find the general solution of :

$$\frac{dy}{dx} = \frac{2y}{x}.$$

SECTION B

5×4=20

II. Short answer type questions :

(i) Answer ANY FIVE questions.

(ii) Each question carries FOUR marks.

11. Find the equation of the circle which cuts orthogonally the circle $x^2 + y^2 - 4x + 2y - 7 = 0$ and having the centre at (2, 3).

12. The line $y = mx + c$ and the circle $x^2 + y^2 = a^2$ intersect at A and B.
If $AB = 2\lambda$, then show that :

$$c^2 = (1 + m^2)(a^2 - \lambda^2).$$

13. Find the equation of the ellipse with focus at (1, -1), $e = \frac{2}{3}$ and directrix as $x + y + 2 = 0$.

14. Find the equations of tangents to the ellipse $2x^2 + 3y^2 = 11$ at the points whose ordinate is 1.

15. Find the foci, eccentricity, equations of the directrix, length of latus rectum of the hyperbola $x^2 - 4y^2 = 4$.
16. Find :

$$\int_0^{2\pi} \sin^4 x \cos^6 x \, dx.$$

17. Solve the differential equation :

$$\cos x \frac{dy}{dx} + y \sin x = \sec^2 x.$$

SECTION C

$5 \times 7 = 35$

III. Long answer type questions :

(i) Answer ANY FIVE questions.

(ii) Each question carries SEVEN marks.

18. Show that the points (9, 1), (7, 9), (-2, 12), (6, 10) are concyclic and find the equation of the circle on which they lie.
19. Show that, four common tangents can be drawn for the circles given by $x^2 + y^2 - 14x + 6y + 33 = 0$ and $x^2 + y^2 + 30x - 2y + 1 = 0$ and find the internal and external centres of similitude.
20. From an external point 'P' tangents are drawn to the parabola $y^2 = 4ax$ and these tangents make angles θ_1, θ_2 with its axis, such that $\cot \theta_1 + \cot \theta_2$ is a constant 'd'. Then show that all such P lie on a horizontal line.

21. Evaluate :

$$\int e^{ax} \sin(bx + c) dx; \quad (a, b, c \in \mathbb{R}; b \neq 0) \text{ on } \mathbb{R}.$$

22. Obtain reduction formula for $I_n = \int \cot^n x dx$; n being a positive integer;
 $n \geq 2$ and deduce the value of $\int \cot^4 x dx$.

23. Find :

$$\int_0^{\pi} x \sin^7 x \cos^6 x dx.$$

24. Solve the differential equation :

$$\frac{dy}{dx} = \frac{2y + x + 1}{2x + 4y + 3}.$$