SHORTEST PATH PROBLEM IN A FUZZY ENVIRONMENT INVOLVING SPECIAL FUZZY NUMBERS

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OUTLINE OF THE DISSERTATION

CHAPTER - 1

INTRODUCTION

- The fuzzy set theory has been proposed in 1965 by Lofti A. Zadeh .
- This theory is a mathematical theory which deals with uncertainty which are common in the natural language.
- Unlike computers, the human reasoning is not binary where everything is either yes(true) or no(false) but deals with imprecise concepts like 'a tall man', 'a moderate temperature' or 'a large profit'. These concepts are ambiguous in the sense that they cannot be sharply defined.
- Thus fuzziness occurs when the boundary of a piece of information is not clear-cut. The word "fuzzy" means "vagueness".

INTRODUCTION

- Shortest Paths are one of the simplest and most widely used concepts in non-fuzzy networks.
- The shortest path problem concentrates on finding the path with minimum distance.
- The fuzzy shortest path problem(FSPP) was first introduced by Dubois and Prade in 1980.
- Many researches have focused on fuzzy shortest path problem in a network, since it is important to many applications such as communications, routing, and transportation.

CHAPTER - 2

PRELIMINARIES

- Basic Definitions
- Arithmetic Operations on fuzzy numbers
- Comparison of fuzzy numbers
- Notations

BASIC DEFINITIONS

Crisp set

$$\chi_A : X \to \{0,1\}, \text{where} \quad \chi_A(x) = \begin{cases} 1 & \text{for } x \in A \\ 0 & \text{for } x \notin A \end{cases}$$ 1

Fuzzy set

2 $$\mu_{\tilde{A}} : X \to [0,1]$$

Fuzzy number

It is a special type of fuzzy set.

$$\mu_{\tilde{A}} : R \to [0,1]$$ 3

BASIC DEFINITIONS

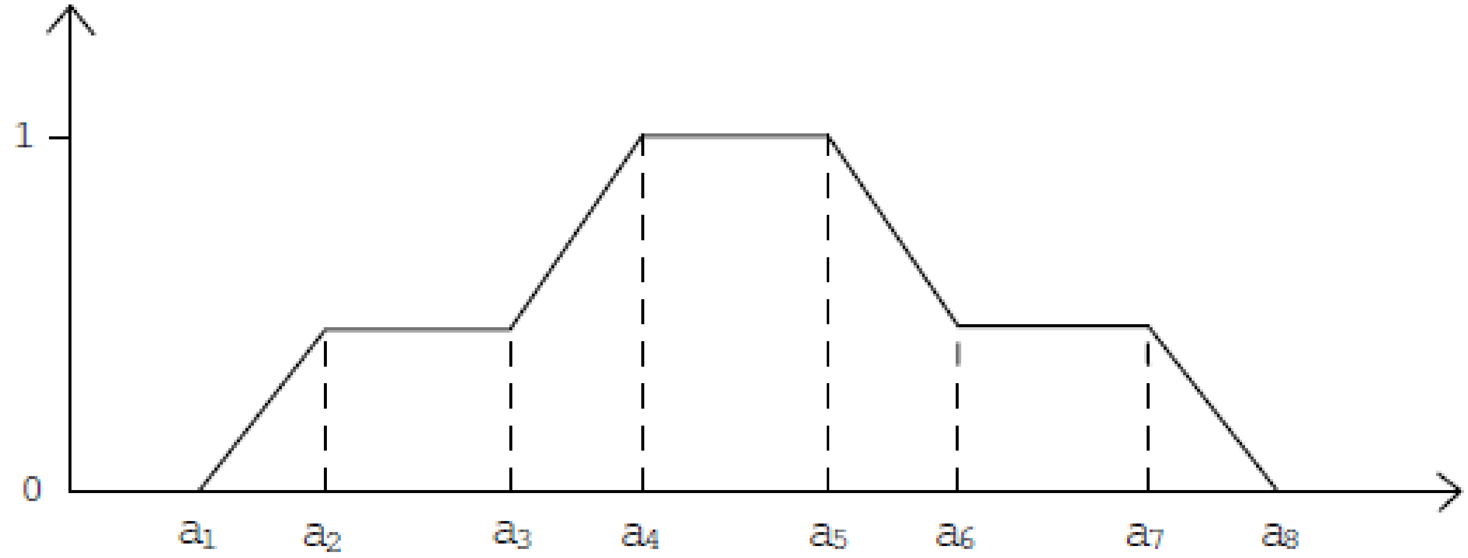
Octagonal fuzzy number

$$\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k, 1)$$ 4

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq a_1, x \geq a_8 \\ k\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \leq x \leq a_2 \\ k, & a_2 \leq x \leq a_3 \\ k + (1-k)\left(\frac{x-a_3}{a_4-a_3}\right), & a_3 \leq x \leq a_4 \\ 1, & a_4 \leq x \leq a_5 \\ k + (1-k)\left(\frac{a_6-x}{a_6-a_5}\right), & a_5 \leq x \leq a_6 \\ k, & a_6 \leq x \leq a_7 \\ k\left(\frac{a_8-x}{a_8-a_7}\right), & a_7 \leq x \leq a_8 \qquad \text{where } 0 < k < 1 \end{cases}$$ 5

BASIC DEFINITIONS

Octagonal fuzzy number

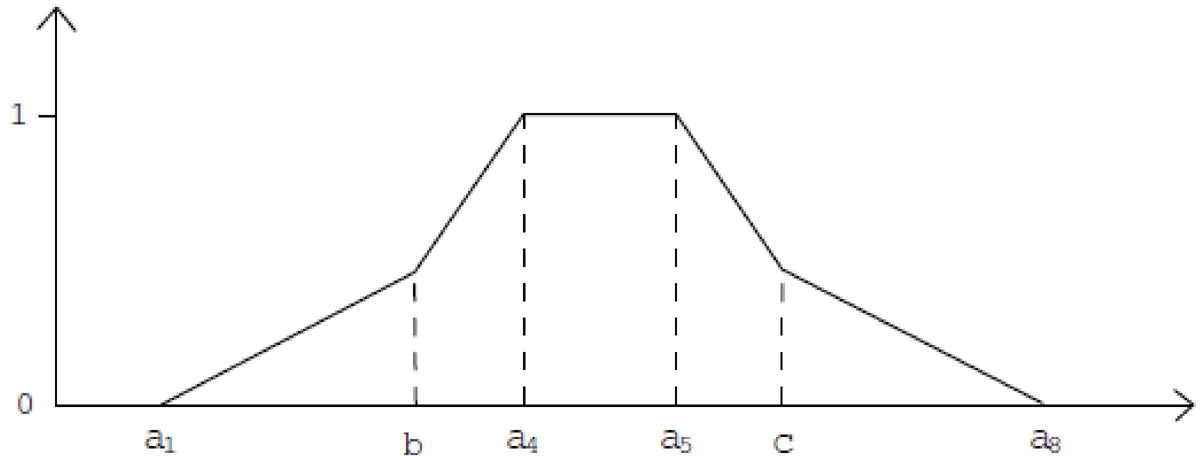


BASIC DEFINITIONS

Hexagonal fuzzy number

When $a_2 = a_3 = b$ and $a_6 = a_7 = c$ in 4

6 $\tilde{A} = (a_1, b, a_4, a_5, c, a_8; k, 1)$

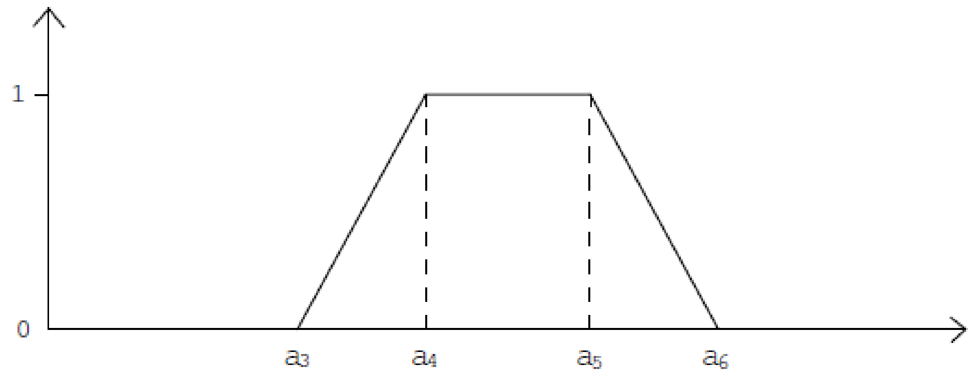


BASIC DEFINITIONS

Trapezoidal fuzzy number

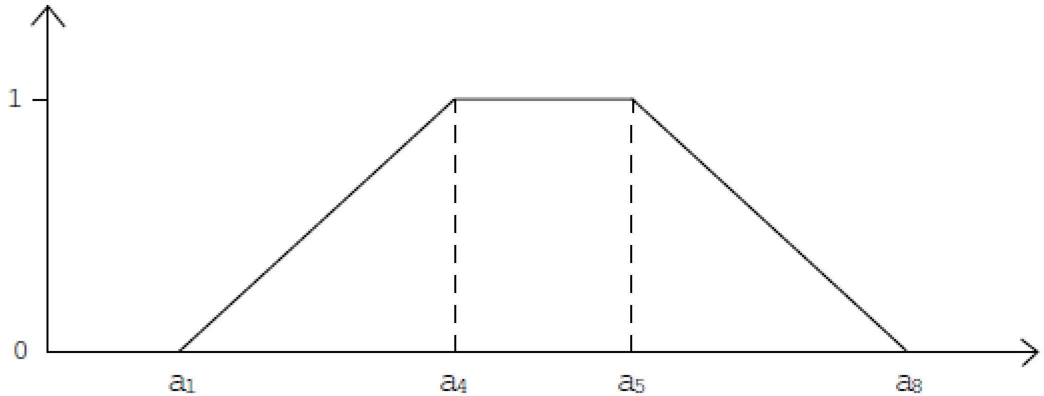
If $k=0$,

7 $\tilde{A} = (a_3, a_4, a_5, a_6; 1)$



If $k=1$,

8 $\tilde{A} = (a_1, a_4, a_5, a_8; 1)$



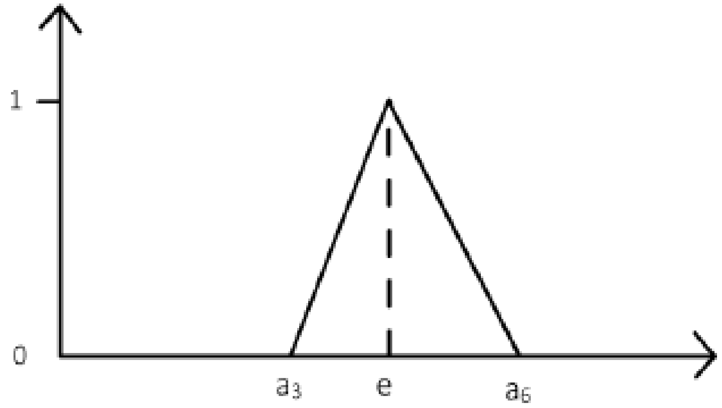
BASIC DEFINITIONS

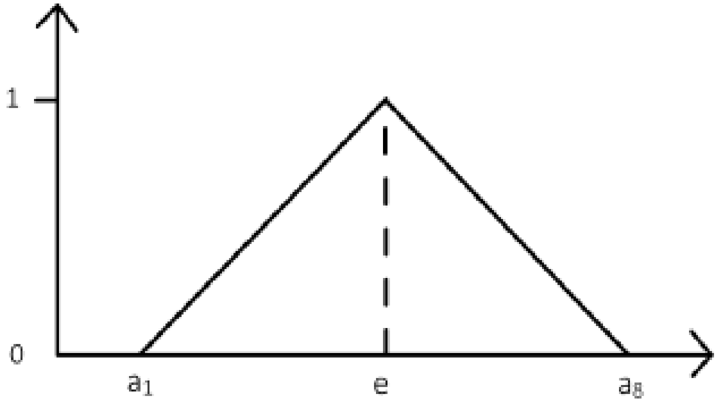
Triangular fuzzy number

If $a_4 = a_5 = e,$

$$\tilde{A} = \begin{cases} (a_3, e, a_6) & \textit{if } k{=}0 \\ (a_1, e, a_8) & \textit{if } k{=}1 \end{cases}$$

9





ARITHMETIC OPERATIONS ON FUZZY NUMBERS

- Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ be two octagonal fuzzy numbers, then

  1. $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8)$
  2. $\tilde{A} \ominus \tilde{B} = (a_1 - b_8, a_2 - b_7, a_3 - b_6, a_4 - b_5, a_5 - b_4, a_6 - b_3, a_7 - b_2, a_8 - b_1)$

- Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6)$ be two hexagonal fuzzy numbers, then

  1. $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6)$
  2. $\tilde{A} \ominus \tilde{B} = (a_1 - b_6, a_2 - b_5, a_3 - b_4, a_4 - b_3, a_5 - b_2, a_6 - b_1)$

ARITHMETIC OPERATIONS ON FUZZY NUMBERS

- Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers, then

  1. $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
  2. $\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

- Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers, then

  1. $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
  2. $\tilde{A} \ominus \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

COMPARISON OF FUZZY NUMBERS

Ranking fuzzy numbers is an important task in analysing fuzzy information in optimization, data mining, decision making and related areas. Unlike real numbers, fuzzy numbers have no natural order; also no ordering technique is conducive to all. Thus significant contributions have been made in ranking fuzzy numbers.

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers,then

$$
\begin{array}{llll}
(i) & \tilde{A} \succ \tilde{B} & \text{if} & \Re(\tilde{A}) \succ \Re(\tilde{B}) \\
(ii) & \tilde{A} \prec \tilde{B} & \text{if} & \Re(\tilde{A}) \prec \Re(\tilde{B}) \\
(iii) & \tilde{A} \approx \tilde{B} & \text{if} & \Re(\tilde{A}) \approx \Re(\tilde{B})
\end{array}
$$

COMPARISON OF FUZZY NUMBERS

Triangular and Trapezoidal fuzzy number

- If $\tilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy number, then

$$\Re(\tilde{A}) = \frac{a_1 + 2(a_2) + a_3}{4} \quad (10)$$

Also If $\tilde{A} = (m, \alpha, \beta)$ be a triangular fuzzy number, then

$$\Re(\tilde{A}) = m + \frac{(\beta - \alpha)}{4} \quad (11)$$

- If $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number, then

$$\Re(\tilde{A}) = \frac{a_1 + a_2 + a_3 + a_4}{4} \quad (12)$$

COMPARISON OF FUZZY NUMBERS

Hexagonal fuzzy number

$$\Re(\tilde{A}) = (\frac{2a_1 + 3a_2 + 4a_3 + 4a_4 + 3a_5 + 2a_6}{18})(\frac{5}{18}) \quad (13)$$

Octagonal fuzzy number

$$\Re(\tilde{A}) = \frac{1}{4}[(a_1 + a_2 + a_7 + a_8)k + (a_3 + a_4 + a_5 + a_6)(1 - k)] \quad (14)$$

NOTATIONS

The notations which will be used throughout the dissertation are given below

$N = \{1, 2, \ldots, n\}$: The set of all nodes in a network.

$Np(j)$: The set of all predecessor nodes of node j.

e_i : The distance between node i and first (source) node.

e_{ij} : The distance between node i and j.

$\tilde{e}_i$ and $\tilde{d}_i$: The fuzzy distance between node i and first (source) node.

$\tilde{e}_{ij}$ and $\tilde{d}_{ij}$: The fuzzy distance between node i and j .

MOTIVATION FACTOR

- ✓ **A.Kumar** and **M.Kaur**, proposed a new algorithm for solving shortest path problem with fuzzy arc lengths.
- ✓ In this thesis, we refer to Kumar and Kaur algorithm wherein the algorithm to obtain the shortest path involving triangular and trapezoidal fuzzy numbers is considered. We have extended the same to hexagonal and octagonal fuzzy numbers. The algorithm is also substantiated using the same numerical illustration invloving hexagonal fuzzy numbers and octagonal fuzzy numbers.

CHAPTER - 3

FSPP INVOLVING OCTAGONAL FUZZY NUMBERS

ALGORITHM

The steps of the algorithm are summarized as follows:

Step:1

Assume $\tilde{d}_1 = (0,0,0,0,0,0,0,0)$ and label the source node (say node 1) as [(0,0,0,0,0,0,0,0),-].

Step:2

Find $\tilde{d}_j$ = minimum { $\tilde{d}_i \oplus \tilde{d}_{ij}$ / $i \in$ Np(j) }; $j \neq 1, j = 2, 3, \ldots$,n.

Step:3

If minimum occurs corresponding to unique value of i i.e., $i = r$ then label node j as $[\tilde{d}_j, r]$. If minimum occurs corresponding to more than one values of i then it represents that there are more than one fuzzy path between source node and node j but fuzzy distance along all paths is $\tilde{d}_j$, so choose any value of i .

ALGORITHM

Step:4

Let the destination node (node n) be labeled as $[\tilde{d}_j, l]$, then the fuzzy shortest distance between source node and destination node is $\tilde{d}_n$.

Step:5

Since destination node is labeled as $[\tilde{d}_n, l]$. So, to find the fuzzy shortest path between source node and destination node, check the label of node l . Let it be $[\tilde{d}_l, p]$, now check the label of node p and so on. Repeat the same procedure until node 1 is obtained.

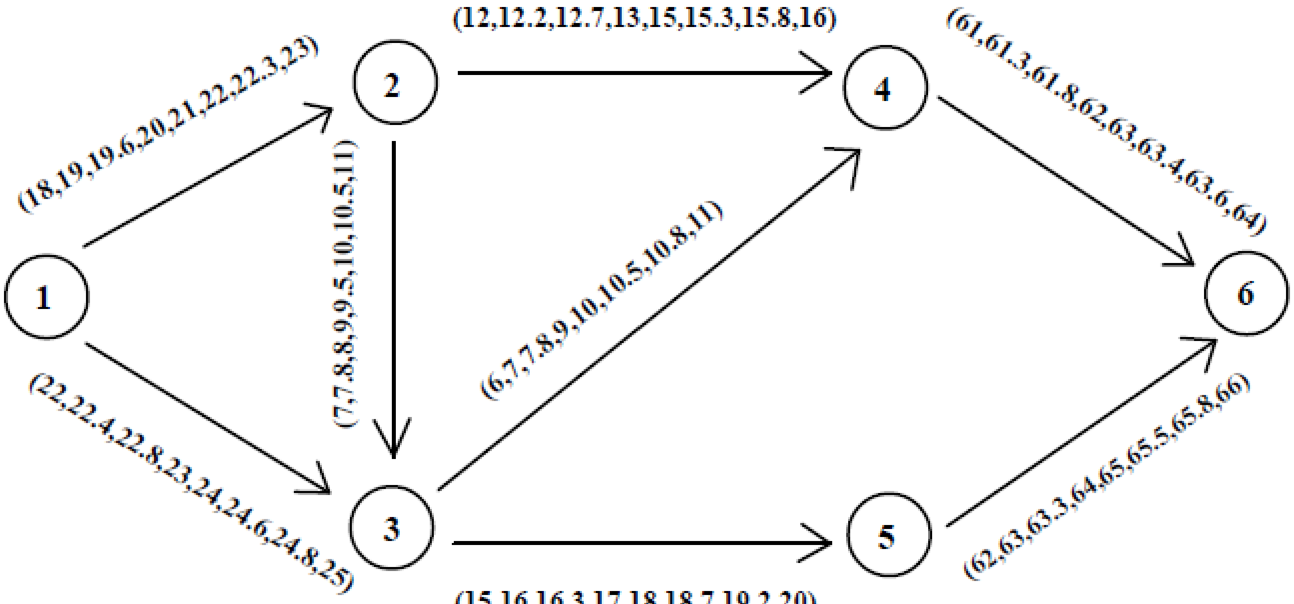
Step:6

Now the fuzzy shortest path can be obtained by combining all the nodes obtained by the Step 5.

NUMERICAL ILLUSTRATION

Solution that goods available in source node 1 need to be stored in a godown viz., node 6 and wherein direct transportation between node 1 to node 6 is not possible as the distance between these two nodes is very long and there is no direct route. Also suppose that the vehicle carrying these goods can be halted at 4 different places or nodes say node 2, node 3, node 4 and node 5 and the distance between any two nodes is given approximately , then the problem is to find the shortest path between the source node and the destination node.

NUMERICAL ILLUSTRATION



<u>Network of the shortest path</u>

NUMERICAL ILLUSTRATION

SOLUTION:

Since,node 6 is the destination node, so n=6.

Assume $\tilde{d}_1 = (0,0,0,0,0,0,0,0)$ and label the source node (say node 1) as [(0,0,0,0,0,0,0,0),-], the values of $\tilde{d}_j$; j= 2,3,4,5,6 can be obtained as follows:

Iteration 1:

Since, only node 1 is the predecessor node of node 2, so putting i=1 and j=2 in step 2, the value of $\tilde{d}_2$ is

$$\begin{aligned}\tilde{d}_2 &= minimum\{\tilde{d}_1 \oplus \tilde{d}_{12}\}\\ &= minimum\{(0,0,0,0,0,0,0,0) \oplus (18,19,19.6,20,21,22,22.3,23)\}\\ &= (18,19,19.6,20,21,22,22.3,23)\end{aligned}$$

Since minimum occurs corresponding to i=1, so label node 2 as

[(18,19,19.6,20,21,22,22.3,23),1].

NUMERICAL ILLUSTRATION

Iteration 2:

The predecessor nodes of the node 3 are node 1 and 2, so putting i=1,2 and j=3 in step 2, the value of $\tilde{d}_3$ is

$$\tilde{d}_3 = minimum\{\tilde{d}_1 \oplus \tilde{d}_{13}, \tilde{d}_2 \oplus \tilde{d}_{23}\}$$

$$= minimum\{(0,0,0,0,0,0,0,0) \oplus (22, 22.4, 22.8, 23, 24, 24.6, 24.8, 25),$$

$$(18, 19, 19.6, 20, 21, 22, 22.3, 23) \oplus (7, 7.8, 8, 9, 9.5, 10, 10.5, 11)\}$$

$$= minimum\{(22, 22.4, 22.8, 23, 24, 24.6, 24.8, 25), (25, 26.8, 27.6, 29, 30.5, 32, 32.8, 34)\}$$

Since $\tilde{A}$= (22,22.4,22.8,23,24,24.6,24.8,25) and $\tilde{B}$=(25,26.8,27.6,29,30.5,32,32.8,34), so using *Ranking Function*,

$$\begin{aligned}\Re(\tilde{A}) &= \frac{1}{4}\{(94.2)(0.3) + (0.7)(94.4)\}\\ &= \frac{1}{4}\{28.26 + 66.08\}\\ &= \frac{1}{4}\{94.34\}\\ &= 23.59\end{aligned}$$

NUMERICAL ILLUSTRATION

$$
\begin{aligned}
\Re(\tilde{B}) &= \frac{1}{4}\{(118.6)(0.3) + (0.7)(151.9)\} \\
&= \frac{1}{4}\{35.58 + 106.33\} \\
&= \frac{1}{4}\{141.91\} \\
&= 35.48
\end{aligned}
$$

Since $\Re(\tilde{A}) < \Re(\tilde{B})$,

so, minimum { (22,22.4,22.8,23,24,24.6,24.8,25), (25,26.8,27.6,29,30.5,32,32.8,34) }

=(22,22.4,22.8,23,24,24.6,24.8,25)

i.e., $\tilde{d}_3 = (22,22.4,22.8,23,24,24.6,24.8,25)$

Since minimum occurs corresponding to i= 1, so label node 6 as

[(22,22.4,22.8,23,24,24.6,24.8,25),1].

NUMERICAL ILLUSTRATION

Iteration 3:

The predecessor nodes of the node 4 are node 2 and 3, so putting i=2,3 and j=4 in step 2, the value of $\tilde{d}_4$ is

$$\tilde{d}_4 = minimum\{\tilde{d}_2 \oplus \tilde{d}_{24}, \tilde{d}_3 \oplus \tilde{d}_{34}\}$$

$$= minimum\{(18, 19, 19.6, 20, 21, 22, 23.3, 23) \oplus (12, 12.2, 12.7, 13, 15, 15.3, 15.8, 16),$$

$$(22, 22.4, 22.8, 23, 24, 24.6, 24.8, 25) \oplus (6, 7, 7.8, 9, 10, 10.5, 10.8, 11)\}$$

$$= minimum\{(30, 31.2, 32.3, 33, 36, 37.3, 38.1, 39), (28, 29.4, 30.6, 32, 34, 35.1, 34.6, 36)\}$$

Since $\tilde{A}$= (30,31.2,32.3,33,36,37.3,38.1,39) and $\tilde{B}$= (28,29.4,30.6,32,34,35.1,34.6,36), so using *Ranking Function*,

$$\Re(\tilde{A}) > \Re(\tilde{B})$$

Since minimum occurs corresponding to i=3, so label node 4 as

[(28,29.4,30.6,32,34,35.1,34.6,36),3].

NUMERICAL ILLUSTRATION

Iteration 4:

Since, only node 3 is the predecessor node of node 5, so putting i=3 and j=5 in step 2, the value of $\tilde{d}_5$ is

$$\begin{aligned}\tilde{d}_5 &= minimum\{\tilde{d}_3 \oplus \tilde{d}_{35}\} \\ &= minimum\{(22, 22.4, 22.8, 23, 24, 24.6, 24.8, 25) \oplus (15, 16, 16.3, 17, 18, 18.7, 19.2, 20)\} \\ &= (37, 38.4, 39.1, 40, 42, 43.3, 44, 45)\end{aligned}$$

Since minimum occurs corresponding to i=3, so label node 5 as

[(37,38.4,39.1,40,42,43.3,44,45),3].

NUMERICAL ILLUSTRATION

Iteration 5:

The predecessor nodes of the node 6 are node 4 and 5, so putting i=4,5 and j=6 in step 2, the value of $\tilde{d}_6$ is

$$\tilde{d}_6 = minimum\{\tilde{d}_4 \oplus \tilde{d}_{46}, \tilde{d}_5 \oplus \tilde{d}_{56}\}$$

$$= minimum\{(28, 29.4, 30.6, 32, 34, 35.1, 34.6, 36) \oplus (61, 61.3, 61.8, 62, 63, 63.4, 63.6, 64),$$

$$(37, 38.4, 39.1, 40, 42, 43.3, 44, 45) \oplus (62, 63, 63.3, 64, 65, 65.5, 65.8, 66)\}$$

$$= minimum\{(89, 90.7, 92.4, 94, 97, 98.5, 98.2, 100), (99, 101.4, 102.4, 104, 107, 108.8, 109.8, 111)\}$$

Since $\tilde{A}$= (92.4,94,97,98.5) and $\tilde{B}$= (102.4,104,107,108.8),

so using *Ranking Function*,

$$\Re(\tilde{A}) < \Re(\tilde{B})$$

Since minimum occurs corresponding to i=4, so label node 6 as

[(89,90.7,92.4,94,97,98.5,98.2,100), 4].

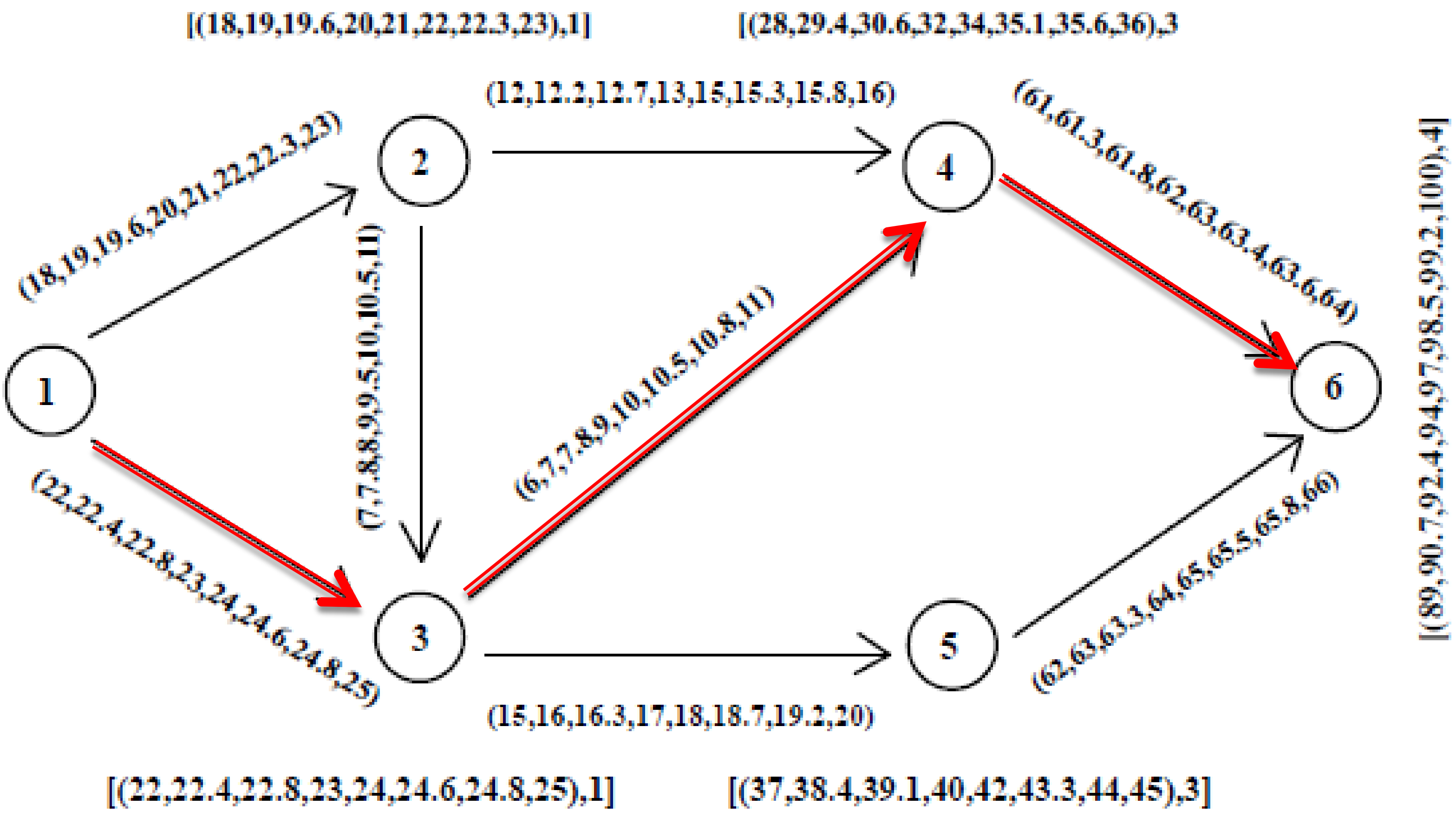
NUMERICAL ILLUSTRATION

The fuzzy shortest distance and the fuzzy shortest path of all the nodes from 1 is shown in the below table

Node No.(j)	$\tilde{d}_j$	Fuzzy shortest path between j^{th} and 1^{st} node
2	(18,19,19.6,20,21,22,22.3,23)	$1 \to 2$
3	(22,22.4,22.8,23,24,24.6,24.8,25)	$1 \to 3$
4	(28,29.4,30.6,32,34,35.1,34.6,36)	$1 \to 3 \to 4$
5	(37,38.4,39.1,40,42,43.3,44,45)	$1 \to 3 \to 5$
6	(89,90.7,92.4,94,97,98.5,98.2,100)	$1 \to 3 \to 4 \to 6$

NUMERICAL ILLUSTRATION

The labeling of each node is shown in figure below:



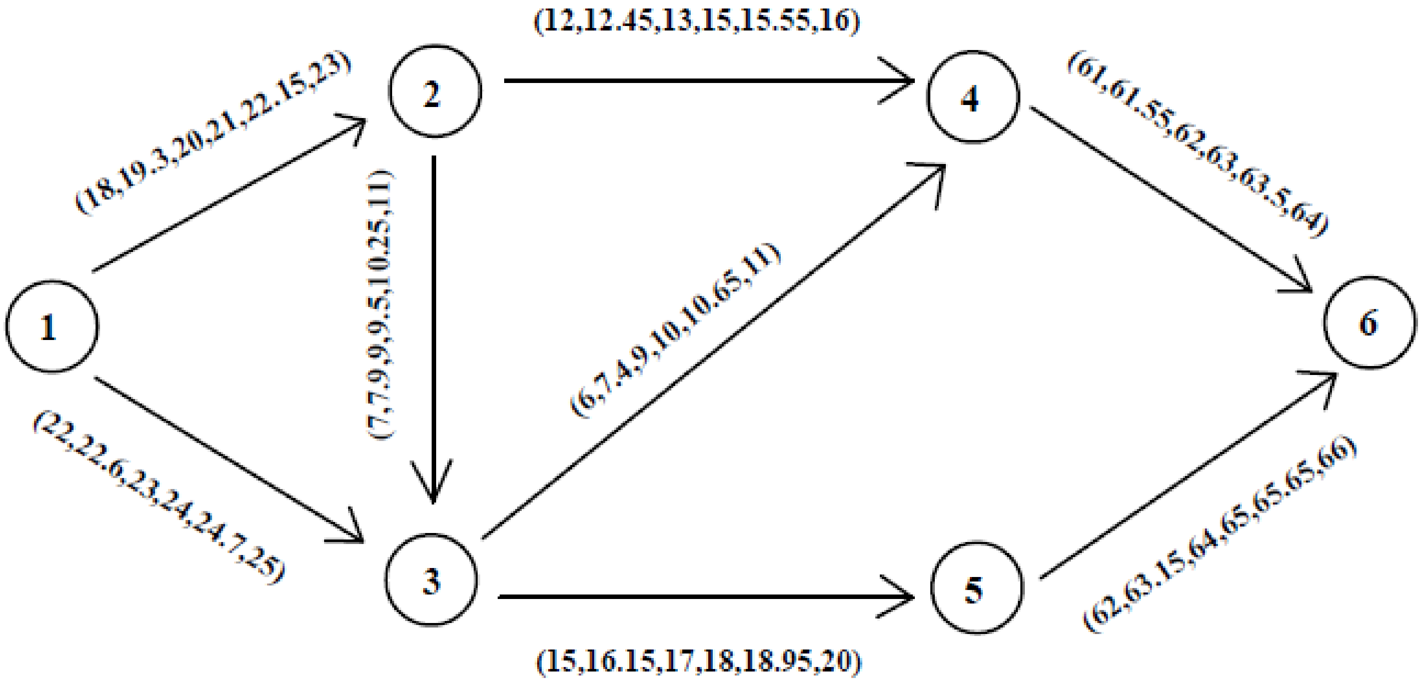
Advantages of the proposed algorithm

- The Fuzzy shortest path and the fuzzy distance of each node from the initial node or the source node can be obtained simultaneously without repeating the algorithm again and again.
- The optimal solution is obtained as an octagonal fuzzy number.
- The techniques of linear programming are not required.
- The knowledge of ranking of octagonal fuzzy numbers and addition and subtraction of octagonal fuzzy numbers is sufficient for the decision maker.
- The proposed algorithm is very easy to understand and to apply.
- There is no need of much knowledge of fuzzy linear programming, Zimmermann approach [31] and crisp linear programming.
- The proposed algorithms can be easily implemented into any programming language.

CHAPTER - 4

FSPP INVOLVING HEXAGONAL FUZZY NUMBERS

NUMERICAL ILLUSTRATION



<u>Network of the shortest path</u>

NUMERICAL ILLUSTRATION

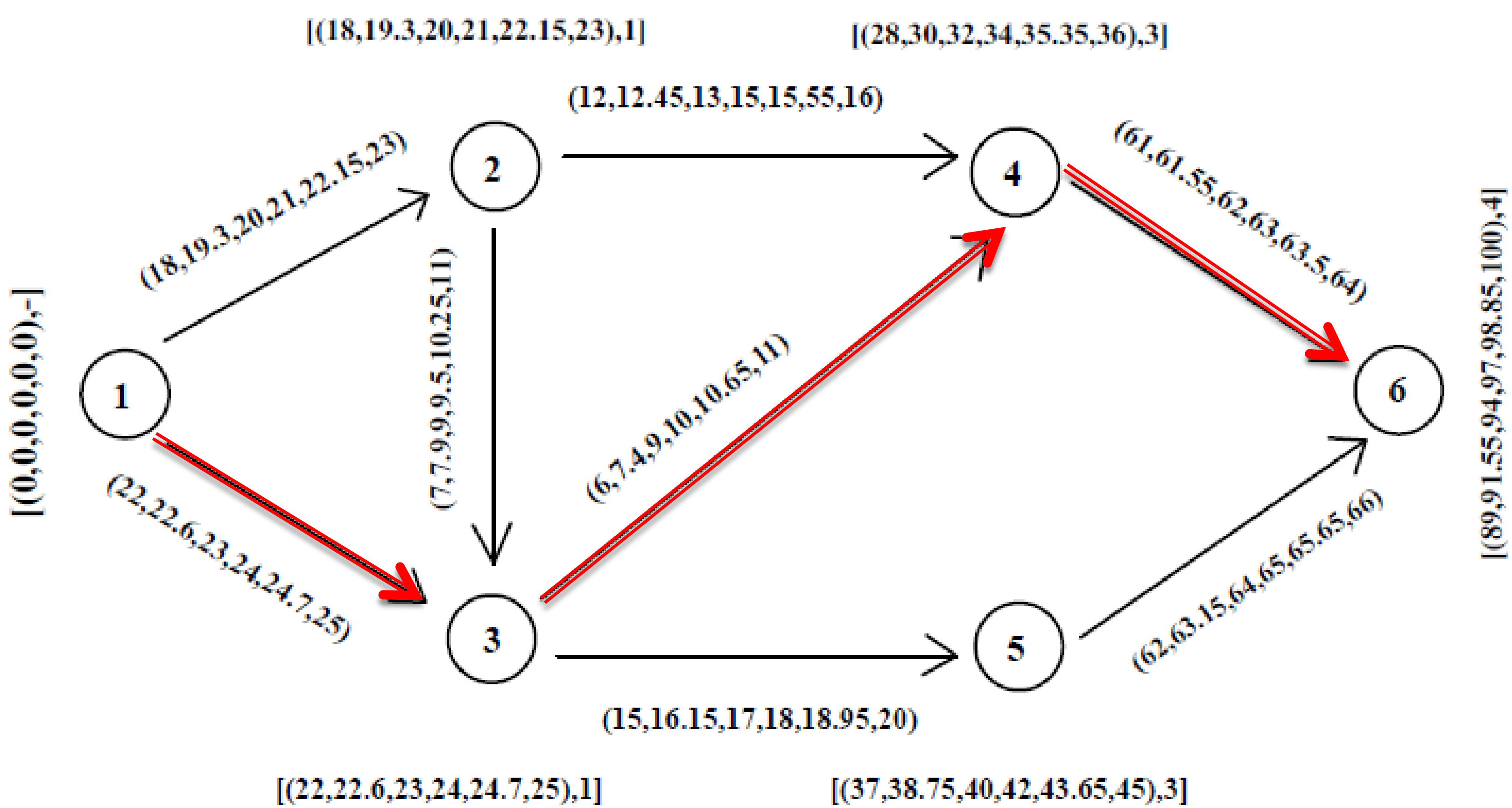
SOLUTION:

The fuzzy shortest distance and the fuzzy shortest path of all the nodes from 1 is shown in the below table

Node No.(j)	$\tilde{d}_j$	Fuzzy shortest path between j^{th} and 1^{st} node
2	(18,19.3,20,21,22.15,23)	$1 \to 2$
3	(22,22.6,23,24,24.7,25)	$1 \to 3$
4	(28,30,32,34,35.35,36)	$1 \to 3 \to 4$
5	(37,38.75,40,42,43.65,45)	$1 \to 3 \to 5$
6	(89,91.55,94,97,98.85,100)	$1 \to 3 \to 4 \to 6$

NUMERICAL ILLUSTRATION

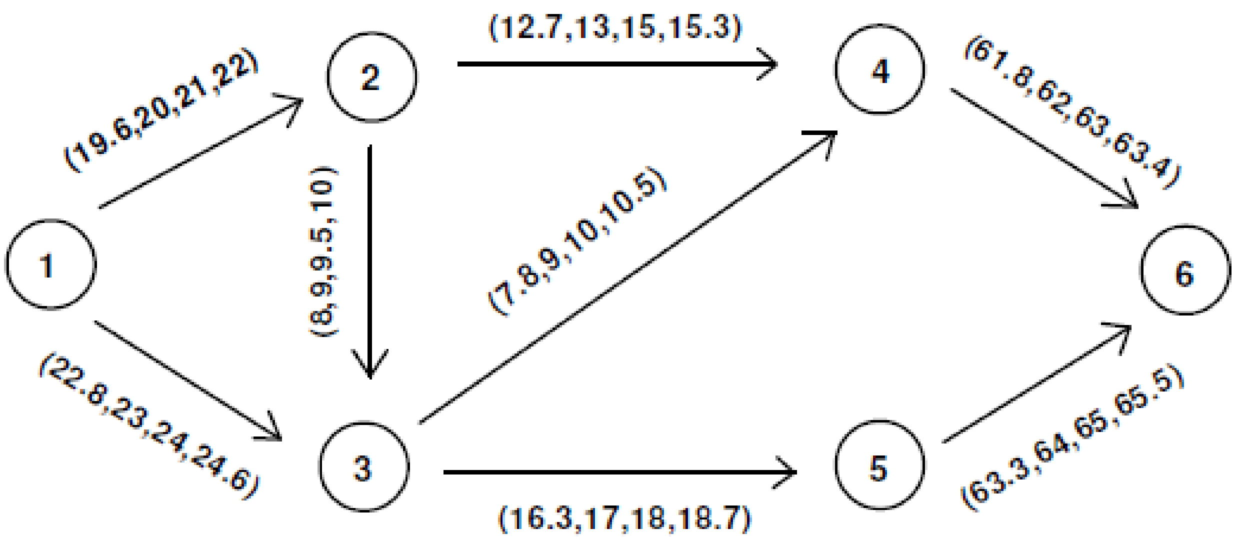
The labeling of each node is shown in figure below:



CHAPTER - 5

FSPP INVOLVING TRAPEZOIDAL FUZZY NUMBERS

NUMERICAL ILLUSTRATION



Network of the shortest path

NUMERICAL ILLUSTRATION

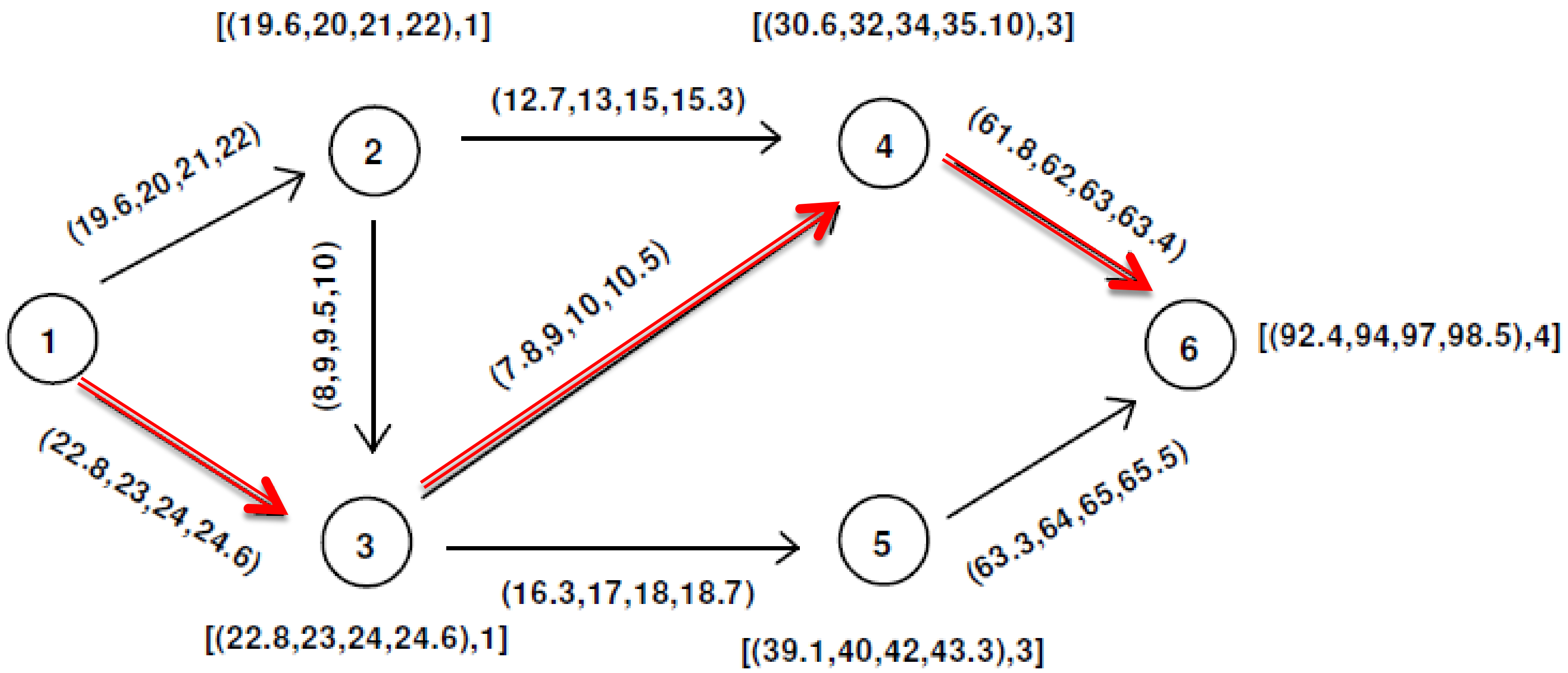
SOLUTION:

The fuzzy shortest distance and the fuzzy shortest path of all the nodes from 1 is shown in the below table

Node No.(j)	$\tilde{d}_j$	Fuzzy shortest path between j^{th} and 1^{st} node
2	(19.6,20,21,22)	$1 \rightarrow 2$
3	(22.8,23,24,24.6)	$1 \rightarrow 3$
4	(30.6,32,34,35.1)	$1 \rightarrow 3 \rightarrow 4$
5	(39.1,40,42,43.3)	$1 \rightarrow 3 \rightarrow 5$
6	(92.4,94,97,98.5)	$1 \rightarrow 3 \rightarrow 4 \rightarrow 6$

NUMERICAL ILLUSTRATION

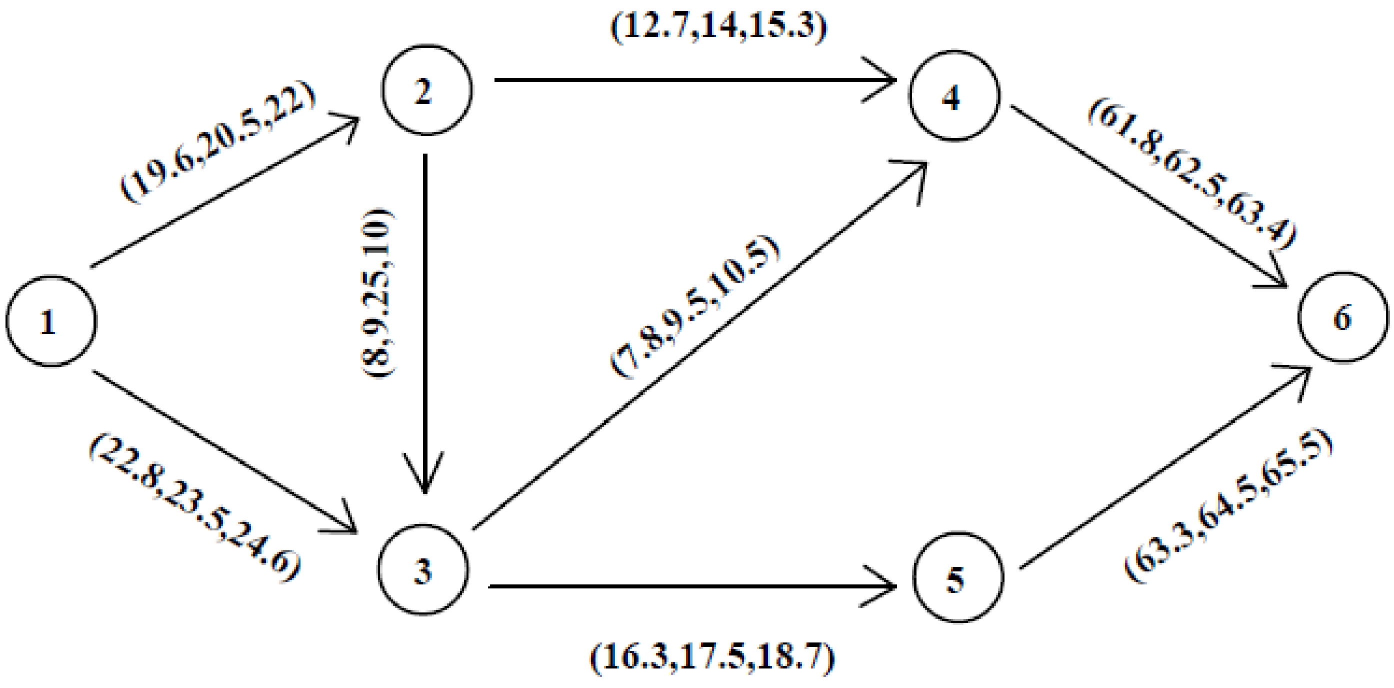
<u>The labeling of each node is shown in figure below:</u>



CHAPTER - 6

FSPP INVOLVING TRIANGULAR FUZZY NUMBERS

NUMERICAL ILLUSTRATION



<u>Network of the shortest path</u>

NUMERICAL ILLUSTRATION

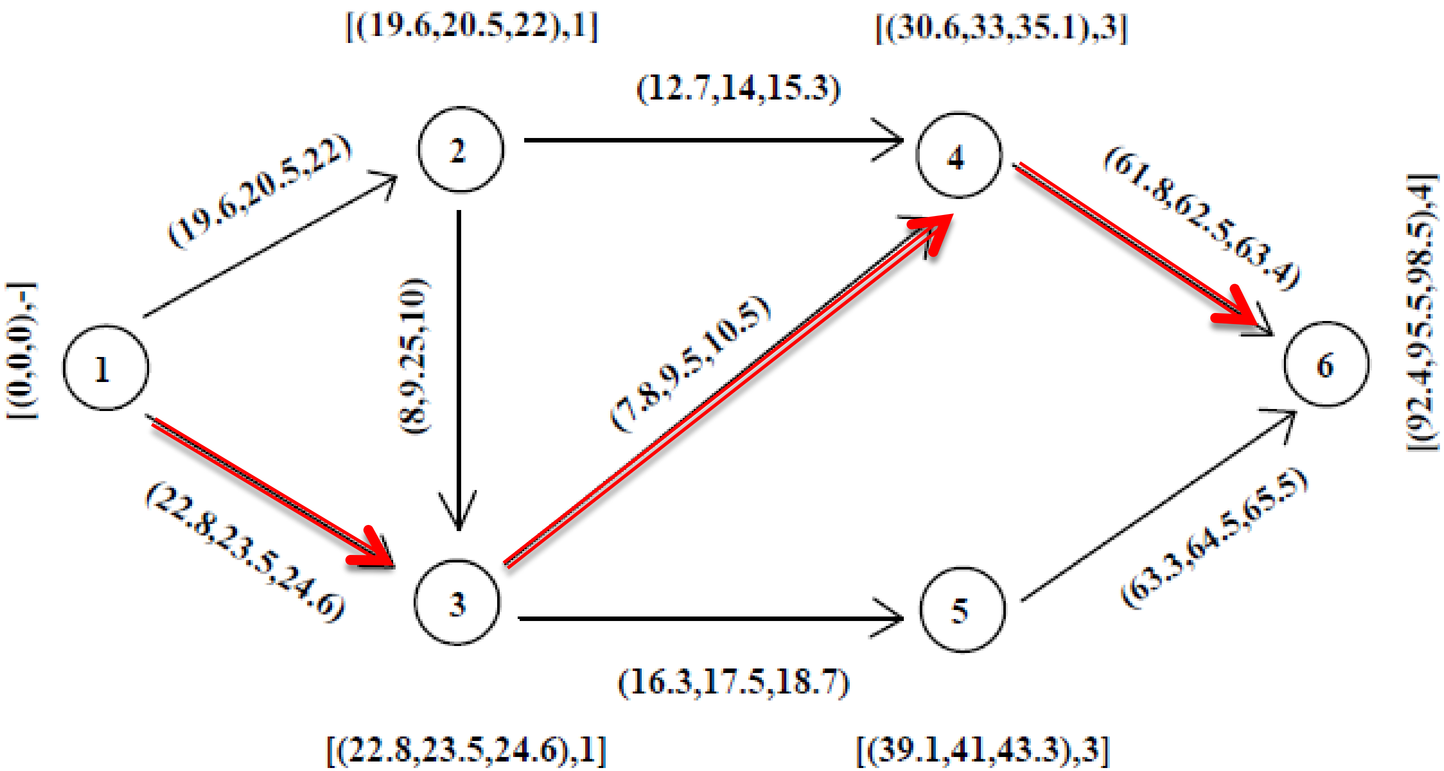
SOLUTION:

The fuzzy shortest distance and the fuzzy shortest path of all the nodes from 1 is shown in the below table

Node No.(j)	$\tilde{d}_j$	Fuzzy shortest path between j^{th} and 1^{st} node
2	(19.6,20.5,22)	$1 \rightarrow 2$
3	(22.8,23.5,24.6)	$1 \rightarrow 3$
4	(30.6,33,35.1)	$1 \rightarrow 3 \rightarrow 4$
5	(39.1,41,43.3)	$1 \rightarrow 3 \rightarrow 5$
6	(92.4,95.5,98.5)	$1 \rightarrow 3 \rightarrow 4 \rightarrow 6$

NUMERICAL ILLUSTRATION

The labeling of each node is shown in figure below:



NUMERICAL ILLUSTRATION

Another method:

The triangular fuzzy numbers in the form of $\langle m, \alpha, \beta \rangle$, i.e., in terms of mean and the left-spreads and right-spreads are as follow:

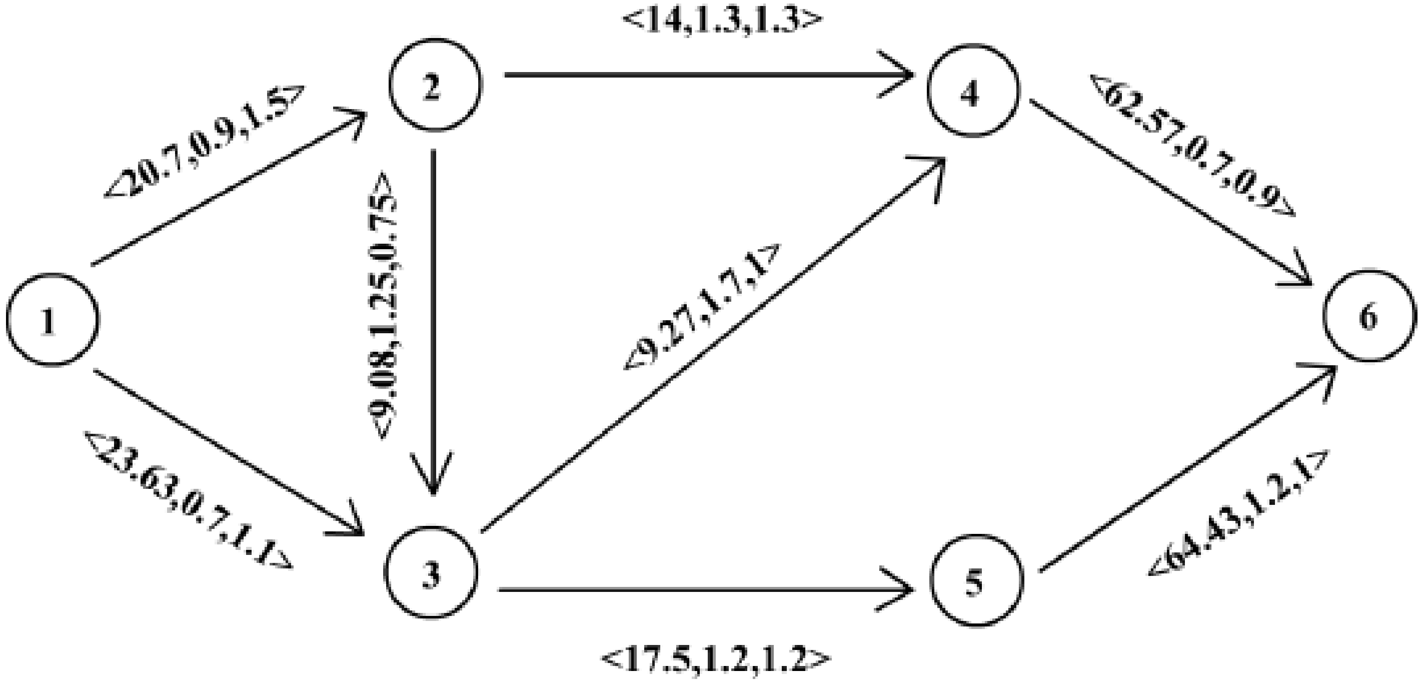
$$\tilde{e}_{12} = \langle 20.7, 0.9, 1.5 \rangle; \qquad \tilde{e}_{13} = \langle 23.63, 0.7, 1.1 \rangle$$

$$\tilde{e}_{23} = \langle 9.08, 1.25, 0.75 \rangle; \qquad \tilde{e}_{24} = \langle 14, 1.3, 1.3 \rangle$$

$$\tilde{e}_{34} = \langle 9.27, 1.7, 1 \rangle; \qquad \tilde{e}_{35} = \langle 17.5, 1.2, 1.2 \rangle$$

$$\tilde{e}_{46} = \langle 62.57, 0.7, 0.9 \rangle; \qquad \tilde{e}_{56} = \langle 64.43, 1.2, 1 \rangle$$

NUMERICAL ILLUSTRATION



Network of the shortest path

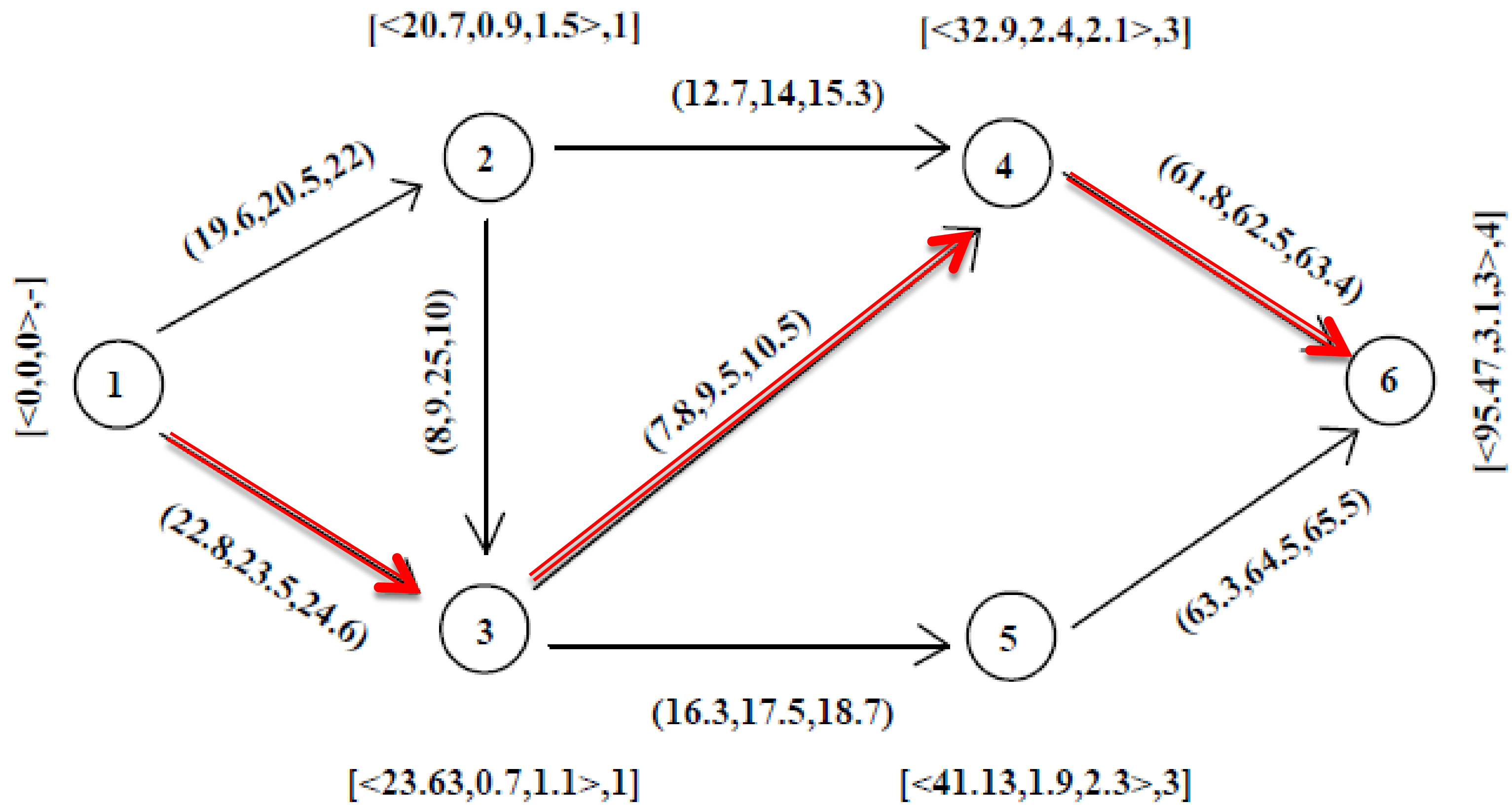
NUMERICAL ILLUSTRATION

The fuzzy shortest distance and the fuzzy shortest path of all the nodes from 1 is shown in the below table

Node No.(j)	$\tilde{e}_j$	Fuzzy shortest path between j^{th} and 1^{st} node
2	$<20.7,0.9,1.5>$	$1 \to 2$
3	$<23.63,0.7,1.1>$	$1 \to 3$
4	$<32.9,2.4,2.1>$	$1 \to 3 \to 4$
5	$<41.13,1.9,2.3>$	$1 \to 3 \to 5$
6	$<95.47,3.1,3>$	$1 \to 3 \to 4 \to 6$

NUMERICAL ILLUSTRATION

The labeling of each node is shown in figure below:



CHAPTER - 7

CONCLUSION

In this thesis a fuzzy network flow problem namely, the fuzzy shortest path problem involving octagonal fuzzy numbers, hexagonal fuzzy numbers, trapezoidal fuzzy numbers and triangular fuzzy numbers were solved and the optimal solutions were obtained as octagonal fuzzy numbers, hexagonal fuzzy numbers, trapezoidal fuzzy numbers and triangular fuzzy numbers which are checked with numerical illustrations. It is found that the shortest path obtained in all the cases remain the same.

THANK YOU