

# Fuzzy Cognitive Maps: Basic Theories and Their Application to Complex Systems

Peter P. Groumpos

**Abstract.** The challenging problem of modeling and controlling complex systems is investigated using Fuzzy Cognitive Maps (FCMs). A mathematical description of FCM models is presented, new construction methods and an algorithm are developed and extensively examined. The issue of modeling the supervisor of large complex systems is addressed and is modeled using a FCM. A manufacturing example is used to prove the usefulness of the proposed method. The problem of Decision Making process in Decision Analysis is considered and analyzed using FCM models. A successful application of FCM theory in a health problem is provided.

**Keywords:** Fuzzy Cognitive Maps, Modeling, Control Systems, Decision Systems.

## 1 Introduction

Most of today systems are characterized as complex systems with high dimension and a variety of variables and factors. It is widely recognized that conventional methods in modeling and controlling modern systems have contributed a lot in the research and on the solution of many control problems. However, their contribution to the solution of the increasingly problems associated with complex dynamical systems has proved to be limited. New methods have been proposed for complex systems that utilize existence knowledge and human experience and will have learning capabilities and advanced characteristics such as failure detection

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and identification qualities. In this chapter Fuzzy Cognitive Maps (FCM) are proposed for modeling and controlling complex systems. The application of FCM may contribute to the effort for more intelligent control methods and for the development of autonomous systems. A Fuzzy Cognitive Map draws a causal picture to represent the model and the behavior of system. The concepts of an FCM interact according to imprecise rules and the operations of complex systems are simulated.

Fuzzy Cognitive Maps are symbolic representation for the description and modeling of the complex system [1]-[3],[6],[16],[17]. They consist of concepts, that illustrate different aspects in the behavior of the system and these concepts interact with each other showing the dynamics of the system. The human experience and knowledge of the operation of the system is used to develop Fuzzy Cognitive Map (FCM), as a result of the method by which it is constructed, i.e., using human experts that know the operation of system and its behavior in different circumstances. An FCM illustrates the system by a graph showing the cause and effect along concepts, and it is a simple way to describe the system's behavior in a symbolic manner, exploiting the accumulated knowledge of the complex system.

Fuzzy Cognitive Map (FCM) have been applied to a variety of scientific areas [4],[10],[12],[13],[15],[20]-[25],[30],[36]-[41]. FCMs have been used to describe and model the behavior of a system and its application in the modeling the supervisor of distributed systems. Fuzzy Cognitive Maps have been used for decision analysis and operation research. The objective of this chapter is to focus on the construction and the use of FCM in modeling complex systems. It will be shown that FCMs are useful to exploit the knowledge and experiences that human have accumulated for years on the operation of a complex system. Such methodologies are crude analogs of approaches that exist in human and animal systems and have their origins in behavioral phenomena related to these beings. So, a FCM represents knowledge in a symbolic manner and relates states, variables, events and inputs in an analogous to beings manner. This methodology can contribute to engineers' intention to construct intelligent systems, since as the more intelligent a system becomes, the more symbolic and fuzzy a representation it utilizes [7]-[12],[37],[42].

## **2 Basic Theories**

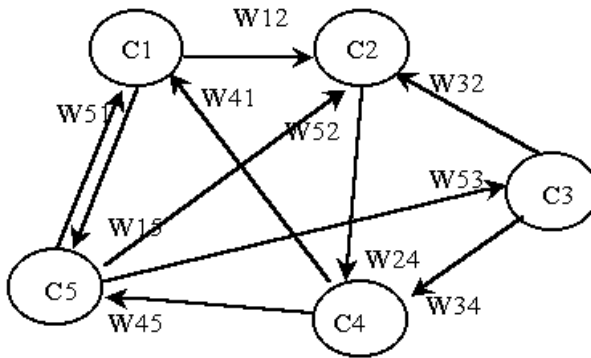
Fuzzy Cognitive Maps (FCMs) consist of concept nodes and weighted arcs, which are graphically illustrated as a signed weighted graph with feed back. Signed weighed arcs, connecting the concept nodes, represent the causal relationship that exists among concepts. In general, concepts of a FCM, represent key-factors and characteristics of the modeled complex system and stand for: events, goals, inputs, outputs, states, variables and trends of the complex system been modeled. This graphic display shows clearly which concepts influences with other concepts and what this degree of influence is.

## 2.1 Fuzzy Cognitive Map Representation

Figure 1 illustrates a simple FCM consisting of five (5) concepts and nine (9) weighed arcs. Thus FCMs are directed graphs capable of modeling interrelationships or causalities existing among concepts. Concept variables and causal relations constitute the fundamental elements of an FCM. Concept variables are represented by nodes, such as  $C_1$ ,  $C_2$ ,  $C_3$ ,  $C_4$  and  $C_5$  in figure 1. Causal variables always depict concept variables at the origin of arrows; effect variables, on the other hand, represent concepts-variables at the terminal points of arrows. For example, looking in figure 1, at  $C_1 \rightarrow C_2$ ,  $C_1$  is said to impact  $C_2$  because  $C_1$  is the causal variable, whereas  $C_2$  is the effect variable. Each concept is characterized by a number  $A_i$  that represents its value and it results from the transformation of the real value of the system's variable, for which this concept stands, in the interval  $[0,1]$ . Causality between concepts allows degrees of causality and not the usual binary logic, so the weights of the interconnections can range in the interval  $[-1,1]$ . Fuzzy Cognitive Map models a system as an one-layer network where nodes can be assigned concept meanings and the interconnection weights represent causal relationships among concepts.

A Fuzzy Cognitive Map is a graph shows the degree of causal relationship - among concepts of the map knowledge expressions and the causal relationships are expressed by and fuzzy weights.

Existing knowledge on the behavior of the system is stored in the structure of nodes and interconnections of the map. Each one of the key-factors of the system. Relationships between concepts have three possible types; a) either express positive causality between two concepts ( $W_{ij} > 0$ ) b) negative causality ( $W_{ij} < 0$ ) and c) no relationship ( $W_{ij} = 0$ ). The value of  $W_{ij}$  indicates how strongly concept  $C_i$  influences concept  $C_j$ . The sign of  $W_{ij}$  indicates whether the relationship between concepts  $C_i$  and  $C_j$  is direct or inverse. The direction



**Fig. 1** A simple Fussy Cognitive Map Drawing

of causality indicates whether concept  $C_i$  causes concept  $C_j$ , or vice versa. These parameters have to be considered when a value is assigned to weight  $W_{ij}$ .

A new formulation for calculating the values of concepts at each time step, of a Fuzzy Cognitive Map, is proposed:

$$A_i^t = f(k_1 \sum_{\substack{j=1 \\ j \neq i}}^n A_j^{t-1} W_{ji} + k_2 A_i^{t-1}) \quad (1)$$

The  $k_1$  expresses the influence of the interconnected concepts in the configuration of the new value of the concept  $A_i$  and  $k_2$  represents the proportion of the contribution of the previous value of the concept in the computation of the new value and the. This new formulation assumes that a concept links to itself with a weight  $W_{ii} = k_2$ .

In this chapter, it is assumed that the influence of the previous value of each concept is high and it is supposed that  $k_1=1$ . This means that the previous value of each concept has a great influence in the determination of the new value. The inclusion of the previous value of each concept in the calculation rule, results in smoother variation on the values of concepts after each recalculation of their value. This will become apparent when the supervisor is modeled using a FCM of section 3. The value  $A_i$  for each concept  $C_i$  is calculated by the following rule:

$$A_i^t = f(\sum_{\substack{j=1 \\ j \neq i}}^n A_j^{t-1} W_{ji} + A_i^{t-1}) \quad (2)$$

Equation (1.2) is the result of equation (1) setting  $k_1 = k_2 = 1$ . Namely,  $A_i^t$  is the value of concept  $C_i$  at time  $t$ ,  $A_i^{t-1}$  the value of concept  $C_i$  at time  $t-1$ ,  $A_j^{t-1}$  the value of concept  $C_j$  at time  $t-1$ , and the weight  $W_{ji}$  of the interconnection from concept  $C_j$  to concept  $C_i$ . The function  $f$  is a threshold function and to squash the result in the interval  $[0,1]$ . Fuzzy Cognitive Maps have discrete nature, at each time step, values of all concepts are recalculated and change according to equation 2. This procedure is called a running cycle of the FCM model and it is very fundamental for the theory of FCM.

Two kinds of threshold functions are used in the Fuzzy Cognitive Map framework, the unipolar sigmoid function, where  $\lambda > 0$  determines the steepness of the continuous function  $f$ :

$$f(x) = \frac{1}{1 + e^{-\lambda x}}$$

When nature of concepts can be negative, their values belong to the interval  $[-1,1]$ , the following function is used:

$$f(x) = \tanh(x)$$

The simplicity of FCM model becomes apparent from its mathematical representation and operation. Suppose that an FCM is consisted by  $n$  concepts. It is mathematically represented by a  $1 \times n$  state vector  $\mathbf{A}$ , which gathers the values of the  $n$  concepts and by an  $n \times n$  weight matrix  $\mathbf{W}$ . Each element  $W_{ij}$  of the matrix  $\mathbf{W}$  indicates the value of the weight  $W_{ij}$  between concept  $C_i$  and  $C_j$ . The diagonal of the matrix is zero since it is assumed that no concept causes itself and thus  $W_{ii} = 0$ .

Equation 2 can be transformed in the following equation, which describes the FCM operation with the compact mathematical model:

$$\mathbf{A}^t = f(\mathbf{A}^{t-1} \mathbf{W} + \mathbf{A}^{t-1}) \quad (3)$$

So, equation (3) computes the new state vector  $\mathbf{A}^t$ , which depends on the previous state of  $\mathbf{A}$  and from the multiplication of the previous, at time  $t-1$ , state vector  $\mathbf{A}^{t-1}$  by the weight matrix  $\mathbf{W}$ . The equation 3 can also be expressed as:

$$\mathbf{A}^t = f(\mathbf{A}^{t-1} \mathbf{W}^{new}) \quad (4)$$

Where, the new weight matrix  $\mathbf{W}^{new}$  is the weight matrix  $\mathbf{W}$  of the Fuzzy Cognitive Map with the entire diagonal elements equal to unit, which means that each concept causes itself with a weight  $W_{ii} = 1$ . This is a new approach, which differs from other representations of Fuzzy Cognitive Map in the literature, where it is assumed that no concept cause itself and the diagonal of the  $\mathbf{W}$  matrix, is zero.

## 2.2 Methods for Constructing Fuzzy Cognitive Maps

The development and construction of Fuzzy Cognitive Map (FCM) have great importance for its use in the modeling of complex systems. Let us remind ourselves that FCM represent the human knowledge on the operation of the system. Experts develop FCMs, using their experience and knowledge on the complex system. Construction methodologies rely on the exploitation of experts' experience on system's model and behavior. Experts determine the number and kind of concepts that consist an FCM and the interrelationships among its concepts. Experts know the main factors that determine the behavior of the complex system, each one of these factors is represented by a concept. Experts according to their experience, determine concepts of FCM that stand for events, actions, goals, values, and trends of the complex system. Experts know which elements of the system influence other elements; for the corresponding concepts they determine the negative or positive effect of one concept to the others, with a fuzzy degree of causation. The determination of the degree of casual relationship among concepts can be improved by the application of learning rules for choosing appropriate weights for the FCM. In this way, an expert decodes his own

knowledge on the behavioral model of the system and transforms this knowledge in a weighted graph.

using

### 2.2.1 Assigning Numerical Weights

Knowledge on the behavior of a complex system is rather subjective and in order to construct a more accurate model of the complex system it is proposed to utilize, the experience of a group of experts. Experts are polled together and they examine the relevant factors that stand as nodes of an FCM. So, they decide the number of concepts, which consist the FCM and what characteristic of the system each concept represents. Then, the experts are individually asked to express the causal relationship among these concepts. The result of this procedure will be a collection of individual FCMs, with the same nodes but different links among concepts or/and different weights of interconnections. The individual FCMs must be combined into one collective FCM and a method to combine the individual maps. A first approach could be the summation of different weight matrixes:

$$\mathbf{W} = f\left(\sum_{k=1}^N \mathbf{W}_k\right) \quad (5)$$

Where  $\mathbf{W}$  is the overall matrix,  $\mathbf{W}_k$  is the individual weight matrix, which each one of the  $N$  experts has developed, and  $f$  is a threshold function, usually a type of the sigmoid function that will transform the sum of weights in the interval  $[-1,1]$ .

It is accepted that experts have different experience and subjective knowledge on the system. Thus it is considered that there are experts of different credibility on the knowledge of the system, and for these experts their contributions on constructing FCMs may be multiplied by a nonnegative ‘credibility’ weight  $b_k$  before combining them with other expert’s opinions.

$$\mathbf{W} = f\left(\sum_{k=1}^N b_k \mathbf{W}_k\right) \quad (6)$$

Where  $b_k$  is the credibility weight for the  $k_{th}$  expert,  $\mathbf{W}_k$  is the weight matrix of  $k_{th}$  expert’s Fuzzy Cognitive Map and  $N$  is the number of the experts. But in this case, another mechanism must be used to determine who and how credibility weights can be assigned to every expert. As an example one expert could be “penalized” with an extremely low or zero credibility weight if the expert’s choice differs from other experts’ average weight choice, by some predetermine rule.

A new advanced algorithm is proposed in order to assign weights for each interconnection and credibility weights for experts. Every expert constructs an individual FCM. Then for each one interconnection of the overall FCM the

corresponding weight from each individual map are collected together and compared according to the following algorithm. In this algorithm, the average value of the proposed weights for each interconnection is used.

First of all, the sign of the proposed weights are examined. If the number of weights with the same sign is less than  $\pi * N$ , this means that it is not very clear among experts the positive or negative causality between two concepts and so they are asked to reassign weights. Otherwise, the procedure continues and the proposed weights are used to determine the weight. Each expert that assigns weight for one interconnection not close enough to the average weight is penalized and the corresponding weight is partially taking into account. This mechanism is implemented using the following algorithm:

*Algorithm 1*

*Step 1: For all the  $N$  experts, set credibility weight  $b_k = 1$*

*Step 2: For  $i, j = 1$  to  $n$*

*Step 3: For each interconnection ( $C_i$  to  $C_j$ ) examine the  $N$  weights  $W_{ij}^k$  that each  $k_{th}$  of the  $N$  experts has assigned.*

*Step 4: IF there are weights  $W_{ij}^k$  with different sign and the number of weights with the same sign is less than  $\pi * N$*

*THEN*

*ask experts to reassign weights for this particular interconnection and go to step 3*

*ELSE*

*take into account the weights of the greater group with the same sign and consider that there are no other weights and penalize the experts who chose "wrong" signed weight with a new credibility weight  $b_k = \mu_1 * b_k$*

*Step 5: For the weights with the same sign, find their average value*

$$W_{ij}^{ave} = \frac{\sum_{k=1}^N b_k W_{ij}^k}{N}$$

*Step 6: IF  $|W_{ij}^{ave} - W_{ij}^k| \geq \omega_1$  THEN consider that there is no weight  $W_{ij}^k$ , penalize the  $k_{th}$  expert  $b_k = \mu_2 * b_k$  and go to step 5*

*Step 7: IF there have not examined all the  $n \times n$  interconnection go to step 2*

*ELSE construct the new weight matrix  $\mathbf{W}$  which has elements the weights  $W_{ij}^{ave}$*

*END.*

### Example 2.1

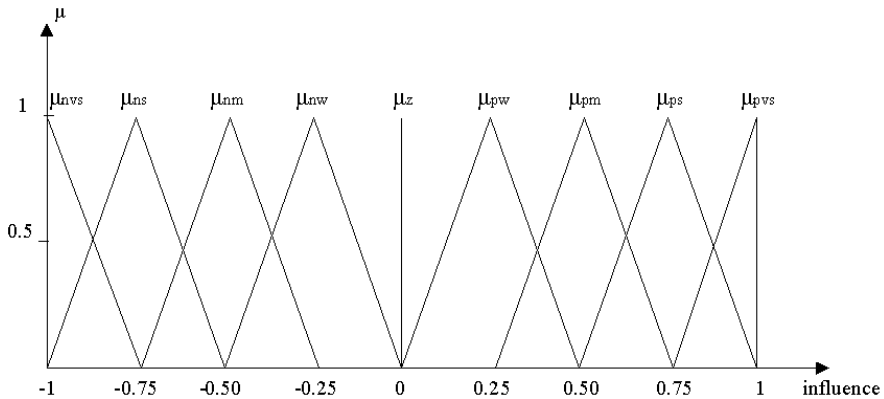
Six experts have constructed six individual FCMs. Experts have suggested the following weights for the one interconnection from concept  $C_i$  to concept  $C_j$  :

$$W_{ij} = [-0.5, 0.6, 0.66, 0.7, 0.65, 0.25].$$

For this example, the requested number of weights with the same sign is  $\pi = 0.8$  and  $\omega_1 = 0.2$  and  $\mu_1 = \mu_2 = 0.9$ . According to step 3 of the algorithm, the majority of experts have assigned positive weights to that interconnection so the 1<sup>st</sup> expert is penalized with credibility weight  $b_1 = \mu_1 * b_1 = 0.9b_1$  and the corresponding weight is dropping out. Then, in step 4 of the algorithm, the average weight is computing  $W_{ij}^{ave} = 0.572$ , and it is compared with other weights, but according to step 6 of the algorithm, the weight suggested by the 6<sup>th</sup> expert with value 0.25 is excluded from the calculation and the 6<sup>th</sup> expert is penalized. The rest of the weights are used to calculate the new average weight. In this case, the chosen weight has the value  $W_{ij}^{ave} = 0.652$  for this particular weight interconnection. The same methodology is used to assign weights for all the interconnections and construct the overall Fuzzy Cognitive Map.

### 2.2.2 Assigning Linguistic Variables for FCM Weights

Another methodology to construct a Fuzzy Cognitive Map that is closer to fuzzy logic is proposed now. Experts are asked to describe the causality among concepts using linguistic notions. Every expert will determine the influence of one concept to the other as “negative” or “positive” and then he will describe the grade of influence with a linguistic variable such as "strong", "weak" and etc [8].



**Fig. 2** Terms of the linguistic variable influence



*Influence* of one concept on another, is interpreted as a linguistic variable taking values in the universe  $U=[-1,1]$  and its term set  $T(\textit{influence})$  could be:

$T(\textit{influence}) = \{\text{negatively very strong, negatively strong, negatively medium, negatively weak, zero, positively weak, positively medium, positively strong, positively very strong}\}$

The semantic rule  $M$  is defined as follows and these terms are characterized by the fuzzy sets whose membership functions are shown in Figure 2:

$M(\text{negatively very strong}) =$  the fuzzy set for "an influence below to -75%" with membership function  $\mu_{nvs}$

$M(\text{negatively strong}) =$  the fuzzy set for "an influence close to -75%" with membership function  $\mu_{ns}$

$M(\text{negatively medium}) =$  the fuzzy set for "an influence close to -50%" with membership function  $\mu_{nm}$

$M(\text{negatively weak}) =$  the fuzzy set for "an influence close to -25%" with membership function  $\mu_{nw}$

$M(\text{zero}) =$  the fuzzy set for "an influence close to 0" with membership function  $\mu_z$

$M(\text{positively weak}) =$  the fuzzy set for "an influence close to 25%" with membership function  $\mu_{pw}$

$M(\text{positively medium}) =$  the fuzzy set for "an influence close to 50%" with membership function  $\mu_{pm}$

$M(\text{positively strong}) =$  the fuzzy set for "an influence close to 75%" with membership function  $\mu_{ps}$

$M(\text{positively very strong}) =$  the fuzzy set for "an influence above to 75%" with membership function  $\mu_{pvs}$

The linguistic variables that describe each interconnection are combined and the overall linguist variable will be transformed in the interval  $[-1,1]$ . A numerical weight for each interconnection will be the outcome of the defuzzifier, where the Center of Gravity method is used to produce this crisp weight [10]. This methodology has the advantage that experts do not have to assign numerical causality weights but to describe the degree of causality among concepts.

### 2.2.3 Synthesizing different Fuzzy Cognitive Maps

A distributed system is considered and for each subsystem a distinct FCM is constructed. Then all FCMs can be combined in one augmented Fuzzy Cognitive Map with a weight matrix  $\mathbf{W}$  for the overall system. The unification of the distinct FCM depends on the concepts of the segmental FCM, if there are no common concepts among different maps; the combined matrix  $\mathbf{W}$  is constructed according to the equation 7 (see below). In this case, there are  $K$  different FCM matrices, with weight matrices  $\mathbf{W}_1$  and the dimension of matrix  $\mathbf{W}$  is  $n \times n$  where  $n$  equals the total number of distinct concepts in all the FCMs.

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_1 & & & \\ & \mathbf{W}_2 & \underline{0} & \\ & \underline{0} & \ddots & \\ & & & \mathbf{W}_k \end{bmatrix} \quad (7)$$

### Example 2.2

It is assumed that there are two Fuzzy Cognitive Maps,  $F_1$  with concepts  $C_1, C_2, C_3$  and  $F_2$  with concepts  $C_4, C_5, C_6$ . Weight matrices for  $F_1$  and  $F_2$  are:

$$\mathbf{W}_1 = \begin{bmatrix} 0 & 0 & W_{13} \\ W_{21} & 0 & 0 \\ W_{31} & W_{32} & 0 \end{bmatrix} \text{ and } \mathbf{W}_2 = \begin{bmatrix} 0 & W_{45} & W_{46} \\ W_{54} & 0 & W_{56} \\ 0 & W_{65} & 0 \end{bmatrix}$$

The augmented weight matrix will be:

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_1 & 0 \\ 0 & \mathbf{W}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & W_{13} & 0 & 0 & 0 \\ W_{21} & 0 & 0 & 0 & 0 & 0 \\ W_{31} & W_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & W_{45} & W_{46} \\ 0 & 0 & 0 & W_{54} & 0 & W_{56} \\ 0 & 0 & 0 & 0 & W_{65} & 0 \end{bmatrix}$$

But, in most cases, the unification is used because there are common concepts among the distinct FCM and the intention is the construction of an enhanced Fuzzy Cognitive Map. In this case, there will be an overlapping between some of the diagonal elements-matrices of matrix  $\mathbf{W}$  in equation 7. Overlapping represent weights of interconnections between concepts that belong to different FCMs. Then, segmental FCMs with common concepts are combined together, calculating new weights for the interconnection between common concepts. If there are more than one common concept between the Fuzzy Cognitive Maps, there will be proposed two or more weights for the same interconnection. In this case, as new weight will be the average of weights  $v$ . Then, equation 7 is implemented to construct the weight matrix of the overall Fuzzy Cognitive Map. It is consisted of  $n$  concepts that correspond to the total number of the different concepts that there are in all the segmental FCMs

### Example 2.3

It is assumed that there are two Fuzzy Cognitive Maps,  $F_1$  with concepts  $C_1, C_2, C_3$  and  $F_2$  with concepts  $C_2, C_3, C_4, C_5$ . Weight matrices for  $F_1$  and  $F_2$  are:

$$\mathbf{W}_1 = \begin{bmatrix} 0 & 0 & W_{13} \\ W_{21} & 0 & 0 \\ W_{31} & W_{32} & 0 \end{bmatrix} \text{ and } \mathbf{W}_2 = \begin{bmatrix} 0 & W_{23} & W_{24} & 0 \\ W_{32} & 0 & W_{34} & W_{35} \\ W_{42} & W_{43} & 0 & 0 \\ W_{52} & W_{53} & W_{54} & 0 \end{bmatrix}$$

The augmented Fuzzy Cognitive Map will have five concepts and its weight matrix will be:

$$\mathbf{W} = \begin{bmatrix} 0 & 0 & W_{13} & 0 & 0 \\ W_{21} & 0 & W_{23} & W_{24} & 0 \\ W_{31} & W_{32}^{ave} & 0 & W_{34} & W_{35} \\ 0 & W_{42} & W_{43} & 0 & 0 \\ 0 & W_{52} & W_{53} & W_{54} & 0 \end{bmatrix}$$

### 2.3 Neural Network Nature of Fuzzy Cognitive Maps

Fuzzy Cognitive Maps have been described as a hybrid methodology, because it utilizes characteristics of fuzzy logic and neural networks. The development and construction of FCMs have shown their fuzzy nature. Learning rules, used in Neural Networks theory, they are used to train the Fuzzy Cognitive Map. Parameter learning of FCM concerns the updating of connection weights among concepts.

The construction of FCM is based on experts who determine concepts and weighted interconnections among concepts. This methodology may lead to a distorted model of the system because human factor is not always reliable. In order to refine the model of the system, learning rules are used to adjust weights of FCM interconnections. The Differential Hebbian learning rule has been proposed to be used in the training of a specific type of FCMs. The Differential Hebbian learning law adjusts the weights of the interconnection between concepts  $t$  grows a positive edge between two concepts if they both increase or both decrease and it grows a negative edge if values of concepts move in opposite directions. Adjusting the idea of differential Hebbian learning rule in the framework of Fuzzy Cognitive Map, the following rule is proposed to calculate the derivative of the weight between two concepts.

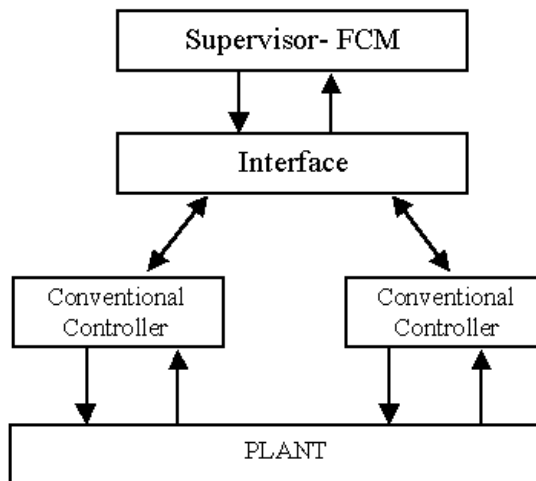
$$w'_{ji} = -w_{ji} + s(A_j^{new})s(A_i^{old}) + s'(A_j^{new})s'(A_i^{old}) \quad (8)$$

$$\text{Where } S(x) = \frac{1}{1 + e^{-\lambda x}}$$

Appropriate learning rules for Fuzzy Cognitive Maps need more investigation. These rules will give FCMs useful characteristics such as the ability to learn arbitrary non-linear mappings, capability to generalize to situations the adaptivity and the fault tolerance capability [4], [13], [47], [48].

### 3 Modeling Supervisors of Complex Systems with a FCM

Complex systems are characterized with high dimension and their dynamics are quite often unknown. A defining characteristic of complex systems in their tendency to self – organize globally as a result of many local interactions. In other words, organization occurs without any central organizing structure or entity. Therefore, conventional techniques cannot easily handle this kind of systems. The application of Fuzzy Cognitive Maps (FCM) for the modeling of the supervisor of complex systems seems to be a prospective methodology. The hierarchical structure of fig. 3 is proposed to model Large Scale complex systems. At the lower level of the structure lies the plant, which is controlled through conventional controllers. These controllers perform the usual tasks and reflect the model of the plant during normal operation conditions using conventional control techniques. The supervisor of the system is modeled as a Fuzzy Cognitive Map (FCM). The first attempt to model a supervisor of a scale complex system utilizing the concept of FCM was made early in 2000. [20]-[25] There is an amount of information that must pass from the lower level to the Supervisor-FCM. So an interface, which will process, transform and communicate information from the lower local controllers to the FCM on the upper level. The Fuzzy Cognitive Map will interact using equation 2; concepts of FCM will have new values that must be transmitted to the conventional controllers. So, the interface will follow the opposite direction. In this way changes on one or more concepts of the FCM could mean change in the value of one or more elements of the system.



**Fig. 3** An FCM – supervisor of a complex system

The model of Fuzzy Cognitive Map can be expanded to include advanced features, such as fault diagnosis, effect analysis or planning and decision making characteristics. Some of the concepts of the FCM could stand for device failure modes, their effects and causes, a subsystem's normal or irregular operation, the functionality of the system, the failures, the system mission, and the ultimate function of the system. So the FCM would represent the failure modes and their effects and the relations among them, that an expert uses to describe the functionality of the system and the failures.

A very interesting quality of FCMs is their ability in predicting and redesigning of the system. This can help the designer in evaluating what, would happen if some parameters of the system have been altered. Another useful characteristic of the FCM is its efficiency in prediction and especially to predict what would be the result of a scenario or what will be the consequences for the whole process if a state changes suddenly. This feature is especially useful for designers of systems to observe the influence of each device separately.

With Fuzzy Cognitive Maps the knowledge and human operator experience is exploited. The human coordinator of a system should know the operation of critical aspects of the whole system and uses a mental model consisted of concepts to describe it. He relates the operation of one subsystem or two different subsystems to a concept or a concept stand for a specific procedure.

FCM models the supervisor and it is consisted of concepts that may represent the irregular operation of some elements of the system, failure mode variables, failure effects variables, failure cause variables, severity of the effect or design variables. Moreover, this FCM will include concepts for determination of a specific operation of the system and it will be used for strategic planning and decision analysis. The supervisor FCM, will represent vital components of the plant and will reflect the operational state of the plant. The development of this FCM requires the integration of several experts opinions in order to construct a FCM with diagnosis and predictive capabilities. We need to point out here that conventional control methods cannot be used to model this supervisor of a complex system, a This approach is best illustrated with the following example.

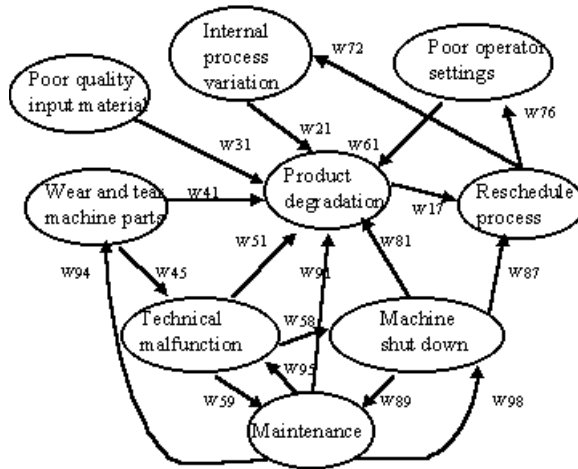
### *Example 3.1*

Four experts working on a chemical industry, which produce refreshments from water, sugar, fruit juice etc, were asked to develop a Fuzzy Cognitive Map. This FCM will be used as a supervisor of the whole plant, which will describe the operation of a process, the final product of the process and the different aspects that determine the quality of the product. Experts developed the Fuzzy Cognitive Map, which is depicted on figure 4. They decided that the most important concept is the quality of the produced product. They developed an FCM around the main concept C1, which represents the "product degradation" of the final product. Then, experts determined other concepts of the real system that influence this concept, so concept C1 depends on:

Concept C2 "the internal variation of the process",

Concept C3 "the poor quality of the input material",

Concept C4 "wear and tear machine parts",



**Fig. 4** Proposed Supervisor – Fuzzy Cognitive Map

Concept C5 “technical malfunction”,

Concept C6 “poor operator settings”,

Concept C7 “reschedule the process”

Concept C8 “machine shut down”.

Concept C9 “maintenance”

Then, the interrelations among concepts were determined with the following logical procedure. The value of concept 1 “degradation of product” increases the need to “reschedule the process” which is presented as concept C7.

Concept C7 decreases the value of concepts C6 “poor operator setting” and concept C2 “Internal process variation”.

Concept C4, which stand for “wear and tear machine parts”, has a positive influences on concept C5 “technical malfunction”.

Concept C5 “technical malfunction” increase the amount of concept C9 “the maintenance” and the amount of concept C8 “the machine shut down”.

Concept C9 “maintenance” decreases the amount of the following concepts: concept C5 “technical malfunction”, concept C8 “the machine shut down” and concept C4 “wear and tear machine parts”.

Concept C8 “machine shut down” increases the amount of concept C7 “reschedule process” and increases the value of concept C9 “maintenance”.

Then, experts were asked to assign values on the interconnections among concepts. Four FCMs were constructed with the same concepts, but with 4 different weights on each interconnection. Then algorithm of section 2.2.1 was implemented and an augmented Fuzzy Cognitive Map was constructed, which is depicted on figure 5 and it is used as supervisor of the plant. The experts decided on the values of weighted arcs,  $W_{ij}$ , figure 4 and is given in figure 5.

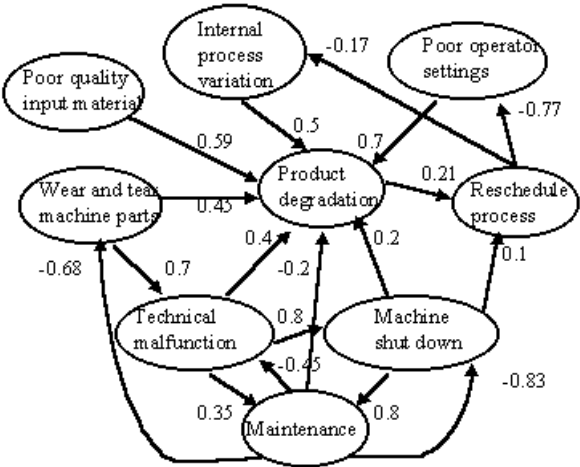


Fig. 5 Supervisor- Fuzzy Cognitive Map with weights

Table 1 The alues of concepts of the supervisor-FCM for 6 simulation steps

C1	C2	C3	C4	C5	C6	C7	C8	C9
0.24	0.48	0.20	0.10	0.15	0.40	0.00	0.07	0.61
0.65	0.50	0.30	0.40	0.45	0.50	0.51	0.40	0.53
0.74	0.48	0.30	0.41	0.51	0.50	0.54	0.48	0.62
0.73	0.48	0.20	0.40	0.51	0.40	0.55	0.47	0.64
0.72	0.48	0.20	0.39	0.50	0.40	0.55	0.47	0.64
0.72	0.48	0.20	0.39	0.50	0.40	0.55	0.47	0.64

For the constructed Fuzzy Cognitive Map, values were assigned to the concepts and the simulation of the FCM starts. Equation 2 is used to calculate the new values of concepts after each step of the FCM. Table 1 gathers the initial values of concepts and their values for six simulation cycles. FCM reaches an equilibrium point and if a new value for one or more concepts comes from the lower level then after a limited number of cycles, FCM will reach another equilibrium point.

The development of the supervisor-FCM that is dedicated to a particular plant depends on the supervisory-coordinator tasks that the user of the overall system requires. A complete Fuzzy Cognitive Map would include a decision making part and a planning part.

4 Decision Analysis and Fuzzy Cognitive Maps

Decision analysis is based on a number of quantitative methods that aid in choosing amongst alternatives. Traditional decision analysis is used to indicate decisions favouring good outcomes even though there is an uncertainty

surrounding the decision itself. Furthermore, the value of each possible outcome of a decision, whether measured in costs and benefits or utility, usually varies.

Over the last years, several approaches have been investigated in the field of Decision Analysis, with the most popular one to be used that of Decision Trees (DT). Some methods combine DT with other machine learning techniques, such as Neural Networks [18] or Bayesian Networks [19]. However, very little work has been reported in combining DT with FCMs. Some research work of this combination has seen the literature the last ten years [42]-[45]. In this chapter the technique of combining a DT with a FCM model in Decision Analysis is presented.

The derived FCM model is subsequently trained using an unsupervised learning algorithm to achieve improved decision accuracy. In this chapter, the C4.5 has been chosen as a typical representative of the decision tree approach [14]. Similarly, the Nonlinear Hebbian Learning (NHL) algorithm is chosen as a representative of unsupervised FCM training.

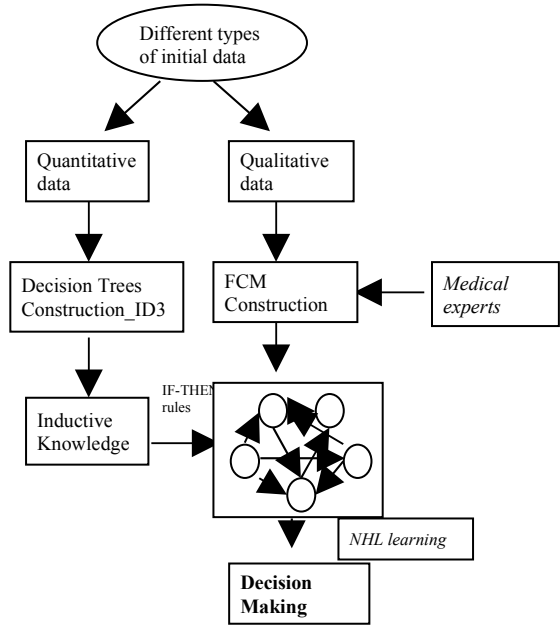
The DT-FCM's function is briefly outlined in Figure 6. If there is a large number of input data, then the quantitative data are used to induce a Decision Tree and qualitative data (through experts' knowledge) are used to construct the FCM model. The FCM's flexibility is enriched by the fuzzification of the strict decision tests (derived fuzzy IF-THEN rules to assign weights direction and values). Finally, the derived FCM model (new weight setting and structure) is trained by the unsupervised NHL algorithm to achieve a decision.

This methodology can be used for three different circumstances, depending on the type of the initial input data: (1) when the initial data are quantitative, the DT generators are used and an inductive learning algorithm produce the fuzzy rules which then are used to update the FCM model construction; (2) when experts' knowledge is available, the FCM model is constructed and through the unsupervised NHL algorithm is trained to calculate the target output concept responsible for the decision line; and (3) when both quantitative and qualitative data are available, the initial data are divided and each data type is used to construct the DTs and the FCMs separately. Then the fuzzy rules induced from the inductive learning restructure the FCM model enhancing it. At the enhanced FCM model the training algorithm is applied to help FCM model to reach a proper decision.

The new technique has three major advantages. First, the association rules derived from the decision trees have a simple and direct interpretation and introduced in the initial FCM model to update its operation and structure. For example, a produced rule can be: If the *variable 1* (input variable) has *feature A* Then the *variable 2* (output variable) has *feature B*.

Second, the procedure that introduces the Decision Tree rules into an FCM also specifies the weight assignment through new cause-effect relationships among the FCM concepts. Third, as will be demonstrated through the experiments, this technique fares better than the best Decision Tree inductive learning technique and the FCM decision tool.





**Fig. 6** The decision making system constructed by Decision Trees and Fuzzy Cognitive Maps

**5 Implementation of the DT-FCM Model for Bladder Tumor Grading**

The above new DT-FCM technique was used by our research team on a number of medical problems [13], [15]. Here, a representative example is presented.

Ninety-two cases of urinary bladder cancer were collected from the archives of the department of pathology of University Hospital of Patras Greece. Histopathologists-experts had diagnosed 63 cases as low-grade and 29 as high-grade using conventional WHO grading system. Following grade diagnosis, each tissue section was evaluated retrospectively, using a list containing eight well documented in the bibliography histopathological criteria essential for tumour grading. The FCM model for tumor grading had been developed and presented analytically in [13]. The FCM grading tool was able to give distinct different values for the majority of high-grade and low-grade cases using a simple Bayesian classifier for the output data. Except the experts' knowledge for determining FCM model, quantitative data for the eight main histopathological features [13,15] were also available and used for constructing DT. Then through the inductive learning procedure, a set of association rules were derived. Some of the best association rules, based on their confidence levels, are given in the Table 2. The necessary If-Then rules were induced and introduced in the FCM model enhancing its initial structure.

**Table 2** Example of Association Rules derived from Decision Trees

Rules	Result/Decision Leaves
Cell-size=uniform, mitosis=absent rate	Grade Low
Cell-distribution=even, nucleoli=inconspicuous	Grade Low
Cell-distribution=clustered, cell-size=pleomorphic	Grade High
Nuclei=uniform, mitosis=absent rate	Grade Low
Cell-size=uniform, cell-number=numerous, nucleoli=inconspicuous	Grade Low

After the development of the DT-FCM model and the determination of specifications for the implementation of the NHL algorithm, the hybrid system was used to examine cases and assigned sensitivity and specificity for grading bladder tumors. The same data set that used in previously proposed FCM-TG model, were also used to evaluate the performance of the DT-FCM methodology in categorizing tumors as low grade or high grade. The results for average sensitivity and average specificity for the ninety two bladder tumour cases were 80% and 90% respectively using the DT-FCM, whereas the resulting accuracies for low grade and high grade cases were 79% and 87.5% through the FCM grading tool [15].

Our obtained results through the implementation of the proposed DT-FCM methodology are very promising and encourage us to continue our effort towards this direction.

Actually, our research group works on improving the medical diagnosis process by different means: (1) introducing a methodology based on FCMs for decision making in complex medical systems where experts' knowledge is available; (2) constructing modular FCMs for characterizing tumor grading; and (3) certain histopathological features-attributes (for example, cell distribution, nuclei, mitosis, necrosis) have been converted into discrete values, although their conceptual vagueness could be quantified by the degree of membership of a numerical value in a fuzzy set. Thus their values would be a user defined finite set of linguistic values.

A research team of the Laboratory for Automation and Robotics of the University of Patras has used extensively FCMs in modeling and analyzing medical problems and has obtained some very interesting results [13],[15],[30],[46], [47], [48].

## 6 Summary and Closing Remarks

In this chapter the analysis of complex systems has been investigated using the exciting and promising models of Fuzzy Cognitive Maps (FCM).

Fundamental mathematical theories of FCM were developed and extensively analyzed. A new algorithm was developed and used to demonstrate the usefulness of the FCM approach in modeling complex systems. Fuzzy Cognitive Map (FCM) theory, a new soft computing approach, utilizes existing experience in the operation of a complex system and combining fuzzy logic and neural networks. For such complex systems it is extremely difficult to describe the entire system with a precise mathematical model. Thus, it is more simple and useful to divide the whole plant in virtual parts and to construct an FCM for each part. The experience of different specialists who can easily judge the variables and states of a small process and then unify these to construct the final system by integrating the different Fuzzy Cognitive Maps into an augmented one have been utilized. This approach represents systems in a graphical way showing the causal relationships between states-concepts and accomplishes the unification of superposing small subsystems. FCMs offer the opportunity to produce better knowledge based on systems applications, addressing the need to handle uncertainties and inaccuracies associated with real world problems.

The issue of modeling the supervisor of a complex system was addressed and analyzed. Then, using the theory of FCM, it was modeled in a hierarchical structure where the plant was controlled using conventional controllers. A simple example from manufacturing field was given demonstrating clearly the usefulness of the proposed approach. The supervisor was modeled with nine (9) concepts and eighteen (18) weighed interconnecting arcs. The concepts of the supervisor – FCM were very interesting features of a manufacturing plant such as: machine shut down, poor operator settings, poor quality input materials, technical malfunction, maintenance and others. The inclusion of all these features as concepts in a supervisor – FCM and ability to run extensive simulation with real data can prove very useful to plant management. Another very interesting feature of this proposed approach is the ability to use several expert opinions, giving the opportunity to predict the degradation of a product. The simulation studies show that after only a few recursive steps the FCM achieves a diagnosis for the desired product. Methods for developing supervisors for manufacturing plants using the theory of FCM is needed and could be the research of future direction in this exciting field.

The basic theories of FCM were then used to address and analyze the difficult problem of Decision Trees (DT) from Medical Decision Making Models. A new integrated system has been developed to assist medical decisions. The new framework has proposed the combination of FCM and Decision Trees in a new integrated DT-FCM system. The performance of the new structure can deal with different kind of input data eliminating numerical errors. A simple medical problem, in which the case of urinary bladder cancer was considered and studied using the proposed DT-FCM structure. The simulation results were very encouraging and had given better results than existing techniques. Future work here should be directed to use extensively the proposed DT- FCM structure to further study similar health problems and compare the results with other today's techniques. Another promising research direction is to further investigate learning (both supervised and unsupervised) techniques in combination with the proposed DT-FCM structure and to be used on a number of challenging and difficult medical problems.

Closing this chapter, it is of interest to raise a basic fundamental question: what is the best way to approach the difficult issue of modeling and controlling complex systems. An issue that has been the subject of extensive investigations the last 40-50 years, especially after the Second World War.

By trying to answer this basic but generic question, a good number of future research directions can be evolved or generated. The Fuzzy Cognitive Map theories, lately have demonstrated, that can provide realistic and useful tools for addressing many problems that our society is confronted with.

## References

- [1] Craiger, J.P., Goodman, D.F., Weiss, R.J., Butler, A.: Modeling Organizational Behavior with Fuzzy Cognitive Maps. *Intern. Journal of Computational Intelligence and Organisations* 1, 120–123 (1996)
- [2] Dickerson, J.A., Kosko, B.: Virtual Worlds as Fuzzy Cognitive Maps. *Presence* 3, 173–189 (1994)
- [3] Hagiwara, M.: Extended Fuzzy Cognitive Maps. In: *Proceedings of IEEE Int. Conference on Fuzzy Systems*, pp. 795–801 (1992)
- [4] Jang, J.S., Sun, C.T., Mizutani, E.: *Neuro-Fuzzy and Soft Computing*. Prentice Hall, Upper Saddle River (1997)
- [5] Kim, H.S., Lee, K.C.: Fuzzy implications of fuzzy cognitive map with emphasis on fuzzy causal relationship and fuzzy partially causal relationship. *Fuzzy Sets and Systems* 97, 303–313 (1998)
- [6] Kosko, B.: Fuzzy Cognitive Maps. *Intern. Journal of Man-Machine Studies* 24, 65–75 (1986)
- [7] Kosko, B.: *Neural Networks and Fuzzy Systems*. Prentice-Hall, Englewood Cliffs (1992)
- [8] Lin, C.T., Lee, C.S.G.: *Neural Fuzzy Systems: A Neuro-Fuzzy Synergism to Intelligent Systems*. Prentice Hall, Upper Saddle River (1996)
- [9] Medsker, L.R.: *Hybrid Intelligent Systems*. Kluwer Academic Publishers, Norwell (1995)
- [10] Nie, J., Linkens, D.: *Fuzzy-Neural Control: principles, algorithms and applications*. Prentice Hall Europe, Hertfordshire (1995)
- [11] Schneider, M., Shnaider, E., Kandel, A., Chew, G.: Automatic construction of FCMs. *Fuzzy Sets and Systems* 93, 161–172 (1998)
- [12] Stylios, C.D., Georgopoulos, V.C., Groumpos, P.P.: Introducing the Theory of Fuzzy Cognitive Maps in Distributed Systems. In: *Proceedings of 12th IEEE Intern. Symposium on Intelligent Control, Istanbul, Turkey*, pp. 55–60 (1997)
- [13] Papageorgiou, E.I., Spyridonos, P., Ravazoula, P., Stylios, C.D., Groumpos, P.P., Nikiforidis, G.: Advanced Soft Computing Diagnosis Method for Tumor Grading. *Artif. Intell. Med.* 36, 59–70 (2006)
- [14] Quinlan, J.R.: *C4.5: Programs for machine learning*. Morgan Kaufmann, San Mateo (1993)
- [15] Papageorgiou, E.I., Spyridonos, P., Ravazoula, P., Stylios, C.D., Groumpos, P.P., Nikiforidis, G.: The Challenge of Using Soft Computing Techniques for Tumor Characterization. In: *Rutkowski, L., Siekmann, J.H., Tadeusiewicz, R., Zadeh, L.A. (eds.) ICAISC 2004. LNCS (LNAI), vol. 3070*, pp. 1031–1036. Springer, Heidelberg (2004)

- [16] Kosko, B.: Verac, “Fuzzy cognitive maps” (1986)
- [17] Dickerson, J.A., Kosko, B.: Virtual Worlds as Fuzzy Cognitive Maps. *Presence* 3, 173–189 (1994)
- [18] D’alche-Buc, F., Zwierski, D., Nadal, J.: Trio learning: a new strategy for building hybrid neural trees. *Neural Syst.* 5(4), 255–274 (1994)
- [19] Janssens, D., Wets, G., Brijs, T., Vanhoof, K., Arentze, T., Timmermans, H.: Integrating Bayesian networks and decision trees in a sequential rule-based transportation model. *Europ. J. Operat. Research* (2005)
- [20] Stylios, C.D., Groumpos, P.P.: The Challenge of modeling Supervisory Systems using Fuzzy Cognitive Maps. *Journal of Intelligent Manufacturing* 9(4), 339–345 (1998)
- [21] Stylios, C.D., Groumpos, P.P.: Fuzzy Cognitive Maps: A model for Intelligent Supervisory Control Systems. *Computers in Industry* 39(3), 229–238 (1999)
- [22] Stylios, C.D., Groumpos, P.P.: A soft computing approach for modeling the supervisor of manufacturing systems. *Journal of Intelligent and Robotics Systems* 26(3-4), 389–403 (1999)
- [23] Stylios, C.D., Groumpos, P.P., Georgopoulos, V.C.: Fuzzy Cognitive Map Approach to Process Control Systems. *J. Advanced Computational Intelligence* 3(5), 409–417 (1999)
- [24] Stylios, C.D., Groumpos, P.P.: Fuzzy Cognitive Maps in Modeling Supervisory Control Systems. *Journal of Intelligent & Fuzzy Systems* 8(2), 83–98 (2000)
- [25] Groumpos, P.P., Stylios, C.D.: Modeling Supervisory Control Systems using Fuzzy Cognitive Maps. *Chaos, Solitons and Fractals* 11(1-3), 329–336 (2000)
- [26] Stylios, C.D., Groumpos, P.P.: Modeling Complex Systems Using Fuzzy Cognitive Maps. *IEEE Transactions on Systems, Man and Cybernetics: Part A Systems and Humans* (IF:0,555) 34(1), 155–162 (2004)
- [27] Stylios, C.D., Groumpos, P.P.: Fuzzy Cognitive Maps: A soft Computing Technique for Intelligent Control. In: *Proceeding 2000 IEEE International Symposium on Intelligent Control*, Patras, Greece, July 17-19, pp. 97–102 (2000)
- [28] Stylios, C.D., Georgoulas, G., Groumpos, P.P.: The Challenge of Using Soft Computing for Decision Support during Labour. In: *Proc. of 23rd Annual International Conference of the IEEE Engineering in Medicine & Biology Society*, Istanbul, Turkey, October 25-28 (2001) (CD-ROM)
- [29] Stylios, C.D., Christova, N., Groumpos, P.P.: A Hierarchical Modeling Technique of Industrial Plants Using Multimodel Approach. In: *Proceeding of 10th IEEE Mediterranean conference on Control and Automation*, Lisbon, Portugal, July 9-12 (2002) (CD-ROM)
- [30] Christova, N., Stylios, C., Groumpos, P.P.: Production Planning For Complex Plants using Fuzzy Cognitive Maps. In: *Proceeding of 7th IFAC Workshop on Intelligent Manufacturing Systems*, Budapest, Hungary, April 6-8, pp. 81–86 (2003)
- [31] Martchenko, A.S., Ermolov, I.L., Groumpos, P.P., Poduraev, J.V., Stylios, C.D.: Investigating Stability Analysis issues for Fuzzy Cognitive Maps. In: *Proc. of 11th IEEE Mediterranean conference on Control and Automation*, Rodos, Greece, June 18-20 (2003)
- [32] Stylios, C.D., Groumpos, P.P.: Using Fuzzy Cognitive Maps to Achieve Intelligence in Manufacturing Systems. In: *Proc. 1st International Workshop on Intelligent Manufacturing Systems*, Lausanne, Switzerland, April 15-17, pp. 85–95 (1998)

- [33] Stylios, C.D., Georgopoulos, V.C., Groumpos, P.P.: Decision Support System for radiotherapy based on Fuzzy Cognitive Maps. In: Int. Conference in Fuzzy logic and Technology, De Montfort University, Leicester, England, September 5-7, pp. 431–434 (2001)
- [34] Glykas, M.: Workflow and Process Management in Printing and Publishing Firms. *International Journal of Information Management* 24(6), 523–538 (2004)
- [35] Xirogiannis, G., Glykas, M.: Intelligent Modeling of e-Business Maturity. *Expert Systems with Applications* 32/2, 687–702 (2007)
- [36] Xirogiannis, G., Stefanou, J., Glykas, M.: A Fuzzy Cognitive Map Approach to Support Urban Design. *Journal of Expert Systems with Applications* 26(2) (2004)
- [37] Xirogiannis, G., Glykas, M.: Fuzzy Cognitive Maps in Business Analysis and Performance Driven Change. *Journal of IEEE Transactions in Engineering Management* 13(17) (2004)
- [38] Xirogiannis, G., Chytas, P., Glykas, M., Valiris, G.: Intelligent impact assessment of HRM to the shareholder value. *Expert Systems with Applications* 35(4), 2017–2031 (2008)
- [39] Sox, J.H.C., Blatt, M.A., Higgins, M.C., Marton, K.I.: *Medical Decision Making*. Butterworths, Boston (1988)
- [40] Krishnan, R., Sivakumar, G., Bhattacharya, P.: Extracting decision trees from trained neural networks. *Pattern Recognition* 32(12), 1999–2009 (1999)
- [41] Heckerman, D., Geiger, D., Chickering, D.M.: Learning Bayesian networks: the combination of knowledge and statistical data. *Machine Learning* 20, 197–243 (1995)
- [42] Podgorelec, V., Kokol, P., Tiglic, S.B., Rozman, I.: Decision Trees: An Overview and Their Use in Medicine. *Journal of Medical Systems* 26(5) (October 2002)
- [43] Papageorgiou, E., Stylios, C., Groumpos, P.: An Integrated Two-Level Hierarchical Decision Making System based on Fuzzy Cognitive Maps (FCMs). *IEEE Trans. Biomed. Engin.* 50(12), 1326–1339 (2003)
- [44] Papageorgiou, E.I., Groumpos, P.P.: A weight adaption method for fine-tuning Fuzzy Cognitive Map causal links. *Soft Computing Journal* 9, 846–857 (2005) doi:10.1007
- [45] Papageorgiou, E.I., Stylios, C.D., Groumpos, P.P.: Unsupervised learning techniques for fine-tuning Fuzzy Cognitive Map causal links. *Intern. Journal of Human-Computer Studies* 64, 727–743 (2006)