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Models of core/periphery structures

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Abstract

A common but informal notion in social network analysis and other fields is the concept of a core/periphery structure. The intuitive conception entails a dense, cohesive core and a sparse, unconnected periphery. This paper seeks to formalize the intuitive notion of a core/periphery structure and suggests algorithms for detecting this structure, along with statistical tests for testing a priori hypotheses. Different models are presented for different kinds of graphs (directed and undirected, valued and nonvalued). In addition, the close relation of the continuous models developed to certain centrality measures is discussed.

Keywords: Core; Periphery; Algorithm

1. Introduction

A common image in social network analysis and other fields is that of the core/periphery structure. The notion is quite prevalent in such diverse fields of inquiry as world systems (Snyder and Kick, 1979; Nemeth and Smith, 1985; Smith and White, 1992), economics (Krugman, 1996) and organization studies (Faulkner, 1987). In the context of social networks, it occurs in studies of national elites and collective action (Laumann and Pappi, 1976; Alba and Moore, 1978), interlocking directorates (Mintz and Schwartz, 1981), scientific citation networks (Mullins et al., 1977; Doreian, 1985), and proximity among Japanese monkeys (Corradino, 1990).

Given its wide currency, it comes as a bit of a surprise that the notion of a core/periphery structure has never been formally defined. The lack of definition means that different authors can use the term in wildly different ways, making it difficult to

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compare otherwise comparable studies. Furthermore, a formal definition provides the basis for statistical methods of testing whether a given dataset has a hypothesized core/periphery structure, and for computational methods of discovering core/periphery structures in data. Without such a definition, we cannot proceed with developing these kinds of tools.

In this paper, we develop two families of core/periphery models, based on intuitive conceptions of the structure. Any formalization of an intuitive concept needs to identify, in a precise way, the essential features of a particular concept. This part of the process involves a certain degree of conceptual clarification and interpretation that can (and many would argue should) be challenged by others. In view of this, we see this paper as a starting point in a methodological debate on what constitutes a core/periphery structure.

2. Intuitive conceptions

One intuitive view of the core/periphery structure is the idea of a group or network that cannot be subdivided into exclusive cohesive subgroups or factions, although some actors may be much better connected than others. The network, to put it another way, consists of just one group to which all actors belong to a greater or lesser extent. This is the sense in which Pattison (1993, p. 97) uses the term. This conception is rooted in the cohesive subsets literature (for a review, see Scott, 1991, or Wasserman and Faust, 1994).

Another intuitive idea is the notion of a two-class partition of nodes (one class is the core and the other is the periphery). In the terminology of blockmodeling, the core is seen as a 1-block, and the periphery is seen as a 0-block. This is the sense in which Breiger (1981) uses the terms. The blocks representing ties between the core and periphery can be either 1-blocks or 0-blocks. In its implications, this conception is quite similar to the ‘‘one-group’’ idea presented above, with the exception that it specifies the character of ties within the periphery as well as within the core.

A third intuitive view of the core/periphery structure is based on the physical center and periphery of a cloud of points in Euclidean space. Given a map of the space, such as provided by multidimensional scaling, nodes that occur near the center of the picture are those that are proximate not only to each other but to all nodes in the network, while nodes that are on the outskirts are relatively close only to the center. This is the view of the core/periphery structure that is implicit in Laumann and Pappi (1976). In its implications, this view is virtually identical to the partition approach described above, as we will discuss in a later section.

As we have phrased them, these intuitive views (particularly the first one) make the assumption that a network cannot have more than one core. However, other ways of thinking about core/periphery structures lead us to think of multiple cores, each with its own periphery. We discuss multiple cores in a companion piece (Everett and Borgatti, in press). In any case, the restriction of a single core is not as limiting as might at first appear, since we can always choose to analyze a subgraph of the network that is thought to contain just one core.

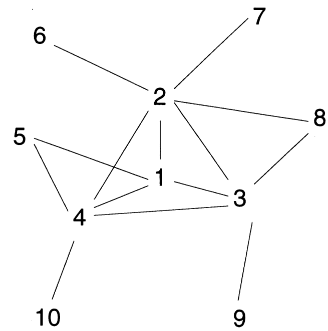

Fig. 1. A network with a core/periphery structure.

We use these intuitive conceptions as the basis for two models of the core/periphery structure: a discrete model and a continuous model. We describe the discrete model first.

3. Discrete model

In this section we explore the idea that the core periphery model consists of two classes of nodes, namely a cohesive subgraph (the core) in which actors are connected to each other in some maximal sense and a class of actors that are more loosely connected to the cohesive subgraph but lack any maximal cohesion with the core.

Consider the graph in Fig. 1, which intuitively seems to have a core/periphery structure. The adjacency matrix for the graph is given in Table 1.

The matrix has been blocked to emphasize the pattern, which is that core nodes are adjacent to other core nodes, core nodes are adjacent to some periphery nodes, and

Table 1
The adjacency matrix of Fig. 1

	1	2	3	4	5	6	7	8	9	10
1		1	1	1	1	0	0	0	0	0
2	1		1	1	0	1	1	1	0	0
3	1	1		1	0	0	0	1	1	0
4	1	1	1		1	0	0	0	0	1
5	1	0	0	1		0	0	0	0	0
6	0	1	0	0	0		0	0	0	0
7	0	1	0	0	0	0		0	0	0
8	0	1	1	0	0	0	0		0	0
9	0	0	1	0	0	0	0	0		0
10	0	0	0	1	0	0	0	0	0	

Table 2
Idealized core/periphery structure

	1	2	3	4	5	6	7	8	9	10
1		1	1	1	1	1	1	1	1	1
2	1		1	1	1	1	1	1	1	1
3	1	1		1	1	1	1	1	1	1
4	1	1	1		1	1	1	1	1	1
5	1	1	1	1		0	0	0	0	0
6	1	1	1	1	0		0	0	0	0
7	1	1	1	1	0	0		0	0	0
8	1	1	1	1	0	0	0		0	0
9	1	1	1	1	0	0	0	0		0
10	1	1	1	1	0	0	0	0	0	

periphery nodes do not connect with other periphery nodes. In blockmodeling terms, the core/core region is a 1-block, the core/periphery regions are (imperfect) 1-blocks, and the periphery/periphery region is a 0-block. We claim that this pattern is characteristic of core/periphery structures and is in fact a defining property. [2]

An idealized version that corresponds to a core/periphery structure of the adjacency matrix is given in Table 2. That this pattern of blocks suggests a core/periphery structure and has been noticed many times (Burt, 1976; White, Boorman and Breiger, 1976; Knoke and Rogers, 1979; Marsden, 1989). The pattern can be seen as a generalization of the maximally centralized graph of Freeman (1979), the simple star (see Fig. 2). In the star, a single node (the center) is connected to all other nodes, which are not connected to each other. To move to the core/periphery image, we simply add duplicates of the center to the graph, and connect them to each other and to the periphery (see Fig. 3).

The patterns in Table 2 and Figs. 2 and 3 are idealized patterns that are unlikely to be actually observed in empirical data. We can readily appreciate that real structures will only approximate this pattern, in that they will have 1-blocks with less than perfect density, and 0-blocks that contain a few ties. A simple measure of how well the real structure approximates the ideal is given by Eq. (1) together with Eq. (2).

$$\rho = \sum_{i,j} a_{ij}\delta_{ij} \tag{1}$$

$$\boldsymbol{\delta}_{ij} = \begin{Bmatrix} 1 \text{ if } c_i = \text{CORE or } c_j = \text{CORE} \\ 0 \text{ otherwise} \end{Bmatrix} \tag{2}$$

In the equations, a_{ij} indicates the presence or absence of a tie in the observed data, c_i refers to the class (core or periphery) that actor i is assigned to, and $\boldsymbol{\delta}_{ij}$ (subsequently

[2] However, in a later section we introduce variations of this pattern that we shall argue are preferable in most circumstances.

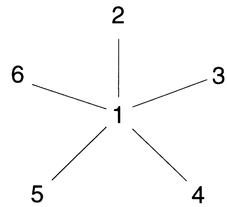

Fig. 2. Freeman's star.

called the *pattern matrix*) indicates the presence or absence of a tie in the ideal image. For a fixed distribution of values, the measure achieves its maximum value when and only when **A** (the matrix of a_{ij}) and **Δ** (the matrix of $\boldsymbol{\delta}_{ij}$) are identical, which occurs when **A** has a perfect core/periphery structure. Thus, a structure is a core/periphery structure to the extent that ρ is large.

Eq. (1) is essentially an unnormalized Pearson correlation coefficient applied to matrices rather than vectors (Hubert and Schultz, 1976; Panning, 1982). A more interpretable and more generally useful measure is the Pearson correlation coefficient itself. [3] For undirected nonreflexive graphs, we define the association measure ρ to be the Pearson correlation coefficient applied to the values found in the upper half of the matrices, diagonal not included. For directed graphs we include the lower half values as well, and for reflexive graphs of any kind we include the diagonal values.

Although simpler measures of similarity are available (e.g., the simple matching coefficient), the correlation coefficient has the benefit of generality, as it works equally well for valued as for nonvalued data, as well as for valued pattern matrices, which we consider later.

A network exhibits a core/periphery structure to the extent that the correlation between the ideal structure and the data is large. However, we need to assume the existence of a partition that assigns each node to either the core or the periphery. In Sections 3.1 and 3.2, we consider, respectively, the case where a partition is given a priori, and the case where we must construct the partition from the data itself.

3.1. Testing a priori partitions

If we obtain a partition of nodes into core and periphery blocks a priori, we can use Eq. (1) as the basis for a statistical test for the presence of a core/periphery structure. This is precisely the QAP test described by Mantel (1967) and Hubert (Hubert and Schultz, 1976; Hubert and Baker, 1978). The test is a permutation test for the independence of two proximity matrices.

[3] At first glance it may seem inappropriate to use the correlation coefficient for dichotomous data since the classical significance test for correlation coefficients demands that the variables follow a bivariate normal distribution in the population. However, we are using the correlation coefficient only to measure association, and will not be using the associated inferential test.

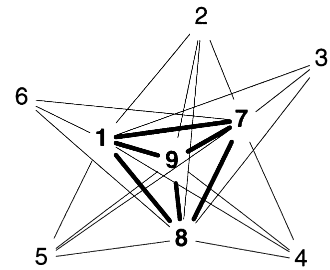

Fig. 3. Core/periphery structure.

As an example, consider testing the naive hypothesis that males in a troop of monkeys — because of their position of physical dominance — would comprise the core of the interaction network, while females would comprise the periphery. Interaction data collected by Linda Wolfe (Borgatti et al., 1999) are shown in Table 3, sorted by sex. The first five monkeys are males, the rest are females. The ideal pattern matrix has the same structure as the matrix in Table 2 but with different dimensions. Note that since

Table 3
Interactions among a troop of monkeys

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
		M	M	M	M	M	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
1	M		2	10	4	5	5	9	7	4	3	3	7	3	2	5	1	4	1	0	1
2	M	2		5	1	3	1	4	2	6	2	5	4	3	2	2	6	3	1	1	1
3	M	10	5		8	9	5	11	7	8	8	14	17	9	11	11	5	9	4	6	5
4	M	4	1	8		4	0	3	4	2	3	5	3	11	4	7	0	4	3	3	0
5	M	5	3	9	4		3	5	7	4	3	5	6	3	4	4	1	2	1	3	3
6	F	5	1	5	0	3		5	2	3	2	2	4	4	3	1	1	2	0	1	2
7	F	9	4	11	3	5	5		5	4	6	3	9	5	5	4	2	6	3	2	2
8	F	7	2	7	4	7	2	5		3	0	3	4	2	1	3	0	1	1	1	0
9	F	4	6	8	2	4	3	4	3		1	3	2	4	5	4	3	4	1	3	2
10	F	3	2	8	3	3	2	6	0	1		4	7	5	5	7	2	2	3	3	2
11	F	3	5	14	5	5	2	3	3	3	4		9	3	4	4	2	4	2	3	1
12	F	7	4	17	3	6	4	9	4	2	7	9		7	7	8	3	7	2	4	3
13	F	3	3	9	11	3	4	5	2	4	5	3	7		8	11	3	8	2	5	3
14	F	2	2	11	4	4	3	5	1	5	5	4	7	8		8	1	5	4	4	1
15	F	5	2	11	7	4	1	4	3	4	7	4	8	11	8		2	5	2	2	1
16	F	1	6	5	0	1	1	2	0	3	2	2	3	3	1	2		6	1	0	1
17	F	4	3	9	4	2	2	6	1	4	2	4	7	8	5	5	6		4	3	3
18	F	1	1	4	3	1	0	3	1	1	3	2	2	2	4	2	1	4		2	1
19	F	0	1	6	3	3	1	2	1	3	3	3	4	5	4	2	0	3	2		6
20	F	1	1	5	0	3	2	2	0	2	2	1	3	3	1	1	1	3	1	6	

the pattern matrix is dichotomous and the data matrix is not, the correlation between them amounts to a test that the average value in the 1-blocks is higher than the average value in the 0-blocks, relative to the variation within blocks. That is, we are implicitly performing an analysis of variance.

The correlation between these two matrices is 0.206 which according to the QAP permutation test is not significant ($p > 0.1$). Thus we conclude that there is no evidence for believing that in this troop of monkeys, the males form a core while the females form a periphery.

3.2. Detecting core / periphery structures in data

We can use the basic approach outlined above as the basis for constructing an algorithm for detecting a core/periphery structure without the benefit of an a priori partition. Using any combinatorial optimization technique such as simulating annealing (Kirkpatrick et al., 1983), Tabu search (Glover, 1989), or genetic algorithm (Goldberg, 1989), we can design a computer program to find a partition such that the correlation between the data and the pattern matrix induced by the partition is maximized. [4] The program we have written uses a genetic algorithm, which is a robust and convenient method, though perhaps not the fastest. For the graph in Fig. 1, the program correctly and reliably identifies the intuitive core/periphery partition (see Table 1), and reports a correlation of 0.475.

An empirical example is provided by Baker (1992), who studied co-citations among social work journals. His data consisted of the number of citations from one journal to another journal during a 1-year period (1985–1986). For our immediate purposes we find it convenient to dichotomize the data. The results of analyzing the data with our genetic algorithm are given in Table 4. The correlation is 0.54, indicating strong but far from perfect fit with the ideal. [5]

It is important to note that the significance tests we presented for testing a priori hypotheses cannot be used to evaluate the core/periphery partitions obtained by the optimization algorithms. This is because the significance tests are based on randomization methods (Edgington, 1980) that count the number of random permutations (or equivalently in this case, partitions) of the data yielding fit statistics as strong as the one actually observed. However, by definition, our algorithms are designed to find the partition that maximizes the fit statistic. Hence, the results would always be significant. As Hubert (1983) puts it, the situation is like sorting all the large numbers in a distribution into one bin and all the small numbers into another, then doing a *t*-test to see if there is a difference in means.

[4] Programs for fitting both the discrete and continuous core/periphery models have been incorporated into the computer package UCINET 5 For Windows (Borgatti et al., 1999).

[5] Reflexive ties were ignored by the algorithm.

Table 4
Core/periphery structure in a citation network

		16 S	7 C	13 J	15 S	17 S	4 C	1 A	8 C	9 F	10 I	11 J	2 A	3 B	14 P	5 C	6 C	12 J	18 S	19 S	20 S
16	SSR	1	1	1	1	1			1				1				1	1	1		1
7	CYSR	1	1		1	1	1			1					1	1	1				1
13	JSWE	1		1	1	1							1	1			1		1	1	1
15	SCW	1	1	1	1	1	1		1	1		1	1			1	1		1	1	1
17	SW	1	1	1	1	1	1	1	1	1	1	1	1	1	1		1		1	1	1
4	CAN		1		1	1	1								1		1				
1	AMH					1		1													
8	CSWJ	1			1	1			1												
9	FR		1		1	1				1											
10	IJSW					1					1										
11	JGSW				1	1						1									
2	ASW	1		1	1	1							1		1						1
3	BJSW			1		1								1					1		
14	PW		1			1	1						1		1		1				
5	CCQ		1		1											1	1				
6	CW	1	1	1	1	1	1								1	1	1				1
12	JSP	1																1			
18	SWG	1		1	1	1								1					1	1	
19	SWHC			1	1	1													1	1	
20	SWRA	1	1	1	1	1							1				1				1

3.3. Additional pattern matrices

The ideal pattern of Table 2 is not the only one that is consistent with the intuitive notion of a core/periphery structure. A more extreme expression of the core/periphery concept is the pattern shown in Table 5 (this is image ''C'' in White, Boorman and Breiger, 1976). Here, the only ties are found among core nodes. All other nodes are isolates. To measure the extent that a graph approximates this version of the core/periphery concept, we can again use correlation to measure fit, but modify the definition of the pattern matrix Δ as follows (note the change of ''or'' to ''and'', as compared with Eq. (2)):

$$\boldsymbol{\delta}_{ij} = \begin{Bmatrix} 1 \text{ if } c_i = \text{CORE and } c_j = \text{CORE} \\ 0 \text{ otherwise} \end{Bmatrix} \tag{3}$$

One problem with Eq. (3) is that part of the intuitive notion of a periphery is that it be somehow related to a core. Yet here the peripheral nodes are complete isolates so it is hard to argue that they are related to the core.

Still another ideal pattern, midway between the patterns given by Eqs. (2) and (3), is the one in which the density of core-to-periphery and periphery-to-core ties is a specified intermediate value between 0 (the density of periphery-to-periphery ties) and 1

Table 5
Alternative ideal core/periphery pattern

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1		1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1		1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1		1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1		1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0		0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0		0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0		0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

(the density of core/core ties). For example, we could decide that the density of core-to-periphery ties should be 0.5.

However, while the density of the core-to-periphery and periphery-to-core ties could be treated as fixed parameters that a core/periphery detecting algorithm would be required to match, it is unlikely that in practice we will have a good reason for choosing one density value over another. A better approach is to treat those off-diagonal regions of the matrix as missing data, so that the algorithm seeks only to maximize density in the core and minimize density in the periphery, without regard for the density of ties between these regions. This is the model we recommend. We formalize the idea as follows (Eq. (4)), where ''.'' indicates a missing value:

$$\boldsymbol{\delta}_{ij} = \begin{Bmatrix} 1 \text{ if } c_i = \text{CORE and } c_j = \text{CORE} \\ 0 \text{ if } c_i = \text{PERIPHERY and } c_j = \text{PERIPHERY} \\ . \text{ otherwise} \end{Bmatrix} \tag{4}$$

Applying this model to the journal co-citation data, we obtain the partition shown in Table 6, which has a correlation of 0.860.

Since in this model no restraints are placed on the density of the core-to-periphery and periphery-to-core blocks, there is no reason why the model cannot handle asymmetric data. The journal co-citation data used above were artificially symmetrized. If we do not symmetrize, the results are as follows Table 7.

Table 6
Alternative core/periphery model

		6 C	7 C	13 J	16 S	15 S	20 S	17 S	4 C	9 F	8 C	1 A	2 A	3 B	14 P	5 C	11 J	12 J	18 S	19 S	10 I
6	CW	1	1	1	1	1	1	1	1						1	1					
7	CYSR	1	1		1	1	1	1	1	1					1	1					
13	JSWE	1		1	1	1	1	1					1	1					1	1	
16	SSR	1	1	1	1	1	1	1			1		1					1	1		
15	SCW	1	1	1	1	1	1	1	1	1	1		1			1	1		1	1	
20	SWRA	1	1	1	1	1	1	1					1								
17	SW	1	1	1	1	1	1	1	1	1	1	1	1	1	1		1		1	1	1
4	CAN	1	1			1		1	1						1						
9	FR		1			1		1		1											
8	CSWJ				1	1		1			1										
1	AMH							1				1									
2	ASW			1	1	1	1	1					1		1						
3	BJSW			1				1						1					1		
14	PW	1	1					1	1				1		1						
5	CCQ	1	1			1										1					
11	JGSW					1		1									1				
12	JSP				1													1			
18	SWG			1	1	1		1						1					1	1	
19	SWHC			1		1		1											1	1	
10	IJSW							1													1

The composition of the core is quite similar to what we found when we had symmetrized the data. However, there are certain notable exceptions. For example, the journal ''CYSR'' moves out of the core. This makes sense because although CYSR has outgoing ties with most of the core, it has only one incoming tie from anywhere. Thus, its relational style is more like a periphery member than a core member; it is in fact a particular type of peripheral member that Burt (1976) has referred to as a ''sycophant''.

It is also worth noting that the density of the bottom left block (periphery to core) is much higher than the density of the top right block (core to periphery). This is consistent with an intuitive notion of coreness associated with directed data. Essentially, we have a prestigious group, the core, that ''nominates'' only other prestigious actors. Then we have a nonprestigious group, the periphery, which also nominates only the prestigious actors. No one nominates nonprestigious actors, including themselves.

The discrete model can also handle valued data, in which case maximizing the correlation between the binary ideal matrix and the valued observed data is equivalent to running a *t*-test for the difference in means between the core-to-core ties and the periphery-to-periphery ties. A valued network has a core/periphery structure to the extent that the difference in means across blocks is large relative to the variation within blocks.

Table 7
Asymmetric core/periphery model
Correlation: 0.826

		6 C	17 S	13 J	16 S	15 S	20 S	2 A	4 C	9 F	8 C	1 A	12 J	3 B	14 P	5 C	11 J	7 C	18 S	19 S	10 I
6	CW	1	1	1	1	1			1						1			1			
17	SW	1	1	1	1	1	1	1							1				1	1	
13	JSWE		1	1	1	1	1	1												1	
16	SSR	1	1	1	1	1	1	1													
15	SCW	1	1	1	1	1	1	1	1	1	1						1		1	1	
20	SWRA	1	1	1	1	1	1														
2	ASW		1	1	1		1	1							1						
4	CAN	1	1			1			1												
9	FR		1			1				1											
8	CSWJ		1		1	1					1										
1	AMH		1									1									
12	JSP				1								1								
3	BJSW		1	1										1							
14	PW	1							1						1						
5	CCQ	1				1										1					
11	JGSW		1			1											1				
7	CYSR	1	1		1	1	1		1	1					1	1		1			
18	SWG		1	1	1	1								1					1	1	
19	SWHC		1			1														1	
10	IJSW		1																		1

An empirical example is provided by the raw citation data (see Table 8). The partition found by the genetic algorithm puts three journals, SSR, SW and SCW, in the core, and all others in the periphery. Ignoring the diagonal, the correlation with the ideal matrix is 0.81.

4. Continuous model

One limitation of the partition-based approach presented above is the excessive simplicity of defining just two classes of nodes: core and periphery. To remedy this, we could introduce a three-class partition consisting of core, semiperiphery, and periphery, as world system theorists have done, or try partitions with even more classes. This approach is feasible, but specifying the ideal blockmodel that best captures the notion of a core/periphery structure is relatively difficult, as there are many reasonable structures to choose from. The problem becomes exponentially more difficult as the number of classes is increased. [6]

[6] However, in cases where theoretical considerations clearly point to one structure over another, this would be a fruitful avenue to explore.

Table 8
Valued citation data

	16 SSR	17 SW	15 SCW	4 CAN	3 BJS	1 AMH	2 ASW	8 CSW	9 FR	10 IJS	11 JGS	12 JSP	13 JSW	14 PW	5 CCQ	6 CW	7 CYS	18 SWG	19 SWH	20 SWR
16 SSR	105	106	36				21	20				7	16			17	14	7		39
17 SW	106	356	124	8	9	3	73	45	9	3	18		58	19		58	28	40	43	44
15 SCW	36	124	149	6			8	47	18		16		21		3	32	8	9	20	18
4 CAN		8	6	109										7		9	12			
3 BJSW		19			95								13					2		
1 AMH		3				13														
2 ASW	21	73	8				70						8	13						20
8 CSWJ	20	45	47					40												
9 FR		9	18						205								4			
10 IJSW		3								6										
11 JGSW		18	16								9									
12 JSP	7											35								
13 JSWE	16	58	21		13		18						104			11		9	7	24
14 PW		19		7			13							9		7	6			
5 CCQ			3												92	12	5			
6 CW	17	58	32	9									11	7	12	187	70			8
7 CYSR	14	28	8	12					4					6	5	70	26			5
18 SWG	7	40	9		2								9					41	9	
19 SWHC		43	20										7					9	86	
20 SWRA	39	44	18				20						24			8	5			40

An alternative approach is to abandon the discrete model altogether in favor of a continuous model in which each node is assigned a measure of ''coreness''. In a Euclidean representation, this would correspond to distance from the centroid of a single point cloud. If we assume that the network data consist of continuous values representing strengths or capacities of relationships, an obvious approach is to continue using correlation to evaluate fit, but define the structure matrix as follows:

$$\boldsymbol{\delta}_{ij} = c_i c_j \tag{5}$$

where $\boldsymbol{C}$ is a vector of nonnegative values indicating the degree of coreness of each node. Thus, the pattern matrix has (a) large values for pairs of nodes that are both high in coreness, (b) middling values for pairs of nodes in which one is high in coreness and the other is not, and (c) low values for pairs of nodes that are both peripheral. Thus, the model is consistent with the interpretation that the strength of tie between two actors is a function of the closeness of each to the center, or perhaps the gregariousness of each actor. This is the same situation found in factor analysis, where the correlations among a set of variables are postulated to be a function of the correlation of each to the latent factor (Nunnally, 1978), and in consensus analysis (Romney et al., 1986), where agreements among pairs of takers of a knowledge test is seen as a function of the knowledge possessed by each one. Thus, when the continuous model fits a given dataset, it provides an extremely parsimonious model of all pairwise interactions.

It should be noted that if the values of $\boldsymbol{C}$ are constrained to 1's and 0's, Eq. (5) reproduces one of the discrete models presented earlier — the one in which there are no ties between the core and the periphery.

As with the partition approach, we can use the basic formulation of the core/periphery model to either estimate coreness empirically, or test a priori hypotheses about core/periphery structures. Sections 4.1 and 4.2 consider each of these in turn.

4.1. Estimating coreness empirically

The objective is to obtain values of $\boldsymbol{C}$ so as to maximize the correlation between the data matrix and the pattern matrix associated with Eq. (5). To accomplish this, we have written a simple computer program using a standard Fletcher–Powell (Press et al., 1989) function maximization procedure. The program simply finds a set of values c_i such that the matrix correlation between $c_i c_j$ and the data matrix is maximized.

As an empirical example, we return to the journal co-citation data provided by Baker (1992), using the data in valued, nondichotomized form, and symmetrized by choosing the larger of a_{ij} and a_{ji}. After calculating coreness for each journal, the correlation between the data and the pattern matrix was 0.917, indicating a good fit of the core/periphery model. We then sorted the rows and columns of the data matrix according to descending values of coreness. The result (Table 9) provides visual confirmation of a basic core/periphery structure, together with a few ties that do not fit the pattern (e.g., journals CAN and CCQ have ''unusually close'' relationships with journals CW and CYSR). It can be seen that the three journals with the highest coreness values, SW, SCW and SSR, are the same journals identified by the discrete model as comprising the core.

Table 9
Citation data sorted by coreness

	SW	SCW	SSR	ASW	CW	JSWE	SWRA	CSWJ	SWHC	CYSR	SWG	JGSW	PW	BJSW	FR	CAN	CCQ	AMH	IJSW	JSP	Core
SW		124	106	73	58	58	44	45	43	28	40	18	19	19	9	8	0	3	3	0	17.5
SCW	124		36	8	32	21	18	47	20	8	9	16	0	0	18	6	3	0	0	0	6.8
SSR	106	36		21	17	16	39	20	0	14	7	0	0	0	0	0	0	0	0	7	5.7
CW	73	8	21		0	18	20	0	0	0	0	0	13	0	0	0	0	0	0	0	3.5
JSWE	58	32	17	0		11	8	0	0	70	0	0	7	0	0	9	12	0	0	0	3.4
ASW	58	21	16	18	11		24	0	7	0	9	0	0	13	0	0	0	0	0	0	3.2
SWRA	44	18	39	20	8	24		0	0	5	0	0	0	0	0	0	0	0	0	0	2.8
SWHC	45	47	20	0	0	0	0		0	0	0	0	0	0	0	0	0	0	0	0	2.7
CSWJ	43	20	0	0	0	7	0	0		0	9	0	0	0	0	0	0	0	0	0	2
SWG	28	8	14	0	70	0	5	0	0		0	0	6	0	4	12	5	0	0	0	1.9
CYSR	40	9	7	0	0	9	0	0	9	0		0	0	2	0	0	0	0	0	0	1.9
JGSW	18	16	0	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0	0	0.9
PW	19	0	0	13	7	0	0	0	0	6	0	0		0	0	7	0	0	0	0	0.9
BJSW	19	0	0	0	0	13	0	0	0	0	2	0	0		0	0	0	0	0	0	0.8
FR	9	18	0	0	0	0	0	0	0	4	0	0	0	0		0	0	0	0	0	0.6
CAN	8	6	0	0	9	0	0	0	0	12	0	0	7	0	0		0	0	0	0	0.5
CCQ	0	3	0	0	12	0	0	0	0	5	0	0	0	0	0	0		0	0	0	0.2
IJSW	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0.1
AMH	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0.1

The matrix of expected values, $\mathbf{\Delta}$, is given as Table 10. Note that because the fit criterion is a correlation coefficient, the absolute values need not resemble the input data in scale: it is only the pattern that matters.

Since $\mathbf{\Delta}$ is constructed as a cross-products matrix, it can be embedded without distortion in a Euclidean space of no more than $N-1$ dimensions. Hence, we can use metric multidimensional scaling procedures (Gower, 1967) to visualize the structure of the matrix. A scaling of the matrix in Table 10 is shown in Fig. 4. It can be seen in the figure that as we consider successively wider concentric circles, centered at the centroid, the average distance among points within the circles increases monotonically with the distance from the center. This is a defining characteristic of a core/periphery structure. It means that in a core/periphery structure, the strength of relationship between any two actors is entirely a function of the extent to which each is associated with the core. [7]

This multiplicative characterization of the core/periphery concept is particularly attractive because it has close links with other mathematical models. Consider, for example, the algorithm we have described for computing $\boldsymbol{C}$ and measuring the fit of the core/periphery model. Let us assume that the data matrix is symmetric, and the values along the diagonal are meaningful. Furthermore, let us allow that instead of maximizing the correlation between the data matrix $\mathbf{A}$ and the pattern matrix $\mathbf{\Delta}$, we are willing to *minimize* the sum of squared differences between the two matrices. Then the vector $\boldsymbol{C}$ we are looking for is the principal eigenvector of $\mathbf{A}$. Besides the theoretical benefits of linking coreness to a well-known mathematical property of matrices, this linkage also means that we can make use of well-known and enormously efficient analytical procedures for finding eigenvectors instead of using optimization algorithms. The use of eigenvectors also suggests an additional measure of fit: the relative size of the principal eigenvalue. [8]

It should also be noted that if the diagonals of the data matrix are not meaningful, the task becomes isomorphic with some forms of common factor analysis, and we can use standard factor analytic procedures such as the MINRES algorithm of Comrey (1962) to estimate the values of $\boldsymbol{C}$. Like the cultural consensus model of Romney et al. (1986), our application of factor analysis is to actors rather than variables, and the coreness scores may be seen as a latent relational profile that all actors resemble to some degree. This factor may be seen as the prime ordering agent in the network so that, aside from the relationship to the core, all associations occur at random. In the language of chaos theory, the coreness vector can be seen as an attractor for each of the actors.

The continuous model also resembles the loglinear model of independence. When independence fits, we have a core/periphery structure, although the converse is not necessarily true. From the point of view of trying to maximize ρ, the difference between the two models is that in the independence model the values for $\boldsymbol{C}$ are constrained to be

[7] However, we will not ordinarily observe this principle to hold perfectly in two-dimensional MDS representations because of high stress: exact representations of core/periphery structures require almost as many dimensions as points. Hence in Fig. 4 there are some pairs of points on the periphery that are too close together given their distance from the core.

[8] It also suggests the possibility of using multiple eigenvectors to analyze networks with multiple cores (Breiger, personal communication); however, this is beyond the scope of this paper.

Table 10
Expected values for co-citation data

	SW	SCW	SSR	CW	JSWE	ASW	SWRA	SWHC	CSWJ	SWG	CYSR	JGSW	PW	BJSW	FR	CAN	CCQ	IJSW	AMH	JSP
SW	307	119	100	59	56	61	50	35	47	32	34	16	16	15	11	9	3	2	2	2
SCW	119	46	39	23	22	24	19	14	18	13	13	6	6	6	4	4	1	1	1	1
SSR	100	39	33	19	18	20	16	12	15	11	11	5	5	5	4	3	1	1	1	1
CW	59	23	19	11	11	12	10	7	9	6	7	3	3	3	2	2	1	0	0	0
JSWE	56	22	18	11	10	11	9	6	9	6	6	3	3	3	2	2	1	0	0	0
ASW	61	24	20	12	11	12	10	7	9	6	7	3	3	3	2	2	1	0	0	0
SWRA	50	19	16	10	9	10	8	6	8	5	5	3	3	2	2	2	1	0	0	0
SWHC	35	14	12	7	6	7	6	4	5	4	4	2	2	2	1	1	0	0	0	0
CSWJ	47	18	15	9	9	9	8	5	7	5	5	2	2	2	2	1	0	0	0	0
SWG	32	13	11	6	6	6	5	4	5	3	4	2	2	2	1	1	0	0	0	0
CYSR	34	13	11	7	6	7	5	4	5	4	4	2	2	2	1	1	0	0	0	0
JGSW	16	6	5	3	3	3	3	2	2	2	2	1	1	1	1	1	0	0	0	0
PW	16	6	5	3	3	3	3	2	2	2	2	1	1	1	1	0	0	0	0	0
BJSW	15	6	5	3	3	3	2	2	2	2	2	1	1	1	1	0	0	0	0	0
FR	11	4	4	2	2	2	2	1	2	1	1	1	1	1	0	0	0	0	0	0
CAN	9	4	3	2	2	2	2	1	1	1	1	1	0	0	0	0	0	0	0	0
CCQ	3	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
IJSW	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
AMH	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
JSP	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

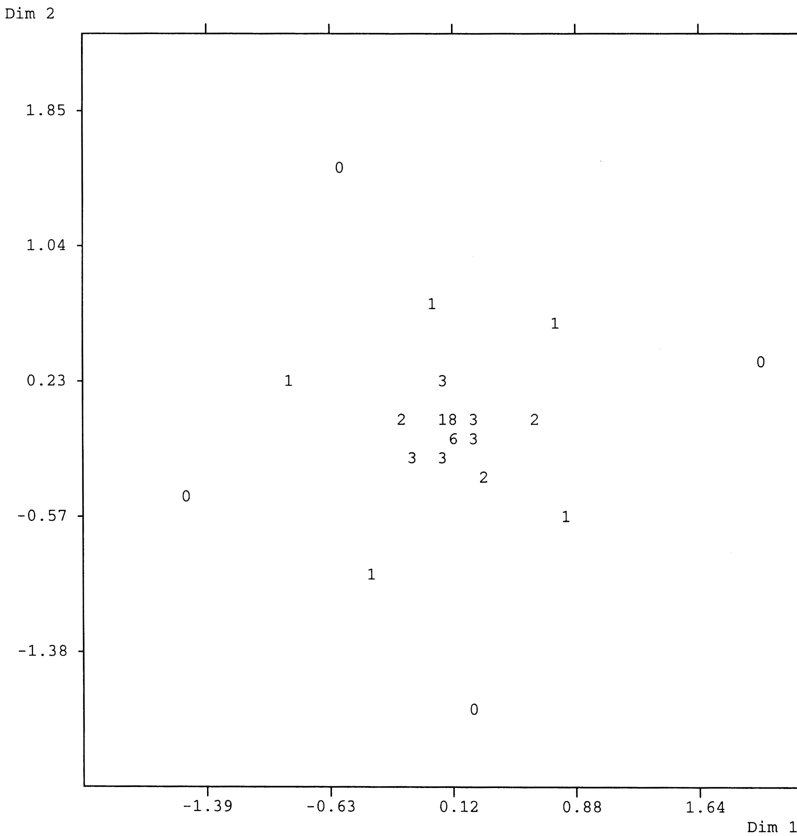

Fig. 4. MDS of core/periphery expected values for co-citation data. Points are labeled by their coreness scores.

row and column marginals, while in the core/periphery model we may use any values that maximize the correlation (not the chi-square nor likelihood ratio statistic) between the expected values and the observed.

The similarity with the model of independence brings up a potentially counterintuitive property of the multiplicative core/periphery model, which is that the conditions of the model are satisfied by networks in which all actors are in the core, as well as networks in which all actors are in the periphery. Hence an adjacency matrix of all 1's is consistent with the core/periphery model, even though no core may appear to exist. [9] The only data that really violate the model are networks that contain distinct, largely exclusive, subgroups. In such networks, actors with high degree need not be connected to each other, as required in a core periphery structure.

[9] Actually, it is the periphery that does not exist — all nodes are in the core.

The multiplicative coreness model clearly applies to valued network data. It is not quite as clear whether it should apply to dichotomous data; the expected values are normally continuous and the data are dichotomous, so the correlation coefficient that measures the fit of the model cannot achieve its maximum value of unity. This does not cause the coreness algorithm any problems, but makes it difficult to evaluate the fit of the model: a correlation of 0.4 may be small under normal circumstances, but not when the maximum possible is 0.5. Unfortunately, without a theory of how ties are generated in the kind of network being studied, it will not usually be possible to calculate the maximum.

An alternative way to formulate the model is to define a threshold value to dichotomize the pattern matrix. For example:

$$\boldsymbol{\delta}_{ij} = f(c_i c_j)$$

$$f(c_i c_j) = \begin{Bmatrix} 1 \text{ if } c_i c_j > t \\ 0 \text{ otherwise} \end{Bmatrix} \tag{6}$$

Thus, the pattern matrix has 1's for pairs of nodes that are both high in coreness and has 0's for pairs of nodes that are both peripheral. Depending on the value of the threshold parameter t, the core/periphery and periphery/core regions contain either all ones, all zeros, or a combination of both (reproducing the models represented by Eqs. (3) and (4)). Note that if the vector $\boldsymbol{C}$ is dichotomous rather than truly continuous, we reproduce the partition models of the previous section. In practice, we can specify t in advance, or estimate it from the data — along with the values of $\boldsymbol{C}$ — so as to maximize the correlation coefficient.

Another approach would be to conceive of the ties as the result of a probabilistic process dependent on $c_i c_j$. The function $f(c_i c_j)$ might be specified as a logistic of the general form

$$\Pr(a_{ij} = 1) = \frac{e^{\alpha + \beta c_i c_j}}{1 + e^{\alpha + \beta c_i c_j}} \tag{7}$$

where α and β are parameters to be estimated. Many variations on Eq. (7) are possible. In general, this approach is aesthetically pleasing, but it is important to remember that without a theory of how ties are formed, there is no reason to choose this particular response function. Again, in a given application it may be possible to choose a particular function with some confidence, but it is doubtful that we can do this in the general case where the nature of dependencies among ties is unknown.

4.2. Coreness and centrality

It could hardly escape notice that the multiplicative core/periphery model, when phrased as an eigenvector, is precisely the measure of centrality of Bonacich (1987). Furthermore, it is closely related to degree — another measure of centrality. The question then arises, is coreness different from centrality or are we simply introducing a new approach to centrality? It is interesting to note in this regard that in the sociological literature, empirical studies of core/periphery structures almost never make use of network centrality measures. For example, in the world systems/dependency literature, several researchers have used blockmodeling (Snyder and Kick, 1979; Nemeth and Smith, 1985; Smith and White, 1992) to classify countries as core, periphery and semiperiphery, but none have used centrality measures.

It is true that all actors in a core are necessarily highly central as measured by virtually any measure (except when the model fits vacuously). However, the converse is not true, as not every set of central actors forms a core. For example, it is possible to collect a set of the *n* most central actors in a network, according to some measure of centrality (say, closeness or degree), and yet find that the subgraph induced by the set contains no ties whatsoever — an empty core. This is because each actor may have high centrality by being strongly connected to different cohesive regions of the graph and need not have any ties to each other.

Our view, then, is that all coreness measures are centrality measures, but the converse is not necessarily true. For example, the betweenness-based measures of centrality (Anthonisse, 1971; Freeman, 1979; Freeman et al., 1991; Friedkin, 1991) will assign high values to actors who are not strongly connected to a core group of people, but who link two otherwise unconnected regions of a network. Coreness measures do not do this.

From a theoretical point of view, the key difference between a centrality measure and a coreness measure is that coreness carries with it a model of the pattern of ties in the network as a whole. The coreness measure is only interpretable to the extent that the model fits. In contrast, a centrality measure is interpretable no matter what the structure of the network. For example, closeness centrality measures the total graph theoretic distance of a node to all others. A node's closeness centrality can be used to predict the time that messages originating at random nodes throughout the network will take to reach that node. The measure holds this interpretation no matter what the structure of the network.

5. Conclusion

This paper sets forth a set of ideal images of core/periphery structures, then develops measures of the extent to which real networks approximate these images. These measures are used as the basis for tests of a priori hypotheses and for optimization algorithms to detect core/periphery structures.

What is missing in this paper is a statistical test for the significance of the core/periphery structures found by the algorithms. We know how well the models fit, but we do not know how easy it is to obtain a fit as good as actually observed by chance alone. To develop such a test, of course, we need additional theory about how network ties are formed — otherwise, we cannot construct a sensible baseline model to compare against. For example, we could assume that ties occur randomly with constant probability equal to the density of the observed network. We could then calculate the chance of obtaining fits as large as actually observed. But that would mean that our data would implicitly be compared with networks that have very different characteristics than our observed network. For instance, our network may show strong reciprocity biases (if *i* chooses *j*, then *j* chooses *i*) because of the nature of the relation being studied. But the random networks do not have this constraint unless we deliberately impose it. Unfortunately, we do not know in general which constraints should be imposed — row and column marginals? Degree of transitivity? Network analysts do not study a homogeneous set of structures. Some researchers study friendship ties among children, others agonistic behavior among primates, still others joint ventures and personnel flows among corporations. Some network data are valued (representing anything like capacities, flows, strengths, costs, probabilities, frequencies, etc), others directed, some have meaningful reflexive ties — in short, network data arise from a variety of social and sampling processes. It seems unlikely that the same baseline models would be appropriate in all these cases. It seems wiser to develop different chance models for every dataset as the need arises. A similar point is made by Friedkin (1991) and Skvoretz (1991).

As a final point for reflection, it is interesting to consider that to fit a core/periphery model is to reduce a complex dyadic variable — a network — to a single attribute of actors. Network researchers tend to disdain ''attribute data'' (Wellman, 1988, p. 31). The complaint is not that we compute from the pattern of network relations a single summary value that describes each actor's position. This is what any centrality measure does and is completely unremarkable. Rather, the core/periphery model says that all ties in the network (error aside) are the result of a single attribute. In effect, this denies the necessity for having collected complex relational data (a matrix), since much simpler data (a vector) contains the same information content. This goes against the grain for network analysts, who like to think that relational data are richer and reveal emergent properties that mere attributes of actors simply cannot capture (e.g., see Wellman, 1988). When the core/periphery model fits, it means that to a certain extent, we do not need to know who is connected to whom. All we need is a single actor attribute. It is the same thing as when we fit the model of independence on a contingency table and find that it fits. As good scientists and structuralists we should be happy to find such a parsimonious description of our data. But, more likely, we are disappointed that nothing more ''interesting'' is going on.

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