Problems:- Date:- 08-01-2020 **29:00**

1) A discrete time signal $x(n)$ is defined as $x[n]$

$$= \begin{cases} 1+\frac{n}{3}, & -3 \le n \le 4 \\ 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

a) Determine its value and sketch the signal $x(n)$.

$1+\frac{n}{3}$, $-3 \le n \le -1$, $\Rightarrow$ at $n=-3$ 0

$n=-2$ $1/3$

$n=-1$ $2/3$

from $0 \le n \le 3$ 1

elsewhere   0

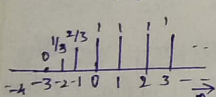
So $x(n) = \{ \dots 0, \frac{1}{3}, \frac{2}{3}, \underset{\uparrow}{1}, 1, 1, 1, 0 \dots \}$

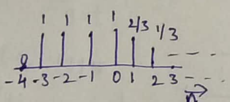
b) Sketch its values and the signals that result if we:

i) First fold $x(n)$ and then delay the resulting signal by four samples.

$x(-n)$ = folded signal for $x(n)$

$x[n]$



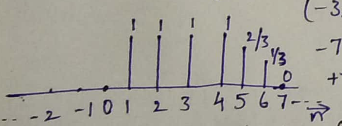


$x(-n+4) \rightarrow$ after delaying.

$(-3 \le n+4 \le 2)$

$-7 \le n \le -2$

$+7 \ge n > -2)$

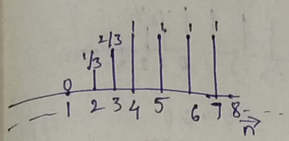


2) First delay $x(n)$ by four samples & then fold the resulting signal.

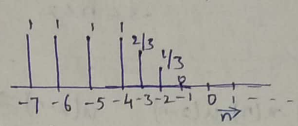
folding $x(n-4) \rightarrow x(-n-4)$

$x(n) \rightarrow$ delay by 4 samples.

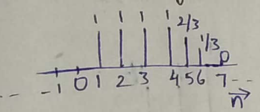
$x(n-4)$



$x(-n-4)$



c) Sketch the signal $x(-n+4)$



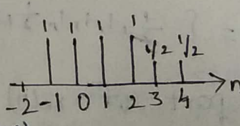
d) compare the results in parts (b) and (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$.

By comparing results in parts (b) & (c) we can say that to get $x(-n+k)$ from $x(n)$ first we need to fold $x(n)$ which results in $x(-n)$ & then we need to shift by k samples to right if $k>0$ (or) to left if $k<0$ results in $x(-n+k)$.

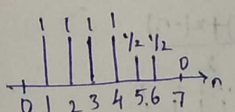
e) Can you express the signal $x(n)$ in terms of signals $\delta(n)$ & $u(n)$?

Yes. $x(n) = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n-1) + u(n) - u(n-4)$

2) A discrete time signal $x(n)$ is shown in the figure. Sketch and label the each of following signals.



a) $x(n-2)$

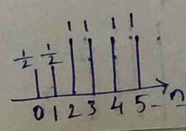


b) $x(4-n)$

$-1 \le 4 - n \le 4$

$-5 \le -n \le 0$

$5 \ge n \ge 0$



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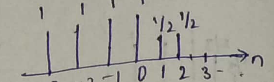
c) $x(n+2)$

$-1 \le n+2 \le 4$

$-3 \le n \le 2$

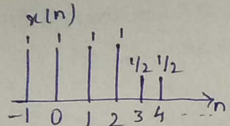


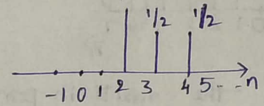
d) $x(n)\,u(2-n)$ $u(2-n) \Rightarrow$ $2-n \ge 0$

$-n \ge 2$

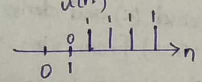
$n > 2$

$x(n)$



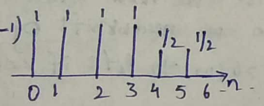


$u(n)$

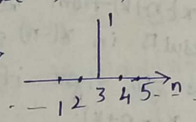


e) $x(n-1)\,\delta(n-3)$

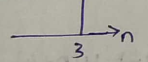
$x(n-1)$



$\Rightarrow$ $x(n-1)\delta(n-3)$



$\delta(n-3)$



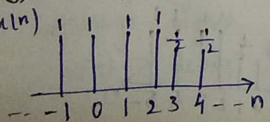
f) $x(n^2) \Rightarrow x(n) = \{x(-2), x(-1), x(0), x(1), x(2), x(3), x(4) \ldots$

$x(n^2) = \{x(4), x(1), x(0), x(1), x(4), x(9), \ldots\ldots\}$

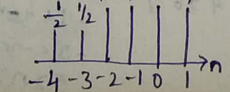
$= \{\frac{1}{2}, 1, 1, 1, \frac{1}{2}, 0, 0 \ldots\}$

g) even part of $x_e(n) = \dfrac{x(n)+x(-n)}{2}$

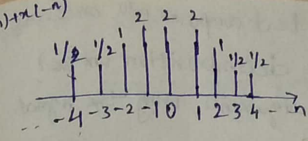
$x(n)$



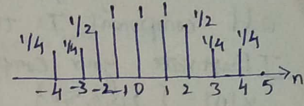
$x(-n)$



$x(n)+x(-n)$

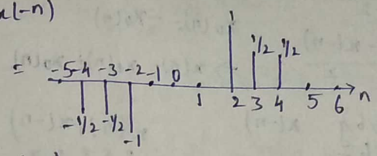


$\dfrac{x(n)+x(-n)}{2}$

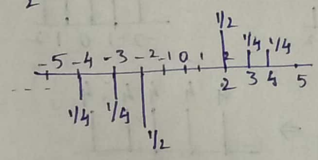


odd part $x_0(n) = \dfrac{x(n)-x(-n)}{2}$

$x(n)-x(-n)$

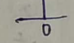


$\dfrac{x(n)-x(-n)}{2}$

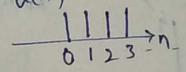


3) a) Show that $\delta(n) = u(n) - u(n-1)$

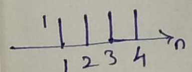
We know $\delta(n) =$



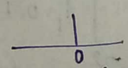
$u(n)$



$u(-n)$



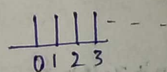
$u(n) - u(n-1)$



$\therefore \delta(n) = u(n) - u(n-1)$

b) $u(n) = \sum_{k=-\infty}^{\infty} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

$u(n) =$



$\Rightarrow \sum_{n=-\infty}^{n} \delta(k) = u(n) = \begin{cases} 0, & n < 0 \\ 1, & n \ge 0 \end{cases}$

$\sum_{k=0}^{\infty} \delta(n-k) = \begin{cases} 0, & n < 0 \\ 1, & n \ge 0 \end{cases}$

4) Show that any signal can be decomposed ... an even & odd component. Is the decomposition unique? Illustrate your compo arguments using the signal.

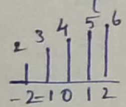
$x[n] = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$

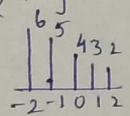
$x_e(n) = \frac{x(n) + x(-n)}{2}$ $\qquad x_e(n) = x_e(-n)$

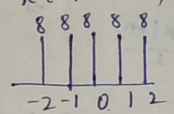
$x_o(n) = -x_o(n)$

$x_o(n) = \frac{x(n) - x(-n)}{2}$ $\qquad \Rightarrow x(n) = x_e(n) + x_o(n)$

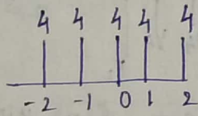
$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$ $\qquad x(-n)$ $\qquad x(n) + x(-n)$



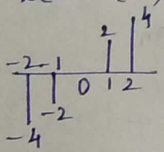


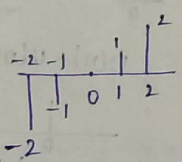


$x_e(n) = \frac{x(n) + x(-n)}{2}$ $\Rightarrow$



$x(n) - x(-n)$ $\qquad x_o(n) = \frac{x(n) - x(-n)}{2}$





5) Show that the energy (power) of a real-valued energy (power) signal is equal to the sum of the energies (powers) of its even and odd components.

To prove that

$\sum_{n=-\infty}^{\infty} x_e(n) + x_o(n) = 0 \Rightarrow \sum_{n=-\infty}^{\infty} x_e(n) + x_o(n) = \sum_{m=-\infty}^{\infty} x_e(-m)\, x_o(-m)$

$\Rightarrow -\sum_{m=-\infty}^{\infty} x_e(m)\, x_o(m)$

$= -\sum_{m=-\infty}^{\infty} x_e(n)\, x_o(n) = \sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n) = 0$

Energies (powers)

$\Rightarrow \sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} [x_e(n) + x_o(n)]^2$

$= \sum_{n=-\infty}^{\infty} x_e^2(n) + x_o^2(n) + 2x_e(n)\cdot x_o(n)$

$= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + 2\sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n)$

$= E_e + E_o + 0$

$E = E_e + E_o$

6) Consider the system $y(n) = T[x(n)] = x(n^2)$

a) Determine if the s/m is time invarient.

given $y(n) = T[x(n)] = x(n^2)$

$x(n-k) \Rightarrow y_1(n) = x[(n-k)^2]$

$= x(n^2 + k^2 - 2nk)$

$x(n-k) \neq y(n-k)$

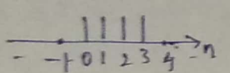
So, the given system is ~~invarient~~ time varient.

b) To clarify the result in part (a) assume that the signal $x(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{otherwise.} \end{cases}$

is applied into the s/m.

1) Sketch the signal $x(n)$

$x(n) = \{ \ldots\ 0, 1, 1, 1, 1, 0, \ldots \}$ (↑ under the first 1)



2) Determine & sketch the signal $y(n) = T[x(n)]$.

$y(n) = T[x(n)] = x(n^2) \Rightarrow \{x(0), x(1), x(2^2), x(3^2), x(4)^2 \ldots\}$

$\Rightarrow \{x(0), x(1), x(4), x(9) \ldots\}$

$y(n) = x(n^2) = \{ \ldots\ 0, 1, 1, 0, 0, 0, \ldots \}$ (↑ at index 0; indices 0 1 2 3 4)

3) sketch the signal $y_2'(n) = y(n-2)$

$y(n-2) = \{ \ldots\ 0, 0, 1, 1, 0, 0, 0, \ldots \}$ (↑ at 0)

4) Sketch the signal $x_2(n) = x(n-2)$

$x(n-2) = \{ \ldots\ 0, 0, 1, 1, 1, 1, 0 \ldots \}$ (↑ at 0; indices 0 1 2 3 4 5 6)

5) Determine and sketch the signal $y_2(n) = T[x_2(n-2)]$

$y_2(n) = T[x_2(n-2)] = \{ \ldots x(0), x(1), x(2), x(3), x(4) \ldots \}$

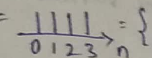
$= \{ \ldots\ 0, 1, 0, 0, 0, 1, 0 \ldots \}$ (↑)

6) Compare the signals $y_2(n)$ and $y(n-2)$. What is your conclusion?

$y_2(n) \neq y(n-2) \Rightarrow$ system is time Varient

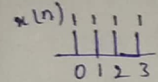
c) Repeat part (b) for the system $y(n) = x(n) - x(n-1)$. Can you use this statement about the time invariance of s/m? why?

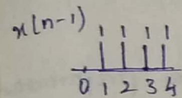
1) $x(n) =$



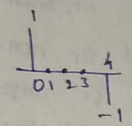
$= \{1, 1, 1, 1\}$ (↑)

2) $y(n) = x(n) - x(n-1)$



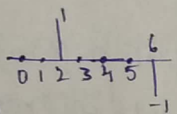


$y(n) \Rightarrow x(n) - x(n-1) =$



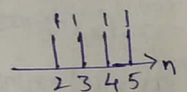
$= \{0, 1, 0, 0, 0, -1\}$

3) $y(n-2) \Rightarrow$



$\Rightarrow \{0, 0, 1, 0, 0, 0, 0, -1\}$ (↑)

4) $x(n-2)$



$\Rightarrow \{0, 0, 1, 1, 1, 1\}$ (↑)

5) $y_2(n) = \{0, 0, 1, 0, 0, 0, -1\}$ (↑)

6) $y_2(n) = y(n-2) \Rightarrow$ system is time invarient.

d) Repeat parts (b) & (c) for the s/m $y(n) = T[x(n)] = n x(n)$

$n \Rightarrow$ integer Values from 0 to n

1) $y(n) = n x(n)$

$x(n) = \{ \ldots\ 0, 1, 1, 1, 1, 0 \ldots \}$ (↑)

2) $y(n) = \{ \ldots\ 0, 1, 2, 3, 4 \ldots \}$ (↑)

3) $y(n-2) = \{ \ldots\ 0, 0, 0, 1, 2, 3, 4 \ldots \}$ (↑)

4) $x(n-2) = \{ \ldots\ 0, 0, 0, 1, 1, 1, 1 \ldots \}$ (↑)

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5) $y_2(n) = T[x(n-2)] = \{ \ldots 0, 0, 2, 3, 4, 5 \ldots \}$

6) $y_2(n) \neq y(n-2) \Rightarrow$ System is time Varient.

7) ① Static or dynamic

2) Linear or Non-linear.

3) Time invarient or varying

4) Causal or non-causal

5) stable or unstable.

Examine the following s/ms w.r.to the properties.

a) $y(n) = \cos[x(n)]$

(i) static (only present i/p). (ii) $y_1(n) = \cos[x_1(n)]$

(iii) $y(n) = \cos[x(n-n_0)]$ $y_2(n) = \cos[x_2(n)]$

$y'(n) = \cos[x(n-n_0)]$ $y(n) = \cos[x_1(n)] + \cos[x_2(n)]$

$\Rightarrow$ Time Varient. $y'(n) = \cos[x_1(n) + x_2(n)]$

(iv) Only present i/p $\rightarrow$ causal $\Rightarrow$ Non-linear.

(V) stable

b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

Dynamic (depends on future values). linear, time invarient, nonCausal (also depends on future value, unstable)

c) $y(n) = x(n) \cos(w_0(n))$

Static, linear, Time variant, Causal, stable

$y(n) = x(n-n_0) \cos(w_0 n + n_0)$

$y'(n) = x(n-n_0) \cos(w_0 n)$

d) $y(n) = x(-n+2)$

Dynamic

at $n=0 \Rightarrow y(0) = x(2)$

future value

$y_1(n) = x_1(-n+2) + x_2(-n+2)$

$y_2(n) = [x + x_2(-n+2)] \Rightarrow x_1(-n+2) + x_2(-n+2)$

Linear

Non Causal, stable, Time invariant

e) $y(n) = \text{Trun}[x(n)]$

Static, non-linear, time invarient, Causal, stable

f) $y(n) = \text{Round}[x(n)]$

Static, non-linear, time invarient, Causal, stable.

g) $y(n) = |x(n)|$

Static, non-linear, time invarient, Causal, state.

h) $y(n) = x(n) \cdot u(n)$

Static, linear, time invarient, causal, stable

i) $y(n) = x(n) + n \cdot x(n+1)$.

Dynamic, linear, time varient, Non-causal, unstable.

j) $y(n) = x(2n)$

Dynamic, linear, time variant, Non-Causal, stable.

k) $y(n) = \begin{cases} x(n); & \text{if } x(n) \geq 0 \\ 0; & \text{if } x(n) < 0 \end{cases}$

Static, linear, time invariant, Non causal, stable

l) $y(n) = x(-n)$

Dynamic, linear, Time invarient, causal, Stable.

m) $y(n) = \text{sign}[x(n)]$

... linear, Time invarient, Causal stable.

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8) The ideal sampling system ... o/p

$x(n) = x_a(nT), \; -\infty < n < \infty$

$x(n) = x_a(nT)$

Static, linear, time variant, non-causal, stable.

9) Let T be an LTI, relaxed and BIBO stable s/m with i/p $x(n)$ and o/p $y(n)$. Show that.

a) If $x(n)$ is periodic with period N. [ie, $x(n) = x(n+N)$] for all $n \geq 0$], the output $y(n)$ tends to a periodic signal with the same period.

$x(n) = x(n+N) \quad \forall n \geq 0$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$y(n+N) = \sum_{k=-\infty}^{n+N} h(k)\, x(n+N-k)$$

$$y(n+N) = \sum_{k=n+1}^{n+N} h(k)\, x(n-k) + \sum_{k=-\infty}^{\infty} h(k) x(n-k)$$

$$y(n+N) = y(n) + \sum_{k=n+1}^{n+N} h(k)\, x(n-k)$$

For BIBO System $\lim_{n\to\infty} |h(n)| = 0$

$$\lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k)\, x(n-k) = 0$$

$$\lim_{n\to\infty} y(n+N) = y(N)$$

$$\therefore \; y(N) = y(n+N)$$

b) If $x(n)$ is bounded and tends to a constant, the output will also tend to a constant.

$x(n) = x_0(n) + a\, u(n)$

$x_0(n) \to$ bounded with $\lim_{n\to\infty} x_0(n) = 0$

$$\Rightarrow y(n) = a \sum_{k=0}^{\infty} h(k) u(n-k) + \sum_{k=0}^{\infty} h(k)\, x_0(n-k) = a\sum_{k=0}^{n} h(k) + y_0(n)$$

$$\Rightarrow \sum_n x_0^2(n) < \infty \Rightarrow \sum_n y_0^2(n) < \infty$$

hence $\lim_{n\to\infty} |y_0(n)| = \infty \; = a\sum_{k=0}^{n} h(k) =$ const.

c) If $x(n)$ is a energy signal, the output $y(n)$ will also be as energy signal.

$$y(n) = \sum_k h(k)\, x(n-k)$$

$$\sum_{-\infty}^{\infty} y^2(n) = \sum_{-\infty}^{\infty} \left[\sum_k h(k) \cdot x(n-k)\right]^2 = \sum_k \sum_l h(k) h(l) \sum_n x(n-k) x(n)$$

but $\sum_n x(n-k) x(n-l) \leq \sum_n x^2(n) \,|h(l)|$

for BIBO stable system $\sum_k |h(k)| < M$

Hence $E_y \leq M^2 E_x$, so that $E_y < 0$ if $E_x < 0$

10) The following i/p-o/p pairs have been observed during the operation of a time invariant system:

$$x_1(n) = \{\underset{\uparrow}{1}, 0, 2\} \xleftrightarrow{T} y_1(n) = \{\underset{\uparrow}{0}, 1, 2\}$$

$$x_2(n) = \{\underset{\uparrow}{0}, 0, 3\} \xleftrightarrow{T} y_2(n) = \{0, 1, 0, 2\}$$

$$x_3(n) = \{\underset{\uparrow}{0}, 0, 0, 1\} \xleftrightarrow{T} y_3(n) = \{1, \underset{\uparrow}{2}, 1\}$$

Can you draw any conclusion regarding the linearity of the s/m. what is the impulse response of the system?

As this is a time - invariant system.

$y_2(n)$ should have only 3 elements and

$y_3(n)$ should have 4 elements.

So, it is non- linear.

11) The following i/p- o/p pairs have been observed during the operation of a linear system?

$$x_1(n) = \{1,2,1\} \xrightarrow{T} y_1(n) = \{1,2,\underset{\uparrow}{-1},0,1\}$$

$$x_2(n) = \{1,-1,-1\} \xrightarrow{T} y_2(n) = \{-1,1,\underset{\uparrow}{0},2\}$$

$$x_3(n) = \{0,1\} \xrightarrow{T} y_3(n) = \{1,\underset{\uparrow}{2},1\}$$

Can you draw any conclusions about the time invariance of this system?

Since $x_1(n)+x_2(n)=\delta(n)$ & system is linear, the impulse response of the s/m is

$$y_1(n)+y_2(n) = \{0,3,\underset{\uparrow}{-1},2,1\}$$

if s/m were time invariant the response of $x_3(n)$ would be $\{\underset{\uparrow}{3},2,1,3,1\}$

12) The only available information about the s/m consist of N i/p- o/p pairs of signals $y_i(n)=T[x_i(n)]$ $= 1,2,\ldots N$.

a) what is the class of i/p signals for which we can determine the o/p, using the information above, if the s/m is known to linear?

Any Linear combination of signal in the form of

$x_i(n)$, $i = 1,2,\ldots N$

because if we take $i=1,3$

$$y_1(n) = x_1(n)$$
$$y_3(n) = x_3(n)$$
$$\Rightarrow y(n) = y_1(n)+y_3(n) = x_1(n)+x_3(n)$$

$$y(n) = x_1(n)+x_3(n)$$

linear

b) same repeat for the s/m is invariant.

Any $x_i(n-k)$ where k is any integer; $i=1,2,\ldots N$

1^{st} replace $n = n-n_0 \Rightarrow x_i(n-n_0-k)$

$x(n)$ by $x(n-n_0) \Rightarrow x_i(n-k-n_0)$

[Time invariant]

13) Show that the necessary and sufficient condition for a relaxed LTI s/ms to be BIBO stable is

$$\sum_{n=-\infty}^{\infty} |h(n)| \leq m_n < \infty \text{ for constant } M_n$$

A system to be BIBO stable only when bounded o/p produce bounded input.

$$y(n) = \sum_k h(k)\, x(n-k)$$

$$|y(n)| = \sum_k |h(k)|\, |x(n-k)|$$

$$= \sum_k |x(n-k)| \leq m_n \text{ [some constant]}$$

so $|y(n)| = M_n \sum_k |h(k)|$

$|y(n)| < \infty$ for all n, if and only if

$\sum_k |h(k)| < \infty$. so $\sum_{n=-\infty}^{\infty} |y(n)|$

$\Rightarrow$ A s/m to be BIBO stable only when bounded i/p and bounded output.

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \; ; \; n \le n-k, \; k \ge 0$$

$$|y(n)| = \sum_{k=-\infty}^{\infty} |h(k)| \, |x(n-k)|$$

as $\sum_{k=-\infty}^{\infty} |x(n-k)| \le M_n$ for some const.

$$|y(n)| = M_n \sum_{k=-\infty}^{\infty} |h(k)| \; ; \; n \le n-k, \; k \ge 0$$

$|y(n)|$ is $< \infty$ if & only if $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$

so $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$.

14) show that

a) A relaxed line s/m is causal if & only if for any i/n $x(n)$ such that $x(n)=0$ for $n<n_0 \Rightarrow y(n)=0$ for $n<n_0$.

If a system is causal o/p depends only on present and pass inputs as $x(n)=0$ for $n<n_0$ the $y(n)$ also became zero for $n<n_0$.

b) A relaxed LTI s/m is causal if and only if $h(n)=0$ for $n<0$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

For finite impulse response:

$h(n)=0$, $n<0$ and $n \ge m$.

so $y(n)$ reduces to $y(n) = \sum_{k=0}^{n-1} h(k)\, x(n-k)$

$$y(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k)$$

15) show that for any real or complex constant a, and any finite integer numbers m and N, we have

a) $$\sum_{n=M}^{N} n = a^n = \begin{cases} \dfrac{a^m - a^{N+1}}{1-a}, & \text{if } a \ne 1 \\ N-m+1, & \text{if } a=1 \end{cases}$$

for $a=1$, $\sum_{n=M}^{N} a^n = N-m+1$

for $a \ne 1$, $\sum_{n=M}^{N} a^n = \dfrac{a^M - a^{N+1}}{1-a}$

$$(1-a)\sum_{n=M}^{N} a^n = a^m + a^{m+1} - a^{m+1} + \dots a^N - a^N - a^{N+1}$$

$$= a^M - a^{N+1}$$

b) show that if $|a|<1$, then

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

for $M=0$, $|a|<1$ and $N \to \infty$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \; ; \; |a|<1$$

16) a) If $y(n) = x(n) * h(n)$, show that $\Sigma_y = \Sigma_x \Sigma_h$ where

$$\Sigma_x = \sum_{n=-\infty}^{\infty} x(n)$$

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$$y(n) = \sum_k h(k)\, x(n-k)$$

$$\sum_n y(n) = \sum_n \sum_k h(k)\, x(n-k)$$

$$\sum_n y(n) = \sum_k h(k) \sum_{n=-\infty}^{\infty} x(n-k)$$

$$\Sigma_n y(n) = \left(\Sigma_k h(k)\right)\left(\Sigma_n x(n)\right)$$

b) Compute the convolution $x(n) * h(n)$ of the following signals and check the corrections of the results by using the test in (a).

1) $x(n) = \{1,2,4\}$, $h(n) = \{1,1,1,1,1\}$

$y(n) = \{1,3,7,7,7,6,4\}$

$\sum_n y(n) = 35$; $\sum_n x(n) = 7$, $\sum_n h(x) = 5$

$\sum_n y(n) = \sum_n x(n) \cdot \sum_n h(n)$

$35 = 7 \times 5$

$35 = 35$

By tabular method,

h(n) \ x(n)	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

2) $x(n) = \{1,2,-1\}$, $h(n) = x(n)$

$x(n) = \{1,2,-1\}$, $h(n) = \{1,2,-1\}$

$y(n) = x(n) * h(n)$

$y(n) = \{1,4,2,-4,1\}$

$\sum_n y(n) = 4$; $\sum_n x(n) = 2$, $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n (x) \sum h(n)$

$4 = 4$.

h(n) \ x(n)	1	2	-1
1	1	2	-1
2	2	4	6
-1	-1	-2	-1

3) $x(n) = \{0,1,-2,3,-4\}$, $h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$

$y(n) = \{0, \frac{1}{2}, -\frac{1}{2}, \frac{3}{2}, -2, 0, -\frac{5}{2}, 2\}$

$\sum_n y(n) = -5$, $\sum_n x(n) = -2$, $\sum_n h(n) = \frac{5}{2}$

$\sum_n y(n) = \sum_n x(n)\, h(n)$

$-5 = -5$

h(n) \ x(n)	0	+1	-2	+3	-4
$\frac{1}{2}$	0	1/2	-1	3/2	-2
$\frac{1}{2}$	0	1/2	-1	3/2	-2
1	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2

4) $x(n) = \{1,2,3,4,5\}$, $h(n) = \{1\}$

$y(n) = \{1,2,3,4,5\}$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$15 = 15(1)$

$15 = 15$

5) $x(n) = \{\underset{\uparrow}{1},2,3\}$, $h(n) = \{0,0,1,1,1,1\}$

$y(n) = \{\underset{\uparrow}{0},0,1,-1,2,2,1,3\}$

$\sum_n y(n) = 8$; $\sum_n x(n) = 2$; $\sum_n h(n) = 4$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$8 = 8$

h(n) \ x(n)	1	-2	3
0	0	0	0
0	0	0	0
1	1	-3	3
1	1	-2	3
1	1	-3	3
1	1	-2	3

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6) $x(n) = \{0, 0, 1, 1, 1, 1\}$; $h(n) = \{1, \underset{\uparrow}{-2}, 3\}$

$y(n) = \{0, \underset{\uparrow}{0}, 1, -1, 2, 2, 1, 3\}$

$h(n) \backslash x(n)$	0	0	1	1	1
1	0	0	1	1	1
-2	0	0	-2	-2	-2
3	0	0	3	3	3

$\sum_n y(n) = 8$, $\sum_n x(n) = 4$ $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n x(n) \sum h(n)$

$8 = 8$

7) $x(n) = \{\underset{\uparrow}{0}, 1, 4, -3\}$, $h(n) = \{\underset{\uparrow}{1}, 0, -1, -1\}$

$y(n) = \{\underset{\uparrow}{0}, 1, 4, -4, -5, -1, 3\}$

$h(n) \backslash x(n)$	0	1	4	-3
1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
-1	0	-1	-4	3

$\sum_n y(n) = -2$; $\sum_n x(n) = -2$; $\sum_n h(n) = 1$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

8) $x(n) = \{\underset{\uparrow}{1}, 1, 2\}$, $h(n) = u(n)$

$y(n) = \{\underset{\uparrow}{1}, 2, 4, 3, 2\}$

$h(n) \backslash x(n)$	1	1	2
1	1	1	2
1	1	1	2
1	1	1	2

$\sum_n y(n) = 12$; $\sum_n x(n) = 4$; $\sum_n h(n) = 3$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$12 = 4 \times 3$

$12 = 12$

9) $x(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$, $h(n) = \{1, -2, -3, 4\}$

$y(n) = \{1, -1, -5, 2, 3, \underset{\uparrow}{-5}, 1, 4\}$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$0 = 0$

$h(n) \backslash x(n)$	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
-4	-4	-4	0	-4	-4

10) $x(n) = \{1, 2, 0, 2, 1\}$, $h(n) = x(n)$

$y(n) = \{1, 4, 4, 4, \underset{\uparrow}{10}, 4, 4, 4, 1\}$

$y(n) \backslash x(n)$	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

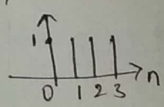
11) $x(n) = \left(\frac{1}{2}\right)^n u(n)$; $h(n) = \left(\frac{1}{4}\right)^n u(n)$

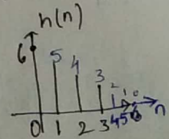
$y(n) = \left[2\left(1/2\right)^n - \left(\frac{1}{4}\right)^n u(n)\right]$

$\sum_n y(n) = 8/3$, $\sum_n h(n) = \frac{4}{3}$ $\sum_n x(n) = 2$

17) Compute and plot convolutions $x(n) * h(n)$ and $h(n) * x(n)$ for the pairs of signal shown below.

a) $x(n)$





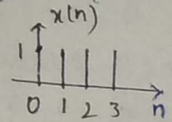
$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$, $h(n) = \{6, 5, 4, 3, 2, 1, 0\}$

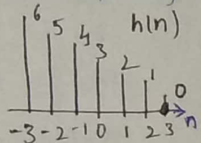
$h(n) \backslash x(n)$	1	1	1
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2
1	1	1	1

$y(n) = x(n) * h(n)$

$y(n) = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$

b)



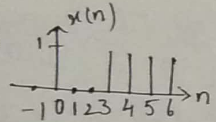


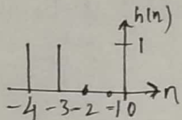
$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$ $h(n) = \{6, 5, 4, 3, \underset{\uparrow}{2}, 1, 0\}$

$y(n) = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$

h(n) \ x(n)	1	1	1
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2
1	1	1	1

c)





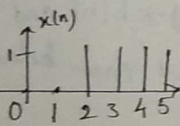
$x(n) = \{0, \underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$

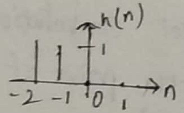
$h(n) = \{1, 1, 0, 0, \underset{\uparrow}{0}\}$

$y(n) = \{0, 0, 0, 1, 2, 2, 2, 1, 0, 0, 0\}$

h(n) \ x(n)	0	0	0	1	1	1
1	0	0	0	1	1	1
1	0	0	0	1	1	1
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

d)





$x(n) = \{0, \underset{\uparrow}{0}, 1, 1, 1, 1\}$ $h(n) = \{1, 1, \underset{\uparrow}{0}, 0, 0\}$

$y(n) = \{0, 0, \underset{\uparrow}{1}, 2, 2, 2, 1, 0, 0, 0\}$

h(n) \ x(n)	0	0	1	1	1	1
1	0	0	1	1	1	1
1	0	0	1	1	1	1
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

18) Determine and sketch the convolution $y(n)$ of the signals.

$x(n) = \begin{cases} \frac{1}{3}n, & 0 \le n \le 6 \\ 0, & \text{elsewhere} \end{cases}$ $h(n) = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{elsewhere.} \end{cases}$

a) graphically.

$x(n) = \{\underset{\uparrow}{0}, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\}$

$h(n) = \{1, 1, 1, 1, 1\}$

$y(n) = \{0, \frac{1}{3}, 1, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$

b) Analytically.

$x(n) = \frac{1}{3} n [u(n) - u(n-7)]$

$h(n) = u(n+2) - u(n-3)$

$y(n) = \frac{1}{3}[n(u(n) - u(n-7)] * u(n+2) - u(n-3)$

$= \frac{1}{3} n u(n) * u(n-2) - \frac{1}{3} n (u(n)) * u(n-3) - \frac{1}{3} n (u(n-7))$

$* \frac{1}{3} u(n+2) + \frac{1}{3} n (u(n-7) * u(n-3)$

20) Consider the following three operations.

a) multiply the integer numbers: 131 and 122.

$131 \times 122 = 15982$

b) Compute the convolution of signals: $\{1, 3, 1\} * \{1, 2, 2\}$

$y(n) = \{1, 5, 9, 8, 2\}$

y(n) \ x(n)	1	2	2
1	1	2	2
3	3	6	6
1	1	2	2

c) multiply the polynomials: $1 + 3z + z^2$ and $1 + 2z + 2z^2$.

$(z^2 + 3z + 1) * (2z^2 + 2z + 1)$

$= 2z^4 + 8z^3 + 9z^2 + 5z + 1$

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d) Repeat part (a) for the numbers 1.31 and 12.2.

1.31 x 12.2 = 15.982

e) Comment on your result.

These are some of the ways to perform convolution

21) Compute the convolution $y(n) * h(n)$ of the following pairs of the signals.

a) $x(n) = a^n u(n)$, $h(n) = b^n u(n)$, when $a \neq b$, and when $a = b$.

$$y(n) = x(n) * h(n)$$

$$= a^n u(n) * b^n u(n)$$

$$= [a^n * b^n]\, u(n)$$

$$y(n) = \sum_{k=0}^{n} a^k u(k)\, b^{n-k}\, u(n-k)$$

$$= b^n \sum_{k=0}^{n} a^k u(k)\, b^{-k}$$

$$= b^n \sum_{k=0}^{n} (ab)^{-k}$$

if $a \neq b$, then $y(n) = \dfrac{b^{n+1} - a^{n+1}}{b-a}\, u(n)$

if $a = b \Rightarrow b^n (n+1)\, u(n)$

b) $x(n) = \begin{cases} 1; & n = -2, 0, 1 \\ 2; & n = -1 \\ 0; & \text{elsewhere} \end{cases}$ $\quad h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$

$x(n) = \{1, 2, 1, 1\}$ $\quad h(n) = \{1, -1, 0, 0, 1, 1\}$

$y(n) = \{1, 1, -1, 0, 0, 3, 3, 2, 1\}$
↑

c) $x(n) = u(n+1) - u(n-4) - \delta(n-5)$

$h(n) = [u(n+2) - u(n-3)] \cdot (3 - |n|)$

$x(n) = \{1, 1, 1, 1, 1, 0, -1\}$
↑

$h(n) = \{1, 2, 3, 2, 1\}$
↑

$y(n) = \{1, 3, 6, 8, 9, 8, 5, 1, -2, -2, 1\}$

h(n) \ x(n)	1	2	1	1
1	1	2	1	1
-1	-1	-2	-1	-1
0	0	0	0	0
0	0	0	0	0
1	1	2	1	1
1	1	2	1	1

h(n) \ x(n)	1	1	1	1	1	0	-1
1	1	1	1	1	1	0	-1
2	2	2	2	2	2	0	-2
3	3	3	3	3	3	0	-3
2	2	2	2	2	2	0	-2
1	1	1	1	1	1	0	-1

d) $x(n) = u(n) - u(n-5)$; $\quad h(n) = u(n-2) - u(n+8) + u(n-11) - u(n-17)$

Sol: $x(n) = \{1, 1, 1, 1, 1\}$
↑

$h'(n) = \{0, 0, 1, 1, 1, 1, 1, 1\}$
↑

$h(n) = h'(n) + h'(n-9)$

$y(n) = y'(n) + y'(n-9)$; where

$y'(n) = \{0, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$

h(n) \ x(n)	1	1	1	1	1
0	0	0	0	0	0
0	0	0	0	0	0
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1

22) Let $x(n)$ be the i/p signal to a discrete time filter with impulse response $h_i(n)$ and let $y_i(n)$ be the corresponding output

a) Compute and sketch $x(n)$ and $y_i(n)$ in the following

Cases using the small scale in all the figures.

$x(n) = \{1, 4, 2, 3, 5, 3, 3, 4, 5, 7, 6, 9\}$

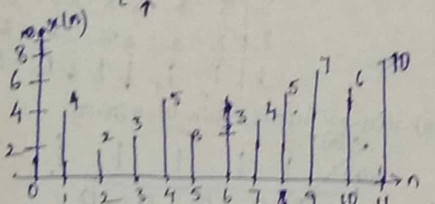
$h_1(n) = \{1, 1\}$ $h_2(n) = \{1, 2, 1\}$, $h_3(n) = \{1/2, 1/2\}$ $h_4(n) = \{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$

$h_5(n) = \{1/4, -1/2, 1/4\}$.

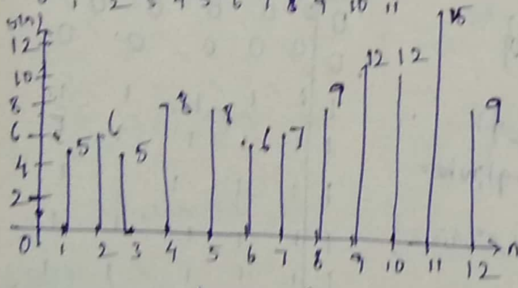
Sketch $x(n), y_1(n), y_2(n)$ on one graph and $x(n), y_3(n)$, $y_4(n), y_5(n)$ on another graph.

$y_1(n) = x(n) * h_1(n)$

$y_1(n) = \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 12, 15, 9\}$
$\uparrow$

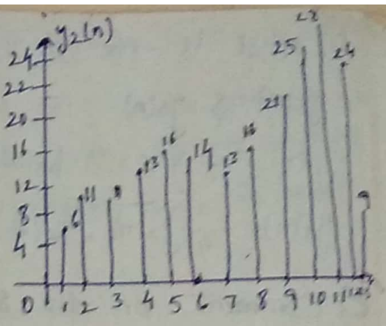


h(n) \ x(n)	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9



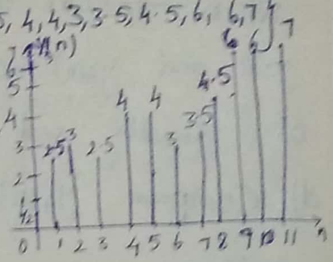
$y_2(n) = x(n) * h_2(n) = \{1, 6, 11, 11, 13, 16, 14, 13, 16, 21, 25, 28, 24, 9\}$

y(n) \ x(n)	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
2	2	8	4	6	10	6	6	8	10	14	12	18
1	1	4	2	3	5	3	3	4	5	7	6	9



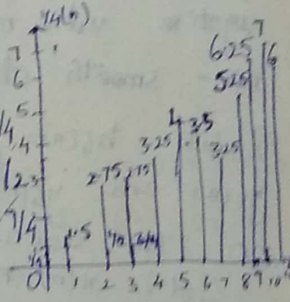
$\Rightarrow y_3(n) = x(n) * h_3(n) = \{1/2, 2.5, 3, 2.5, 4, 4, 3, 3.5, 4.5, 6, 6.5, 7.5, 4.5\}$

h(n) \ x(n)	1	4	2	3	5	3	3	4	5	7	6	9
1/2	1/2	2	1	3/2	5/2	3/2	3/2	2	5/2	7/2	3	9/2
1/2	1/2	2	1	3/2	5/2	3/2	3/2	2	5/2	7/2	3	9/2

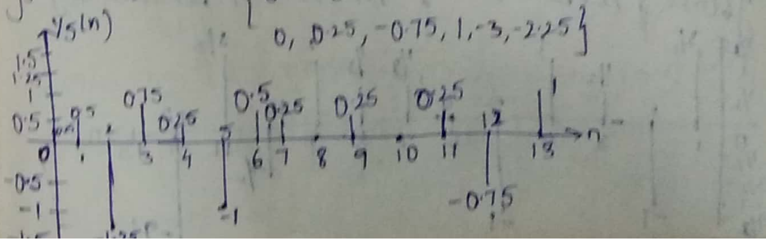


$y_4(n) = x(n) * h_4(n) = \{0.25, 1.5, 2.75, 2.75, 3.25, 4, 3.5, 3.25, 5.25, 6.25, 7, 6, 2.25\}$

h(n) \ x(n)	1	4	2	3	5	3	3	4	5	7	6	9
1/4	1/4	1	1/2	3/4	5/4	3/4	3/4	1	5/4	7/4	3/2	9/4
1/2	1/2	2	1	3/2	5/2	3/2	3/2	2	5/2	7/2	3	9/2
1/4	1/4	1	1/2	3/4	5/4	3/4	3/4	1	5/4	7/4	3/2	9/4



$y_5(n) = x(n) * h_5(n) = \{0.25, 0.5, -1.25, 0.75, 0.25, -1, 0.5, 0.25, 0.25, 0, 0.25, -0.75, 1, -3, -2.25\}$



b) What is the diff. b/w $y_1(n)$ and $y_2(n)$ and b/w $y_3(n)$ & $y_4(n)$.

$y_3(n) = \frac{1}{2} y_1(n)$; $h_3(n) = \frac{1}{2} h_1(n)$

$y_4(n) = \frac{1}{4} y_2(n)$; $h_4(n) = \frac{1}{4} h_2(n)$

c) Comment on the smoothness of $y_2(n)$ & $y_4(n)$. Which factors affect the smoothness?

$y_2(n)$ and $y_4(n)$ are smoother than $y_1(n)$ because of smaller scale factor.

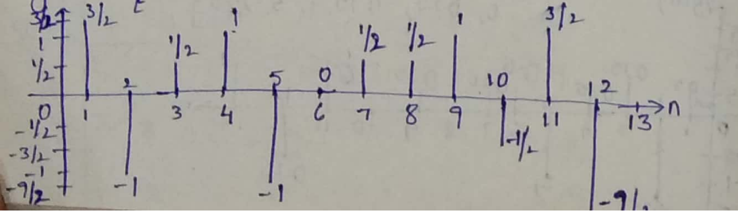
d) Compare $y_4(n)$ with $y_5(n)$. What is the difference can you explain it?

$y_4(n)$ results in smoother output than $y_5(n)$. The negative value of $h_5(0)$ is responsible for the non-smooth characteristics of $y_5(n)$.

e) Let $h_6(n) = \{\frac{1}{2}, -\frac{1}{2}\}$ compute $y_6(n)$. Sketch $x(n)$, $y_2(n)$ and $y_6(n)$ on the same figure and comment on the results.

$y_6(n) = x(n) * h_6(n)$

$y_6(n) = \{\underset{\uparrow}{\frac{1}{2}}, \frac{3}{2}, -1, \frac{1}{2}, 1, -1, 0, \frac{1}{2}, \frac{1}{2}, 1, -\frac{1}{2}, \frac{3}{2}, -\frac{9}{2}\}$



$y_2(n)$ is more smaller than $y_6(n)$.

23) Express the output $y(n)$ of a linear time-invariant system with impulse response $h(n)$ in terms of its step response $s(n) = h(n) * u(n)$ and the input $x(n)$.

We can express $\delta(n) = u(n) - u(n-1)$

$h(n) = h(n) * \delta(n)$

$= h(n) * [u(n) - u(n-1)]$

$= h(n) * u(n) - h(n) * u(n-1)$

$= s(n) - s(n-1)$

then $y(n) = h(n) * x(n)$

$= [s(n) - s(n-1)] * x(n)$

$= s(n) * x(n) - s(n-1) * x(n)$

24) The discrete time system $y(n) = ny(n-1) + x(n), n \geq 0$ is at rest [ie., $y(-1) = 0$], check if the s/m is LTI and BIBO stable.

$y(n) = ny(n-1) + x(n), \quad n \geq 0$

$y_1(n) = ny_1(n-1) + x_1(n)$

$y_2(n) = ny_2(n-1) + x_2(n)$

$\oplus \Rightarrow y(n) = ny_1(n-1) + x_1(n) + ny_2(n-1) + x_2(n)$

$y(n) = ny(n-1) + x(n)$

$x(n) = ny(n-1) + \ldots$

$x(n) = ax_1(n) + bx_2(n)$

$y(n) = ay_1(n) + by_2(n)$

Hence the system is linear

$\Rightarrow y(n-1) = (n-1)\,y(n-2) + x(n-1)$

delayed $\Rightarrow y(n-1) = n\,y(n-2) + x(n-1)$

So the system is time variant.

$\Rightarrow$ If $x(n) = u(n)$, then $|x(n)| \le 1$, for this bounded input, output is $y(0)=0$, $y(1)=2$, $y(2)=5$ ...... unbounded

So system is unstable.

25) Consider the signal $\delta(n) = a^n u(n)$; $0<a<1$

a) Show that any sequence $x(n)$ can be decomposed as $x(n) = \sum_{n=-\infty}^{\infty} C_k \delta(n-k)$ and express C_k in terms of $x(n)$.

$\delta(n) = \gamma(n) - a\gamma(n-1)$

$\delta(n-k) = \gamma(n-k) - a\gamma(n-k-1)$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\left[\gamma(n-k) - a\gamma(n-k-1)\right]$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k)\,\gamma(n-k-1)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k-1)\,\gamma(n-k)$$

$$= \sum_{k=-\infty}^{\infty}\left[x(k) - ax(k-1)\right]\gamma(n-k)$$

Thus $C_k = x(k) - ax(k-1)$

b) Use the property of linearity and time invariance to express the output $y(n) = T[x(n)]$ in terms of the input $x(n)$ and the signal $g(n) = T[\gamma(n)]$, where $T[\;]$ is an LTI s/m

$$y(n) = T[x(n)]$$

$$= T\left[\sum_{k=-\infty}^{\infty} C_k\,\gamma(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} C_k\,T\left[\gamma(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} C_k\,g(n-k)$$

c) Express the impulse response $h(n) = T[\delta(n)]$ in terms of $g(n)$.

$h(n) = T[\delta(n)]$

$h(n) = T[\gamma(n) - a\gamma(n-1)]$

$= g(n) - ag(n-1)$

26) Determine the zero i/p-response of the s/m described by the second order differential equation

$(x(n) - 3y(n-1) - 4y(n-2) = 0$

with $x(n) = 0$

$-3y(n-1) - 4y(n-2) = 0 \quad (\div (-3))$

$y(n-1) + \frac{4}{3} y(n-2) = 0$

at $n=0$

$y(-1) = -\frac{4}{3} y(-2)$

at $n=1$ $y(0) = -\frac{4}{3} y(-1) = \left(\frac{-4}{3}\right)^2 y(-2)$

$y(1) = \left(\frac{-4}{3}\right)^3 y(-2)$

$\vdots$

$y(k) = \left(\frac{-4}{3}\right)^{k+2} y(-2)$

→ zero i/p response.

27) Determine the particular solution of the difference equation $y(n) = 5/6\ y(n-1) - 1/6\ y(n-2) + x(n)$ when the forcing function is $x(n) = 2^n u(n)$

$y(n) = \frac{5}{6} y(n-1) - 1/6\ y(n-2) + x(n)$

$x(n) = y(n) - 5/6\ y(n-1) + 1/6\ y(n-2)$

Characteristic equation is

$\lambda^2 - \frac{5}{6}\lambda + 1/6 = 0; \quad \lambda = 1/2, 1/3$

So $y_h(n) = C_1 (1/2)^n + C_2 (1/3)^n$

$x(n) = 2^n u(n) \quad y_p(n) = k[2^n] u(n)$

so $k(2^n) u(n) - k(5/6)(2^{n-1}) u(n-1) + k(1/6)(2^{n-2}) u(n-2)$
$= 2^n u(n)$

for $n=2$

$4k - \frac{5k}{3} + \frac{k}{6} = 4$

$k = 8/5$

Total solution is

$y_p(n) + y_h(n) = y(n)$

$y(n) = \frac{8}{5}(2^n) u(n) + C_1 (1/2)^n u(n) + C_2 (1/3)^n u(n)$

Assume $y(-2) = y(-1) = 0$ so $y(0) = 1$

then $y(1) = \frac{5}{6} y(0) + 2 = 17/6$

So $\frac{8}{5} + C_1 + C_2 = 1$

$C_1 + C_2 = 3/5 \rightarrow (1)$

$\frac{16}{5} + \frac{1}{2} C_1 + \frac{1}{3} C_2 = 17/6$

$3C_1 + 2C_2 = -11/5 \rightarrow (2)$

By solving (1) & (2)

$C_1 = -1, C_2 = 2/5$

So, the total solution is

$y(n) = \left[\frac{8}{5}(2)^n - (1/2)^n + \frac{2}{5}(1/3)^n\right] u(n)$

28) In the given equation $y(n) = (-a_1)^{n+1} y(-1) + \frac{(1-(-a_1)^{n+1})}{1+a_1}$

for $n \geq 1$ separate o/p sequence $y(n)$ into the transient response and the steady state response. plot these responses for $a_1 = -0.9$.

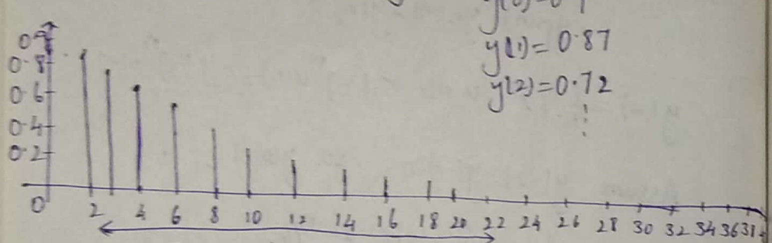
at $y(-1)=1$

The given equation $y(n) = (-a)^{n+1} + \frac{(1-(-a)^{n+1})}{1+a}$

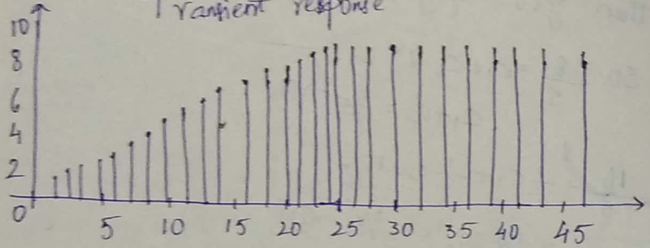
$y(n) = y_{zi}(n) + y_{zs}(n)$

= Transient + steady state

$y(0)=0.9$
$y(1)=0.87$
$y(2)=0.72$



Transient response



Steady state response

29) Determine the impulse response for the cascade of two linear time invarient systems having impulse responses.

$h_1(n) = a^n[u(n) - u(n-N)]$ and $h_2(n) = [u(n) - u(n-m)]$

$h(n) = h_1(n) * h_2(n)$

$= \sum_{k=-\infty}^{\infty} a^k [u(k)u(k-N)][u(n-k) - u(n-k-m)]$

$= \sum_{k=-\infty}^{\infty} a^k u(k)\, u(n-k) - \sum_{k=-\infty}^{\infty} a^k u(k)\, u(n-k-m) -$

$\sum_{k=-\infty}^{\infty} a^k u(k-N)\, u(n-k) + \sum_{k=-\infty}^{\infty} a^k u(k-N)\, u(n-k-m)$

$= \left(\sum_{k=0}^{n} a^k - \sum_{k=0}^{n-M} a^k\right) - \left(\sum_{k=N}^{n} a^k - \sum_{k=N}^{n-m} a^k\right)$

$h(n) = 0$

30) Determine the response $y(n), n \geq 0$ of the system described by the 2nd order difference equation.

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

to the input $x(n) = 4^n u(n)$.

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

Characteristic equation is

$\lambda^2 - 3\lambda - 4 = 0$

$\lambda = 4, -1$

so $\lambda_h(n) = C_1 4^n + C_2(-1)^n$

$x(n) = 4^n u(n)$

$y_p(n) = kn\, 4^n u(n)$

$kn 4^n u(n) - 3k(n-1)4^{n-1} u(n-1) - 4k(n-2)4^{n-2} u(n-2)$

$= 4^n u(n) + 2(4)^{n-1} u(n-1)$

for $n=2$ $\quad k(32-12) = 4^2 + 8 = 24 \rightarrow k = 6/5$

The total solution is

$y(n) = y_p(n) + y_h(n)$

$= \left[\frac{6}{5} n 4^n + c_1 4^n + c_2(-1)^n\right] u(n)$

to find c_1 and c_2, Let $y(-2)$ & $y(-1) = 0$ then

$y(0) = 1$

$y(1) = 3y(0) + 4 + 2 = 9$

$c_2 + c_1 = 1 \rightarrow ①$

$\frac{24}{5} + 4c_1 - c_2 = 9 \Rightarrow 4c_1 - c_2 = \frac{21}{5} \rightarrow ②$

from ① & ②

$c_1 = \frac{26}{25}$ & $c_2 = \frac{-1}{25}$

so $y(n) = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25}(-1)^n\right] u(n)$

31) Determine the impulse response of the following causal system $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

sol: characteristic equation $\lambda^2 - 3\lambda - 4 = 0$

$\lambda = -4, 1$

$y_h(n) = c_1 4^n + c_2(-1)^n$

$x(n) = \delta(n)$

$y(0) = 1$ and $y(1) - 3y(0) = 2$

$y(1) = 5$

so $c_1 + c_2 = 1 \rightarrow ①$

$4c_1 - c_2 = 5 \rightarrow ②$

from ① & ② $c_1 = 6/5$ & $c_2 = -1/5$

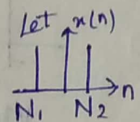
$\therefore h(n) = \left[\frac{6}{5} 4^n - \frac{1}{5}(-1)^n\right] u(n)$

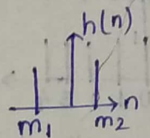
32) Let $x(n)$, $N_1 \le n \le N_2$ and $h(n)$, $M_1 \le n \le M_2$ be two finite duration signals.

a) Determine the range $L_1 \le n \le L_2$ of their convolution in terms of N_1, N_2, M_1 and M_2.

$L_1 = N_1 + M_1$

$L_2 = N_2 + M_2$

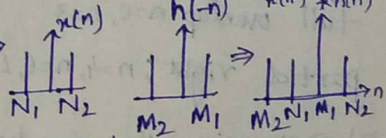
Let




b) Determine the limits of the cases of partial overlap from the left, full over lap and partial overlap from right. For convenience, assume that h(n) has shortest duration than x(n).

partial over lap from left.

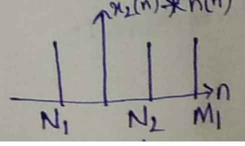
$\rightarrow x(n) * h(n) \Rightarrow$



LOW $N_1 + m_1$ & high $m_2 + N_1 - 1$

It fully over lap then $N_1 + m_2$ (Low) & high $N_2 + m_1$

partial overlap from right.



Low ⇒ $N_2 + m_1 + 1$

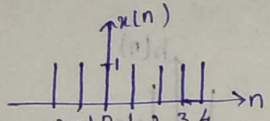
high ⇒ $N_2 + m_2$

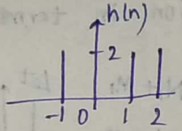
if fully overlapped high $N_2 + m_1$; Low $= N_1 + m_2$

c) Illustrate the validity of your result by computing the convolution of the signal $x(n)$

$= \begin{cases} 1, & -2 \le n \le 4 \\ 0, & \text{else where} \end{cases}$ $\quad h(n) = \begin{cases} 2, & -1 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$

$x(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\}$ $\quad h(n) = \{2, \underset{\uparrow}{2}, 2, 2\}$



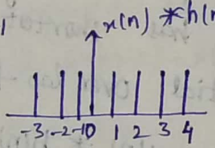


$N_1 = -2 \quad N_2 = 4$ $\quad m_1 = -1, \; m_2 = 2$

partial overlap from left

Low $N_1 + m_1 = -3$

high $m_2 + N_1 - 1 = 2 - 2 - 1 = -1$



full overlap $n = 0, n = 3$

partial right; $n = 4, n = 6, L_2 = 6$

33) Determine the impulse response and the unit step response the systems described by the difference equation.

a) $y(n) = 0.6\, y(n-1) - 0.08\, y(n-2) + x(n)$

$x(n) = y(n) - 0.6\, y(n-1) - 0.08\, y(n-2)$

characteristic equation

$\lambda^2 - 0.6\lambda + 0.08 = 0$

$\lambda = 1/2, \; 1/5$

$y_h(n) = C_1 (1/5)^n + C_2 (2/5)^n$

Impulse response $x(n) = \delta(n)$ with $y(0) = 1$

$y(1) - 0.6\, y(0) = 0 \Rightarrow y(1) = 0.6$

so $C_1 + C_2 = 1 \rightarrow ①$

$\frac{1}{5} C_1 + \frac{2}{5} C_2 = 0.6 \rightarrow ②$

from ① & ② $C_1 = -1, \; C_2 = 3$

$\therefore h(n) = \left[-(1/5)^n + 2(2/5)^n \right] u(n)$

step response $x(n) = u(n)$

$\delta(n) = \sum_{k=0}^{n} y_h(n-k), \; n \ge 0$

$= \sum_{k=0}^{n} \left[2(2/5)^{n-k} - (1/5)^{n-k} \right]$

$= 2\left(\frac{2}{5}\right)^n \sum_{k=0}^{n} (5/2)^k - \left(\frac{1}{5}\right)^n \sum_{k=0}^{n} (5)^k$

$= \left\{ \left[2(2/5)^n \left((5/2)^{n+1} - 1 \right) \right] - (1/5)^k \left(5^{n+1} - 1 \right) \right\} u(n)$

b) $y(n) = 0.7\, y(n-1) - 0.1\, y(n-2) + 2x(n) - x(n-2)$

$2x(n) - x(n-2) = y(n) - 0.7\, y(n-1) + 0.1\, y(n-2)$

characteristic equation,

$\lambda^2 - 0.7\lambda + 0.1 = 0$

$\lambda = 1/2, \; 1/5$

$y_h(n) = C_1\left(\tfrac{1}{2}\right)^n + C_2\left(\tfrac{1}{5}\right)^n$

Impulse response $x(n) = \delta(n)$ $y(0)=2$

$y(1) - 0.7\,y(0) = 0 \Rightarrow y(1) = 1.4$

$C_1 + C_2 = 2$

$\frac{1}{2}C_1 + \frac{1}{5}C_2 = 7/5 \rightarrow ①$

$C_1 + \frac{2}{5}C_2 = 14/5 \rightarrow ②$

Solving 1 & 2

$C_1 = \frac{10}{3}, \quad C_2 = -4/3$

So, $h(n) = \left[\frac{10}{3}\left(\tfrac{1}{2}\right)^n - \frac{4}{3}\left(\tfrac{1}{5}\right)^n\right] u(n)$

Step response $s(n) = \sum_{k=0}^{n} h(n-k)$

$$= \frac{10}{3}\sum_{k=0}^{n}\left(\tfrac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\tfrac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\tfrac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\tfrac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left[\left(\frac{1}{2}\right)^n \left(2^{n+1}-1\right)u(n)\right] - \frac{4}{3}\left[\left(\frac{1}{5}\right)^n \left(5^{n+1}-1\right)\right]u(n)$$

34) Consider a s/m with impulse response $h(n) = \begin{cases} (1/2)^n, & 0 \le n \le 4 \\ 0, & \text{else where} \end{cases}$. Determine the i/p $x(n)$ for $0 \le n \le \infty$ that will generate the o/p sequence $y(n) = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0\}$

$h(n) = \{1, 1/2, 1/4, 1/8, 1/16 \ldots\}$

$y(n) = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0 \ldots\}$

$y(0) = x(0)\,h(0)$

$y(0) = x(0)\cdot 1 \Rightarrow x(0) = 1$

$y(1) = x(1) + h(1)\,x(0)$

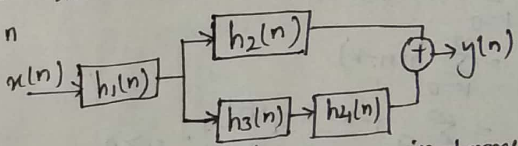
$2 = x(1) + \frac{1}{2}(1) \Rightarrow x(1) \Rightarrow 3/2$

$y(2) = x(2) + h(2)\,x(1) + h_1\,x(0)$

$2.5 = x(2) + 1/4\,(3/2) + 1/2\,(1)$

So $x(n) = \left\{1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \frac{3}{2} \ldots\right\}$

35) Consider the inter connection of LTI s/ms as shown in



a) Express the overall impulse response in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$, $h_4(n)$

$h(n) = h_1(n) * \left[h_2(n) - \{h_3(n) * h_4(n)\}\right]$

b) Determine $h(n)$ when $h_1(n) = \{1/2, 1/4, 1/2\}$

$h_2(n) = h_3(n) = (n+1)\,u(n)$

$h_4(n) = \delta(n-2)$

$h_3(n) * h_4(n) = (n+1)\,u(n) * \delta(n-2)$

$= (n+1)u(n-2) = (n+1)\,u(n-2)$

$h_2(n) - \left[h_3(n) * h_4(n)\right] = (n+1)\,u(n) - (n+1)\,u(n-2)$

$= 2u(n) - \delta(n)$

$h_1(n) = \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)$

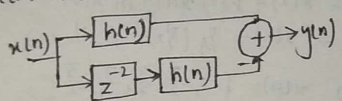
$h(n) = \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)\right] * \left[2u(n) - \delta(n)\right]$

$= -\frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$

c) Determine the response of the s/m in part (b) if $x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3)$

$$x(n) = \{1, 0, \underset{\uparrow}{0}, 3, 0, -4\}$$

36) Consider the s/m fig with $h(n) = a^n u(n), -1 < a < 1$. Determine the response $y(n)$ of the s/m to the excitation. $x(n) = u(n+5) - u(n-10)$



$s(n) = u(n) * h(n)$

$s(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$

$= \sum_{k=0}^{\infty} h(n-k)$

$= \sum_{k=0}^{\infty} a^{n-k} = \frac{a^{n+1}-1}{a-1}; \ n \geq 0$

for $x(n) = u(n+5) - u(n-10)$ then

$\delta(n+5) - \delta(n-10) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10)$

from given figure $y(n) = x(n) * h(n) - x(n) * h(n-2)$

$y(n) = \frac{a^{n+6}-1}{a-1} u(n-5) - \frac{a^{n-9}-1}{a-1} u(n-10) -$

$\frac{a^{n+4}-1}{a-1} u(n+3) + \frac{a^{n-11}-1}{a-1} u(n-12)$

37) Compute and sketch step response of the s/m.

$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$

$h(n) = \left[\frac{u(n) - u(n-M)}{M}\right]$

$s(n) = \sum_{k=-\infty}^{\infty} u(k)\, h(n-k)$

$= \sum_{k=0}^{n} h(n-k) = \begin{cases} \frac{n+1}{M}, & n < M \\ 1, & n \geq M \end{cases}$

38) Determine the range of values of the parameter a for which the linear time-invarient system with impulse response

$h(n) = \begin{cases} a^n; & n \geq 0, \ n \text{ even} \\ 0, & \text{otherwise} \end{cases}$

$\sum_{k=-\infty}^{\infty} |h(n)| = \sum_{\substack{n=0 \\ n=\text{even}}}^{\infty} |a|^n$

$= \sum_{n=0}^{\infty} |a|^{2n} = \frac{1}{1-|a|^2}$

stable if $|a| < 1$

39) Determine the response of the system with impulse response $h(n) = a^n u(n)$, to i/p signal $x(n) = u(n) - u(n-10)$.

$h(n) = a^n u(n)$

$y_1(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$

$= \sum_{k=0}^{n} a^{n-k}$

$= a^m \sum_{k=0}^{n} a^{-k} = \frac{1-a^{n+1}}{1-a} u(n)$

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$$y(n) = y_1(n) - y_1(n-10)$$

$$= \frac{1}{1-a}\left[(1-a^{n+1})u(n) - (1-a^{n-9})u(n-10)\right]$$

41) Determine the response of s/m characterized by the impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n)$ to the input signals.

a) $x(n) = 2^n u(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k 2^{n-k}$$

$$= 2^n \sum_{k=0}^{n} \left(\frac{1}{4}\right)^k$$

$$= 2^n \left[1 - \left(\frac{1}{4}\right)^{n+1}\right]\left(\frac{4}{3}\right)$$

$$= \frac{2}{3}\left[2^{n+1} - \left(\frac{1}{2}\right)^{n+1}\right] u(n)$$

b) $x(n) = u(-n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=0}^{\infty} h(k) = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k = 2, \; n<0$$

$$y(n) = \sum_{k=n}^{\infty} h(k)$$

$$= \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^k$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k - \sum_{k=0}^{n-1} \left(\frac{1}{2}\right)^k$$

$$= 2 - \left(\frac{1-\left(\frac{1}{2}\right)^n}{1/2}\right) = 2(1/2)^n, \; n \geq 0$$

42) Three s/ms with impulse responses $h(n) = \delta(n) - \delta(n-1)$, $h_2(n) = h(n)$ and $h_3(n) = u(n)$, are connected in cascade.

a) What is the impulse response, $h_c(n)$ of the overall system.

$$h_c(n) = h_1(n) * h_2(n) * h_3(n)$$

$$= [\delta(n) - \delta(n-1)] * u(n) * h(n)$$

$$= [u(n) - u(n-1)] * h(n)$$

$$= \delta(n) * h(n) = h(n)$$

b) Does the order of interconnection affect the overall s/m?

No

43) a) Prove and explain graphically the diff b/w the relations $x(n)\,\delta(n-n_0) = x(n_0)\,\delta(n-n_0)$ and $x(n) * \delta(n-n_0) = x(n-n_0)$

$x(n)\,\delta(n-n_0) = x(n_0)$. Thus only the value of $x(n)$ at $n = n_0$ is of interest.

$x(n) * \delta(n-n_0) = x(n-n_0)$. Thus, we obtain shifted version of $x(n)$ sequence

b) Show ~~that~~ a discrete-time system, which is described by a convolution summation, is LTI & relaxed.

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= h(n) * x(n)$$

linearity: $x_1(n) \rightarrow y_1(n) = h(n) * x_1(n)$

$x_2(n) \to y_2(n) = h(n) * x_2(n)$

$= \alpha h(n) * x_1(n) + \beta h(n) * x_2(n)$

$= \alpha y_1(n) + \beta y_2(n)$

Time invariance:

$x(n) \to y_1(n) = h(n) * x(n)$

$x(n-n_0) \to y_1(n) = h(n) * x(n-n_0)$

$= \sum_{K} h(K)\, x(n-n_0-k)$

$= y(n-n_0)$

e) what is the impulse response of the system described by $y(n) = x(n-n_0)$

Ans:- $h(n) = \delta(n-n_0)$.

45) Compute the zero state response of the system described by the difference equation $y(n) + \frac{1}{2} y(n-1) = x(n) + 2x(n-2)$ to the input $x(n) = \{1, 2, 3, 4, 2, 1\}$ by solving the difference equation recursively

$y(n) = -\frac{1}{2} y(n-1) + x(n) + 2x(n-2)$

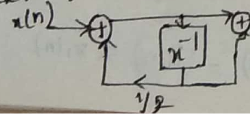
at $y(-2) = -\frac{1}{2} y(-3) + x(-2) + 2x(-4) = 1$

$y(-1) = -\frac{1}{2} y(-2) + x(-1) + 2x(-3) = 3/2$

$y(0) = -\frac{1}{2} y(-1) + 2x(-2) + x(0) = \frac{17}{4}$

$y(1) = -\frac{1}{2} y(0) + x(1) + 2x(-1) = \frac{47}{8}$

47) Consider the discrete time shown in fig



a) compute the 10 first samples of its impulse response

b) Find the i/p-o/p relation.

c) Apply the i/p $x(n) = \{1, 1, 1, \ldots\}$ and compute the first 10 samples of the output

d) Compute the first 10 samples of the output for the i/p given in part (c) by using convolution

e) Is the system causal? Is it stable?

a) $x(n) = \{\underset{\uparrow}{1}, 0, 0 \ldots\}$

$y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

$y(0) = x(0) = 1$

$y(1) = \frac{1}{2} y(0) + x(1) + x(0) = 3/2$

$y(2) = \frac{1}{2} y(1) + x(2) + x(1) = 3/4$. Thus we obtain

$y(n) = \{1, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16} \ldots\}$

b) $y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

c) as in part (a) we obtain

$y(n) = \{1, \frac{5}{2}, \frac{13}{4}, \frac{29}{8}, \frac{61}{16} \ldots\}$

d) $y(n) = u(n) * h(n)$

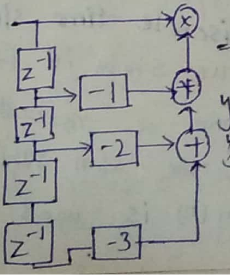
$= \sum_{K} u(K)\, h(n-k)$

$= \sum_{k=0}^{n} h(n-k)$

$y(0) = h(0) = 1$

$y(1) = h(0) + h(1) = 5/2$

$y(2) = h(0) + h(1) + h(2) = \frac{13}{4}$.



e) from part (a), $h(n)=0$ for $n<0 \Rightarrow$ The s/m is Causal. $\sum_{n=0}^{\infty}|h(n)| = 1+\frac{3}{2}(1+\frac{1}{2}+\frac{1}{4}+\cdots) = 4 \Rightarrow$ s/m is stable

48) Consider the s/m described by the diff. equ.

$y(n) = ay(n-1)+bx(n)$.

a) Determine b in terms of a so that

$$\sum_{n=-\infty}^{\infty} h(n) = 1$$

b) Compute the zero-state step response $s(n)$ of the s/m and choose b so that $s(\infty)=1$.

c) Compare the values of b obtained in parts (a) and (b). when did you notice?

a) $y(n) = ay(n-1)+bx(n)$

$h(n) = ba^n u(n)$

$$\sum_{n=0}^{\infty} h(n) = \frac{b}{1-a} = 1.$$

$b = 1-a$.

b) $s(n) = \sum_{k=0}^{n} h(n-k)$

$$= b\left[\frac{1-a^{n+1}}{a+1}\right]u(n)$$

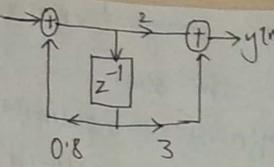
$$s(\infty) = \frac{b}{1-a} = 1$$

$b = 1-a$.

c) $b=1-a$ in both the cases.

49) A discrete time s/m is realised by the structure show in fig P2.49

a) Determine a realization for its inverse s/m, that is, the s/m which produces $x(n)$ as an o/p when $y(n)$ is used as an i/p.



a) $y(n) = 0.8y(n-1)+2x(n)+3x(n-1)$

$y(n)-0.8y(n-1) = 2x(n)+3x(n-1)$

The characteristic equation is

$$\lambda - 0.8 = 0$$

$$\lambda = 0.8$$

$$y_h(n) = c(0.8)^n$$

Let us first consider the response of the s/m.

$y(n) - 0.8y(n-1) = x(n)$

to $x(n)=\delta(n)$. since $y(0)=1$, it follows that $c=1$. Then, the impulse response of the original s/m is,

$$h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1)$$

$$= 2\delta(n) + 4.6(0.8)^{n-1} u(n-1)$$

b) The inverse s/m is characterized by the difference equation.

$$x(n) = -1.5x(n-1) + \tfrac{1}{2}y(n) - 0.4y(n-1).$$

50) Consider the discrete time s/m shown below

a) Compute the first six values of the impulse response of a s/m.

b) compute the first six values of the zero-state step response of the s/m.

c) Determine an analytical expression for the impulse response of the s/m.

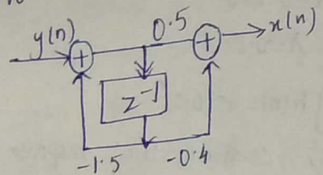
$y(n) = 0.9\,y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$

$y(n) - 0.9\,y(n-1) = x(n) + 2x(n-1) + 3x(n-2)$

a) for $x(n) = \delta(n)$, we have

$y(0)=1,\ y(1)=2.9,\ y(2)=5.61, y(3)=5.049, y(4)=4.544$

$y(5) = 4.090$



b) $s(0) = y(0) = 1$

$s(1) = y(0) + y(1) = 3.91$

$s(2) = y(0) + y(1) + y(2) = 9.51$

$s(3) = y(0) + y(1) + y(2) + y(3) = 14.56$

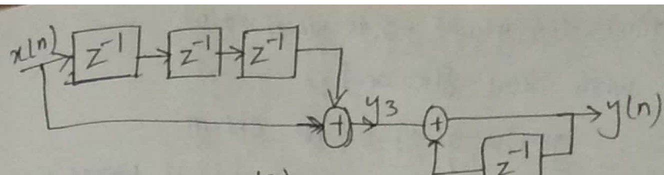
$s(4) = \sum_{0}^{4} y(n) = 19.10$

$s(5) = \sum_{0}^{5} y(n) = 23.19$

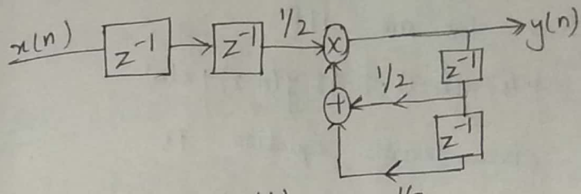
c) $h(n) = (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2} u(n-2)$

$= \delta(n) + 2.9\,\delta(n-1) + 5.61\,(0.9)^{n-2} u(n-2)$

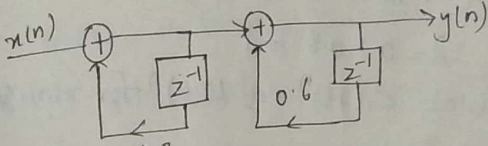
51) Determine & sketch the impulse response of the following systems for $n = 0, 1, \ldots, 9$)



(a)



(b)



a) fig P2.51(a) b) fig P2.51(b) c) fig P2.51(c)

d) classify the s/ms above as FIR or IIR.

e) find an explicit expression for the impulse response of the s/m in part(c).

a) $y(n) = \frac{1}{3}x(n) + \frac{1}{3}x(n-3) + y(n-1)$

for $x(n) = \delta(n)$, we have

$h(n) = \left\{ \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \ldots \right\}$

b) $y(n) = \frac{1}{2}y(n-1) + \frac{1}{8}y(n-2) + \frac{1}{2}x(n-2)$

with $x(n) = \delta(n)$ and

$y(-1) = y(-2) = 0$, we obtain

$h(n) = \left\{ 0, 0, \frac{1}{2}, \frac{1}{4}, \frac{3}{16}, \frac{1}{8}, \frac{11}{128}, \frac{15}{256}, \frac{41}{1024} \ldots \right\}$

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c) $y(n) = 1.4\, y(n-1) - 0.48\, y(n-2) + x(n)$

with $x(n) = \delta(n)$ and

$y(-1) = y(-2) = 0$ we obtain

$h(n) = \{1, 1.4, 1.48, 1.4, 1.2496, 1.0774, 0.9086 \ldots\}$

d) All three s/ms are IIR.

e) $y(n) = 1.4\, y(n-1) - 0.48\, y(n-2) + x(n)$

The characteristic equation is

$\lambda^2 - 1.4\lambda + 0.48 = 0$ hence

$\lambda = 0.8, 0.6$ and

$y_h(n) = c_1 (0.8)^n + c_2 (0.6)^n$ for $x(n) = \delta(n)$,

we have,

$c_1 + c_2 = 1$ and

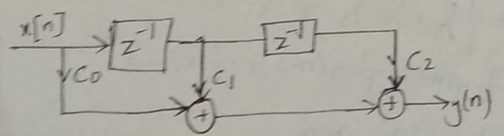
$0.8 c_1 + 0.6 c_2 = 1.4$

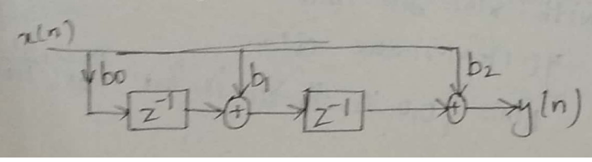
$c_1 = 4$

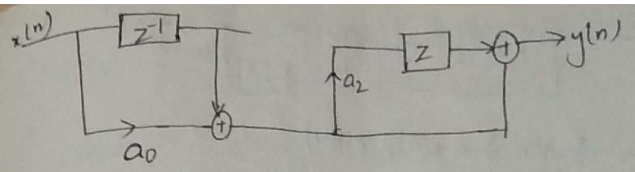
$c_2 = -3$

$h(n) = [4(0.8)^n - 3(0.6)^n]\, u(n)$

2.52) Consider s/ms shown below.







a) determine & sketch their impulse response $h_1(n)$, $h_2(n)$ & $h_3(n)$.

Sol:- $h_1(n) = C_0 \delta[n] + C_1 \delta[n-1] + C_2 \delta[n-2]$

$h_2(n) = b_2 \delta[n] + b_1 \delta[n-1] + b_0 \delta[n-2]$

$h_3(n) = a_0 \delta(n) + (a_1 + a_0 a_2)\, \delta(n-1) + a_1 a_2 \delta[n-2]$

b) Is it possible to choose the coefficients of these s/ms in such a way that.

$h_1(n) = h_2(n) = h_3(n)$.

Sol:- The only question is whether

$h_3(n) = h_2(n) = h_1(n)$

Let $a_0 = C_0$

$a_1 + a_2 C_0 = C_1$

$a_2 a_1 = C_2$

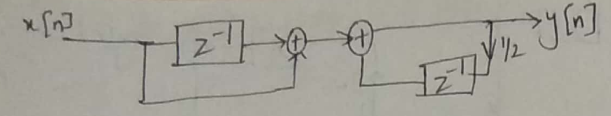
Hence $\frac{C_2}{a_2} + a_2 C_0 - C_1 = 0$

$C_0 a_2^2 - C_1 a_2 + C_2 = 0$

For $C_0 \neq 0$ the quadratic eq. has real solution if and only if $C_1^2 - 4 C_0 C_2 \geq 0$

53) Consider the s/m shown in figure.

a) Determine the impulse response $h[n]$.



$y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

for $y(n) - \frac{1}{2} y(n-1) = \delta(n)$ the solution is

$$h(n) = \left(\tfrac{1}{2}\right)^n u[n] + \left(\tfrac{1}{2}\right)^{n-1} u[n-1]$$

b) Show that $h(n)$ is equal to the convolution of the following signals

$h_1(n) = \delta(n) + \delta(n-1)$

$h_2(n) = \left(\frac{1}{2}\right)^n u[n]$

Sol:- $h_1(n) * [\delta(n) + \delta(n-1)] = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u[n-1]$.

54) Compare and sketch the convolution $y_i[n]$ and correlation $r_i[n]$ sequences for the following pair of signals & comment on the results obtained.

a) $x_1[n] = \{\underset{\uparrow}{1}, 2, 4\}$ $h_1(n) = \{1, 1, 1, 1, 1\}$

Sol.-

	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

Convolution

$y(n) = \{1, 3, 7, 7, 7, 6, 4\}$

Correlation

$\therefore r_1[n] = \{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$

b) $x_2(n) = \{\underset{\uparrow}{0}, 1, -2, 3, -4\}$ $h_2(n) = \{1/2, 1, 2, 1, \frac{1}{2}\}$

Convolution $y_1(n) = \{\frac{1}{2}, \underset{\uparrow}{0}, \frac{3}{2}, -2, \frac{1}{2}, -6, -5/2, -2\}$

Correlation: $r_1(n) = \{\frac{1}{2}, \underset{\uparrow}{0}, \frac{3}{2}, -2, \frac{1}{2}, -6, -5/2, 2\}$

Note that $y_2[n] = r_2[n]$, because $h_2[-n] = h_2[n]$

c) $x_3[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $h_3(n) = \{4, 3, 2, 1\}$

Convolution, $y_3(n) = \{4, 11, 20, 30, 20, 11, 4\}$

Correlation, $r_3(n) = \{1, 4, 10, 20, 25, 24, 16\}$

d) $x_4(n) = \{1, 2, 3, 4\}$ $h_4[n] = \{1, 2, 3, 4\}$.

Sol:- Convolution $y_4[n] = \{1, 4, 10, 20, 25, 24, 16\}$

correlation $r_4(n) = \{4, 11, 20, 30, 20, 11, 4\}$

Note that $h_3[-n] = h_4[n+3]$

hence $r_3(n) = y_4(n+3)$

and $h_4(-n) = h_3(n+3)$

$\Rightarrow r_4[n] = y_3[n+3]$

55) The zero-state response of a causal LTI s/m to the i/p $x[n] = \{1, 3, 3, 1\}$ is $y[n] = \{1, 4, 6, 4, 1\}$. Determine its impulse response.

Sol:- the length of $h[n]$ is 2

$h[n] = \{h_0, h_1\}$

$h_0 = 1$

$3h_0 + h_1 = 4$

$\Rightarrow h_0 = 1, h_1 = 1$

56) prove by direct substitution of equivalence of eqn $w[n] = -\sum_{k=1}^{N} a_k w[n-k] + x[n]$ and $y(n) = \sum_{k=0}^{M} b_k w[n-k]$ to the II structure which describe the direct form.

to the relation $y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$

which describes the direct form I structure.

Sol:- $y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k] \rightarrow ①$

$W[n] = -\sum_{k=1}^{N} a_k \, w[n-k] + x[n] \rightarrow ②$

$y[n] = \sum_{k=0}^{M} b_k \, w[n-k] \rightarrow ③$

from eq ② we obtain.

$x[n] = w[n] + \sum_{k=1}^{N} a_k \, w[n-k] \rightarrow ④$

By substituting eq③ for $y(n)$ and eq④ into ① we get LHS = RHS.

57) Determine the response $y[n]$, $n \geq 0$ of the s/m described by the second order diff. equation

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

where i/p is $x[n] = (-1)^n u(n)$ & the initial condition are $y(-1) = y(-2) = 0$.

$y(-n) - 4y(n-1) + 4y(n-2) = x(n) - x[n-1]$

The characteristic equation is

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda = 2, 2.$

Hence $y_h(n) = C_1 2^n + C_2 n 2^n$

The particular solution is

$y_p[n] = K(-1)^n u[n]$.

Substituting this solution into the diff. equation we obtain $K(-1)^n u[n] - 4K(-1)^{n-1} u[n-1] + 4K(-1)^{n-2} u(n-2)$

$= (-1)^n u(n) - (-1)^{n-1} u(n-1)$.

for $n=2$, $K[1+4+4] = 2 \Rightarrow K = 2/9$. The total solution is

$y(n) = \left[C_1 2^n + C_2 n 2^n + 2/9 (-1)^n\right] u(n)$

from the initial conditions we obtain.

$y[0] = 1$, $y[1] = 2$.

Then $C_1 + 2/9 = 1$

$\Rightarrow C_1 = 7/9$

$2C_1 + 2C_2 - 2/9 = 2$

$\frac{14}{9} - \frac{2}{9} + 3C_2 = 2 \Rightarrow 2C_2 = 2 - \frac{12}{9}$

$= 6/9$

$C_2 = 3/9 = 1/3$.

58) Determine the impulse response $h[n]$ for the s/m described by the second order diff. equation

$y[n] - 4y[n-1] + 4y[n-2] = x[n] - x[n-1]$.

Sol:- from prev. problem

$h[n] = \left[C_1 2^n + C_2 n 2^n\right] u(n)$

with $y(0) = 1$, $y(1) = 3$, we have

$C_1 = 1$, $2C_1 + 2C_2 = 3$.

$C_2 = 1/2$

This, $h[n] = \left[2^n + \frac{1}{2} n 2^n\right] u(n)$

with $y(0) = 1$, $y(1) = 3$, we have $C_1 = 1$

$2C_1 + 2C_2 = 3$

$\Rightarrow C_2 = 1/2$

Thus, $h[n] = \left[2^n + \frac{1}{2} n 2^n\right] u(n)$

59) Show that any discrete-time signal $x(n)$ can be expressed by $x[n] = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u[x-k]$ Where

$u(n-k)$ is a unit step delayed by k units in time, that is $u(n-k) = \begin{cases} 1, & n \geq k \\ 0 & \text{otherwise.} \end{cases}$

$$x(n) = x(n) * \delta(n)$$
$$= x(n) * [u(n) - u(n-1)]$$
$$= [x(n) - x(n-1)] * u(n)$$
$$x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u(n-k)$$

60) Show that the output of an LTI sm can be expressed in terms of its unit step response $\delta(n)$ as follows.

$$y(n) = \sum_{k=-\infty}^{\infty} [\delta(k) - \delta(k-1)] x(n-k)$$
$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] \delta(n-k)$$

Let $h(n)$ be the impulse response of the sm.

$$\delta(k) = \sum_{m=-\infty}^{k} h(m)$$

$$h(k) = \delta(k) - \delta(k-1)$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) x(n-k)$$

$$= \sum [\delta(k) - \delta(k-1)] x(n-k)$$

61) Compute the correlation seq. $r_{xx}(l)$ and $r_{xy}(l)$ for the following signal sequences.

$$x(n) = \begin{cases} 1, & n_0 - N \leq n \leq n_0 + N \\ 0 & \text{otherwise.} \end{cases}$$

$$y(n) = \begin{cases} 1, & -N \leq n \leq N \\ 0, & \text{Otherwise.} \end{cases}$$

$x(n) = \begin{cases} 1, & n_0 - N \leq n \leq n_0 + N \\ 0, & \text{otherwise} \end{cases}$ $\quad y(n) = \begin{cases} 1, & -N \leq n \leq N \\ 0, & \text{otherwise} \end{cases}$

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

The range of non-zero values of $r_{xx}(l)$ is determined by

$$n_0 - N \leq n \leq n_0 + N$$
$$n_0 - N \leq n - l \leq n_0 + N$$

which implies

$$-2N \leq l \leq 2N$$

for a given shift l, the no. of terms in the summation for which both $x(n)$ and non-zero is $2N+1-|l|$ and the value of each term is 1. Hence

$$r_{xx}(l) = \begin{cases} 2N+1-|l|, & -2N \leq l \leq 2N \\ 0, & \text{otherwise.} \end{cases}$$

for $r_{xy}(l)$ we have

$$r_{xy}(l) = \begin{cases} 2N+1-|l-n_0|, & n_0 - 2N \leq l \leq n_0 + 2N \\ 0, & \text{otherwise.} \end{cases}$$