Random variable and Probability distribution

Random variables:

In a random experiment if a real variable is associated with every experiment then it is called random variable In other words a random variable is a function that assigns a real no. to every sample ~~space~~ point, in the sample space of a random experiment.

Random variables are usually denoted by x, y, z ... and it may be noted that diffrent random variables may be associated with the same sample space S.

The set of all real nos of a random variable x is called Range of x.

Eg: 1) while tossing a coin suppose value 1 is associated with outcome head and 0 is associated with outcome tail. Then we have the sample space

$$S = \{H, T\} \quad \text{Range of } x = \{0, 1\}$$

where x is random variable and $X(H) = 1$ $\quad X(T) = 0$

2) Suppose a coin is tossed twice we shall associate 2 diffrent random variables X & Y as follows when sample space $S = \{HH, HT, TH, TT\}$

X = No of heads in the out come

Then

outcome	HH	HT	TH	TT
X	2	1	1	0

Range of $X = \{0, 1, 2\}$

Suppos Y be number of tails in the outcome

Then

outcome	HH	HT	TH	TT
Y	0	1	1	2

Range of $Y = \{0, 1, 2\}$

<u>Eg 3</u> Let the random experiment be throwing a pair of die and the sample space S associated with it is the set of all pairs of numbers chosen from 1 to 6

ie $S = \{(x, y) \mid x, y$ being set of nos $1, 2, 3, 4, 5, 6\}$

To each out-come (x, y) of S let us associate a random variable $X = x + y$

Then range of $X = \{2, 3, \ldots 12\}$ corresponds to 36 out comes in S ~~an~~ namely

$(1, 1)$ $(1, 2)$ - - - $(6, 6)$

we have $X(1, 1) = 2$ $X(1, 2) = 3$ . . $X(6, 6) = 12$

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Discrete and continuous random variables

If a random variable takes finite number of values then it is called a discrete random variable

If a random variable takes non countable infinite no. of values then it is called continuous random variable.

Eg: 1) Tossing a coin and observing outcomes → discrete

2) Tossing a die and observing numbers of the face → discrete

3) Conducting a survey of life of electric bulbs – continuous

4) weiht of objects on earth – continous.

Discrete Probability distributions

For each value x_i of a discrete random variable X we assign a real no $P(x_i)$ such that

(i) $P(x_i) \geq 0$ (ii) $\sum_i P(x_i) = 1$

then the function $P(x)$ is called probability function

If the probability that X takes value x_i is P_i

then $P(X = x_i) = P_i$ or $P(x_i)$

The set $[x_i, P(x_i)]$ is called discrete probability distribution. The function $P(x)$ is called probability density function (p.d.f) or probability mass function (p.m.f)

The distribution function $F(x)$ defined by

$F(x) = P(X \le x) = \sum_{i=1}^{n} P(x_i)$ is called the cumulative distribution function (c.m.f)

We have mean $\bar{x} = \dfrac{\sum_i f_i x_i}{\sum_i f_i}$

Variance $\sigma^2 = \dfrac{\sum_i f_i (x_i - \bar{x})^2}{\sum_i f_i}$

We can as well as define mean & variance of discrete prob distribution. In this case $P(x_i)$ corresponds to f_i and $\sum_i f_i = \sum_i P(x_i) = 1$

$\therefore$ Mean of discrete prob distribution

$\mu = \sum_i x_i P(x_i)$

Variance $V = \sum_i (x_i - \mu)^2 P(x_i)$

Std deviation $\sigma = \sqrt{V}$

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Prob: A coin is tossed twice. A random variable X represent the number of heads turning up. Find discrete prob distribution of X. Also find its mean and variance.

Ans: $S = \{HH, HT, TH, TT\}$ Let X represents random variable corresponds to heads. Then range $X = \{0, 1, 2\}$

$P(HH) = 1/4 \quad P(HT) = 1/4 \quad P(TH) = 1/4 \quad P(TT) = 1/4$

$P(x=0$, no head$) = 1/4 \quad P(x=1$, ie 1 head$) = 1/4 + 1/4 = 1/2$

$P(x=2$, ie 2 heads$) = 1/4$

∴ Discrete prob distribution

$X = x_i$	0	1	2
$P(x_i)$	$1/4$	$1/2$	$1/4$

Mean $\mu = \sum_i x_i P(x_i) = 0 \times 1/4 + 1 \times 1/2 + 2 \times 1/4 = 1$

Variance $V = \sum_i (x_i - \mu)^2 P(x_i)$

$= (0-1)^2 \times 1/4 + (1-1)^2 \times \frac{1}{2} + (1-2)^2 \times 1/4$

$= 1/4 + 0 + 1/4 = 1/2$

Prob: A random experiment of tossing a die is performed twice. Random variable X represents the sum of two numbers turning up. Find prob distribution mean variance & S.D

Ans: $S = \{ (x, y)$ where $x = 1, 2, \cdots 6 \quad y = 1, 2, \cdots 6 \}$

ie $S = \{ (1,1)\ (1,2) \cdots (6,6) \}$

The set of values of the random variables X defined as sum of two numbers

$\therefore X = \{ 2, \cdots 12 \}$.

Then

Elements of S	$X = x_i$	total cases
1. (1,1)	$x_1 = 2$	1
2) (1,2) (2,1)	$x_2 = 3$	2
3) (1,3)(3,1) (2,2)	$x_3 = 4$	3
4) (1, 4) (4,1) (3,2) (2,3)	$x_4 = 5$	4
5) (1,5) (5,1) (2,4) (4,2) (3,3)	$x_5 = 6$	5
6) (1,6) (6,1) (2,5) (5,2) (3,4) (4,3)	$x_6 = 7$	6
7) (2,6) (6,2) (3,5) (5,3) (4,4)	$x_7 = 8$	5
8) (3,6) (6,3) (4,5) (5,4)	$x_8 = 9$	4
9) (4,6) (6,4) (5,5)	$x_9 = 10$	3
10) (5,6) (6,5)	$x_{10} = 11$	2
11) (6,6)	$x_{11} = 12$	1

$P(x_1) = \frac{1}{36} \qquad P(x_2) = \frac{2}{36} \cdots \qquad P(x_{11}) = \frac{1}{36}$

$X = x_i$	2	3	4	5	6	7	8	9	10	11	12
$P(x_i)$	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$

$\mu = \sum_i x_i P(x_i) = 7$

Variance: $\sum_i (x_i - \mu)^2 P(x_i) = \frac{35}{6}$

$S.D = \sqrt{V} = \sqrt{\frac{35}{6}}$

Prob: Find value of k such that following represent prob distribution. Hence find mean & variance. Also find $P(x \le 1)$, $P(x > 1)$ & $P(-1 < x \le 2)$

x_i	-3	-2	-1	0	1	2	3
$P(x_i)$	k	$2k$	$3k$	$4k$	$3k$	$2k$	k

Ans: We have $\sum P(x_i) = 1$

$\therefore k + 2k + 3k + 4k + 3k + 2k + k = 1$ $16k = 1$ $\therefore k = 1/16$

$\therefore x_i$	-3	-2	-1	0	1	2	3
$P(x_i)$	$1/16$	$2/16$	$3/16$	$4/16$	$3/16$	$2/16$	$1/16$

$\mu = \sum x_i P(x_i) = 0$

Variance $V = \sum (x_i - \mu)^2 P(x_i) = \frac{1}{16}(9 + 8 + 3 + 0 + 3 + 8 + 9)$

$= 5/2$

$SD = \sqrt{5/2}$

$P(x_i \le 1) = P(-3) + P(-2) + P(-1) + P(0) + P(1) = 13/16$

$P(x > 1) = P(2) + P(3) = 3/16$ $P(-1 \le x \le 2) = 9/16$

Prob: ~~To~~ Find value of k for following p.d.f.

Also find $P(x \geq 5)$ $P(3 < x \leq 6)$

x_i	0	1	2	3	4	5	6
$P(x_i)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Ans: $\sum_i P(x_i) = 1$ $\therefore$ $49k = 1$ $\therefore k = \frac{1}{49}$

$\therefore P(x \geq 5) =$ $P(5) + P(6) = \frac{24}{49}$

$P(3 < x \leq 6) =$ $P(4) + P(5) + P(6) = \frac{33}{49}$

Prob: A random variable x has following prob function for various values of x

x_i	0	1	2	3	4	5	6	7
$p(x_i)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find k Find $P(x < 6)$, $P(x \geq 6)$ $P(3 < x \leq 6$

Also find pdf of x

Ans: We have $\sum_i P(x_i) = 1$

$\therefore$ $10k^2 + 9k = 1$ or $10k^2 + 9k - 1 = 0$

$\therefore k = 1/10$ $k = -1$

If $k = -1$ then $P(x_i)$ becomes -ve which is not possible

$\therefore$ ~~$P(x_i)$~~ $= k = 1/10$

$\therefore$ Pdf is

x_i	0	1	2	3	4	5	6	7
$P(x_i)$	0	$1/10$	$1/5$	$1/5$	$3/10$	$1/100$	$1/50$	$\frac{17}{100}$

$\therefore P(x<6) = P(0)+P(1)+P(2)+P(3)+P(4)+P(5) = 0.81$

$P(x \geq 6) = P(6)+P(7) = 0.19$

$P(3<x\leq 6) = P(4)+P(5)+P(6) = 0.33$

Distribution function of X is $f(x) = P(X \leq x)$

x	0	1	2	3	4	5	6	7
$f(x)$	0	0.1	0.3	0.5	0.8	0.81	0.83	1

Prob: A random variable x takes values $-3,-2,-1,0,1,2,3$ such that $P(x=0) = P(x<0)$ and $P(x=-3) = P(x=-2) = P(x=-1) = P(x=1) = P(x=2) = P(x=3)$ Find prob distribution

Ans: Let the prob distribution be $[X, P(X)]$

X	-3	-2	-1	0	1	2	3
$p(x)$	P_1	P_2	P_3	P_4	P_5	P_6	P_7

By data given $P(X=0) = P(X<0)$ ~~$\Rightarrow P_4 = P_1+P$~~

$\Rightarrow P(X=0) = P(X=-1)+P(X=-2)+P(X=-3)$

ie $P_4 = P_3+P_2+P_1$ — ① Also by given data

$P_1 = P_2 = P_3 = P_5 = P_6 = P_7$ — ②

Further we must have

$P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 = 1$ — (3)

From (1) & (2) $\quad 3P_1 = P_4$

using (2) in (3) we get

$6P_1 + P_4 = 1$ but $P_4 = 3P_1$

$\therefore 9P_1 = 1 \quad \therefore P_1 = 1/9 \qquad P_4 = 3/9 = 1/3$

$\therefore X$	-3	-2	-1	0	1	2	3
$P(x)$	$1/9$	$1/9$	$1/9$	$1/3$	$1/3$	$1/9$	$1/9$.

Prob: A sealed box contains dozen of apples it was found that 3 apples are perished obtain prob distribution of number of apples perished when 2 apples drawn at random. Find mean & variance of this distribution.

Ans: Let x be the number of perished apples. Since 2 were drawn we have $x = 0, 1, 2$

Since 2 out of 12 (dozen) can be selected in $12C_2$ ways out of which 3 were perished and 9 are good apples.

$\therefore P(x=0) =$ Prob of getting 0 perished apple $= \dfrac{3C_0 \times 9C_2}{12C_2}$

$= 6/11$

$P(X=1)$ = Probability of getting 1 perished apple

$$= \frac{3C_1 \times 9C_1}{12C_2} = \frac{9}{22}$$

$P(x=2)$ = Prob of getting 2 perished apple

$$= \frac{3C_2 \times 9C_0}{12C_2} = 1/22$$

$\therefore X = x_i$	0	1	2
$P(x_i) = p_i$	$6/11$	$9/22$	$1/22$

$$\mu = \Sigma_i x_i \, P(x_i) = 1/2$$

$$\text{Variance} = \Sigma_i x^2 p_i - \mu^2 = \frac{15}{44}$$

Prob: If a random variable x takes values 1, 2, 3, 4 such that $2P(x=1) = 3P(x=2) = P(x=3) = 5P(x=4)$ Find prob distribution of x

Ans: Let the distribution as follows

x_i	1	2	3	4
$P(x_i)$	P_1	P_2	P_3	P_4

By data $2P_1 = 3P_2 = P_3 = 5P_4$

Also we have $P_1 + P_2 + P_3 + P_4 = 1$

we have $P_2 = \frac{2}{3} P_1$ $P_3 = 2P_1$ $P_4 = \frac{2}{5} P_1$

$\therefore P_1 + \frac{2}{3} P_1 + 2P_1 + \frac{2}{5} P_1 = 1$

$\frac{61}{15} P_1 = 1 \Rightarrow P_1 = \frac{15}{61}$ $\therefore P_2 = \frac{10}{61}$ $P_3 = \frac{30}{61}$ $P_4 = \frac{6}{61}$

$\therefore f(x) = \sum_{i=1}^{x} P(x_i)$ as follows

x_i	1	2	3	4
$P(x_i)$	$15/61$	$10/61$	$\frac{30}{61}$	$6/61$
$f(x)$	$15/61$	$25/61$	$55/61$	$61/61 = 1$

Formula: $a + ar + ar^2 + \cdots = \frac{a}{1-r}$

Prob: A random variable X has $P(x) = 2^{-x}$ $x = 1, 2 \ldots$

S.T $P(x)$ is prob fn Also find $P(X \text{ even})$

$P(X$ divisible by 3) and $P(X \geq 5)$

Ans: $P(x) = 2^{-x}$ $\therefore P(x) \geq 0$ for all x

$\sum_i P(x_i) = \sum_{x=1}^{\infty} \frac{1}{2^x} = \frac{1}{2} + \frac{1}{2^2} + \cdots = \frac{1/2}{1 - 1/2} = \frac{1/2}{1/2} = 1$

$\therefore P(x) = \frac{1}{2^x}$ is prob fn

Case(1) $P(X\text{-even}) = \sum_{x=2,4,6\ldots} P(x) = \frac{1}{2^2} + \frac{1}{2^4} + 1/2^6 + \ldots$

$$= \frac{\frac{1}{2^2}}{1-1/2^2} = \frac{1/4}{3/4} = 1/3$$

Case(2) $P(X \text{ divisible by } 3) = \sum_{x=3,6,9} \frac{1}{2^x} = 1/2^3 + 1/2^6 + 1/2^9 + \ldots$

$$= \frac{\frac{1}{2^3}}{1-1/2^3} = 1/7$$

Cas(3) $P(x \geq 5) = 1 - P(x<5)$

$$= 1 - \sum_{x=1}^{4} P(x) = 1 - \left(\frac{1}{2} + 1/2^2 + 1/2^3 + 1/2^4\right)$$

$$= \frac{1}{16}$$

Prob: If X is a discrete random variable for $1, 2, 3 \ldots$

$p(x) = \frac{1}{2}\cdot\left(\frac{2}{3}\right)^x$ Fin $P(X$ being an odd no$)$

Ans: $\sum P(x_i) = \sum_{x=1}^{\infty} \frac{1}{2}\cdot\left(\frac{2}{3}\right)^x$

$$= 1/2\left[\frac{2}{3} + \left(2/3\right)^2 + \ldots\right]$$

$$= 1/2\left[\frac{\frac{2}{3}}{1-2/3}\right] = 1 \quad \therefore P(x) \text{ is a prob fn}$$

Now $P(X \text{ being odd no}) = \sum_{x=1,3,5\ldots}' P(x) = \sum_{x=1,3,5\ldots}' \frac{1}{2}\cdot\left(\frac{2}{3}\right)^x$

$= \frac{1}{2}\left[\frac{2}{3} + \left(\frac{2}{3}\right)^3 + \ldots\right]$

$= \frac{1}{2}\left[\frac{2/3}{1-(2/3)^2}\right] = \frac{3}{5}$

Prob: X is a discrete random variable having $p(x)$ defined as follows

$$p(x) = \begin{cases} \frac{x}{15} & 1 \le x \le 5 \\ 0 & x > 5 \end{cases}$$

S.T $p(x)$ is a prob function.

Also find (i) $P(X=1 \text{ or } 2$ ~~(ii) $P(\frac{1}{2} < X < \frac{5}{2})$~~ and ~~$P(X>1)$~~

Ans: the prob distribution for is as follows

x	1	2	3	4	5
$p(x)$	$1/15$	$2/15$	$3/15$	$4/15$	$5/15$

$\therefore p(x) \geq 0$ & $\sum' p(x) = 1$

$P(X=1 \text{ or } X=2) = P(X=1) + P(X=2)$

$= 1/15 + 2/15 = 3/15 = 1/5$

~~$P(\frac{1}{2} < X < \frac{3}{2})$ and $P(X>1)$~~ =

Prob: The range of a random variable $X = \{1, 2, \ldots n\}$ and prob of X are kx find value of k and find mean & variance of p.d.f.

Ans: We have

x_i	1	2	---	n
$P(x_i) = P_i$	k	$2k$	---	nk

We have

$\therefore P(x) \geq 0 \qquad \Sigma_i P(X) = 1$

$\therefore k + 2k + \ldots + nk = 1$

$k(1 + 2 + \ldots + n) = 1 \quad \therefore k \dfrac{(n)(n+1)}{2} = 1$

$$k = \frac{2}{n(n+1)}$$

Mean $\mu = \Sigma_i x_i P(x_i)$

$= k + 2 \times 2k + 3 \times 3k + \ldots + n \times nk$

$= k(1 + 2^2 + \ldots + n^2)$

$= k \dfrac{n(n+1)(2n+1)}{6} = \dfrac{\cancel{n} \cdot 2}{\cancel{n(n+1)}} \times \dfrac{\cancel{n(n+1)}(2n+1)}{\cancel{6}\,3}$

$= \dfrac{2n+1}{3}$

Variance $V = \Sigma_i x_i^2 P_i - \mu^2$

$= 1^2 k + 2^2 \cdot 2k + \ldots + n^2 \cdot nk - \mu^2$

$= k(1^3 + \ldots + n^3) - \mu^2$

$$= 18 \frac{n^2(n+1)^2}{4} - \left(\frac{2n+1}{3}\right)^2$$

$$= \frac{2}{n(n+1)} \cdot \frac{n^2(n+1)^2}{4} - \frac{(2n+1)^2}{9}$$

$$= \frac{n^2+n-2}{18}$$

Bernoulli's Theorem:

A random experiment with only two possible outcomes categorized as sucess and failure is called a Bernoulli's trail.

Theorem:

Let p be prob of sucess & q be prob of failure Then prob of x sucess in n trails is equal to $nCx\ p^x q^{n-x}$

Eg: Suppose a coin is tossed 3 times and we wish to find prob of getting 2 heads Then we have HHT, THH, HTH

$\therefore$ Prob $= 3/8$

Using above thm

$n = 3$ (no of tosses) $\quad p = 1/2 \quad q = 1 - 1/2 = 1/2$

$x = 2$ (2 heads)

$\therefore nC_x \, p^x q^{n-x} = 3C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1 = \frac{3}{8}$

Discrete Probability distributions

1. Binomial Distribution

Let p be prob of success and q be prob of failure then prob success in n trials is given by

$$P(x) = nC_x \, p^x q^{n-x}$$

We have following prob distribution of $(x, P(x))$

x	0	1	2	...	...	n
$P(x)$	q^n	$nC_1 q^{n-1} p$	$nC_2 q^{n-2} p^2$			p^n

Mean $\mu = np$

Variance $= npq$

S.D $\sigma = \sqrt{npq}$.

Prob: Find binomial prob distribution for which mean $= 2$ variance $= \frac{4}{3}$

Ans:

From the formula $\mu = 2$ $\quad V = \frac{4}{3}$

$np = 2 \quad npq = \frac{4}{3}$

$\therefore \frac{npq}{np} = \frac{\frac{4}{3}}{2} = \frac{4}{6} = \frac{2}{3} \quad \therefore q = \frac{2}{3}$

$p = 1 - \frac{2}{3} = \frac{1}{3}$

$\therefore np = 2 \quad \therefore n \times \frac{1}{3} = 2 \quad \therefore n = 6$

$\therefore P(x) = {}^nC_x\, p^x\, q^{n-x}$

x	0	1	2	3
$P(x)$	$(2/3)^6$	$6C_1\, 1/3\, (2/3)^5$	$6C_2 (1/3)^2 (2/3)^4$	$6C_3 (1/3)^3 (2/3)^3$

4	5	6
$6C_4 (1/3)^4 (2/3)^2$	$6C_5 (1/3)^5 (2/3)$	$(1/3)^6$

<u>Prob</u>: When a coin is tossed 4 times find prob of getting (1) Exactly one head (2) at most 3 heads (3) atleast 2 heads.

Ans: $p = 1/2 \quad q = 1/2 \quad n = 4$

$P(x) = {}^nC_x\, p^x\, q^{n-x} = 4C_x (0.5)^x (0.5)^{4-x}$

(i) $P(1\text{ head}) = P(x=1) = 4C_1 (0.5)^1 (0.5)^3 = 0.25$

(ii) $P(\text{at most 3 heads}) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= 4C_0 (0.5)^0 (0.5)^4 + 4C_1 (0.5)^1 (0.5)^3 +$
$4C_2 (0.5)^2 (0.5)^2 + 4C_3 (0.5) (0.5)^3$

$= 0.9375$

(iii) $P(\text{Atleast 2 heads}) = 1 - (P(x=0) + P(x=1))$

$= \underline{0.6875}$

Prob' Prob that a pen manufactured from a factory to be defective is $\frac{1}{10}$. If 12 such pens are manufactured what is the prob that

(i) Exactly 2 are defective (ii) at least 2 are defective (iii) none of them is defective

Ans' $p = \frac{1}{10} = 0.1$, $q = 1 - p = 0.9$ $n = 12$

$$P(x) = nC_x \, p^x \, q^{n-x} = 12C_x \, (0.1)^x \, (0.9)^{12-x}$$

(1) Exactly 2 defective $P(x=2) = 12C_2 (0.1)^2 (0.9)^{12-2}$

$= 0.2301$

(2) Atleast 2 are defective

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - [\ 12C_0 (0.1)^0 (0.9)^{12} + 12C_1 (0.1)^1 (0.9)^{12-1}]$

$= 0.341$

(3) Prob no defective $= P(x=0) = 12C_0 (0.1)^0 (0.9)^{12}$

$= 0.2824$

Prob The prob that a person aged 60 years will live upto 70 yrs is 0.65 what is prob that out of 10 persons aged 60 at least 7 of them live up to 70

Ans: $p = ~~60~~\ 0.65 \quad q = 0.35 \quad n = 10$

$P(x) = 10C_x\, (0.65)^x\, (0.35)^{10-x}$

we have to find $P(x \geq 7)$

$= P(7) + P(8) + P(9) + P(10)$

$= 10C_7\, (0.65)^7\, (0.35)^3 + 10C_8\, (0.65)^8\, (0.35)^2 + \cancel{10C_9}$

$10C_9\, (0.65)^9\, (0.35) + (0.65)^{10} = 0.5138$

Prob: The number of telephone lines ~~by~~ busy at an instant of time is a binomial variate with prob 0.1 that a line is busy. If 10 lines are chosen at random, what is the prob that (i) no line is busy (ii) all lines are busy (iii) at least one line is busy (iv) at most 2 lines are busy

Pf: Let x denote the number of lines busy

$p = 0.1 \quad q = 1 - 0.1 = 0.9 \quad n = 10$

$P(x) = nC_x\, p^x\, q^{n-x} = 10C_x\, (0.1)^x\, (0.9)^{10-x}$

(1) no line busy $= P(0) = (0.9)^{10} = 0.3487$

(2) all lines busy $= P(10) = (0.1)^{10}$

(3) at least one line busy $= 1 - P(0) = 0.6513$

(4) At most 2 lines busy $= P(0) + P(1) + P(2)$
$= (0.9)^{10} + (0.9) \ldots = 0.9298$

Prob: In a quiz of answering "yes" or "no" what is prob of guessing 6 answers correct out of 10 questions? Also find the prob of the same if there are 4 options for correct answer

Ans: Let x denote the correct answer and we have in the first case

$p = 0.5 \qquad q = 0.5 \qquad n = 10$

$P(x) = 10C_x (0.5)^x (0.5)^{10-x} = 10C_x \left(\frac{1}{2}\right)^{10}$

$\therefore P(x \geq 6) = \frac{1}{2^{10}} \left[10C_6 + 10C_7 + 10C_8 + 10C_9 + 10C_{10}\right]$

$= 0.377$

In 2nd case $\quad p = \frac{1}{4} \quad q = \frac{3}{4}$

$P(x) = 10C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{10-x} = \frac{1}{4^{10}} \left[3^{10-x}\ 10C_x\right]$

$\therefore P(x \geq 6) = P(6) + P(7) + P(8) + P(9) + P(10) = 0.019$

Prob: If the mean & std deviation of no of correctly answered questions in a test of 4096 stud are 2.5 and $\sqrt{1.875}$. Find no of students answering (i) 8 or more questions (ii) 2 or less (iii) 5 questions.

Ans: ~~AP=~~ mean $\mu = np = 2.5$

$S.D = \sqrt{npq} = \sqrt{0.1875} \quad \therefore npq = 1.875$

$\therefore 2.5q = 1.875 \quad \therefore q = 0.75$

$p = 1 - q = 0.25$

$\therefore np = 2.5 \quad \therefore n = 10$

Let x denote correctly answered questions

$p(x) = {}^nC_x p^x q^{n-x} = {}^{10}C_x (0.25)^x (0.75)^{10-x}$

$\therefore p(x) = \frac{1}{4^{10}} \left[{}^{10}C_x \, 3^{10-x} \right]$

Since estimate needed for 4096 students we have

$4096\, p(x) = \frac{4096}{4^{10}} \left[{}^{10}C_x \, 3^{10-x} \right] = \frac{2^{12}}{2^{20}} \left[{}^{10}C_x \, 3^{10-x} \right]$

$= \frac{1}{256} \, {}^{10}C_x \, 3^{10-x} = f(x) \text{ [say]}$

(1) $f(8) + f(9) + f(10) = \frac{1}{256} \left[{}^{10}C_8 \, 3^2 + {}^{10}C_9 \, 3 + 1 \right]$

$= 1.703 \cong 2$

(2) $f(2) + f(1) + f(0) = \frac{1}{256} \left[{}^{10}C_2 \, 3^8 + {}^{10}C_1 3^9 + 3^{10} \right]$

$= 2153$

(3) $f(5) = \frac{1}{256} \, {}^{10}C_5 \, 3^5 = 239$

Prob: An air line knows that ~~0~~5% of the ppl making reservations will turn up. Consequently their policy to sell 52 tickets to a flight that can hold only 50 passengers. What is the prob that there will be a seat for ~~at~~ every passenger who turns up.

Ans: $p = 0.05 \quad q = 0.95 \quad n = 52$

$$p(x) = 52C_x (0.05)^x (0.95)^{52-x}$$

A seat is assured for every passenger who turns up if ~~there~~ no. of passengers who fail to turn up is more than or equal to 2

$$\therefore p(x \geq 2) = 1 - p(x < 2)$$

$$= 1 - [p(x=0) + p(x=1)]$$

$$= 1 - [(0.95)^{52} + 52 \times 0.05 (0.95)^{51}]$$

Prob: In 800 families with 5 children each how many of them expected to have

(i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys

(iv) at most 2 girls assuming prob for girls & boys equal

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p = prob of having boy = 0.5

q = prob of having girl = 0.5

$$p(x) = nC_x \; p^x \; q^{n-x}$$

$$= 5C_x (0.5)^x (0.5)^{5-x} = \frac{1}{2^5} 5C_x = \frac{5C_x}{32}$$

Since we have to calculate for 800 families

$$800\,P(x) = 800 \frac{5C_x}{32} = 25 \times 5C_x = f(x)$$

(1) $f(3) = 5C_3 \times 25 = 250$

(2) $f(0) = 5C_0 \times 25 = 25$

(3) $f(2) + f(3) = 25 \times 5C_2 + 25 \times 5C_3 = 500$

(4) At most 2 girls means

families can have 5 boys, 4 boys and 1 girl

or 3 boys and 2 girl

$f(5) + f(4) + f(3) = 25 \times 16 = 400$

Prob: Five dice were thrown 96 times and the number of times an odd number actually turned out in the experiment given Fit a binomial distribution to this data and find expected frequencies

No of dice showing 1 or 3 or 5	0	1	2	3	4	5
Observed freq	1	10	24	35	18	8

Ans: Let p be the prob of getting 1 or 3 or 5 $= 3/6 = 1/2$

$\therefore q = 1/2$

Hence x denote the number of times odd no turns out

$$P(x) = {}^nC_x \, p^x q^{n-x} = {}^5C_x (0.5)^x (0.5)^{5-x}$$

$$= \frac{1}{2^5} \, {}^5C_x$$

This is the binomial prob distribution function.

Since 5 dice were thrown 96 times expected freq. are obtained from $f(x) = 96\,P(x)$ for $x = 0, 1, 2 \ldots 5$

$$\therefore f(x) = 96 \times \frac{1}{2^5} \, {}^5C_x = 3 \times {}^5C_x$$

Hence $f(0) = 3 \times {}^5C_0 = 3$ $\quad f(1) = 15$ $\quad f(2) = 30$

$f(3) = 30$ $\quad f(4) = 15$ $\quad f(5) = 3$

Prob: 4 coins are tossed 100 times and following result were obtained. Fit a binomial distribution for the data and calculate theoretical freq.

No of heads	0	1	2	3	4
Freq	5	29	36	25	5

Ans: Let x denote the number of heads and f the corresponding frequency since data is in the form of freq distribution we first calculate mean

$$\mu = \frac{\sum f x}{\sum f} = \frac{0+29+72+75+20}{100} = 1.96$$

But $\mu = np \Rightarrow 1.96 = 4p \therefore p = 0.49 \quad q = 0.51$

Binomial distribution of prob fn is

$$P(x) = nC_x \, p^x q^{n-x} = 4C_x (0.49)^x (0.51)^{4-x}$$

Since 4 coins tossed 100 times

$$F(x) = 100\, P(x) = 100 \times 4C_x (0.49)^x (0.51)^{4-x}$$

$x = 0, 1, 2, 3$

$$F(0) = 100 \times 4C_0 (0.49)^0 (0.51)^{4-0} = 7$$

$F(1) = 26 \quad F(2) = 37 \quad F(3) = 24 \quad F(4) = 6$

Prob: A lot contains 1% of defective items what should be the number of items in a random sample so that the prob of finding atleast one defective item is 0.75?

Ans: Let p be the prob of defective item

By data $p = 0.01 \quad q = 0.99$

If x denotes a defective item

$$p(x) = n_{C_x}\, p^x q^{n-x} = n_{C_x} (0.01)^x (0.99)^{n-x}$$

We need to find n such that prob finding atleast one defective item is ≥ 0.75. That is to find n such that

$$p(x \geq 1) \geq 0.75$$

$$\therefore 1 - p(x < 1) \geq 0.75$$

$$1 - p(0) \geq 0.75 \qquad p(0) = n_{C_0} (0.01)^0 (0.99)^{n-0}$$

$$1 - (0.99)^n \geq 0.75 \qquad = (0.99)^n$$

$$\therefore 0.25 \geq 0.99^n$$

$$\therefore n \leq \frac{\log 0.25}{\log 0.99} = 137.935 \cong 138$$

<u>Prob</u>: Prob that a shooter hits a target is $1/3$. How many times he should shoot so that prob of hitting target is more than $3/4$

Ans:- $p = 1/3 \quad q = 2/3$

$$p(x) = n_{C_x}\, p^x q^{n-x} = n_{C_x} (1/3)^x (2/3)^{n-x}$$

$$p(x \geq 1) > 3/4$$

$$1 - p(x < 1) > 3/4$$

$$1 - p(0) > 3/4$$

$$1 - (2/3)^n > 3/4 \qquad \therefore (2/3)^n < 1/4$$

$$\therefore n \cong 4$$

Prob: S.T for a binomial distribution

$$p(x+1) = \frac{n-x}{x+1}\frac{p}{q}\,p(x)$$

Pf: $p(x) = nC_x\, p^x q^{n-x}$

$$p(x+1) = nC_{x+1}\, p^{x+1} q^{n-(x+1)}$$

$$= \frac{n!}{(x+1)!\,[(n-(x+1))]!}\, p^{x+1} q^{n-(x+1)}$$

$$= \frac{n!\,(n-x)}{(x+1)!\,x!\,(n-x)\,(n-x-1)!}\, p^{x+1} q^{n-(x+1)}$$

$$= \frac{n-x}{x+1}\cdot\frac{n!}{x!\,(n-x)!}\, p\, p^x\, \frac{q^{n-x}}{q}$$

$$= \frac{n-x}{x+1}\,\frac{p}{q}\,\frac{n!}{x!\,(n-x)!}\, p^x q^{n-x}$$

$$= \frac{n-x}{x+1}\cdot\frac{p}{q}\, nC_x\, p^x q^{n-x}$$

$$p(x+1) = \frac{n-x}{x+1}\,\frac{p}{q}\,p(x)$$

Poisson Distribution:

Poisson distribution is regarded as the limiting case of Binomial distribution when $n \to \infty$ and p is prob of sucess is very small ($p \to 0$) so that np tends to fixed finite constant m

The prob ~~dist~~ distribution fn of Poisson distribn or poisson prob function and x is called poisson variable

$$p(x) = \frac{m^x e^{-m}}{x!}$$

Mean(μ) = $m = np$

variance = v = m

$SD(\sigma) = \sqrt{v} = \sqrt{m}$

Prob: Fit a poisson distribution for following data

x	0	1	2	3	4
f	122	60	15	2	1

Ans: mean $\mu = \frac{\sum f x}{f} = \frac{0+60+30+6+4}{200} = 0.5$

we have $\mu = m$

Poission distribution is

$$p(x) = \frac{m^x e^{-m}}{x!} \Rightarrow \quad \& \quad f(x) = 200\, P(x)$$

$$= 200 \times \frac{(0.5)^x e^{-0.5}}{x!}$$

$$f(x) = \frac{121.3\,(0.5)^x}{x!}$$

For $x = 0, 1, 2, 3, 4$

$f(0) = 121$ $\quad f(1) = 61$ $\quad f(2) = 15$ $\quad f(3) = 3$ $\quad f(4) = 0$

Prob' The number of accidents per day as recorded in a textile industry over a period of 400 days given. Fit a poisson distribution and calculate theoritical frequencies.

x	0	1	2	3	4	5
f	173	168	37	18	3	1

Ans'

mean $\mu = \frac{\Sigma f_i x_i}{\Sigma f} = \frac{0 + 168 + 34 + 54 + 12 + 5}{400} = m$

$= 0.7825$

$$P(x) = \frac{m^x e^{-m}}{x!}$$

$f(x) = 400\,P(x)$ $\quad = \frac{400\,(0.7825)^x\, e^{-0.7825}}{x!}$

$$= \frac{182.9\,(0.7825)^x}{x!}$$

$f(0) = 183$ $\quad f(1) = 143$ $\quad f(2) = 56$ $\quad f(3) = 15$

$f(4) = 3$ $\quad f(5) = 0$

Prob: In a certain factory turning razor blade a small prob of $\frac{1}{500}$ for any blade to be defective The blades are supplied in packets of 10 Use Poission distribution to calculate approximate no. of packets containing (1) no defective (2) one defective (3) two defective in a consignment of 10,000 packets.

Ans: $p = 0.002$ $\quad m = np = 10 \times 0.002 = 0.02$

We have $P(x) = \dfrac{m^x e^{-m}}{x!} = \dfrac{e^{-0.02} \times (0.02)^x}{x!}$

$f(x) = 10000\, P(x) = \dfrac{9802 (0.2)^x}{x!}$

(1) $f(0) = 9802$ $\quad$ (2) $f(1) = 196$ $\quad$ (3) $f(2) = 2$

Prob: The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean 3. Out of 1000 taxi drivers find approximately number of drivers with (1) no accidents (2) more than 3 accidents

Ans: By $\mu = 3$ $\quad P(x) = \dfrac{m^x e^{-m}}{x!} = \dfrac{e^{-3} 3^x}{x!}$

$f(x) = 1000\, P(x)$

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$$f(x) = 1000 \frac{3^x e^{-3}}{x!} = \frac{50 \times 3^x}{x!}$$

(i) We have $f(0) = \frac{50 \times 3^0}{0!} = 50$

(ii) More than 3 accidents $= P(x > 3) = 1 - P(x \leq 3)$

$= 1 - [P(0) + P(1) + P(2) + P(3)]$

$= 1 - \left[e^{-3} \left(\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!} \right) \right] = 0.35$

$\therefore$ No of drives with more than 3 accidents $= 1000 \times 0.35 = \underline{\underline{350}}$

<u>Prob:</u> 2% of fuses manufactured in a company are found defective find prob that box of 200 fuses contain (1) no defective fuses (2) 3 or more defective fuses

Ans: $p = 0.02$ $\quad \mu = np = 200 \times 0.02 = 4$

$$P(x) = \frac{m^x e^{-m}}{x!} \Rightarrow P(x) = \frac{4^x e^{-4}}{4!} = \frac{0.0183\, 4^x}{x!}$$

(1) $P(0) = 0.0183$ $\quad$ 2) $P(x \geq 3) = 1 - P(x \leq 3)$

$= \underline{0.72621}$

Prob: A communication channel receives independent pulses at the rate 12 pulses per microsecond. The prob of transmission error is 0.001 for each micro second. Compute prob that (1) no error during micro second (2) one error per microsecond (3) at least one error per micro second (4) 2 errors. at most 2 errors.

Ans: mean $\mu = np = 12 \times 0.001 = 0.012$

$$P(x) = \frac{m^x e^{-m}}{x!} = \frac{(0.012)^x e^{-0.012}}{x!}$$

(1) $P(0) = 0.988072$ (2) $P(1) = 0.01186$

(3) At least one err $= 1 - P(0) = 1 - 0.988072 = 0.01193$

(4) $P(2) = 0.000071$

(5) At most 2 errors $= 0.99998\ldots74 \cong 1$

Prob: The prob of a poisson variate taking values 3 & 4 are equal. Calculate the prob of variate taking values 0 & 1

Ans: We have $P(x) = \frac{m^x e^{-m}}{x!}$ and $P(3) = P(4)$

$$\frac{m^3 e^{-m}}{3!} = \frac{m^4 e^{-4}}{4!} \Rightarrow \frac{m^3}{6} = \frac{m^4}{24} \Rightarrow m = 4$$

$$\therefore P(x) = \frac{e^{-4} \cdot 4^x}{x!} \quad P(0) = \frac{1}{e^4} \quad P(1) = \frac{4}{e^4}$$

Prob: If x follows poisson law such that $p(x=2) = \frac{2}{3} p(x=1)$ Then fin $p(x=0)$ & $p(x=3)$

$$p(x) = \frac{m^x e^{-m}}{x!}$$

$$p(2) = \frac{m^2 e^{-m}}{2!} \qquad p(1) = \frac{m^1 e^{-1}}{1!}$$

$$\therefore \frac{m^2 e^{-m}}{2!} = \frac{2}{3} \frac{m e^{-1}}{1!} \Rightarrow \frac{m^2}{2} = \frac{2m}{3} \Rightarrow m = \frac{4}{3}$$

$$\therefore p(x) = \frac{e^{-4/3} \left(\frac{4}{3}\right)^x}{x!} \qquad p(x=0) = 0.2626$$

$$p(x=3) = 0.10414$$

Continious Probability Distributions

Defn: If for every x belonging to the range of a continious random variable X we assign a real number $f(x)$ satisfying the conditons

(i) $f(x) \geq 0$ (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Then $f(x)$ is called a continious prob funcn or probability density fuction. (p.d.f)

Exponential Distribution.

The ~~ets~~ continious prob distribution having p.d.f $f(x)$ given by

$$f(x) = \begin{cases} a e^{-ax} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

is known as the exponential distribution.

Mean and standard deviation of exponential Distribution

Mean $(\mu) = \frac{1}{\alpha}$ S.D $= \frac{1}{\alpha}$

Normal Distribution:

The continuous prob distribution ~~fun~~ having prob. density function $f(x)$ given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$ where $-\infty < x < \infty$, $-\infty < \mu < \infty$, $\sigma > 0$

is known as normal distribution.

Mean of Normal Distribution $= \mu$

Variance $= \sigma^2$

(39) = 36

Prob: Which of the following is prob density function

(1) $f_1(x) = \begin{cases} 2x & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$

(2) $f_2(x) = \begin{cases} 2x & -1<x<1 \\ 0 & \text{otherwise} \end{cases}$

(3) $f_3(x) = \begin{cases} |x| & |x|\le 1 \\ 0 & \text{otherwise} \end{cases}$

(4) $f_4(x) = \begin{cases} 2x & 0<x\le 1 \\ 4-4x & 1<x<2 \\ 0 & \text{otherwise} \end{cases}$

Ans ~~Clearly $f(x)\geq 0$~~

Conditions for pdf are $f(x)\geq 0$ & $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(1) Clearly $f(x)\geq 0$ $\int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 2x\,dx = 1$

$\therefore$ $f_1(x)$ is pdf

(2) when $x<0$ i.e when $-1<x<0$

we get $f(x) = 2x<0$ $\therefore$ not pdf

(3) $f_3(x) = |x|\geq 0$

$\int_{-\infty}^{\infty} f_3(x)\,dx = \int_{-1}^{1}|x|\,dx = \int_{-1}^{0} -x\,dx + \int_0^1 x\,dx$

$= \underline{1}$

$\therefore$ pdf

(40)

(4) $f_4(x) = 2x > 0$ for $0 < x \leq 1$

& $f_4(x) = 4 - 4x$ $\quad 1 < x < 2$

when $x = 1.5$ (say) then

$f_4(1.5) = 4 - 6 = -2 < 0$ $\therefore$ not pdf

Prob' Find c such that

$$f(x) = \begin{cases} \frac{x}{6} + c & 0 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$ 18 a

pdf in ~~..~~ Also find $P(1 \leq x \leq 2)$

Ans' we have $f(x) \geq 0$ $\therefore$ $c \geq 0$

Also $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_0^3 \left(\frac{x}{6} + c\right) dx = 1$

$\left.\frac{x^2}{12} + cx\right|_0^3 = 1$

$\frac{9}{12} + 3c = 1$

$3c = 1 - \frac{9}{12} = \frac{3}{12}$ $\quad c = \frac{1}{12}$

$$P(1 \le x \le 2) = \int_1^2 f(x)\,dx$$

$$= \int_1^2 \left(\frac{x}{6} + \frac{1}{12}\right) dx$$

$$= \frac{x^2}{12} + \frac{x}{12}\Big|_1^2 = \underline{\underline{1/3}}$$

Prob: Find k such that

$$f(x) = \begin{cases} kx^2 & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$ is a pdf

Also find (1) $p(1 < x < 2)$ (2) $p(x \le 1)$

(3) $p(x > 1)$ (4) mean & variance

Ans: we have $f(x) \ge 0$ if $k \ge 0$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_0^3 kx^2\,dx = 1$$

$$\Rightarrow \frac{kx^3}{3}\Big|_0^3 = 1$$

$$\Rightarrow 9k = 1$$

$$k = 1/9$$

(1) $P(1 < x < 2) = \int_1^2 f(x)\,dx = \int_1^2 \frac{1}{9}x^2\,dx = \underline{\frac{7}{27}}$

(2) $P(x \le 1) = \int_0^1 f(x)\,dx = \frac{1}{27}$

(3) $P(x > 1) = \int_1^3 f(x)\,dx = 6/27$

(4) Mean $\mu = \int_{-\infty}^{\infty} x f(x)\, dx = \int_0^3 x \frac{x^2}{9}\, dx = \frac{9}{4}$

(5) $V = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2 = \frac{27}{80}$

Prob: Find k such that

$$f(x) = \begin{cases} kx e^{-x} & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$ is pdf

Find the mean?

Ans: we have $f(x) \geq 0$ Then $k \geq 0$

Also $\int_{-\infty}^{\infty} f(x)\, dx = 1 \Rightarrow \int_0^1 kx e^{-x}\, dx = 1$

$\Rightarrow k = \frac{e}{e-2}$

Mean $\mu = \int_{-\infty}^{\infty} x f(x)\, dx = \frac{2e-5}{e-2}$

Cummulative distribution function:

If $f(x)$ is p.d.f then $F(x) = \int_{-\infty}^{x} f(x)\, dx$ is call cummulative distribution function (c.d.f)

Prob: Find c.d.f for

(1) $f(x) = \begin{cases} 6x - 6x^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

(2) $f(x) = \begin{cases} \frac{x}{4} e^{-x/2} & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$

(3) Exponential distribution.

Ans: 1) $F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{x} f(x)\,dx$

$= 0 + \int_{0}^{x} (6x - 6x^2)\,dx = 3x^2 - 2x^3$

$\therefore$ c.d.f. $F(x) = 3x^2 - 2x^3 \quad 0 \le x \le 1$

(2) $F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{0}^{x} \frac{x}{4} e^{-x/2}\,dx$

$$= 1 - e^{-x/2} - \frac{x}{2} e^{-x/2}$$

(3) $f(x) = \begin{cases} \alpha e^{-\alpha x} & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$

$F(x) = \int_{-\infty}^{0} f(x)\,dx = \int_{0}^{x} \alpha e^{-\alpha x}\,dx = 1 - e^{\alpha x}$

Prob: If c.d.f is

$$F(x) = \begin{cases} 0 & x \le 1 \\ c(x-1)^4 & 1 \le x \le 3 \\ 1 & x > 3 \end{cases}$$

Find c & p.d.f

Ans: $f(x) = \frac{d}{dx} F(x)$

$$\therefore f(x) = \begin{cases} 0 & x \le 1 \\ 4c(x-1)^3 & 1 \le x \le 3 \\ 0 & x > 0 \end{cases}$$

$f(x) \geq 0$ for $c > 0$ and $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\therefore \int_1^3 4c(x-1)^3\,dx = 1 \Rightarrow c(x-1)^4 \Big|_1^3 = 1$$

$$16c = 1 \quad \therefore c = 1/16$$

$$\therefore f(x) = \begin{cases} 0 & x \le 1 \\ \frac{(x-1)^3}{4} & 1 \le x \le 3 \\ 0 & x > 3 \end{cases}$$

Prob: A random variable has density function

$f(x) = \frac{k}{1+x^2}$ $\quad -\infty < x < \infty$ Find k & evaluate

(1) $P(x \geq 0)$ (2) $P(0 < x < 1)$

Ans: we have $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} \frac{k}{1+x^2}\,dx = 1$$

$$\Rightarrow 2\int_{0}^{\infty} \frac{k\,dx}{1+x^2} = 1$$

$$\Rightarrow 2k \tan^{-1}(x)\Big|_0^{\infty} = 1$$

$$\Rightarrow 2k\left[\frac{\pi}{2} - 0\right] = 1$$

$$\Rightarrow k = \frac{1}{\pi}$$

Now $P(x \geq 0) = \int_0^{\infty} f(x)\,dx$

$$= \frac{1}{\pi}\int_0^{\infty} \frac{1}{1+x^2}\,dx = \frac{1}{2}$$

$$P(0 < x < 1) = \frac{1}{\pi}\int_0^1 \frac{dx}{1+x^2} = \frac{1}{\pi}\left[\tan^{-1}(1) - 0\right] = \frac{1}{4}$$

Prob: The kilometer run (in thousand of k.m s) without any sort of problem in respect of a certain vehicle is a random variable having pdf

$$f(x) = \begin{cases} \frac{1}{40} e^{-\frac{x}{40}} & x \geq 0 \\ 0 & x \leq 0 \end{cases}$$

Find probability that the vehicle is trouble free

(1) at least 25000 km (2) At most 25000 km

(3) between 16000 to 32000 km

Ans

let x be the random variable representing km in multiple of 1000 regarding trouble free run by vehicle

(1) To find $P(x \geq 25)$

$$P(x \geq 25) = 1 - P(x < 25)$$

$$= 1 - \int_0^{25} \frac{1}{40} e^{-\frac{x}{40}} dx$$

$$= e^{-5/6}$$

(2) To find $P(x \leq 25)$

$$P(x \leq 25) = \int_0^{25} \frac{1}{40} e^{-x/40} dx = 1 - e^{-5/8}$$

(3) To find $P(16 \leq x \leq 32)$

$$= \int_{16}^{32} \frac{1}{40} e^{-x/40} dx$$

$$= -e^{-4/5} + e^{-2/5}$$

Prob' If x is an exponential variate with mean 3 find

(1) $P(x > 1)$ (2) $P(x < 3)$

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<u>Ans</u>: pdf of exponential distribution is

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

The mean of the distribution $\frac{1}{\alpha}$

$\therefore \frac{1}{\alpha} = 3 \qquad \alpha = 1/3$

(i) $\therefore P(x > 1) = 1 - P(x \leq 1)$

$$= 1 - \int_0^1 f(x)\, dx$$

$$= 1 - \int_0^1 \frac{1}{3} e^{-\frac{1}{3}x}\, dx = e^{-1/3}$$

$\therefore P(x > 1) = e^{-1/3} = 0.7163$

(ii) $P(x < 3) = \int_0^3 f(x)\, dx$

$$= \int_0^3 \frac{1}{3} e^{-x/3}\, dx$$

$$= -\left[e^{-x/3}\right]\Big|_0^3 = 0.6321$$

<u>Prob</u>: If x is a exponential variate with mean 5 then find

(1) $P(0 < x < 1)$ (2) $P(-\infty < x < 10)$

(3) $P(x \leq 0 \text{ or } x \geq 1)$

Ans: $f(x) = \alpha e^{-\alpha x} \quad 0 < x < \infty$

Given mean $\frac{1}{\alpha} = 5 \quad \alpha = 1/5$

$\therefore f(x) = \frac{1}{5} e^{-x/5} \quad 0 < x < \infty$

(1) $P(0 < x < 1) = \int_0^1 f(x)\,dx$

$= \frac{1}{5} \int_0^1 \frac{1}{5} e^{-x/5}\,dx = 0.1813$

(2) $P(-\infty < x < 10) = \int_{-\infty}^{0} f(x)\,dx + \int_0^{10} f(x)\,dx$

$= 0 + \int_0^{10} \frac{1}{5} e^{-x/5}\,dx = 0.8647$

(3) $P(x \leq 0 \text{ or } x \geq 1) = P(x \leq 0) + P(x \geq 1)$

$= 0 + \int_1^{\infty} f(x)\,dx$

$= \int_1^{\infty} \frac{1}{5} e^{-x/5}\,dx = e^{-0.2} = 0.8187$

Prob: length of Telephone conversation in a booth has been an exponential distribution and found on an avarge of 5 min. Find a prob that a random call made from this booth (i) ends in less than 5 min (ii) between 5 and 10 min.

49

Ans: $f(x) = \alpha e^{-\alpha x}$

Mean $= \frac{1}{\alpha} = 5 \quad \therefore \alpha = \frac{1}{5}$

(1) $P(x<5) = \int_0^5 \frac{1}{5} e^{-x/5}\, dx = 0.6321$

(2) $P(5<x<10) = \int_5^{10} \frac{1}{5} e^{-x/5}\, dx = \underline{0.2325}$

Prob: The sales per day in a shop is exponentially with the avarage sales amounting to Rs. 100 and net profit 8%. Find prob that net profit exceeds Rs 30 in two consecutive days

Ans: $f(x) = \alpha e^{-\alpha x} \quad x \geq 0$

Here $\frac{1}{\alpha} = 100 \quad \therefore \alpha = 0.01$

$\therefore f(x) = 0.01\, e^{-0.01x}$

Let A be the amount for which profit is 8%.

$\therefore A \times 0.08 = 30 \quad \therefore A = \frac{30}{0.08} = 375$

$\therefore$ Prob that profit exceeding Rs. 30

$= 1 - \text{Prob}(\text{Profit} \leq 30)$

$= 1 - \text{Prob}(\text{sales} \leq 375)$

$= 1 - \int_0^{375} 0.01\, e^{-0.01x} = e^{-3.75}$

The prob that it repeates on consitetive day

$= \cancel{e^{-0.375} \times e^{-0.33}} = e^{-3.75} \times e^{-3.75} = 0.00055$

Prob: In a certain town the duration of a shower is exponentially distributed with mean 5 min. What is the prob that the shower will be last for (i) 10 min or more (ii) less than 10 min. (iii) bet 10 & 12 min.

Ans: ~~α~~ = Mean $\frac{1}{\alpha} = 5$ $\therefore \alpha = \frac{1}{5} = 0.2$

(i) $P(x \geq 10) = \int_{10}^{\infty} \frac{1}{5} e^{-x/5} dx = 0.1353$

(ii) $P(x < 10) = \int_{0}^{10} \frac{1}{5} e^{-x/5} dx = 0.8647$

(iii) $P(10 < x < 12)$ $\int_{10}^{12} \frac{1}{5} e^{-x/5} dx = 0.0446$

Normal Distribution

Mean of N.D = μ varience = σ

Standred normal variate $z = \frac{x-\mu}{\sigma}$

$P(z \geq 0) = 0.5$

Prob: If x is a normal variate with mean 30 and S.D 5 find prob that

(i) $26 \leq x \leq 40$ (ii) $x \geq 45$

Ans: $z = \frac{x-\mu}{\sigma}$ $\mu = 30$ $\sigma = 5$

$\therefore z = \frac{x-36}{5}$

(1) To find $P(26 \le x \le 40)$

If $x = 26$ $z = -0.8$ If $x = 40$ $z = 2$

$\therefore P(-0.8 \le x \le 2)$

$= P(-.8 \le z \le 0) + P(0 \le z \le 2)$

$= P(0 \le z \le 0.8) + P(0 \le z \le 2)$

$= \phi(0.8) + \phi(2)$

$= 0.2881 + 0.4772 = 0.7653$

(2) $P(x \ge 45)$ $x = 45$ $\therefore z = 3$

$\therefore P(z \ge 3) = P(z \ge 0) - P(z \le 3)$

$= 0.5 - \phi(3)$

$= 0.5 - 0.4987 = 0.0013$

Prob: If x is normally distributed with mean 12 SD 4 find $P(x \ge 20)$ $P(x \le 20)$

Ans: $z = \frac{x-\mu}{\sigma} = \frac{x-12}{4}$

(1) If $x = 20$ then $z = 2$

$\therefore$ we need to find $P(z \ge 2)$

Now $P(Z \geq 2) = P(Z \geq 0) - P(Z \leq 2)$

$= 0.5 - \phi(2)$

$= 0.5 - 0.4772 = 0.0228$

(2) $P(x \leq 20) = P(Z \leq 2) = P(-\infty \leq Z \leq 0)$

$+ P(0 \leq Z \leq 2)$

$= 0.5 + \phi(2)$

$= 0.5 + 0.4772 = 0.9772$

Prob: The marks of 1000 students in an examination follows normal distribution with mean 70 and SD 5 Find number of students whose marks will be (1) less than 65 (2) more than 75 (3) between 65 & 75

Ans: Let x represent mark of student

By data $\mu = 70$ $\sigma = 5$ $z = \frac{x-\mu}{\sigma} = \frac{x-70}{5}$

(1) If $x = 65 \Rightarrow z = -1$

we have to find $P(z < -1) = P(z > 1)$

$= P(z \geq 0) - P(0 < z < 1)$

$= 0.5 - \phi(1) = 0.5 - 0.3413$

$= 0.1587$

$\therefore$ No of stud scoring less than 65 $= 1000 \times 0.1587$

$= 159$

(ii) $x = 75 \Rightarrow z = 1$ $\therefore P(Z > 1) = P(Z \geq 0) - P(0 < Z < 1)$

$= 0.5 - \phi(1) = 0.5 - 0.3413 = 0.1587$

$\therefore$ No of stud scoring above $75 = 1000 \times 0.1587 = \underline{159}$

(iii) $P(-1 < z < 1)$

$= 2P(0 < z < 1)$

$= 2\phi(1) = 2 \times 0.3413 = 0.6826$

$\therefore$ No of stud scoring 65 & 75 $= \underline{683}$

Prob: In a test on electric bulbs it was found that the life time of a particular brand distributed normally with an avarage of 2000 hours and SD of 60 hours. If a firm purchases 2500 bulbs find number of bulbs that are likely to last for (i) more than 2100 hrs (2) less than 1950 hrs (iii) between 1900 & 2100 hrs.

5

Ans: $\mu = 2000$ $\sigma = 60$

$z = \frac{x - \mu}{\sigma} = \frac{x - 2000}{60}$

(1) To find $P(x > 2000)$

$x = 2000 \quad z = \frac{100}{60} = 1.67$

$\therefore P(x > 2000) = P(z > 1.67)$

$= P(z \geq 0) - P(0 < z < 1.67)$

$= 0.5 - \phi(1.67)$

$= 0.5 - 0.4525$

$= \underline{0.0475}$ $\therefore$ out of 2800 we have

$= 2800 \times 0.0475 = 119$

(2) $P(x < 1950)$

$x = 1950 \quad \therefore z = -\frac{5}{6} = -0.83$

$\therefore P(x < 1950) = P(z < -0.83)$

$= P(z > 0.83)$

$= P(z \geq 0) - P(0 < z < 0.83)$

$= 0.5 - \phi(0.83)$

$= 0.2033$

$\therefore$ out of 2800 weight $= 2800 \times 0.2033 = 508$

(iii) $P(1900 < x < 2100)$

If $x = 1900 \quad z = -1.67 \qquad x = 2100 \quad z = 1.67$

$\therefore P(1900 < x < 2100) = P(-1.67 < z < 1.67)$

$= 2P(0 < z < 1.67)$

$= 2 \times 0.4525 = 0.905$

$\therefore$ out of $2800 = 2800 \times 0.905 = \underline{2263}$

Prob: In a normal distribution 31% of items are under 45 and 8% of items are above 64 find S.D of the distribution

Ans: B.y data $P(x<45) = 0.31$ $\quad P(x>64) = 0.08$

we have $z = \frac{x-\mu}{\sigma}$ $\quad \mu \to$ mean
$\sigma \to$ S.D.

when $x=45$ $\quad z = \frac{45-\mu}{\sigma} = z_1$ (say)

$x = 64$ $\quad z = \frac{65-\mu}{\sigma} = z_2$ (say)

we have $P(z<z_1) = 0.31$ $\quad P(z>z_2) = 0.08$

ie $0.5 + \phi(z_1) = 0.31$ $\quad 0.5 - \phi(z_2) = 0.08$

$\Rightarrow \phi(z_1) = -0.19$ $\quad \phi(z_2) = 0.42$

From table

$0.1915 \cong 0.19 = \phi(.5)$ $\quad 0.4192 \cong 0.42 = \phi(1.4)$

$\therefore \phi(z_1) = -\phi(0.5)$ $\quad \phi(z_2) = \phi(0.14)$

$\therefore z_1 = -0.5$ $\quad z_2 = 1.4$

$\therefore \frac{45-\mu}{\sigma} = -0.5$ $\quad \frac{64-\mu}{\sigma} = 1.4$

$\mu - 0.5\sigma = 45$ $\quad \mu + 1.4\sigma = 64$

$\therefore \mu = 50 \quad \sigma = 10$

Prob: In an examination 7% of students score less than 35% marks and 89% of the students score less than 60% marks. Find μ & σ if marks are normally distributed.

Ans: Let μ & σ be mean & SD of the distributions

From data $P(x<35) = 0.07 \qquad P(x<60) = 0.89$

We have $z = \dfrac{x-\mu}{\sigma}$

When $x = 35 \qquad z = \dfrac{35-\mu}{\sigma} = z_1$

$x = 60 \qquad z = \dfrac{60-\mu}{\sigma} = z_2$

We have $\quad P(z<z_1) = 0.07 \qquad P(z<z_2) = 0.89$

ie $0.5 + \phi(z_1) = 0.07 \qquad 0.5 + \phi(z_2) = 0.89$

$\therefore \phi(z_1) = -0.43 \qquad \phi(z_2) = 0.39$

Then $z_1 = -1.4757 \qquad z_2 = 1.2263$

$\therefore \mu = 48.65 \qquad \sigma = 9.25$

Obtain the equation of normal prob distribution curve for following data (57)

x Variable	6	7	8	9	10	11	12
f freq.	3	6	9	13	8	5	4

<u>Ans</u>: The eqn of normal prob curve is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

$$\mu = \frac{\Sigma_i f x}{\Sigma_i f} = \frac{6\times3 + 7\times6 + 8\times9 + 9\times13 + 10\times8 + 11\times5 + 12\times4}{3+6+9+13+8+5+4}$$

$$= 9$$

$$\sigma^2 = \frac{\Sigma_i f x^2}{\Sigma_i f} - \mu^2$$

$$= \frac{3\times36 + 6\times49 + 9\times64 + 13\times81 + 8\times100 + 5\times121 + 4\times144}{48}$$

$$= 2.5833 \qquad \therefore \sigma = 1.609$$

$$\therefore f(x) = \frac{1}{1.609\sqrt{2\pi}} e^{-(x-9)^2/2\times2.5833}$$

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