

## Statistics4 –session 18----assignment

### Problem 1

#### Null Hypothesis

The two categorical variables are independent

#### Alternative Hypothesis

The two categorical variables are dependent

#### Chi-Square Test Statistic

$$\chi^2 = \sum (O - E)^2 / E$$

where  $O$  represents the observed frequency.  $E$  is the expected frequency under the null hypothesis and computed by:

$$E = \text{row total} \times \text{column total} / \text{sample size}$$

We will compare the value of the test statistic to the critical value of  $\chi^2$  with degree of freedom =  $(r - 1)(c - 1)$ , and reject the null hypothesis.

Here's the table of expected counts:

	High School	Bachelors	Masters	Ph.d.	Total
Female	50.886	49.868	50.377	49.868	201
Male	49.114	48.132	48.623	48.132	194
Total	100	98	99	98	395

So, working this out,  $\chi^2 = (60 - 50.886)^2 / 50.886 + \dots + (57 - 48.132)^2 / 48.132 = 8.006$

The critical value of  $\chi^2$  with 3 degree of freedom is 7.815. Since  $8.006 > 7.815$ , therefore we reject the null hypothesis and conclude that the education level depends on gender at a 5% level of significance.

### Problem 2

Sample means ( $\bar{x}$ ) for the groups: = 48.2, 35.4, 69.8

Intermediate steps in calculating the group variances:

[[1]]

value mean deviations sq deviations

1	51	48.2	2.8	7.84
2	45	48.2	-3.2	10.24
3	33	48.2	-15.2	231.04

4	45	48.2	-3.2	10.24
5	67	48.2	18.8	353.44

[[2]]

	value	mean	deviations	sq deviations
1	23	35.4	-12.4	153.76
2	43	35.4	7.6	57.76
3	23	35.4	-12.4	153.76
4	43	35.4	7.6	57.76
5	45	35.4	9.6	92.16

[[3]]

	value	mean	deviations	sq deviations
1	56	69.8	-13.8	190.44
2	76	69.8	6.2	38.44
3	74	69.8	4.2	17.64
4	87	69.8	17.2	295.84
5	56	69.8	-13.8	190.44

Sum of squared deviations from the mean (SS) for the groups:

[1] 612.8 515.2 732.8

$\text{Var1} = 612.85 - 1 = 153.2$   $\text{Var1} = 612.85 - 1 = 153.2$

$\text{Var2} = 515.25 - 1 = 128.8$   $\text{Var2} = 515.25 - 1 = 128.8$

$\text{Var3} = 732.85 - 1 = 183.2$   $\text{Var3} = 732.85 - 1 = 183.2$

$\text{MSerror} = 153.2 + 128.8 + 183.23 = 155.07$   $\text{MSerror} = 153.2 + 128.8 + 183.23 = 155.07$  *Note: this is just the average within-group variance; it is not sensitive to group mean differences!*

Calculating the remaining error (or within) terms for the ANOVA table:

$\text{dferror} = 15 - 3 = 12$   $\text{dferror} = 15 - 3 = 12$

$\text{SSerror} = (155.07)(15 - 3) = 1860.8$   $\text{SSerror} = (155.07)(15 - 3) = 1860.8$

Intermediate steps in calculating the variance of the sample means:

Grand mean  $(\bar{x} - \text{grand} \bar{x} - \text{grand}) = 48.2 + 35.4 + 69.83 = 51.13$   $48.2 + 35.4 + 69.83 = 51.13$

group	mean	grand mean	deviations	sq deviations
	48.2	51.13	-2.93	8.58
	35.4	51.13	-15.73	247.43

69.8	51.13	18.67	348.57
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Sum of squares (SSmeans)=604.58(SSmeans)=604.58

Varmeans=604.583-1=302.29Varmeans=604.583-1=302.29

MSbetween=(302.29)(5)=1511.45MSbetween=(302.29)(5)=1511.45 *Note: This method of estimating the variance IS sensitive to group mean differences!*

Calculating the remaining *between* (or *group*) terms of the ANOVA table:

dfgroups=3-1=2dfgroups=3-1=2

SSgroup=(1511.45)(3-1)=3022.9SSgroup=(1511.45)(3-1)=3022.9

Test statistic and critical value

F=1511.45155.07=9.75F=1511.45155.07=9.75

Fcritical(2,12)=3.89Fcritical(2,12)=3.89

Decision: reject H0 Decision: reject H0

ANOVA table

source	SS	df	MS
group	3022.9	2	1511.45
error	1860.8	12	155.07
total	4883.7		

Effect size

$\eta^2=3022.94883.7=0.62\eta^2=3022.94883.7=0.62$

APA writeup

$F(2, 12)=9.75, p < 0.05, \eta^2=0.62.$

### Problem 3

Calculate Variance of first set

Total Inputs (N) =(10,20,30,40,50)

Total Inputs (N)=5

Mean (xm)=  $(x_1+x_1+x_2...x_n)/N$

Mean (xm)= 150/5

Means(xm)= 30

SD=sqrt(1/(N-1)\*((x<sub>1</sub>-xm)<sup>2</sup>+(x<sub>2</sub>-xm)<sup>2</sup>+..+(x<sub>n</sub>-xm)<sup>2</sup>))  
 =sqrt(1/(5-1)((10-30)<sup>2</sup>+(20-30)<sup>2</sup>+(30-30)<sup>2</sup>+(40-30)<sup>2</sup>+(50-30)<sup>2</sup>))  
 =sqrt(1/4((-20)<sup>2</sup>+(-10)<sup>2</sup>+(0)<sup>2</sup>+(10)<sup>2</sup>+(20)<sup>2</sup>))  
 =sqrt(1/4((400)+(100)+(0)+(100)+(400)))  
 =sqrt(250)

$$=15.8114$$

$$\text{Variance} = \text{SD}^2$$

$$\text{Variance} = 15.8114^2$$

$$\text{Variance} = 250$$

Calculate Variance of second set

For 5, 10, 15, 20, 25:

Total Inputs(N) = (5, 10, 15, 20, 25)

Total Inputs(N) = 5

Mean (xm) =  $(x_1 + x_2 + x_3 \dots x_N) / N$

Mean (xm) =  $75 / 5$

Means (xm) = 15

$$\text{SD} = \sqrt{1/(N-1) * ((x_1 - x_m)^2 + (x_2 - x_m)^2 + \dots + (x_n - x_m)^2)}$$

$$= \sqrt{1/(5-1) * ((5-15)^2 + (10-15)^2 + (15-15)^2 + (20-15)^2 + (25-15)^2)}$$

$$= \sqrt{1/4 * ((-10)^2 + (-5)^2 + (0)^2 + (5)^2 + (10)^2)}$$

$$= \sqrt{1/4 * ((100) + (25) + (0) + (25) + (100))}$$

$$= \sqrt{62.5}$$

$$= 7.9057$$

$$\text{Variance} = \text{SD}^2$$

$$\text{Variance} = 7.9057^2$$

$$\text{Variance} = 62.5$$

To calculate F Test

F Test = (variance of 10, 20, 30, 40, 50) / (variance of 5, 10, 15, 20, 25)

$$= 250 / 62.5$$

$$= 4.$$

The F Test value is 4.