A STUDY OF OPEN JACKSON NETWORK IN OPERATIONS RESEARCH WITH FUZZY APPROACH

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OUTLINE OF THE DISSERTATION

CHAPTER - 1

INTRODUCTION

INTRODUCTION

- The fuzzy set theory has been proposed in 1965 by Lofti A. Zadeh .
- This theory is a mathematical theory which deals with uncertainty which are common in the natural language.
- Unlike computers, the human reasoning is not binary where everything is either yes(true) or no(false) but deals with imprecise concepts like ‘a tall man’, ‘a moderate temperature’ or ‘a large profit’. These concepts are ambiguous in the sense that they cannot be sharply defined.
- Thus fuzziness occurs when the boundary of a piece of information is not clear-cut. The word “fuzzy” means “vagueness”.

INTRODUCTION (Continued…)

- The Jackson network is a class of queueing network where the equilibrium distribution is particularly simple to compute as the network has a product-form solution.
- The networks were first identified by James R. Jackson.
- A Jackson network consists of a number of nodes, where each node represents a queue in which the service rate can be both node-dependent(different nodes have different service rates) and state-dependent (service rates change depending on queue lengths).
- In this dissertation we are introducing fuzzy in Jackson network i.e., all the arrivals are in the range [0,1].

CHAPTER - 2

PRELIMINARIES

BASIC DEFINITIONS

Crisp set

$$\chi_A : X \to \{0,1\}, \text{where} \quad \chi_A(x) = \begin{cases} 1 & \text{for } x \in A \\ 0 & \text{for } x \notin A \end{cases}$$ 1

Fuzzy set

2 $$\mu_{\tilde{A}} : X \to [0,1]$$

Fuzzy number

It is a special type of fuzzy set.

$$\mu_{\tilde{A}} : R \to [0,1]$$ 3

PRE-REQUISITES

Membership function of triangular fuzzy number

$$\tilde{A} = (a, m, b; 1)$$

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(m-a)}, & a \le x < m \\ 1, & x = m \\ \frac{(b-x)}{(b-m)}, & m < x \le b \\ 0, & otherwise \end{cases}$$

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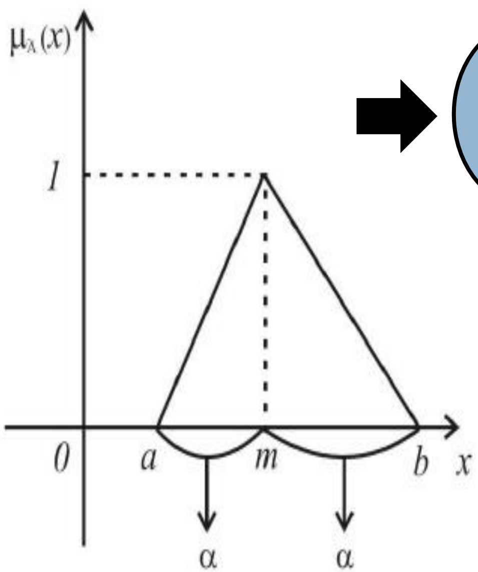
PRE-REQUISITES

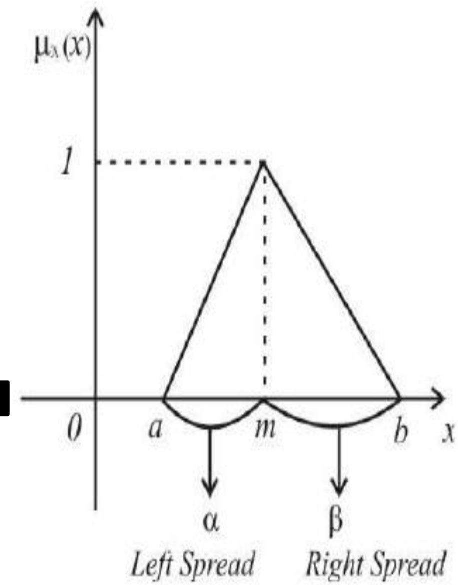
Defuzzification formulae for triangular fuzzy number

a) Mean Measure (MM) of triangular fuzzy number $\tilde{A} = (a, m, b; 1)$



5 $MM(\tilde{A}) = \frac{(a+b)}{2} = R(\tilde{A})$

6 $LRM(\tilde{A}) = \frac{1}{2}\left[m + \frac{(a+b)}{2}\right] = R(\tilde{A})$



b) Left-Right Measure (LRM) of triangular fuzzy number $A = (a, m, b; 1)$

PRE-REQUISITES

Operations on triangular fuzzy numbers

Let $\tilde{A} = (a_1, a_2, a_3)$ *and* $\tilde{B} = (b_1, b_2, b_3)$ *be two triangular fuzzy numbers, then*

- $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
- $\tilde{A} \ominus \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$
- $\tilde{A} \otimes \tilde{B} = \{\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3)\}$

CHAPTER - 3

FUZZY JACKSON NETWORK

Necessary conditions for Fuzzy Jackson Network

A network of m interconnected queues is known as a Jackson network if it meets the following conditions:

1. Here the external arrivals are fuzzy events.
2. If the network is open, any external arrivals to node i form a Poisson process.
3. All service times are exponentially distributed and the service discipline at all queues is first-come , first-served.
4. A customer completing service at queue i will either move to some new queue j with probability $\tilde{P}_{ij}$ or leave the system with probability $1 - \sum_{j=1}^{m} \tilde{P}_{ij}$, which for an open network, is non-zero for some subset of the queues.
5. The utilization of all of the queues is less than one.

Mathematical Formulation for Fuzzy Jackson Network

In an open network, jobs arrive from outside following a poisson process with fuzzy rate $\bar{\alpha} > 0$. Each arrival is independently routed to node j with probability $\bar{p}_{0j} \geq 0$ and $\sum_{j=1}^{J} \bar{P}_{0j} = 1$. Upon service completion at node i, a job may go to another node j with probability $\bar{p}_{ij}$ or leave the network with probability $\bar{p}_{i0} = 1 - \sum_{j=1}^{J} \bar{P}_{ij}$.

$i.e$

$$P(\bar{A}) = \begin{cases} \bar{p}_{0j} \geq 0 \text{and} \sum_{j=1}^{J} \bar{P}_{0j} = 1 \\ \bar{p}_{i0} = 1 - \sum_{j=1}^{J} \bar{P}_{0j} \\ \bar{p}_{ij} \end{cases}$$

Hence

$$\bar{\lambda}_i = \bar{\alpha} \otimes \bar{p}_{0i} + \sum_{j=1}^{J} \bar{\lambda}_j \otimes \bar{P}_{ji}$$

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where $\bar{\alpha} \otimes \bar{p}_{0i}$ is the arrival rate of customers to i coming from outside the system, and as $\bar{\lambda}_j$ is the rate of which customers depart server j (rate in must equal rate out), $\bar{\lambda}_j \otimes \bar{P}_{ji}$ is the arrival rate to i of those coming from server j.

CHAPTER - 4

NUMERICAL EXAMPLE

NUMERICAL EXAMPLE

Solution for fuzzy jackson open network using three methods

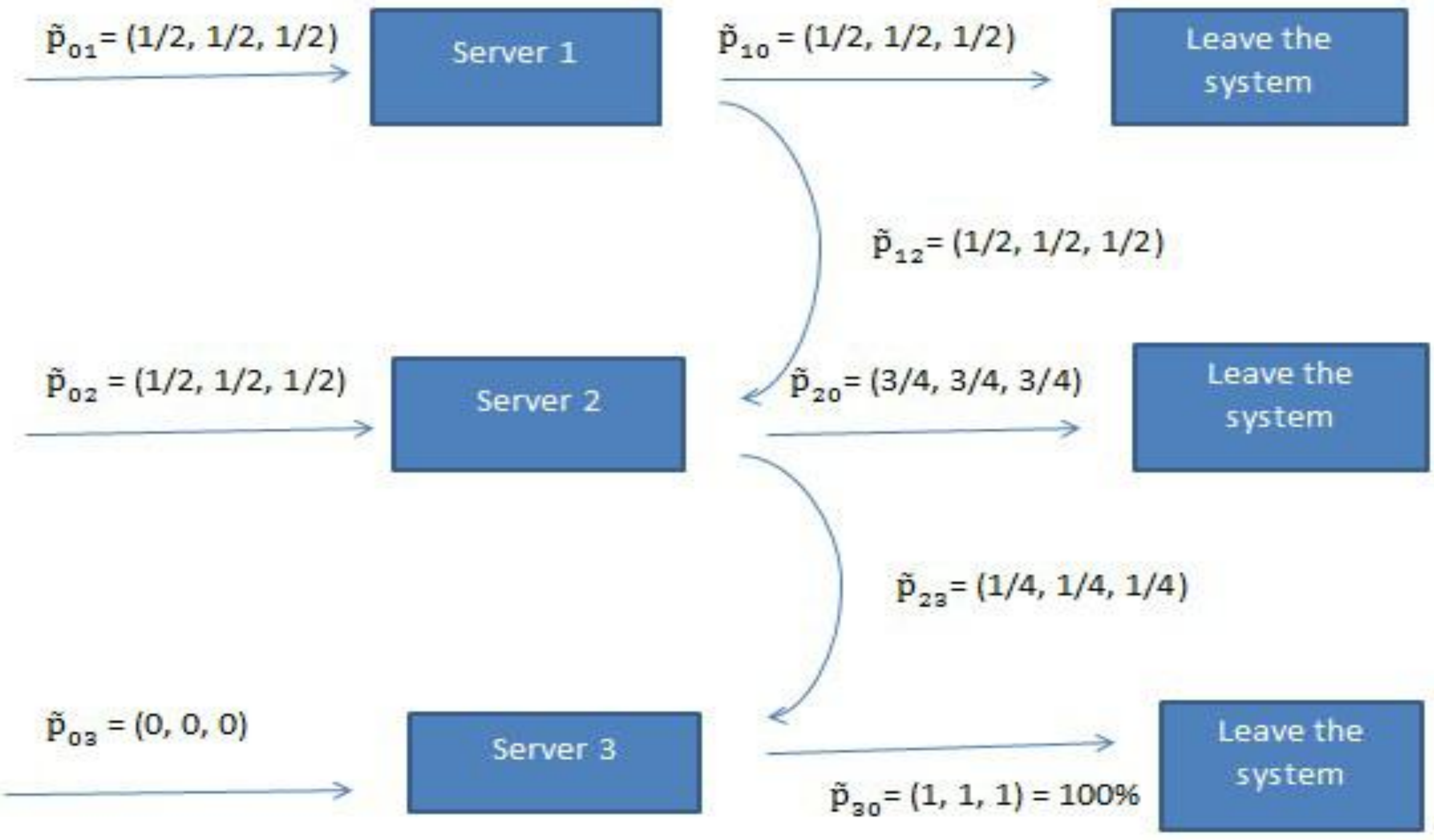
1. Solving with fuzzy number
2. Solving fuzzy number in k stages
3. Solving with Defuzzified fuzzy number

BALANCED TRIANGULAR FUZZY NUMBER

Consider a system of three servers where customers arrive from outside following a fuzzy poisson process with rate $\tilde{\alpha} = (2,4,6), \tilde{\alpha} > 0$ *. Each arrival is independently routed to node j, (j=1,2,3) with probability* $P_{01} = P_{02} = \frac{1}{2}, P_{03} = 0$.

A customer upon completion of service at server 1 is equally likely to go to server 2 or to leave the system $(i.e., P_{11}, P_{12})$ *where as a departure from server 2 will go 25 % of the time to server 1 and will depart the system otherwise. At server 3, after customers completing the service will leave the system.*

BALANCED TRIANGULAR FUZZY NUMBER



BALANCED TRIANGULAR FUZZY NUMBER

Method 1: Using Fuzzy Number

We know that,

the Fuzzy Jackson's flow balance equations are

$$\tilde{\lambda}_i = \tilde{\alpha} \otimes \tilde{p}_{0i} \oplus \sum_{j=1}^{3} \tilde{\lambda}_j \otimes \tilde{P}_{ji}$$

For i=1, we get

$$\begin{aligned}
\tilde{\lambda}_1 &= (2,4,6) \otimes \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \oplus \tilde{\lambda}_1 \otimes \tilde{p}_{11} \oplus \tilde{\lambda}_2 \otimes \tilde{p}_{21} \oplus \tilde{\lambda}_3 \otimes \tilde{p}_{31} \\
&= (1,2,3) \oplus \tilde{\lambda}_1 \otimes (0,0,0) \oplus \tilde{\lambda}_2 \otimes (0,0,0) \oplus \tilde{\lambda}_3 \otimes (0,0,0) \\
&= (1,2,3) \oplus (0,0,0) \\
&= (1,2,3)
\end{aligned}$$

Proceeding like this for ***i***=2 and ***i***=3, we get

$$\tilde{\lambda}_2 = (1.5, 3, 4.5)$$

$$\tilde{\lambda}_3 = (0.38, 0.75, 1.13)$$

BALANCED TRIANGULAR FUZZY NUMBER

Method 2: Using Fuzzy Number

Here k=3, since it is a triangular fuzzy number.

Given

$$\tilde{\alpha} = (2, 4, 6)$$

Stage: 1	**Stage: 2**	**Stage: 3**
$\alpha_1 = 2$	$\alpha_2 = 4$	$\alpha_3 = 6$
we get	we get	we get
$\lambda_{11} = 1$	$\lambda_{12} = 2$	$\lambda_{13} = 3$
$\lambda_{21} = 1.5$	$\lambda_{22} = 3$	$\lambda_{23} = 5.25$
$\lambda_{31} = 0.38$	$\lambda_{32} = 0.94$	$\lambda_{33} = 1.31$

Therefore the total arrival rates are

$$\tilde{\lambda}_1 = (\lambda_{11}, \lambda_{12}, \lambda_{13}) = (1, 2, 3)$$

$$\tilde{\lambda}_2 = (\lambda_{21}, \lambda_{22}, \lambda_{23}) = (1.5, 3, 4.5)$$

$$\tilde{\lambda}_3 = (\lambda_{31}, \lambda_{32}, \lambda_{33}) = (0.38, 0.75, 1.13)$$

BALANCED TRIANGULAR FUZZY NUMBER

Method 3: Using Defuzzified Fuzzy Number

Given

$$\tilde{\alpha} = (2, 4, 6)$$

Then *Mean Measure (MM) of triangular fuzzy number*

$$m = \frac{(a+b)}{2} = MM(\tilde{A})\ \alpha = \frac{2+6}{2} = \frac{8}{2} = 4$$

For i=1, we get $\lambda_1 = 2$

For i=2, we get $\lambda_2 = 3$

For i=3, we get $\lambda_3 = 0.75$

Verification:

Let $\tilde{\lambda}_1 = (1,2,3)$, then $\lambda_1 = 2$

Let $\tilde{\lambda}_2 = (1.5, 3, 4.5)$, then $\lambda_2 = 3$

Let $\tilde{\lambda}_3 = (0.38, 0.75, 1.13)$, then $\lambda_3 = 0.76$

In all three methods, we got the same total arrival rates. So we can use any one method to find the total arrival rates in Fuzzy open Jackson Network.

CHAPTER - 5

APPLICATION OF FUZZY JACKSON NETWORK

APPLICATION

Queueing theory applied in our day to day life:

- Queues are very familiar in our daily life.
- Earlier days [i.e, before the arrivals of Website or Internet], queue formed by our physical presence. That is, if we want to buy something for our need, we have to go straight away to that required place.
- After some years, when website arrived, we got queues in computer system.
- Nowadays, internet is developed so much. Human's life is running with the help of internet. Since internet has more facilities like ordering dress, books, food, home appliances, etc., from wherever we are and however we are, human lives became very much easier.
- Everything in today's life will be done with internet. Queue forming there but not for long time as normal queueing system.

APPLICATION

Role of internet in our daily life:

- Entertainment- movies, music, games and books.
- Online banking
- Shopping
- Sending and storing files(data)
- Social networking
- Communication

APPLICATION

<u>Queue forms in online process:</u>

- ***Internet***

  Global system of interconnected computer networks.

- ***Servers***

  Accept limited no.of arrivals at a time in internet, except selected arrivals other arrivals have to wait.

  example:

  Tatkal Scheme

APPLICATION

- Online queue is forming based on speed of the internet.
- In internet queueing system, whose internet is faster will be served first even he is the last comer, whose internet is slower will be served after the first one.
- Survey

What is required to use the internet faster ?

1. Signal
2. Internet Speed
3. Weather condition must be good

From survey, we couldn't get the accurate answer. So vaguely we move on to analyze the service time and service rate. Applying fuzzy open Jackson network in online process, we can find the total arrival rate for each server in the online queueing system but we cannot know the service rate because we don't know the service time of each arrival. If we know the service time, we can find the service rate.

CHAPTER - 6

CONCLUSION

CONCLUSION

- ✓ Many researches have focused on Queueing Theory, since it aims to predict the queue lengths and waiting time.
- ✓ Also many researches are interested to work in Jackson network because it has multi-queues in which customers are of different types.
- ✓ In this dissertation we were newly introduced fuzzy Jackson network to apply in online process.
- ✓ In online process queue forms based on server's capacity. Using probability, probability data's are well defined in discrete.

CONCLUSION (Continued…)

- ✓ In online, server data processing cannot be defined using probability as it may vary. But using fuzzy any number of arrivals to the server can be defined. In many servers, the service time in the process cannot be studied using probability.
- ✓ To satisfy this, we introduced the concept of fuzzy in it which will exhibit better understanding.
- ✓ The fuzzy approach to the queueing theory will explore into new avenues of perspective and approach of problems. The limitations and constrains in the probabilistic approach can be sorted out with fuzzy logic.

CONCLUSION (Continued…)

- Fuzzy tools enhances and helps in better understanding and progressive in nature. As in this dissertation, more applications of server problems can be analyzed and helps in the growth of the queueing theory, Operations Research in large.

THANK YOU