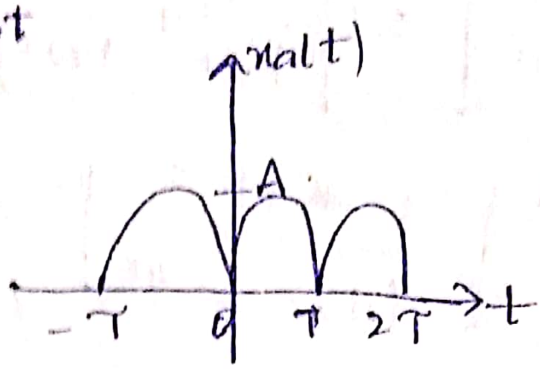
4th chapter solution

4.1 $x_a(t) = \sum_{k=-\infty}^{\infty} C_k e^{j2\pi k f_0 t}$



Let $f_0 = \frac{1}{T}$

$$= \sum_{k=-\infty}^{\infty} C_k e^{j2\pi k \frac{t}{T}}$$

$$C_k = \frac{1}{T}\int_0^T A \sin\left(\frac{\pi t}{T}\right) e^{-j2\pi k t/T} dt$$

$$= \frac{1}{2j\cdot T}\int_0^T A\left(e^{\frac{j\pi t}{T}} - e^{\frac{-j\pi t}{T}}\right) e^{-j2\pi k t/T} dt$$

$$= \frac{A}{2jT}\int_0^T \left(e^{j\pi\frac{t}{T}} \cdot e^{-j2\pi k\frac{t}{T}}\right) - \left(e^{-\frac{j\pi t}{T}} \cdot e^{-j2\pi k\frac{t}{T}}\right) dt$$

$$= \frac{A}{2j\cdot T}\int_0^T e^{j\pi(1-2k)\frac{t}{T}} - e^{-j\pi(1-2k)t/T} dt$$

$$= \frac{A}{2jT}\left[\int_0^T e^{j\pi\left(1-2k\frac{t}{T}\right)} dt - \int_0^T e^{-j\pi(1-2k)\frac{t}{T}} dt\right]$$

$$= \frac{A}{2j\cdot T}\left[\frac{e^{j\pi(1-2k)\frac{t}{T}}}{j\pi(1-2k)\frac{1}{T}}\right]_0^T - \left[\frac{e^{-j\pi(1-2k)\frac{t}{T}}}{-j\pi(1-2k)\frac{1}{T}}\right]_0^T$$

$$= \frac{A}{2jT}\left[\frac{e^{j\pi(1-2k)} - e^{0}}{j\pi(1-2k)\cdot\frac{1}{T}} - \frac{e^{-j\pi(1+2k)} - e^{0}}{-j\pi(1+2k) - 1/T}\right]$$

$$= \frac{A}{2jT}\cdot\frac{T}{j\pi}\left[\frac{-1-1}{(1-2k)} + \frac{(-1-1)}{(1+2k)}\right]$$

$$= \frac{\cancel{2}A}{\cancel{2}\pi}\left[\cancel{2}\left[\frac{1}{1-2k} + \frac{1}{1+2k}\right]\right]$$

$$= \frac{A}{\pi}\left[\frac{1+\cancel{2k}+1-\cancel{2k}}{1+\cancel{2k}-\cancel{2k}-4k^2}\right] = \frac{2A}{\pi(1-4k^2)}$$

$$x_a(t) = \int_{-\infty}^{\infty} x_a(t)\cdot e^{-j2\pi ft}\,dt$$

$$= \int_{-\infty}^{\infty}\sum_{n=-\infty}^{\infty} C_k e^{j2\pi k f_0 t}\cdot e^{-j2\pi ft}\cdot dt$$

$$= \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty} C_k e^{-j2\pi(F-kF_0)t}\,dt$$

$$= \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty} C_k e^{-j2\pi\left(F-\frac{tr}{T}\right)}dt$$

$$= \sum_{k=-\infty}^{\infty} C_k\int_{-\infty}^{\infty} e^{-j2\pi\left(F-\frac{k}{T}\right)t}\,dt$$

$$= \sum_{k=-\infty}^{\infty} C_k\,\delta\left(F-\frac{tr}{T}\right)$$

b) $$P_k = \frac{1}{T}\int_0^T x_a^2(t)\,dt$$

$$= \frac{1}{T}\int_0^T\left(A\sin\frac{\pi t}{T}\right)^2 dt$$

$$= \frac{A^2}{T}\int_0^T \sin^2\frac{\pi t}{T}\,dt$$

$$= \frac{A^2}{T}\int_0^T \frac{1-\cos 2\cdot\left(\frac{\pi}{T}\right)\cdot t}{2}$$

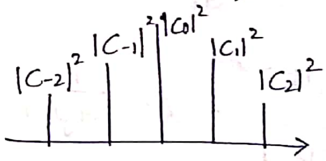
$$= \frac{A^2}{T}\int_0^T \frac{1}{2} - \frac{\cos 2\left(\frac{\pi}{T}\right)t}{2}$$

$$= \frac{A^2}{T}\left[\frac{T}{2} - \left[\frac{\cos 2(\pi)}{2} - 0\right]\right]$$

$$= \frac{A^2}{T}\cdot\frac{T}{2} - 0$$

$$= \frac{A^2}{2}$$

c)



d) $$P_x = \sum_{k=-\infty}^{\infty}|C_k|^2$$

$$= \sum_{k=-\infty}^{\infty}\left(\frac{2A}{\pi(1-4k^2)}\right)^2$$

$$= \frac{4A^2}{\pi^2}\sum_{k=-\infty}^{\infty}\frac{1}{(4k^2-1)^2}$$

$$= \frac{4A^2}{\pi^2}\left[\frac{1}{(4k^2-1)^2}\Big|_{k=0} + 2\sum_{k=0}^{\infty} \frac{1}{(4k^2-1)^2} \right]$$

$$= \frac{4A^2}{\pi^2}\left[1 + \frac{2}{3^2} + \frac{2}{15^2} + \cdots \right]$$

$\leftarrow$ Consider some values.

$$= \frac{4A^2}{\pi^2}[1.231]$$

$$= 0.498\,A^2$$

$$\simeq 0.5A^2 = \frac{A^2}{2}.$$

2) (a) $x_a(t) = \begin{cases} Ae^{-at} & t \geq 0 \\ 0, & t < 0 \end{cases}$

$$X_a(F) = \int_0^{\infty} Ae^{-at}\, e^{-j2\pi Ft}\, dt.$$

$$= A\int_0^{\infty} e^{-(a+j2\pi fj)t}\, dt.$$

$$= A\left[\frac{e^{-(a+2\pi fj)t}}{-(a+2\pi fj)}\right]_0^{\infty} = A\cdot\frac{1}{a+j2\pi f}$$

$$= \frac{A}{a+j2\pi f}$$

$$|X_a(F)| = \frac{A}{\sqrt{a^2+(2\pi f)^2}} \qquad \angle X_a(F) = -\tan^{-1}\left(\frac{2\pi f}{a}\right)$$

b) $x_a(t) = Ae^{-a|t|}$

$$X_a(F) = \int_{-\infty}^{\infty} Ae^{-a|t|}\, e^{-j2\pi ft}\, dt.$$

$$= \int_{-\infty}^{0} Ae^{at}\, e^{-j2\pi ft} + \int_0^{\infty} Ae^{-at}\, e^{-j2\pi ft}.$$

$$= \int_0^{\infty} Ae^{-at}\, e^{j2\pi ft} + \int_0^{\infty} Ae^{-at}\, e^{-j2\pi ft}.$$

$$= A\cdot\int_0^{\infty} e^{-(-j2\pi f+a)t} + A\int_0^{\infty} e^{-(a+j2\pi f)t}.$$

$$= A\cdot\left[\frac{e^{-(a-j2\pi f)t}}{-(a-j2\pi f)}\right]_0^{\infty} + A\left[\frac{e^{-(a+j2\pi f)t}}{-(a+j2\pi f)}\right]_0^{\infty}.$$

$$= A\cdot\left[\frac{1}{a-j2\pi f}\right] + A\left[\frac{1}{a+j2\pi f}\right]$$

$$= \frac{Aa + \cancel{Aj2\pi f} + Aa - \cancel{Aj2\pi f}}{a^2-(j2\pi f)^2}$$

$$= \frac{2aA}{a^2+(2\pi f)^2}$$

$$|X_a(F)| = X_a(F)$$

$$\angle X_a(F) = -\tan^{-1}\left(\frac{0}{\text{some value}}\right)$$

$$= 0$$

4.3 $x(t) = \begin{cases} 1 - \frac{|t|}{T}, & |t| \leq T \\ 0, & \text{elsewhere.} \end{cases}$

$$X_a(F) = \int_{-T}^{0} \left(1+\frac{t}{T}\right) e^{-j2\pi F t}\, dt + \int_{0}^{T} \left(1-\frac{t}{T}\right) e^{-j2\pi F t}\, dt.$$

Let $y(t) = \left\{ \frac{1}{T} ; \; -T \le t \le T \right.$

$$Y(F) = \int_{-T}^{0} \frac{1}{T} e^{-j2\pi f t}\, dt + \int_{0}^{T} \frac{-1}{T} e^{-j2\pi f t}\, dt.$$

$$= -2 \frac{\sin^2 \pi f T}{j\pi f T}.$$

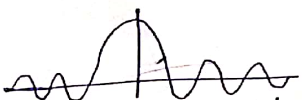
$$X(F) = \frac{1}{j2\pi f} Y(f) = T \left(\frac{\sin \pi f T}{\pi f T}\right)^2$$

$$|X(F)| = T \left(\frac{\sin \pi f T}{\pi f T}\right)^2$$

$\angle X_a(F) = 0$

$T\left(\frac{\sin \pi f T}{\pi f T}\right)^2$ Let $f = t/T$

$$\left(\frac{\sin \pi t}{\pi t}\right)^2 = \text{sinc}^2(t)$$



b) $$C_K = \frac{1}{T_p} \int_{-T_p/2}^{T_p/2} x(t)\, e^{-j2\pi k t/T_p}\, dt.$$

$$= \frac{1}{T_p} \int_{-T}^{0} (1+t/T) e^{-j2\pi k t/T_p} \cdot dt + \int_{0}^{T} (1-t/T) e^{-j2\pi k t/T_p}\, dt$$

$$= \frac{T}{T_p} \left[\frac{\sin \pi k T/T_p}{\pi k T/T_p}\right]^2$$

c) Using the results in parts a and b show that

$$C_K = \left(\frac{1}{T_p}\right) x_a\left(\frac{k}{T_p}\right)$$

$$\frac{1}{T_p} \cdot x_a\left(\frac{k}{T_p}\right)$$

$$\frac{1}{T_p} \cdot T \left(\frac{\sin \pi \frac{k}{T_p} \cdot T}{\pi \cdot \frac{k}{T_p} \cdot T}\right)^2$$

$$\frac{T}{T_p} \left(\frac{\sin \pi k T/T_p}{\pi k T/T_p}\right)^2 = C_k.$$

$$\therefore C_k = \frac{1}{T_p} x_a\left(\frac{k}{T_p}\right)$$

4) (a) 1 0 1 2 3 2 1 0 1

N = 6.

$$C_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N}$$

$$= \frac{1}{6} \sum_{n=0}^{5} x(n) e^{-j2\pi k n/6}$$

for $n=0 \rightarrow x(0) e^{-j2\pi k (0)/6} = 3 \times 1 = 3$

$n=1 \rightarrow x(1) e^{-j2\pi k(1)/6} = 2e^{-j2\pi k(1)/6}$

$n=2 \rightarrow x(2) e^{-j\frac{2\pi k}{6}(2)} = e^{-j\frac{2\pi k}{3}}$

$n=3 \rightarrow x(3) e^{-j2\pi k(3)/6} = 0$

$n=4 \rightarrow x(4) e^{-j2\pi k(4)/6} = 1 \cdot e^{-j\frac{2\pi k}{3}}$

$n=5 \rightarrow x(5) e^{-j2\pi k 5/6} = 2 \cdot e^{-j10\pi k/6}$

$$= \frac{1}{6}\left[3+2e^{-j\frac{\pi 2k}{6}} + e^{-j\frac{2\pi k}{3}} + 0 + e^{-j\frac{4\pi k}{3}} + 2e^{-j\frac{10\pi k}{6}}\right]$$

for $k=0$

$$= \frac{1}{6}[3+2+1+0+1+2]$$

$$= \frac{1}{6}[9] = 9/6$$

for $k=1$

$$= \frac{1}{6}\left[3+2e^{-j\frac{\pi 2}{6}} + e^{-j\frac{2\pi}{3}} + 0 + e^{-j\frac{4\pi}{3}} + 2e^{-j\frac{10\pi}{6}}\right]$$

$$= \frac{1}{6}\left[3+2e^{-j\frac{\pi}{3}} + e^{-j\frac{2\pi}{3}} + 0 + e^{-j\frac{4\pi}{3}} + 2e^{-j\frac{5\pi}{3}}\right]$$

$$= \frac{1}{6}\left[3+2(\cos(\pi/3) - j\sin(\tfrac{\pi}{3})) + \cos(\tfrac{2\pi}{3}) - j\sin(\tfrac{2\pi}{3}) + \cos(\tfrac{4\pi}{3}) - j\sin(\tfrac{4\pi}{3}) + 2\left(\cos\frac{5\pi}{3} - j\sin\frac{5\pi}{3}\right)\right]$$

$= 4/6$

IIIy for $k=2;\ C_2=0$

$k=3;\ C_3 = 1/6$

$k=4;\ C_4=0$

$k=5;\ C_5 = 4/6$

1 b) $P_t = \frac{1}{6}\sum_{n=0}^{5}|x(n)|^2$

$$= \frac{1}{6}[1^2+0^2+1^2+2^2+3^2+2^2] = \frac{1}{6}[1+1+4+9+4] = 19/6$$

$P(f) = \sum_{n=0}^{5}|c(n)|^2$

$$= \left(\frac{9}{6}\right)^2 + \left(\frac{4}{6}\right)^2 + 0^2 + \left(\frac{1}{6}\right)^2 + 0^2 + \left(\frac{4}{6}\right)^2 = \frac{114}{36} = 19/6$$

4.5) (a) $x(n) = 2+2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$

$$= 2+2\left[\frac{e^{\frac{j\pi n}{4}} + e^{-\frac{j\pi n}{4}}}{2}\right] + \frac{e^{\frac{j\pi n}{2}} + e^{-\frac{j\pi n}{2}}}{2} + \frac{1}{2}\left[\frac{e^{\frac{j3\pi n}{4}} + e^{-\frac{j3\pi n}{4}}}{2}\right]$$

$$= 2 + e^{\frac{j\pi n}{4}} + e^{-\frac{j\pi n}{4}} + \frac{1}{2}e^{\frac{j\pi n}{2}} + \frac{1}{2}e^{-\frac{j\pi n}{2}} + \frac{1}{4}e^{\frac{j3\pi n}{4}} + \frac{1}{4}e^{-\frac{j3\pi n}{4}}$$

$N=8$

$$C_k = \frac{1}{8}\sum_{n=0}^{7} x(n)e^{-j\pi k\frac{n}{4}}$$

$$x(n) = \left\{\frac{11}{2},\ 2+\frac{3}{4}\sqrt{2},\ 1,\ 2-\frac{3}{4}\sqrt{2},\ \frac{1}{2},\ 2-\frac{3}{4}\sqrt{2},\ 1,\ 2+\frac{3}{4}\sqrt{2}\right\}$$

$C_0 = 2,\quad C_1 = C_7 = 1,\ C_2 = C_6 = \frac{1}{2},\ C_3 = C_5 = \frac{1}{4},\ C_4 = 0.$

b) $\sum_{n=0}^{7}(C_k)^2$

$$= \left[2^2+1^2+1^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{4}\right)^2+\left(\frac{1}{4}\right)^2\right]$$

$$= \left[4+2+\frac{1}{2}+\frac{1}{8}\right] = \frac{32+16+4+1}{8} = 53/8$$

4.6) (a) $x(n) = 4\sin\frac{\pi(n-2)}{3}$

$$4\left[\frac{e^{\frac{j\pi(n-2)}{3}} - e^{-\frac{j\pi(n-2)}{3}}}{2j}\right]$$

$$= 4\left[e^{3j\pi(n-2)} - e^{-3j\pi(n-2)}\right]$$

$N=6$

$$C_k = \frac{1}{6}\sum_{n=0}^{5} x(n)\, e^{-\frac{j2\pi kn}{6}}$$

$$= \frac{4}{6}\sum_{n=0}^{5} \sin\frac{2\pi(n-2)}{6} e^{-2j\pi kn/6}$$

$$= \frac{1}{\sqrt{3}}\left[-e^{-j2\pi k/3} - e^{-j\pi k/3} + e^{-j\pi k/3} + e^{-j2\pi k/3} \right]$$

$$= \frac{1}{\sqrt{3}}(-j2)\left[\sin\frac{2\pi k}{6} + \sin\frac{\pi k}{3}\right] e^{-j2\pi k/3}$$

Hence $C_0 = 0,\ C_1 = -j2e^{-j2\pi/3},\ C_2 = C_3 = C_4 = 0, C_5 = C_1^*$

and $|C_1| = |C_5| = 2,\ |C_0| = |C_2| = |C_3| = |C_4| = 0$

and $\angle C_1 = \pi + \pi/2 - \frac{2\pi}{3} = \frac{5\pi}{6}$

$\angle C_5 = \frac{-5\pi}{6}$

$\angle C_0 = \angle C_2 = \angle C_3 = \angle C_4 = 0$

b) $x(n) = \cos\frac{2\pi n}{3} + \sin\frac{2\pi}{5} n \Rightarrow N = 15$

$C_k = C_{1k} + C_{2k}$

where C_{1k} is the DTFS coefficients of $\cos\frac{2\pi n}{3}$

and C_{2k} is the DTFS coefficients of $\sin\frac{2\pi n}{5}$.

But $\cos\frac{2\pi n}{3} = \frac{1}{2}\left[e^{j2\pi n/3} + e^{-j2\pi n/3}\right]$

Hence,

$$C_{1k} = \begin{cases} \frac{1}{2}, & k = 5, 10 \\ 0, & \text{otherwise.} \end{cases}$$

similarly, $\sin\frac{2\pi n}{5} = \frac{1}{2j}\left(e^{j2\pi n/5} - e^{-j2\pi n/5}\right)$

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Hence,

$$c_{1K} = \begin{cases} \frac{1}{2}, & K = 5, 10 \\ 0, & \text{otherwise} \end{cases}$$

Similarly,

$$\sin \frac{2\pi n}{5} = \frac{1}{2j}\left(e^{j2\pi n/5} - e^{-j2\pi n/5}\right)$$

Hence,

$$c_{2K} = \begin{cases} \frac{1}{2j}, & K = 3 \\ \frac{-1}{2j}, & K = 12 \\ 0, & \text{otherwise} \end{cases}$$

$$\therefore c_K = c_{1K} + c_{2K} = \begin{cases} \frac{1}{2j}, & K = 3 \\ \frac{1}{2}, & K = 5 \\ \frac{1}{2}, & K = 10 \\ \frac{-1}{2j}, & K = 12 \\ 0, & \text{otherwise} \end{cases}$$

c) $x(n) = \cos \frac{2\pi n}{3} \sin \frac{2\pi n}{5}$.

Sol: $x(n) = \cos \frac{2\pi n}{3} \sin \frac{2\pi n}{5}$

$$= \frac{1}{2}\sin \frac{16\pi n}{15} - \frac{1}{2}\sin \frac{4\pi n}{15}$$

Hence, $N = 15$, Following the same method in as in (b) above, we find that

$$c_K = \begin{cases} \frac{-1}{4j}, & K = 2, 7 \\ \frac{1}{4j}, & K = 8, 13 \\ 0, & \text{otherwise} \end{cases}$$

d) $x(n) = \{ \ldots, -2, -1, \underset{\uparrow}{0}, 1, 2, -2, -1, 0, 1, 2, \ldots \}$

Sol: $N = 5$

$$c_K = \frac{1}{5}\sum_{n=0}^{4} x(n) e^{-j2\pi nK/5}$$

$$= \frac{1}{5}\left(e^{-j2\pi K/5} + 2e^{-j4\pi K/5} - 2e^{-j6\pi K/5} - e^{-j8\pi K/5}\right)$$

$$= \frac{2j}{5}\left[-\sin\left(\frac{2\pi K}{5}\right) - 2\sin\left(\frac{4\pi K}{5}\right)\right]$$

$\therefore c_0 = 0$

$$c_1 = \frac{2j}{5}\left[-\sin\left(\frac{2\pi}{5}\right) + 2\sin\left(\frac{4\pi}{5}\right)\right]$$

$$c_2 = \frac{2j}{5}\left[\sin\left(\frac{4\pi}{5}\right) - 2\sin\left(\frac{2\pi}{5}\right)\right]$$

$c_3 = -c_2$

$c_4 = -c_1$.

e) $x(n) = \{ \ldots, -1, 2, \underset{\uparrow}{1}, 2, -1, 0, -1, 2, 1, 2, \ldots \}$

Sol: $N = 6$

$$c_K = \frac{1}{6}\sum_{n=0}^{5} x(n) e^{-j2\pi nK/6}$$

$$= \frac{1}{6}\left[1 + 2e^{-j\pi K/3} - e^{-j2\pi K/3} - e^{-j4\pi K/3} + 2e^{-j5\pi K/3}\right]$$

$$= \frac{1}{6}\left[1 + 4\cos\left(\frac{\pi K}{3}\right) - 2\cos\left(\frac{2\pi K}{3}\right)\right]$$

$\therefore c_0 = \frac{1}{2}, \quad c_1 = \frac{2}{3}, \quad c_2 = 0, \quad c_3 = \frac{-5}{6}, \quad c_4 = 0, \quad c_5 = \frac{2}{3}$.

(f) $x(n) = \{ \ldots, 0, 0, \underset{\uparrow}{1}, 1, 0, 0, 0, 1, 1, 0, 0, \ldots \}$

Sol: $N = 5$

$$c_K = \frac{1}{5}\sum_{n=0}^{4} x(n) e^{-j2\pi nK/5}$$

$$= \frac{1}{5}\left(1 + e^{-j2\pi K/5}\right)$$

$$= \frac{2}{5}\cos\left(\frac{\pi K}{5}\right) e^{-j\pi K/5}$$

$\therefore c_0 = \frac{2}{5}, \quad c_1 = \frac{2}{5}\cos\left(\frac{\pi}{5}\right)e^{-j\pi/5}, \quad c_2 = \frac{2}{5}\cos\left(\frac{2\pi}{5}\right)e^{-j2\pi/5}$

$c_3 = \frac{2}{5}\cos\left(\frac{3\pi}{5}\right)e^{-j3\pi/5}, \quad c_4 = \frac{2}{5}\cos\left(\frac{4\pi}{5}\right)e^{-j4\pi/5}$.

g) $x(n) = 1, \; -\infty < n < \infty$.

Sol: $N = 1, \quad c_K = x(0) = 1 \quad \text{or} \quad c_0 = 1$

h) $x(n) = (-1)^n, \; -\infty < n < \infty$.

Sol: $N = 2$

$$c_K = \frac{1}{2}\sum_{n=0}^{1} x(n) e^{-j\pi nK}$$

$$= \frac{1}{2}\left(1 - e^{-j\pi K}\right)$$

$\Rightarrow c_0 = 0, \quad c_1 = 1$.

7] Determine the periodic signals $x(n)$, with fundamental period $N=8$, if their fourier coefficients are given by:

a] $c_K = \cos\frac{K\pi}{4} + \sin\frac{3K\pi}{4}$.

Sol: $x(n) = \sum_{k=0}^{7} c_K e^{j2\pi nK/8}$

Note that if $c_K = e^{j2\pi pK/8}$, then

$$\sum_{K=0}^{7} e^{j2\pi pK/8} e^{j2\pi nK/8} = \sum_{n=0}^{7} e^{\frac{j2\pi(p+n)K}{8}}$$

$= 8, \quad p=-n$

$= 0, \quad p \neq -n$

Since $c_K = \frac{1}{2}\left[e^{j2\pi K/8} + e^{-j2\pi K/8}\right] + \frac{1}{2j}\left[e^{\frac{j6\pi K}{8}} - e^{\frac{-j6\pi K}{8}}\right]$

we have $x(n) = 4\delta(n+1) + 4\delta(n-1) - 4j\delta(n+3) + 4j\delta(n-3)$,

$-3 \leq n \leq 5$.

b] $c_K = \begin{cases} \sin\frac{K\pi}{3}, & 0 \leq K \leq 6 \\ 0, & K=7 \end{cases}$

Sol: $c_0 = 0,\ c_1 = \frac{\sqrt{3}}{2},\ c_2 = \frac{\sqrt{3}}{2},\ c_3 = 0,\ c_4 = -\frac{\sqrt{3}}{2},\ c_5 = -\frac{\sqrt{3}}{2},$

$c_6 = c_7 = 0$

$x(n) = \sum_{k=0}^{7} c_K e^{j2\pi nK/8}$

$= \frac{\sqrt{3}}{2}\left[e^{j\pi n/4} + e^{j2\pi n/4} - e^{j4\pi n/4} - e^{j5\pi n/4}\right]$

$= \sqrt{3}\left[\sin\frac{\pi n}{2} + \sin\frac{\pi n}{4}\right] e^{\frac{j\pi(3n-2)}{4}}$

c] $c_K = \{\ldots, 0, \frac{1}{4}, \frac{1}{2}, 1, \underset{\uparrow}{2}, 1, \frac{1}{2}, \frac{1}{4}, 0, \ldots\}$

Sol: $x(n) = \sum_{K=-3}^{4} c_K e^{j2\pi nK/8}$

$= 2 + e^{j\pi n/4} + e^{-j\pi n/4} + \frac{1}{2}e^{j\pi n/2} + \frac{1}{2}e^{-j\pi n/2} + \frac{1}{4}e^{j3\pi n/4} + \frac{1}{4}e^{-j3\pi n/4}$.

$= 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$.

8] Two DT signals, $S_K(n)$ and $S_l(n)$, are said to be orthogonal over an interval $[N_1, N_2]$ if

$$\sum_{n=N_1}^{N_2} S_K(n) S_l^*(n) = \begin{cases} A_K, & K=l \\ 0, & K \neq l \end{cases}$$

if $A_K = 1$, the signals are called orthonormal.

a) prove the relation

$$\sum_{n=0}^{N-1} e^{j2\pi Kn/N} = \begin{cases} N, & K=0, \pm N, \pm 2N, \ldots \\ 0, & \text{otherwise.} \end{cases}$$

Sol: If $K = 0, \pm N, \pm 2N, \ldots$

$$\sum_{n=0}^{N-1} e^{j2\pi Kn/N} = \sum_{n=0}^{N-1} 1 = N$$

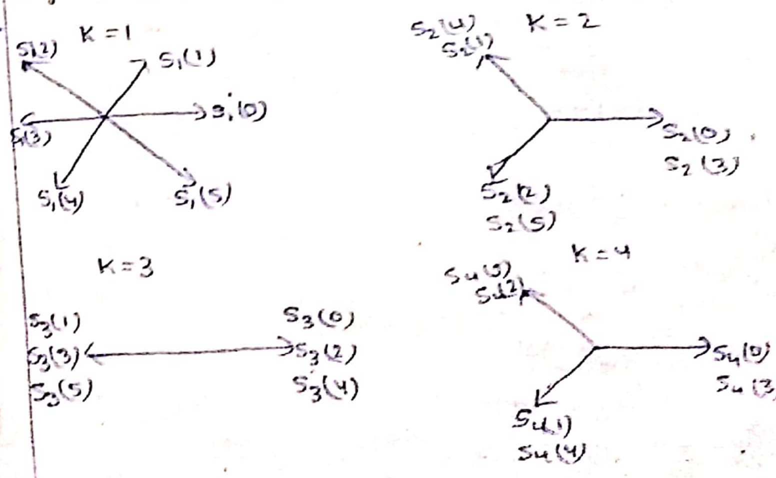
If $K \neq 0, \pm N, \pm 2N, \ldots$

$$\sum_{n=0}^{N-1} e^{j2\pi Kn/N} = \frac{1-e^{j2\pi K}}{1-e^{j2\pi K/N}}$$

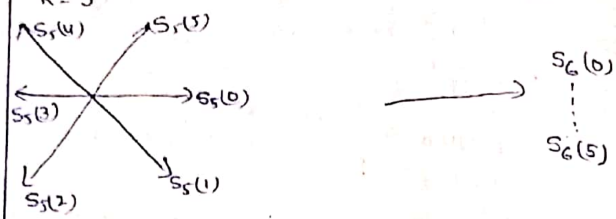
$= 0$

b) show that the harmonically related signals $S_K(n) = e^{j(2\pi/N)Kn}$ are orthogonal over any interval of length N.

Sol:



K = 5 K = 6



$$\sum_{n=0}^{N-1} S_K(n) S_i^*(n) = \sum_{n=0}^{N-1} e^{j2\pi kn/N} e^{-j2\pi in/N}$$

$$= \sum_{n=0}^{N-1} e^{j2\pi(k-i)n/N}$$

$$= N, \quad K = i$$

$$= 0, \quad K \neq i$$

$\therefore S_K(n)$ are orthogonal.

9] Compute the Fourier transform of the following signals

a] $x(n) = u(n) - u(n-6)$

Sol: $x(n) = u(n) - u(n-6)$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{5} e^{-j\omega n}$$

$$= \frac{1 - e^{-j6\omega}}{1 - e^{-j\omega}}$$

b] $x(n) = 2^n u(-n)$

Sol:- $x(n) = 2^n u(-n)$

$$X(\omega) = \sum_{n=-\infty}^{0} 2^n e^{-j\omega n}$$

$$= \sum_{m=0}^{\infty} \left(\frac{e^{j\omega}}{2}\right)^m$$

$$= \frac{2}{2 - e^{j\omega}}$$

c] $x(n) = \left(\frac{1}{4}\right)^n u(n+4)$

Sol: $$X(\omega) = \sum_{n=-4}^{\infty} \left(\frac{1}{4}\right)^n e^{-j\omega n}$$

$$= \left(\sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^m e^{-j\omega m}\right) 4^4 e^{j4\omega}$$

$$= \frac{4^4 e^{j4\omega}}{1 - \frac{1}{4} e^{-j\omega}}$$

d] $x(n) = (\alpha^n \sin \omega_0 n) u(n), \quad |\alpha| < 1$

Sol: Note that $\sum_{n=-\infty}^{\infty} |x(n)| = \sum_{n=-\infty}^{\infty} |\alpha|^n |\sin \omega_0 n|$

suppose that $\omega_0 = \frac{\pi}{2}$, so that $|\sin \omega_0 n| = 1$

$$\sum_{n=-\infty}^{\infty} |\alpha|^n = \sum_{n=-\infty}^{\infty} |x(n)| \to \infty$$

$\therefore$ FT does not exist.

f] $x(n) = \begin{cases} 2 - \left(\frac{1}{2}\right)n, & |n| \le 4 \\ 0, & \text{elsewhere} \end{cases}$

Sol: $$X(\omega) = \sum_{n=-4}^{4} x(n) e^{-j\omega n}$$

$$= \sum_{n=-4}^{4} \left(2 - \left(\frac{1}{2}\right)n\right) e^{-j\omega n}$$

$$= \frac{2e^{j4\omega}}{1 - e^{-j\omega}}$$

$$= -\frac{1}{2}\left[-4e^{j4\omega} + 4e^{-j4\omega} - 3e^{j3\omega} + e^{-j3\omega} - 2e^{j2\omega} + 2e^{-j2\omega} - e^{j\omega} + e^{-j\omega}\right]$$

$$= \frac{2e^{j4\omega}}{1 - e^{-j\omega}} + j\left[4\sin 4\omega + 3\sin 3\omega + 2\sin 2\omega + \sin\omega\right].$$

g] $x(n) = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$

Sol: $$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= -2e^{j2\omega} - e^{j\omega} + e^{j\omega} + 2e^{-j2\omega}$$

$$= -2j\left[2\sin 2\omega + \sin\omega\right]$$

h] $x(n) = \begin{cases} n(2M + 1 - |n|), & |n| \le M \\ 0, & |n| > M \end{cases}$

Sketch the magnitude and phase spectra for parts (a), (f) and (g).

Sol:- $X(\omega) = \sum_{n=-M}^{M} x(n) e^{-j\omega n}$

$$= A\sum_{n=-M}^{M}(2M+1-|n|)e^{-j\omega n}$$

$$=(2M+1)A + A\sum_{k=1}^{M}(2M+1-k)(e^{-j\omega k}+e^{j\omega k})$$

$$=(2M+1)A+2A\sum_{k=1}^{M}(2M+1-k)\cos\omega k$$

10) Determine the signals having the following F.T.

a) $X(\omega) = \begin{cases} 0, & 0\le|\omega|\le\omega_0 \\ 1, & \omega_0<|\omega|\le\pi \end{cases}$

Sol:- $x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}\,d\omega$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\omega_0} e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{\omega_0}^{\pi} e^{j\omega n}\,d\omega$$

$$x(0) = \frac{1}{2\pi}(\pi-\omega_0) + \frac{1}{2\pi}(\pi-\omega_0)$$

$$= \frac{\pi-\omega_0}{\pi}$$

For $n\neq 0$, $\int_{-\pi}^{-\omega_0} e^{j\omega n}\,d\omega = \frac{1}{jn}e^{j\omega n}\Big|_{-\pi}^{-\omega_0}$

$$= \frac{1}{jn}\left(e^{-j\omega_0 n} - e^{-j\pi n}\right)$$

$$\int_{\omega_0}^{\pi} e^{j\omega n}\,d\omega = \frac{1}{jn}e^{j\omega n}\Big|_{\omega_0}^{\pi}$$

$$= \frac{1}{jn}\left(e^{j\pi n} - e^{j\omega_0 n}\right)$$

Hence, $x(n) = -\frac{\sin n\omega_0}{n\pi}$, $n\neq 0$.

b) $X(\omega) = \cos^2\omega$

Sol: $X(\omega) = \cos^2\omega$

$$= \left(\frac{1}{2}e^{j\omega} + \frac{1}{2}e^{-j\omega}\right)^2$$

$$= \frac{1}{4}\left(e^{j2\omega} + 2 + e^{-j2\omega}\right)$$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}\,d\omega$$

$$= \frac{1}{8\pi}\left[2\pi\delta(n+2) + 4\pi\delta(n) + 2\pi\delta(n-2)\right]$$

$$= \frac{1}{4}\left[\delta(n+2) + 2\delta(n) + \delta(n-2)\right]$$

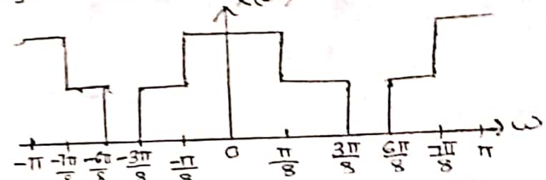
c) $X(\omega) = \begin{cases} 1, & \omega_0 - \delta\omega/2 \le |\omega| \le \omega_0 + \delta\omega/2 \\ 0, & \text{elsewhere} \end{cases}$

Sol: $x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}\,d\omega$

$$= \frac{1}{2\pi}\int_{\omega_0-\delta\omega/2}^{\omega_0+\delta\omega/2} e^{j\omega n}\,d\omega$$

$$= \frac{2}{\pi}\delta\omega\left(\frac{\sin(n\delta\omega/2)}{n\delta\omega/2}\right)e^{jn\omega_0}$$

d) the signal shown in Fig.



Sol:- $x(n) = \frac{1}{2\pi}\mathrm{Re}\left\{\int_0^{\pi/8} 2e^{j\omega n}\,d\omega + \int_{\pi/8}^{3\pi/8} e^{j\omega n}\,d\omega + \int_{6\pi/8}^{7\pi/8} e^{j\omega n}\,d\omega + \int_{7\pi/8}^{\pi} e^{j\omega n}\,d\omega\right\}$

$$= \frac{1}{\pi}\left[\int_0^{\pi/8} 2\cos\omega n\,d\omega + \int_{\pi/8}^{3\pi/8}\cos\omega n\,d\omega + \int_{6\pi/8}^{7\pi/8}\cos\omega n\,d\omega + \int_{7\pi/8}^{\pi} 2\cos\omega n\,d\omega\right]$$

$$= \frac{1}{n\pi}\left[\sin\frac{7\pi n}{8} + \sin\frac{6\pi n}{8} - \sin\frac{3\pi n}{8} - \sin\frac{\pi n}{8}\right]$$

11) consider the signal

$x(n) = \{1, 0, -1, \underset{\uparrow}{2}, 3\}$ with FT

$X(\omega) = X_R(\omega) + j(X_I(\omega))$. Determine & sketch the signal $y(n)$ with FT $Y(\omega) = X_I(\omega) + X_R(\omega)e^{j2\omega}$.

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Sol: $x_e(n) = \frac{x(n)+x(-n)}{2}$

$= \{\frac{1}{2}, 0, 1, \underset{\uparrow}{2}, 1, 0, \frac{1}{2}\}$

$x_o(n) = \frac{x(n)-x(-n)}{2}$

$= \{\frac{1}{2}, 0, -2, \underset{\uparrow}{0}, 2, 0, \frac{1}{2}\}$

then $X_R(\omega) = \sum_{n=-3}^{3} x_e(n) e^{-j\omega n}$

$jX_I(\omega) = \sum_{n=-3}^{3} x_o(n) e^{-j\omega n}$

Now, $Y(\omega) = X_I(\omega) + X_R(\omega) e^{j2\omega}$.

$\therefore y(n) = F^{-1}\{X_I(\omega)\} + F^{-1}\{X_R(\omega) e^{j2\omega}\}$

$= -jx_o(n) + x_e(n+2)$

$= \{\frac{1}{2}, 0, 1-\frac{j}{2}, 2, 1+\frac{j}{2}, \underset{\uparrow}{0}, \frac{1}{2}-j2, 0, \frac{j}{2}\}$

12a)

Sol: $x(n) = \frac{1}{2\pi}\left[\int_{8\pi/10}^{9\pi/10} e^{j\omega n} d\omega + \int_{-9\pi/10}^{-8\pi/10} e^{j\omega n} d\omega + 2\int_{9\pi/10}^{\pi} e^{j\omega n} d\omega + 2\int_{-\pi}^{-9\pi/10} e^{j\omega n} d\omega\right]$

$= \frac{1}{2\pi}\left[\frac{1}{jn}\left(e^{j9\pi n/10} - e^{-j9\pi n/10} - e^{j8\pi n/10} + e^{-j8\pi n/10}\right) + \frac{2}{jn}\left(-e^{j9\pi n/10} + e^{-j9\pi n/10} + e^{j\pi n} - e^{-j\pi n}\right)\right]$

$= \frac{1}{n\pi}\left(\sin \pi n - \sin 8\pi n/10 - \sin 9\pi n/10\right)$

$= -\frac{1}{n\pi}\left[\sin 4\pi n/5 + \sin 9\pi n/10\right]$

b)

Sol: $x(n) = \frac{1}{2\pi}\int_{-\pi}^{0} X(\omega) e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{0}^{\pi} X(\omega) e^{j\omega n} d\omega$

$= \frac{1}{2\pi}\int_{-\pi}^{0}\left(\frac{\omega}{\pi}+1\right) e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{0}^{\pi} \frac{\omega}{\pi} e^{j\omega n} d\omega$

$= \frac{1}{2\pi}\left[\frac{\omega}{jn\pi} e^{j\omega n}\Big|_{-\pi}^{\pi} + \frac{e^{j\omega n}}{jn}\Big|_{-\pi}^{0}\right]$

$= \frac{1}{\pi n} \sin \frac{\pi n}{2} e^{-jn\pi/2}$

c)

Sol: $x(n) = \frac{1}{2\pi}\int_{\omega_c-\frac{\omega}{2}}^{\omega_c+\frac{\omega}{2}} 2e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{-\omega_c-\frac{\omega}{2}}^{-\omega_c+\frac{\omega}{2}} 2e^{j\omega n} d\omega$

$= \frac{1}{\pi}\left(\frac{1}{jn\pi} e^{j\omega n}\Big|_{\omega_c-\frac{\omega}{2}}^{\omega_c+\frac{\omega}{2}} + \frac{e^{j\omega n}}{jn}\Big|_{-\omega_c-\frac{\omega}{2}}^{-\omega_c+\frac{\omega}{2}}\right)$

$= \frac{2}{\pi n}\left(\frac{e^{j(\omega_c+\frac{\omega}{2})n} - e^{j(\omega_c-\frac{\omega}{2})n} + e^{-j(\omega_c-\frac{\omega}{2})n} - e^{-j(\omega_c+\frac{\omega}{2})n}}{2j}\right)$

$= \frac{2}{\pi n}\left[\sin(\omega_c+\frac{\omega}{2})n - \sin(\omega_c-\frac{\omega}{2})n\right]$

13)

Sol: $x_1(n) = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$

$X_1(\omega) = \sum_{n=0}^{M} e^{-j\omega n}$

$= \frac{1-e^{-j\omega(M+1)}}{1-e^{-j\omega}}$

$x_2(n) = \begin{cases} 1, & -M \le n \le -1 \\ 0, & \text{otherwise} \end{cases}$

$X_2(\omega) = \sum_{n=-M}^{-1} e^{-j\omega n}$

$= \sum_{n=1}^{M} e^{j\omega n}$

$= \frac{1-e^{j\omega M}}{1-e^{j\omega}} e^{j\omega}$

$X(\omega) = X_1(\omega) + X_2(\omega)$

$= \frac{1+e^{j\omega} - e^{j\omega} - 1 - e^{-j\omega(M+1)} - e^{j\omega(M+1)} + e^{j\omega M} + e^{-j\omega M}}{2-e^{-j\omega}-e^{j\omega}}$

$= \frac{2\cos\omega M - 2\cos\omega(M+1)}{2-2\cos\omega}$

$= \frac{2\sin(\omega M+\frac{\omega}{2})\cos\frac{\omega}{2}}{2\sin^2\frac{\omega}{2}}$

$$= \frac{\sin\left(M+\frac{1}{2}\right)\omega}{\sin\left(\frac{\omega}{2}\right)}$$

14)

a) $X(0) = \sum_n x(n) = -1$

b) $\angle X(\omega) = \pi$ for all ω

c) $x(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\,d\omega$ Hence, $\int_{-\pi}^{\pi} X(\omega)\,d\omega = 2\pi x(0) = -6\pi$

d) $X(\pi) = \sum_{n=-\infty}^{\infty} x(n)e^{-jn\pi} = \sum_n (-1)^n x(n) = -3-4-2 = -9$

e) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega = 2\pi \sum_n |x(n)|^2 = (2\pi)(19) = 38\pi$

15)

$$c = \frac{\sum_{n=-\infty}^{\infty} n\,x(n)}{\sum_{n=-\infty}^{\infty} x(n)}$$

a) Express c in terms of $X(\omega)$.

Sol: $X(\omega) = \sum_n x(n)e^{-j\omega n}$

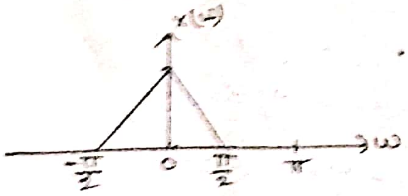
$X(0) = \sum_n x(n)$

$$\left.\frac{dX(\omega)}{d\omega}\right|_{\omega=0} = -j\sum_n n\,x(n)e^{-j\omega n}\Big|_{\omega=0}$$

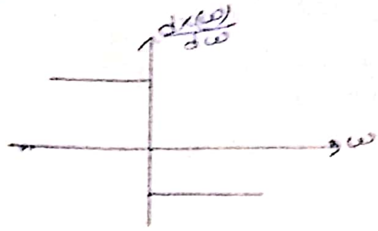
$$= -j\sum_n n\,x(n)$$

$$\therefore\ c = \frac{\left. j\frac{dX(\omega)}{d\omega}\right|_{\omega=0}}{X(0)}$$

b) Compute c for the signal $x(n)$ whose FT is shown in Fig.



Sol: $X(0) = 1$, $\therefore\ c = \frac{0}{1} = 0$.



16)

Sol: $x_1(n) = a^n u(n)$

$$\xleftrightarrow{F} \frac{1}{1-ae^{-j\omega}}$$

Now suppose that

$$x_k(n) = \frac{(n+k-1)!}{n!\,(k-1)!}a^n u(n)$$

$$\xleftrightarrow{F} \frac{1}{(1-ae^{-j\omega})^k}$$

holds. then

$$x_{k+1}(n) = \frac{(n+k)!}{n!\,k!}a^n u(n) = \frac{n+k}{k}x_k(n)$$

$$X_{k+1}(\omega) = \frac{1}{k}\sum_n n\,x_k(n)e^{-j\omega n} + \sum_n x_k(n)e^{-j\omega n}$$

$$= \frac{1}{k}j\frac{dX_k(\omega)}{d\omega} + X_k(\omega)$$

$$= \frac{ae^{-j\omega}}{(1-ae^{-j\omega})^{k+1}} + \frac{1}{(1-ae^{-j\omega})^k}$$

17)

a) $\sum_n x^*(n)e^{-j\omega n} = \left(\sum_n x(n)e^{-j(-\omega)n}\right)^* = X^*(-\omega)$

b) $\sum_n x^*(-n)e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x^*(n)e^{j\omega n} = X^*(\omega)$

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(b) $\sum_n y(n) e^{-j\omega n} = \sum_n x(n) e^{-j\omega n} - \sum_n x(n-1) e^{-j\omega n}$

$Y(\omega) = X(\omega) - X(\omega) e^{-j\omega}$

$= (1 - e^{-j\omega}) X(\omega)$

(d) $y(n) = \sum_{k=-\infty}^{n} x(k)$

$= y(n) - y(n-1)$

$= x(n)$

Hence, $X(\omega) = (1 - e^{-j\omega}) Y(\omega)$

$\Rightarrow Y(\omega) = \frac{X(\omega)}{1 - e^{-j\omega}}$

(e) $Y(\omega) = \sum_n x(2n) e^{-j\omega n}$

$= \sum_n x(n) e^{-j\frac{\omega}{2} n}$

$= X\left(\frac{\omega}{2}\right)$

(f) $Y(\omega) = \sum_n x\left(\frac{n}{2}\right) e^{-j\omega n}$

$= \sum_n x(n) e^{-j2\omega n}$

$= X(2\omega)$.

(k)

(a) $X_1(\omega) = \sum_n x(n) e^{-j\omega n}$

$= e^{j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega}$

$= 1 + 2\cos\omega + 2\cos 2\omega$

(b) $X_2(\omega) = \sum_n x_2(n) e^{-j\omega n}$

$= e^{j4\omega} + e^{j2\omega} + 1 + e^{-j2\omega} + e^{-j4\omega}$

$= 1 + 2\cos 2\omega + 2\cos 4\omega$

(c) $X_3(\omega) = \sum_n x_3(n) e^{-j\omega n}$

$= e^{j6\omega} + e^{j3\omega} + 1 + e^{-j3\omega} + e^{-j6\omega}$

$= 1 + 2\cos 3\omega + 2\cos 6\omega$

(d) $X_2(\omega) = X_1(2\omega)$ and $X_3(\omega) = X_1(3\omega)$

(e) If $x_k(n) = \begin{cases} x\left(\frac{n}{k}\right), & \frac{n}{k} \text{ an integer} \\ 0, & \text{otherwise} \end{cases}$

then, $X_k(\omega) = \sum_{n,\ \frac{n}{k} \text{ an integer}} x_k(n) e^{-j\omega n}$

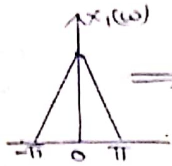
$= \sum_n x(n) e^{-jk\omega n}$

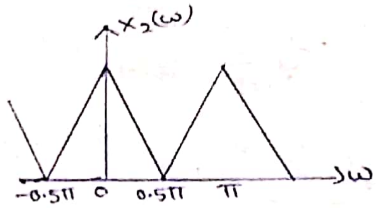
$= X(k\omega)$.

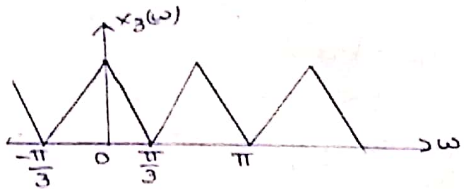
(l)

(a) $x_1(n) = \frac{1}{2}\left(e^{j\pi n/4} + e^{-j\pi n/4}\right) x(n)$

$X_1(\omega) = \frac{1}{2}\left[X\left(\omega - \frac{\pi}{4}\right) + X\left(\omega + \frac{\pi}{4}\right)\right]$







b) $x_2(n) = \frac{1}{2j}\left(e^{j\pi n/2} + e^{-j\pi n/2}\right) x(n)$

$X_2(\omega) = \frac{1}{2j}\left[X\left(\omega - \frac{\pi}{2}\right) + X\left(\omega + \frac{\pi}{2}\right)\right]$

c) $x_3(n) = \frac{1}{2}\left(e^{j\pi n/2} + e^{-j\pi n/2}\right) x(n)$

$X_3(\omega) = \frac{1}{2}\left[X\left(\omega - \frac{\pi}{2}\right) + X\left(\omega + \frac{\pi}{2}\right)\right]$

d) $x_4(n) = \frac{1}{2}\left(e^{j\pi n} + e^{-j\pi n}\right) x(n)$

$X_4(\omega) = \frac{1}{2}\left[X(\omega - \pi) + X(\omega + \pi)\right]$

$= X(\omega - \pi)$

20)

Sol:- $c_k^y = \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-j2\pi k n/N}$

$= \frac{1}{N} \sum_{n=0}^{N-1} \left[\sum_{l=-\infty}^{\infty} x(n - lN)\right] e^{-j2\pi k n/N}$

$= \frac{1}{N} \sum_{l=-\infty}^{\infty} \sum_{m=-lN}^{N-1-lN} x(n) e^{-j2\pi k(m+lN)/N}$

But

$\sum_{l=-\infty}^{\infty} \sum_{m=-lN}^{N-1-lN} x(m) e^{-j\omega(m+lN)} = X(\omega)$

$\therefore c_k^y = \frac{1}{N} X\left(\frac{2\pi k}{N}\right).$

21)

Sol:- Let $x_N(n) = \frac{\sin \omega_c n}{\pi n}, \quad -N \le n \le N$

$= x(n)\, w(n)$

where $x(n) = \frac{\sin \omega_c n}{\pi n}, \quad -\infty \le n \le \infty$

$w(n) = 1, \quad -N \le n \le N$

$= 0, \quad$ otherwise

then $\frac{\sin \omega_c n}{\pi n} \xleftrightarrow{F} X(\omega)$

$= 1, \quad |\omega| \le \omega_c$

$= 0, \quad$ otherwise

$X_N(\omega) = X(\omega) * W(\omega)$

$= \int_{-\pi}^{\pi} X(\theta)\, W(\omega - \theta)\, d\theta$

$= \int_{-\omega_c}^{\omega_c} \frac{\sin (2N+1)(\omega - \theta)/2}{\sin (\omega - \theta)/2}\, d\theta$

22)

a) $X_1(\omega) = \sum_n x(2n+1) e^{-j\omega n}$

$= \sum_k x(k) e^{-j\omega k/2} e^{j\omega/2}$

$= X\left(\frac{\omega}{2}\right) e^{j\omega/2}$

$= \frac{e^{j\omega/2}}{1 - a e^{j\omega/2}}$

b) $X_2(\omega) = \sum_n x(n+2) e^{\pi n/2} e^{-j\omega n}$

$= -\sum_k x(k) e^{-jk(\omega + j\pi/2)} e^{j2\omega}$

$= -X\left(\omega + \frac{j\pi}{2}\right) e^{j2\omega}$

c) $X_3(\omega) = \sum_n x(-2n) e^{-j\omega n}$

$= -\sum_k x(k) e^{-jk\omega/2}$

$= X\left(-\frac{\omega}{2}\right).$

d) $X_4(\omega) = \sum_n \frac{1}{2}\left(e^{j0.3\pi n} + e^{-j0.3\pi n}\right) x(n) e^{-j\omega n}$

$= \frac{1}{2} \sum_n x(n) \left[e^{-j(\omega - 0.3\pi) n} + e^{-j(\omega + 0.3\pi) n}\right]$

$= \frac{1}{2}\left[X(\omega - 0.3\pi) + X(\omega + 0.3\pi)\right]$

e) $X_5(\omega) = X(\omega)\left[X(\omega)e^{-j\omega}\right] = X^2(\omega)e^{-j\omega}$

f) $X_6(\omega) = X(\omega)\,X(-\omega)$

$$= \frac{1}{(1-ae^{-j\omega})(1-ae^{j\omega})}$$

$$= \frac{1}{(1-2a\cos\omega + a^2)}$$

23)

a) $Y_1(\omega) = \sum_n y_1(n)e^{-j\omega n}$

$$= \sum_{n,\,n\ \text{even}} x(n)e^{-j\omega n}$$

The FT $X_1(\omega)$ can easily be obtained by combining the results of (b) and (c).

b) $y_2(n) = x(2n)$

$$Y_2(\omega) = \sum_n y_2(n)e^{-j\omega n}$$

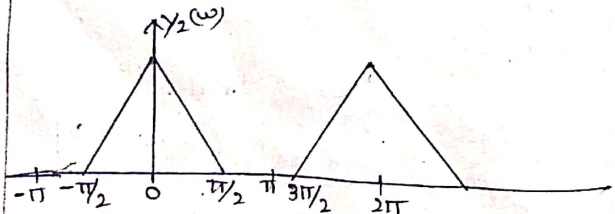
$$= \sum_n x(2n)e^{-j\omega n}$$

$$= \sum_n x(m)e^{-j\omega m/2}$$

$$= X\left(\frac{\omega}{2}\right).$$

c) $y_3(n) = \begin{cases} x(n/2), & n \text{ even} \\ 0, & \text{otherwise} \end{cases}$

$$Y_3(\omega) = \sum_n y_3(n)e^{-j\omega n}$$



$$= \sum_{n\,\text{even}} x(n/2)e^{-j\omega n}$$

$$= \sum_m x(m)e^{-j2\omega m}$$

$$= X(2\omega)$$

We now return to part (a). Note that $y_1(n)$ may be expressed as

$$y_1(n) = \begin{cases} y_2(n/2), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

Hence, $Y_1(\omega) = Y_2(2\omega)$.