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A fuzzy system for evaluating students' learning achievement

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ABSTRACT

The proper system for evaluating the learning achievement of students is the key to realizing the purpose of education. In recent years, several methods have been presented for applying the fuzzy set theory in the educational grading systems. In this paper, we propose a method for the evaluation of students' answerscripts using a fuzzy system. The proposed system applies fuzzification, fuzzy inference, and defuzzification in considering the difficulty, the importance and the complexity of questions. The transparency, objectivity, and easy implementation of the proposed fuzzy system provide a useful way to automatically evaluate students' achievement in a more reasonable and fairer manner.

1. Introduction

Evaluation of students' learning achievement is the process of determining the performance levels of individual students in relation to educational objectives. A high quality evaluation system certifies, provides grounds for individual improvement, and ensures that all students receive fair grading so as not to limit students' present and future opportunities. Thus, the system should regularly be reviewed and improved to ensure that it is precise, fair, and beneficial to all students. Hence, the evaluation system needs the transparency, objectivity, logical reasoning, and easy computer implementation which could be provided by the fuzzy logic system.

Since its introduction in 1965 by Zadeh (1965), the fuzzy set theory has been widely used in solving problems in various fields, and recently in educational grading systems. Biswas (1995) presented two methods for the evaluation of students' answerscripts using fuzzy sets and a matching function: a fuzzy evaluation method and a generalized fuzzy evaluation method. Chen and Lee (1999) presented two methods for applying fuzzy sets to overcome the problem of giving two different fuzzy marks to students with the same total score which could arise from Biswas' method. Echauz and Vachtsevanos (1995) proposed a fuzzy logic system for translating traditional scores into letter-grades. Law (1996) built a fuzzy structure model for an educational grading system with its algorithm to aggregate different test scores in order to produce a single score for individual students. He also proposed a method to build the membership functions (MFs) of several linguistic values with different weights. Wilson, Karr, and Freeman (1998) presented an automatic grading system based on fuzzy rules and genetic algorithms. Ma and Zhou (2000) proposed a fuzzy set approach to assess the outcomes of student-centered learning using the evaluation of their peers and lecturer. Wang and Chen (2008) presented a method for evaluating students' answerscripts using fuzzy numbers associated with degrees of confidence of the evaluator. From the previous studies, it can be found that fuzzy numbers, fuzzy sets, fuzzy rules, and fuzzy logic systems have been used for various educational grading systems.

Weon and Kim (2001) developed an evaluation strategy based on fuzzy MFs. They pointed out that the system for students' achievement evaluation should consider the three important factors of the questions given to students: the difficulty, the importance, and the complexity. Weon and Kim used singleton functions to describe the factors of each question reflecting the individual effect of the three factors, but not the collective effect. Bai and Chen (2008b) pointed out that the difficulty factor is a very subjective parameter and may cause an argument concerning fairness in evaluation.

Bai and Chen (2008a) proposed a method to automatically construct the grade MFs of fuzzy rules for evaluating student's learning achievement. Bai and Chen (2008b) proposed a method for applying fuzzy MFs and fuzzy rules for the same purpose. To solve the subjectivity of the difficulty factor in Weon and Kim's method (2001), they obtained the level of difficulty as a function of the accuracy of the student's answerscript and the time consumed to answer the questions. However, their method still has the subjectivity problem, since the results in scores and ranks are heavily dependent on the values of several weights which are determined by the subjective knowledge of domain experts.

In this paper, as an improved alternative to Bai and Chen's method (2008b), we propose a fuzzy logic evaluation system considering the importance, the difficulty, and the complexity of questions based on Mamdani's fuzzy inference (Mamdani, 1974) and

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center of gravity (COG) defuzzification. The transparency and objective nature of the fuzzy logic system make it easy to understand and explain the results of evaluation, and thus to persuade students who are skeptical or not satisfied with the evaluation results.

The paper is organized as follows. In Section 2, we consider the same structure of evaluating students' learning achievement as Bai and Chen's (2008b) and introduce their solution method using fuzzy MFs and fuzzy rules. In Section 3, we propose a three node fuzzy evaluation system. The procedure consists of fuzzification, inference, and defuzzification. In Section 4, through an example, the procedure of the proposed system is explained and its result is compared with Bai and Chen's. Conclusions are drawn in Section 5.

2. A review of evaluation methods using membership functions and fuzzy rules

In this paper, we consider the same situation and example as in Bai and Chen's (2008b). Assume that there are n students to answer m questions. Accuracy rates of students' answerscripts (student's scores in each question divided by the maximum score assigned to this question) are the basis for evaluation. We get an accuracy rate matrix of dimension $m \times n$,

$$A = [a_{ij}], \quad m \times n,$$

where $a_{ij} \in [0,1]$ denotes the accuracy rate of student j on question i. Time rates of students (time consumed by a student to solve a question divided by the maximum time allowed to solve this question) is another basis to be considered in the evaluation. We get a time rate matrix of dimension $m \times n$,

$$T = [t_{ij}], \quad m \times n,$$

where $t_{ij} \in [0,1]$ denotes the time rate of student j on question i. We are given a grade vector

$$G = [g_i], \quad m \times 1,$$

where $g_i \in [1,100]$ denotes the assigned maximum score of question i satisfying

$$\sum_{i=1}^{m} g_i = 100.$$

Based on the accuracy rate matrix A and the grade vector G, we obtain the original total score vector of dimension $n \times 1$,

$$S = A^{\mathrm{T}} G = [s_j], \quad n \times 1, \tag{1}$$

where $s_j \in [0,100]$ is the total score of student j. The "classical" ranks of students are then obtained by sorting the element values of S in descending order.

Example. Assume that 10 students are given an examination of 5 questions and the accuracy rate matrix, the time rate matrix, and the grade vector are given as follows (Bai & Chen, 2008b):

$$A = \begin{bmatrix} 0.59 & 0.35 & 1 & 0.66 & 0.11 & 0.08 & 0.84 & 0.23 & 0.04 & 0.24 \\ 0.01 & 0.27 & 0.14 & 0.04 & 0.88 & 0.16 & 0.04 & 0.22 & 0.81 & 0.53 \\ 0.77 & 0.69 & 0.97 & 0.71 & 0.17 & 0.86 & 0.87 & 0.42 & 0.91 & 0.74 \\ 0.73 & 0.72 & 0.18 & 0.16 & 0.5 & 0.02 & 0.32 & 0.92 & 0.9 & 0.25 \\ 0.93 & 0.49 & 0.08 & 0.81 & 0.65 & 0.93 & 0.39 & 0.51 & 0.97 & 0.61 \end{bmatrix},$$

$$T = \begin{bmatrix} 0.7 & 0.4 & 0.1 & 1 & 0.7 & 0.2 & 0.7 & 0.6 & 0.4 & 0.9 \\ 1 & 0 & 0.9 & 0.3 & 1 & 0.3 & 0.2 & 0.8 & 0 & 0.3 \\ 0 & 0.1 & 0 & 0.1 & 0.9 & 1 & 0.2 & 0.3 & 0.1 & 0.4 \\ 0.2 & 0.1 & 0 & 1 & 1 & 0.3 & 0.4 & 0.8 & 0.7 & 0.5 \\ 0 & 0.1 & 1 & 1 & 0.6 & 1 & 0.8 & 0.2 & 0.8 & 0.2 \end{bmatrix},$$

$$G^{\mathrm{T}} = [10 \quad 15 \quad 20 \quad 25 \quad 30].$$

In this paper, V^{T} denotes the transpose of vector V. □

The importance of the questions is an important factor to be considered. We have l levels of importance to describe the degree of importance of each question in the fuzzy domain. The domain expert determines the importance matrix of dimension $m \times l$

$$P = [p_{ik}], \quad m \times l,$$

where $p_{ik} \in [0,1]$ denotes the membership value (degree of the membership) of question i belonging to the importance level k. In this paper, five levels (fuzzy sets) of importance ($l = 5$) are used; $k = 1$ for linguistic term "low", $k = 2$ for "more or less low", $k = 3$ "medium", $k = 4$ for "more or less high", and $k = 5$ for "high". Their MFs are shown in Fig. 1. We note that the same five fuzzy sets are applied to the accuracy, the time rate, the difficulty, the complexity, and the adjustment of questions. Once crisp values are given for the importance of questions by a domain expert, the values of p_{ik}'s are obtained by the fuzzification.

The complexity of the questions which indicates the ability of students to give correct answers is also an important factor to be considered. The domain expert determines the fuzzy complexity matrix of dimension $m \times l$,

$$C = [c_{ik}], \quad m \times l,$$

where $c_{ik} \in [0,1]$ denotes the membership value of question i belonging to the complexity level k.

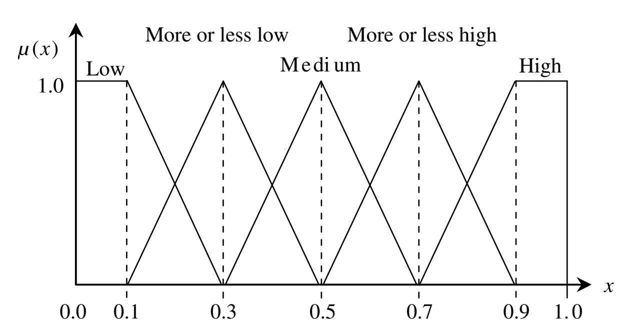

Fig. 1. Fuzzy membership functions of the five levels.

Example. For the above example we get the following by the domain expert:

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0.33 & 0.67 & 0 & 0 \\ 0 & 0 & 0 & 0.15 & 0.85 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0.07 & 0.93 & 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0.85 & 0.15 & 0 & 0 \\ 0 & 0 & 0.33 & 0.67 & 0 \\ 0 & 0 & 0 & 0.69 & 0.31 \\ 0.56 & 0.44 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0.3 & 0 \end{bmatrix}.$$

Total score is then obtained by formula (1) as

$$S^{\mathrm{T}} = \begin{array}{c} \begin{array}{cccccccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & S_9 & S_{10} \end{array} \\ \left[\begin{array}{cccccccccc} 67.60 & 54.05 & 38.40 & 49.70 & 49.70 & 48.80 & 46.10 & 52.30 & 85.95 & 49.70 \end{array}\right] \end{array}$$

and thus the classical ranks of the students become

$$S_9 > S_1 > S_2 > S_8 > S_4 = S_5 = S_{10} > S_6 > S_7 > S_3,$$

where $S_a > S_b$ means score of student a is higher than score of student b, and $S_a = S_b$ means their scores are equal. □

Bai and Chen's method (2008b) uses three steps to evaluate students' answerscripts. In the first step, the crisp values of accuracy and time, respectively, are obtained as the average accuracy rate vector of dimension $m \times 1$,

$$\overline{A} = [a_{i\bullet}], \quad m \times 1,$$

where $a_{i\bullet}$ denotes the average accuracy rate of question i which is obtained by

$$a_{i\bullet} = \sum_{j=1}^{n} a_{ij}/n, \tag{2}$$

and the average time rate vector of the same dimension,

$$\overline{T} = [t_{i\bullet}], \quad \mathrm{m} \times 1,$$

where $t_{i\bullet}$ denotes the average time rate of question i which is obtained by

$$t_{i\bullet} = \sum_{j=1}^{n} t_{ij} \Big/ n. \tag{3}$$

Next, by fuzzification, we obtain the fuzzy accuracy rate matrix of dimension $m \times l$,

$$FA = [fa_{ik}], \quad m \times l,$$

where $fa_{ik} \in [0,1]$ denotes the membership value of the average accuracy rate of question i belonging to level k, and the fuzzy time rate matrix of dimension $m \times l$,

$$FT = [ft_{ik}], \quad m \times l,$$

where $ft_{ik} \in [0,1]$ denotes the membership value of the average time rate of question i belonging to level k, respectively.

Example. In the above example, we get by formula (2) and formula (3), respectively,

$$\overline{A}^{\mathrm{T}} = [0.45 \quad 0.31 \quad 0.711 \quad 0.47 \quad 0.637],$$
$$\overline{T}^{\mathrm{T}} = [0.57 \quad 0.48 \quad 0.31 \quad 0.50 \quad 0.57].$$

Based on the fuzzy MFs in Fig. 1, we obtain the fuzzy accuracy rate matrix and the fuzzy time rate matrix as

$$FA = \begin{bmatrix} 0 & 0.25 & 0.75 & 0 & 0 \\ 0 & 0.95 & 0.05 & 0 & 0 \\ 0 & 0 & 0 & 0.945 & 0.055 \\ 0 & 0.15 & 0.85 & 0 & 0 \\ 0 & 0 & 0.315 & 0.685 & 0 \end{bmatrix},$$

$$FT = \begin{bmatrix} 0 & 0 & 0.65 & 0.35 & 0 \\ 0 & 0.1 & 0.9 & 0 & 0 \\ 0 & 0.95 & 0.05 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0.65 & 0.35 & 0 \end{bmatrix}. \qquad \square$$

In the second step, based on the fuzzy accuracy rate matrix, FA, the fuzzy time rate matrix, FT, and the fuzzy rules, $\Re_D$, given in the form of IF-THEN rules, we obtain the fuzzy difficulty matrix of dimension $m \times l$,

$$D = [d_{ik}], \quad m \times l,$$

where $d_{ik} \in [0,1]$ denotes the membership value of the difficulty of question i belonging to level k. When the level of accuracy, l_A, and the level of time, l_T, are given, the level of difficulty, l_D, is determined by the given fuzzy rules,

$$l_D = \Re_D(l_A, l_T).$$

Denote the relative weights of the accuracy rate and the time rate, respectively, by w_A and w_T ($w_A + w_T = 1$), which are determined by a domain expert. The value of d_{ik} is obtained by

$$d_{ik} = \max_{\{(l_A, l_T) | \Re_D(l_A, l_T) = k\}} \{w_A \cdot fa_{i,l_A} + w_T \cdot ft_{i,l_T}\}. \tag{4}$$

Next, based on the fuzzy difficulty matrix, D, and the fuzzy complexity matrix, C, with their relative weights, w_D and w_C ($w_D + w_C = 1$), respectively, given the fuzzy rules, $\Re_E$, we obtain the effort (which is the answer-cost in Bai and Chen's (2008b)) matrix of dimension $m \times l$, in the same manner as we obtained the difficulty matrix above

$$E = [e_{ik}], \quad m \times l,$$

where $e_{ik} \in [0,1]$ denotes the membership value of the effort to answer question i belonging to level k, which is a measure of effort required by students to answer question i.

Next, based on the fuzzy effort matrix, E, and fuzzy importance matrix, P, with their relative weighs, w_E and w_P ($w_E + w_P = 1$), respectively, given the fuzzy rules, $\Re_W$, we obtain the adjustment matrix of dimension $m \times l$

$$W = [w_{ik}], \quad m \times l,$$

where $w_{ik} \in [0,1]$ denotes the membership value of the adjustment of question i belonging to level k.

Then we use the following formula to obtain the adjustment vector,

$$\overline{W} = [w_{i\bullet}], \quad m \times 1,$$

where $w_{i\bullet} \in [0,1]$ denotes the final adjustment value required by question i obtained by

$$w_{i\bullet} = \frac{0.1 \times w_{i1} + 0.3 \times w_{i2} + 0.5 \times w_{i3} + 0.7 \times w_{i4} + 0.9 \times w_{i5}}{0.1 + 0.3 + 0.5 + 0.7 + 0.9}, \tag{5}$$

where 0.1, 0.3, 0.5, 0.7 and 0.9 are the centers of the fuzzy MFs shown in Fig. 1.

We note that Bai and Chen's method, formula (4), has no background of fuzzy set theory. We also note that different pairs of values of weights, which are subjectively determined, in w_A and w_T, w_D and w_C, and w_E and w_P, result in the different matrices, D, E, and W in the above.

Example. Assume that we are given the fuzzy rule base for $\Re_D$, and $\Re_E$ in Table 1a and b, respectively, and $\Re_W$ in Table 1b.

The difficulty of question 1 in level 4 ($l_D = 4$), for example, is obtained from $\Re_D$ (1,2), $\Re_D$ (1,3), $\Re_D$ (2,3), $\Re_D$ (2,4), $\Re_D$ (3,4), $\Re_D$ (3,5), and $\Re_D$ (4,5). When $w_A = 0.6$ and $w_T = 0.4$, from formula (4)

$$\begin{aligned}
d_{14} &= \max_{\{(l_A,l_T)|\Re_D(l_A,l_T)=4\}} \{0.6 \cdot fa_{1,l_A} + 0.4 \cdot ft_{1,l_T}\} \\
&= \max_{\{(1,2),(1,3),(2,3),(2,4),(3,4),(3,5),(4,5)\}} \{0.6 \cdot fa_{1,l_A} + 0.4 \cdot ft_{1,l_T}\} \\
&= \max\{(0.6 \cdot 0 + 0.4 \cdot 0), (0.6 \cdot 0 + 0.4 \cdot 0.65), \\
&\qquad (0.6 \cdot 0.25 + 0.4 \cdot 0.65), (0.6 \cdot 0.25 + 0.4 \cdot 0.35), \\
&\qquad (0.6 \cdot 0.75 + 0.4 \cdot 0.35), (0.6 \cdot 0.75 + 0.4 \cdot 0), \\
&\qquad (0.6 \cdot 0 + 0.4 \cdot 0)\} \\
&= \max\{0, 0.26, 0.41, 0.29, 0.59, 0.45, 0\} \\
&= 0.59.
\end{aligned}$$

By obtaining all d_{ij}'s in the same manner, we obtain the fuzzy difficulty matrix

$$D = \begin{bmatrix} 0 & 0.45 & 0.71 & 0.59 & 0.15 \\ 0.04 & 0.57 & 0.61 & 0.93 & 0.57 \\ 0.567 & 0.947 & 0.567 & 0.567 & 0 \\ 0 & 0.51 & 0.91 & 0.51 & 0.09 \\ 0.411 & 0.671 & 0.551 & 0.411 & 0.14 \end{bmatrix}.$$

In the same manner, when $w_D = 0.7$ and $w_C = 0.3$, we obtain the fuzzy effort matrix

Table 1
Fuzzy rule bases to infer the difficulty, the effort, and the adjustment

(a) Difficulty

Accuracy	Time rate 1	2	3	4	5
1	3	4	4	5	5
2	2	3	4	4	5
3	2	2	3	4	4
4	1	2	2	3	4
5	1	1	2	2	3

(b) Cost

Difficulty	Complexity 1	2	3	4	5
1	1	1	2	2	3
2	1	2	2	3	4
3	2	2	3	4	4
4	2	3	4	4	5
5	3	4	4	5	5

1: "Low", 2: "more or less low", 3: "medium", 4: "more or less high", and 5: "high".

$$E = \begin{bmatrix} 0.315 & 0.752 & 0.668 & 0.497 & 0.413 \\ 0.399 & 0.651 & 0.651 & 0.852 & 0.651 \\ 0.663 & 0.663 & 0.87 & 0.756 & 0.49 \\ 0.525 & 0.805 & 0.637 & 0.637 & 0.357 \\ 0.47 & 0.68 & 0.596 & 0.498 & 0.288 \end{bmatrix}.$$

Also, when $w_E = 0.5$ and $w_P = 0.5$, we obtain the fuzzy adjustment matrix

$$W = \begin{bmatrix} 0.376 & 0.376 & 0.658 & 0.876 & 0.750 \\ 0.365 & 0.661 & 0.661 & 0.761 & 0.426 \\ 0.331 & 0.435 & 0.756 & 0.860 & 0.803 \\ 0.903 & 0.819 & 0.679 & 0.403 & 0.319 \\ 0.340 & 0.805 & 0.763 & 0.714 & 0.249 \end{bmatrix}.$$

The adjustment vector $\overline{W}$ is obtained by formula (5) as

$$\overline{W}^{\mathrm{T}} = [0.706 \quad 0.592 \quad 0.747 \quad 0.497 \quad 0.552]. \qquad \square$$

In the last third step, we adjust the original ranks of students. We note that Bai and Chen (2008b) applied their method only to the students with the same total score. Assume that there are q students with an equal total score. Let us regroup these students and renumber them from student 1 to student q, and also the original accuracy rate a_{ij} correspondingly. The sum of differences (SOD) for students with the same total score is computed by

$$SOD_j = \sum_{k=1}^{q} \sum_{i=1}^{m} (a_{ij} - a_{ik}) \cdot \hat{G}, \tag{6}$$

where $\hat{G} = [\hat{g}_i], \quad \hat{g}_i = 0.5 + w_{i\bullet}$.

New ranks of students are then obtained by sorting the element values of SOD in descending order.

Example. For this example, three students S_4, S_5, and S_{10} got the same total score ($q = 3$). After rearranging the original matrix A, we get

$$\begin{array}{c} \begin{matrix} S_4 & S_5 & S_{10} \end{matrix} \\ \begin{bmatrix} 0.66 & 0.11 & 0.24 \\ 0.04 & 0.88 & 0.53 \\ 0.71 & 0.17 & 0.74 \\ 0.61 & 0.5 & 0.25 \\ 0.81 & 0.65 & 0.61 \end{bmatrix} \end{array}.$$

The SOD vector of students with the same total score is obtained by formula (6) as

$$SOD = [\overset{S_4}{3.27} \quad \overset{S_5}{-5.46} \quad \overset{S_{10}}{2.19}]$$

with the new ranks being

$$S_4 > S_{10} > S_5.$$

Thus, the new ranks of all students become

$$S_9 > S_1 > S_2 > S_8 > S_4 > S_{10} > S_5 > S_6 > S_7 > S_3.$$

When we apply this method to all students, we obtain the following SOD vector:

$$SOD = [\overset{S_1}{139.8} \quad \overset{S_2}{-7.9} \quad \overset{S_3}{-145.1} \quad \overset{S_4}{-41.8} \quad \overset{S_5}{-70.9} \quad \overset{S_6}{-51.8} \quad \overset{S_7}{-72.9} \quad \overset{S_8}{-41.7} \quad \overset{S_9}{337.8} \quad \overset{S_{10}}{-45.4}]$$

with the new ranks being

$S_9 > S_1 > S_2 > S_8 > S_4 > S_{10} > S_6 > S_5 > S_7 > S_3.$ □

3. Three node fuzzy logic evaluation system

Bai and Chen's method (2008b) has seven weights, two for each of the main steps and one for the grade adjustment step, which are determined subjectively by domain experts. Quite different ranks can be obtained depending on their values. By using their method, the examiners could not easily verify how new ranks are obtained and could not persuade skeptical students. Naturally, there is no method to determine the optimum values of weights. Also, the membership values in Bai and Chen's method do not satisfy the concept of the fuzzy set. To reduce the degree of subjectivity in this method and provide a theory of fuzzy set, we propose a system applying the most commonly used Mamdani's fuzzy inference mechanism (Mamdani, 1974) and center of gravity (COG) defuzzification technique. The same situations and, for the sake of comparison, the same numerical example of Bai and Chen's (2008b) will be used.

The proposed system can be represented as a block diagram of fuzzy logic systems as shown in Fig. 2. Bai and Chen's (2008b) can be considered as a simple and specific case of the block diagram. The system consists of three nodes: the difficulty node, the effort node, and the adjustment node. Each node of the system behaves like a fuzzy logic controller (FLC in Fig. 2) with two scalable inputs and one output, as in Fig. 3. It maps a two-to-one fuzzy relation by inference through a given rule base. The inputs to the system, in the left part of the figure, are given either by examination results or domain experts. The inputs are fuzzified based on the defined levels (fuzzy sets) in Fig. 1. In the first node, both inputs are given by examination results, whereas in the later nodes, one input is the output of its previous node and the other is given by a domain expert. The output of each node can be in the form of a crisp value (defuzzified) or in the form of linguistic variables (MFs). Each node has two scale factors (SFs in Fig. 2), and we can adjust the effects of inputs by varying the values of the scale factors. In this paper, we let both scaling facots have the same value of unity assuming the equal influence of each input on the output.

In our proposed method, each fuzzy node proceeds in three steps.

Step 1 (Fuzzification). In this step, inputs are converted into membership values of the fuzzy sets shown in Fig. 1. Triangular MF is commonly used because of its simplicity and easy computation.

Step 2 (Inference). In this step, inference is performed based on the given rule base, in the form of IF-THEN rules. We use Mamdani's max–min inference mechanism which is the most commonly used inference mechanism to produce fuzzy sets for defuzzification (1974). In Mamdani's max–min mechanism, implication is modeled by means of the minimum operator, and the resulting output MFs are combined using the maximum operator.

The inference mechanism can be written into the form

$$\alpha_{ik} = \max_{\{(l_1,l_2)|\Re(l_1,l_2)=k\}} \{\min(fa_{i,l_1}, ft_{i,l_2})\}, \tag{7}$$

where α_{ik} is the output of inference of question i in fuzzy set k. We obtain a matrix of dimension $m \times l$,

$$\alpha = [\alpha_{ik}], \quad m \times l,$$

where α_{ik} denotes the fire-strength of question i in fuzzy level k.

Step 3 (Defuzzification). In this step, fuzzy output values are converted into a single crisp value or final decision. In this paper, the COG method is applied. The crisp value of question i is obtained by

$$y_i = \int x \cdot \mu(x)\mathrm{d}x \Big/ \int \mu(x)\mathrm{d}x, \tag{8}$$

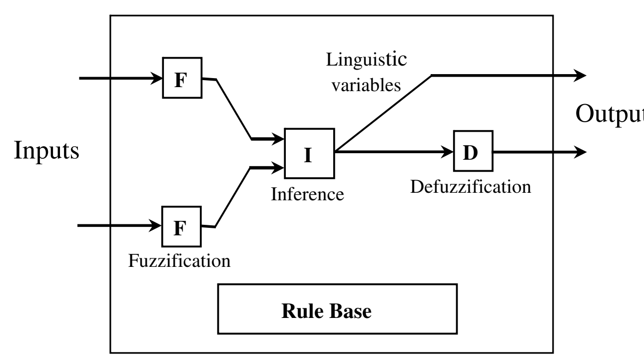

Fig. 3. Representation of node process as a fuzzy logic controller.

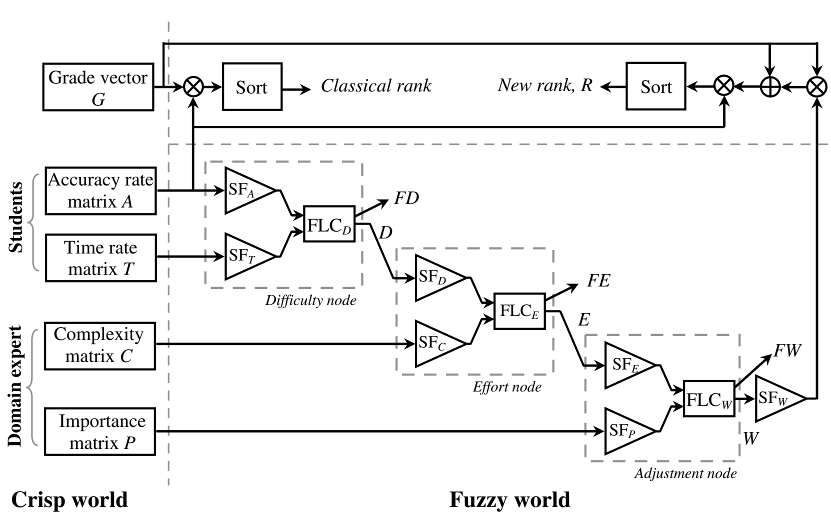

Fig. 2. Block diagram of the three node fuzzy evaluation system.

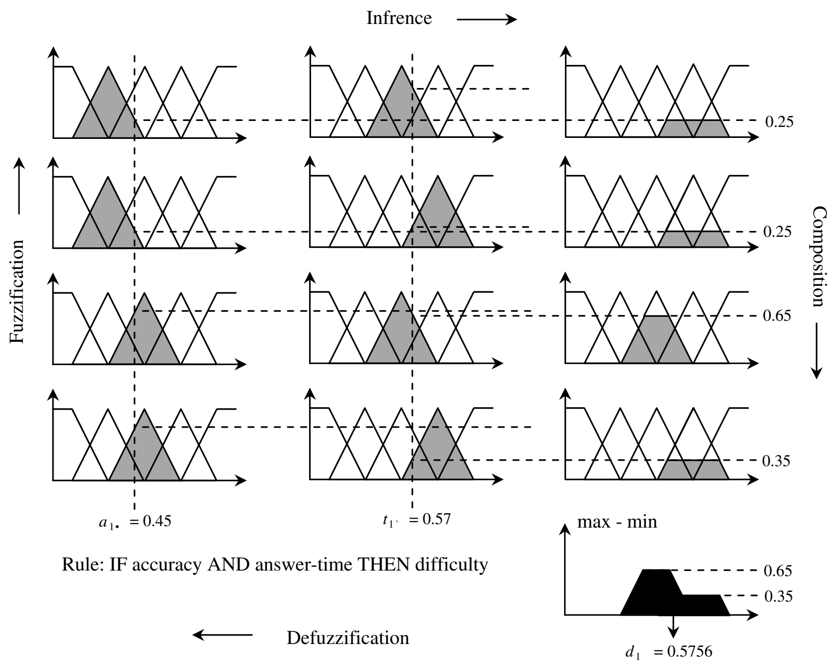

Fig. 4. An illustration of fuzzification, Mamdani's max–min inference, and COG for the difficulty of question 1.

where integrals are taken over the entire range of the output and $\mu(x)$ is obtained from step 2. By taking the COG, a reasonable crisp value can be obtained. The proposed system has been implemented using the Fuzzy Logic Toolbox™ 2.2.7 by MathWorks (http://www.mathworks.com/products/fuzzylogic).

Each of the three nodes follows the above scheme. The difficulty node has two inputs of the accuracy rate and the time rate, and one output of the difficulty. The effort node has two inputs of the difficulty and the complexity, and one output of the effort. The adjustment node has two inputs of the effort and the importance, and one output of the adjustment.

The adjustment vector, W, is then used to obtain the adjusted grade vector of dimension $m \times 1$,

$$\widetilde{G} = [\widetilde{g}_i], \quad m \times 1,$$

where $\widetilde{g}_i$ is the adjusted grade of question i,

$$\widetilde{g}_i = g_i \cdot (1 + w_{i\bullet}), \tag{9}$$

and $w_{i\bullet}$ is the average adjustment of question i. Then, the value is scaled to its total grade (i.e., 100) by using the formula

$$\widetilde{g}_i = \widetilde{g}_i \cdot \sum_{j=1}^{m} g_j \Big/ \sum_{j=1}^{m} \widetilde{g}_j. \tag{10}$$

Finally, we obtain the adjusted total scores of students by

$$\widetilde{S} = A^{\mathrm{T}}\widetilde{G}. \tag{11}$$

New ranks of students are then obtained by sorting element values of $\widetilde{S}$ in descending order.

4. An example with comparison of methods

At the difficulty node, in step 1, the same fuzzy matrices, FA and FT are obtained by fuzzifying the averages of A and T, respectively, as in Section 2. The procedure of our method deviates from that of Bai and Chen's (2008b), beginning from the point of obtaining the difficulty matrix, D.

In step 2, based on the rule base in Table 1a, the output of Mamdani's fuzzy inference mechanism for question 1 in level 4 (fuzzy set "more or less high"), for example, is obtained by formula (7) as follows:

$$\begin{aligned}
\alpha_{14} &= \max_{\{(l_1,l_2)|\Re_D(l_1,l_2)=4\}} \{\min(fa_{1,l_1}, ft_{1,l_2})\} \\
&= \max_{\{(1,2),(1,3),(2,3),(2,4),(3,4),(3,5),(4,5)\}} \{\min(fa_{1,l_1}, ft_{1,l_2})\} \\
&= \max\{\min(0,0), \min(0,0.65), \min(0.25,0.65), \\
&\quad \min(0.25,0.65), \min(0.75,0.35), (0.75,0), \min(0,0)\} \\
&= \max\{0,0,0.25,0.25,0.35,0,0\} \\
&= 0.35.
\end{aligned}$$

By the same procedure, we obtain the inference output, the fire-strength difficulty matrix, as

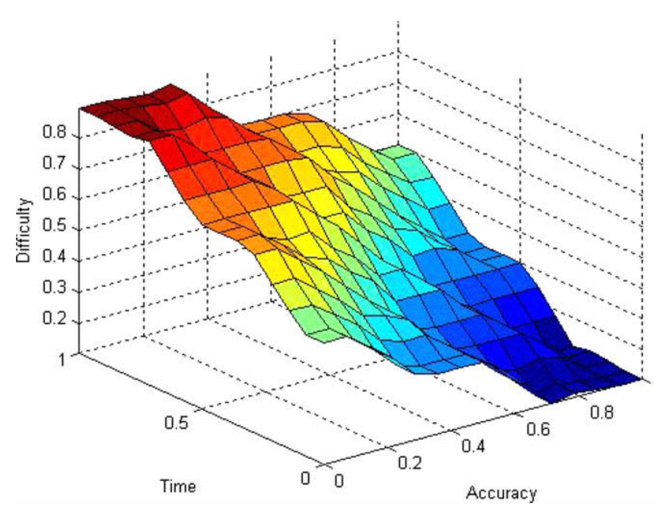

Fig. 5. Surface view of the rule base in Table 1a for the difficulty.

$$\alpha_D = \begin{bmatrix} 0 & 0 & 0.65 & 0.35 & 0 \\ 0 & 0.05 & 0.1 & 0.9 & 0 \\ 0.055 & 0.945 & 0 & 0 & 0 \\ 0 & 0 & 0.85 & 0.15 & 0 \\ 0 & 0.65 & 0.35 & 0.315 & 0 \end{bmatrix}.$$

In Step 3, we use COG to obtain the crisp value of the difficulty of question 1 by formula (8) as 0.576, as illustrated in Fig. 4. Likewise, we obtain the crisp values of other questions as

$$D^{\mathrm{T}} = [0.576 \quad 0.653 \quad 0.299 \quad 0.538 \quad 0.456].$$

The surface view of the relation of the rule base in Table 1a is shown in Fig. 5.

At the next effort node, in step 1, the crisp values of D obtained at the previous node are fuzzified to obtain the fuzzy difficulty matrix as

$$FD = \begin{bmatrix} 0 & 0 & 0.622 & 0.378 & 0 \\ 0 & 0 & 0.236 & 0.764 & 0 \\ 0.003 & 0.997 & 0 & 0 & 0 \\ 0 & 0 & 0.811 & 0.189 & 0 \\ 0 & 0.221 & 0.779 & 0 & 0 \end{bmatrix}.$$

In step 2, based on the rule base given in Table 1b, we obtain the inference output by formula (7), the fire-strength effort matrix, as

$$\alpha_E = \begin{bmatrix} 0 & 0.622 & 0.378 & 0.15 & 0 \\ 0 & 0 & 0.236 & 0.67 & 0 \\ 0 & 0.003 & 0.994 & 0.31 & 0 \\ 0 & 0.56 & 0.189 & 0 & 0 \\ 0 & 0.221 & 0.7 & 0.3 & 0 \end{bmatrix}.$$

In step 3, the crisp values are obtained as

$$E^{\mathrm{T}} = [0.424 \quad 0.642 \quad 0.568 \quad 0.354 \quad 0.514].$$

The surface view of the rule base in Table 1b is shown in Fig. 6.

At the final adjustment node, in step 1, we get the fuzzy counter part of crisp values obtained in the previous step 3 at the difficulty node as

$$FE = \begin{bmatrix} 0 & 0.38 & 0.62 & 0 & 0 \\ 0 & 0 & 0.289 & 0.711 & 0 \\ 0 & 0 & 0.661 & 0.339 & 0 \\ 0 & 0.733 & 0.267 & 0 & 0 \\ 0 & 0 & 0.931 & 0.069 & 0 \end{bmatrix}.$$

In step 2, based on the rule base in Table 1b, we obtain the inference output by formula (7), the fire-strength adjustment matrix as

$$\alpha_W = \begin{bmatrix} 0 & 0 & 0 & 0.62 & 0 \\ 0 & 0.289 & 0.33 & 0.67 & 0 \\ 0 & 0 & 0 & 0.661 & 0.339 \\ 0.733 & 0.267 & 0 & 0 & 0 \\ 0 & 0.07 & 0.93 & 0.069 & 0 \end{bmatrix}.$$

In step 3, we get the crisp values of adjustment as

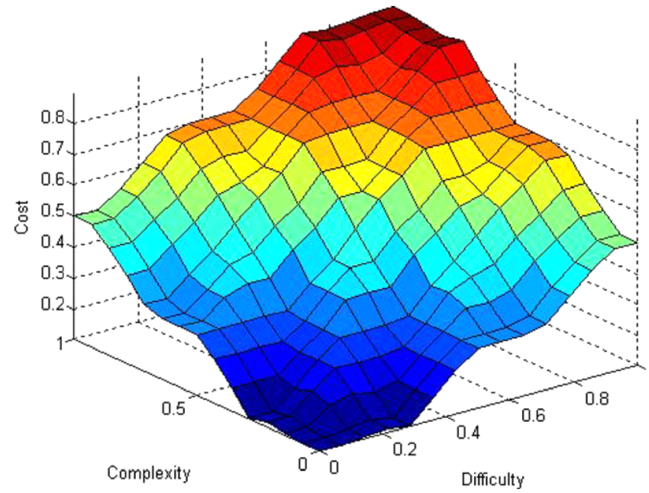

Fig. 6. Surface view of the rule base in Table 1b for the effort and the adjustment.

$$W^{\mathrm{T}} = [0.7 \quad 0.552 \quad 0.749 \quad 0.177 \quad 0.5].$$

Finally, we get the fuzzified adjustment matrix as

$$FW = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0.742 & 0.258 & 0 \\ 0 & 0 & 0 & 0.754 & 0.246 \\ 0.617 & 0.383 & 0 & 0 & 0 \\ 0 & 0.002 & 0.998 & 0 & 0 \end{bmatrix}.$$

Now, the adjusted grades are obtained using formula (9) as

$$\widetilde{G}^{\mathrm{T}} = [17 \quad 23.272 \quad 34.829 \quad 29.415 \quad 44.990].$$

After scaling to the total score of the manuscript using formula (10), we have

$$\widetilde{G}^{\mathrm{T}} = [11.371 \quad 15.566 \quad 23.296 \quad 19.675 \quad 30.092].$$

The new total scores of the students are then obtained using formula (11) as

$$\widetilde{S}^{\mathrm{T}} = \begin{array}{c} \begin{matrix} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & S_9 & S_{10} \end{matrix} \\ [\begin{matrix} 67.15 & 53.17 & 42.10 & 52.19 & 48.31 & 51.81 & 48.47 & 49.27 & 85.23 & 51.49 \end{matrix}] \end{array}$$

with the new ranks being

$S_9 > S_1 > S_2 > S_4 > S_6 > S_{10} > S_8 > S_7 > S_5 > S_3$.

Also, when our method is applied to only students with the same total score (S_4, S_5, and S_{10} in the example), the adjusted total scores will be

$$\hat{S}^{\mathrm{T}} = \begin{array}{c} \begin{matrix} S_4 & S_5 & S_10 \end{matrix} \\ [\begin{matrix} 52.19 & 48.31 & 51.49 \end{matrix}] \end{array},$$

yielding the same ranks as Bai and Chen's method:

$S_4 > S_{10} > S_5$.

Table 2
Ranks obtained by the three methods

Method	1	2	3	4	5	6	7	8	9	10
Classical	9	1	2	8	4=	5=	10	6	7	3
Bai & Chen (3)	9	1	2	8	4	10	5	6	7	3
Fuzzy (3)	9	1	2	8	4	10	5	6	7	3
Bai & Chen (10)	9	1	2	8	4	10	6	5	7	3
Fuzzy (10)	9	1	2	4	6	10	8	7	5	3

(3): Applied to 3 students with equal score, (10): applied to 10 students.

We understand that Bai and Chen (2008b) applied their method only to students with equal total scores, not to all students. Adjusting the original scores and ranks already determined through the original grade, G, by the adjusted grade, $\hat{G}$ determined after examination and through adjustment, may not be desirable. However, the adjustment is often necessary by reflecting the degrees of difficulty, the importance and the complexity recognized after the results of examinations. In the real case of the academic ability test of Korea, a nation wide test held by the Korean government, the equivalent of the US' SAT (Scholastic Aptitude Test), to which about 700,000 high school seniors apply yearly, the original scores of students are adjusted by reflecting the distribution of scores. When Bai and Chen's method would be applied to all students, the total scores would be given as

$$\hat{S}^{\mathrm{T}} = A^{\mathrm{T}}\hat{G} = [\overset{S_1}{74.04}\ \overset{S_2}{59.27}\ \overset{S_3}{45.56}\ \overset{S_4}{55.88}\ \overset{S_5}{52.97}\ \overset{S_6}{54.88}\ \overset{S_7}{52.77}\ \overset{S_8}{55.89}\ \overset{S_9}{93.85}\ \overset{S_{10}}{55.52}]$$

with the ranks being

$S_9 > S_1 > S_2 > S_8 > S_4 > S_{10} > S_6 > S_5 > S_7 > S_3.$

The ranking orders using the classical method, Bai and Chen's method and our proposed method are summarized in Table 2. From which we note the following.

When Bai and Chen's method is applied to all students, the original ranks

$S_8 > S_4 = S_5 = S_{10} > S_6 > S_7,$

will be changed to

$S_8 > S_4 > S_{10} > S_6 > S_5 > S_7,$

whereas applied the proposed method, the original ranks will be changed to

$S_4 > S_6 > S_{10} > S_8 > S_7 > S_5,$

which displays a wider degree of change.

When Bai and Chen's method is applied to three students with equal total scores, the original ranks

$S_4 = S_5 = S_{10},$

will be changed to

$S_4 > S_{10} > S_5,$

and applied the proposed method, the original ranks will be changed to the same results. However, we note that the results will vary from one example to another.

5. Conclusions

In this paper, we proposed a fuzzy logic system for the evaluation of students' learning achievement. The proposed method considers the importance, the complexity, and the difficulty of the questions given to students, as factors of evaluation. The system has been represented as a block diagram of three fuzzy logic controllers. Each fuzzy logic controller generates an output from two inputs using Mamdani's max–min inference mechanism and the COG defuzzification.

The system can be a method used to overcome the subjectivity and the lack of theoretical foundation in previous studies. The transparency and the theoretically verified reasoning of the fuzzy logic system make it easy to interpret scores and explain to students why certain scores have been adjusted. The system inherently has a kind of feedback function to reflect subtle variations in the values of weights which is determined by domain expert knowledge. The system is implemented by using Fuzzy Logic Toolbox™ 2.2.7 by MathWorks (http://www.mathworks.com/products/fuzzylogic).

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