FTP, FAP AND FTSP UNDER FUZZY ONE POINT AND FUZZY ZERO POINT METHOD

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OUTLINE OF THE PROJECT

CHAPTER - 1

INTRODUCTION

- The fuzzy set theory has been proposed in 1965 by Lofti A. Zadeh.
- This theory is a mathematical theory which deals with uncertainty which are common in the natural language.
- Unlike computers, the human reasoning is not binary where everything is either yes(true) or no(false) but deals with imprecise concepts like ‘a tall man’, ‘a moderate temperature’ or ‘a large profit’. These concepts are ambiguous in the sense that they cannot be sharply defined.
- Thus fuzziness occurs when the boundary of a piece of information is not clear-cut. The word “fuzzy” means “vagueness”.

PRE-REQUISITES

Crisp set

$$\chi_A : X \to \{0,1\}, \text{where} \quad \chi_A(x) = \begin{cases} 1 & \text{for } x \in A \\ 0 & \text{for } x \notin A \end{cases}$$

1

Fuzzy set

2

$$\mu_{\tilde{A}} : X \to [0,1]$$

Support of a fuzzy set

$$supp\left(\tilde{A}\right) = \{x \in X | \mu_{\tilde{A}}(x) > 0\}$$

3

PRE-REQUISITES

Height of a fuzzy set ($h(\tilde{A})$)

$\text{If } h(\tilde{A}) = 1, \text{ the fuzzy set is normal.}$

$\text{It is called subnormal if } h(\tilde{A}) < 1 .$

Fuzzy number

It is a special type of fuzzy set.

$$\mu_{\tilde{A}} : R \to [0, 1]$$

PRE-REQUISITES

Membership function of triangular fuzzy number $\tilde{A} = (a, m, b; 1)$

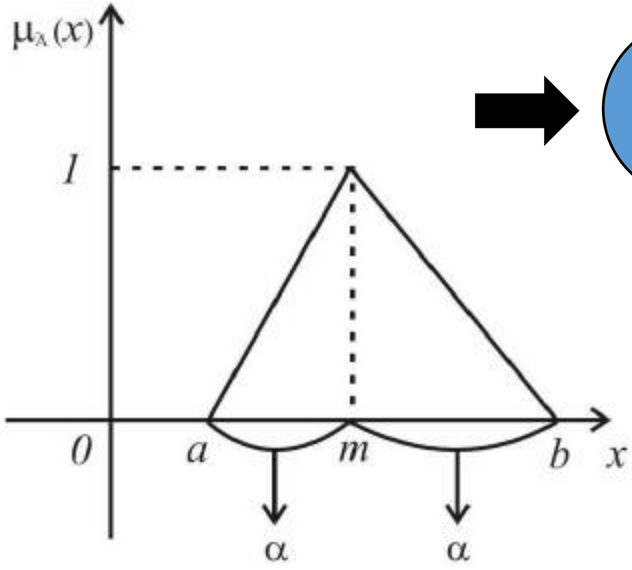
$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(m-a)}, & a \leq x < m \\ 1, & x = m \\ \frac{(b-x)}{(b-m)}, & m < x \leq b \\ 0, & otherwise \end{cases}$$

5

PRE-REQUISITES

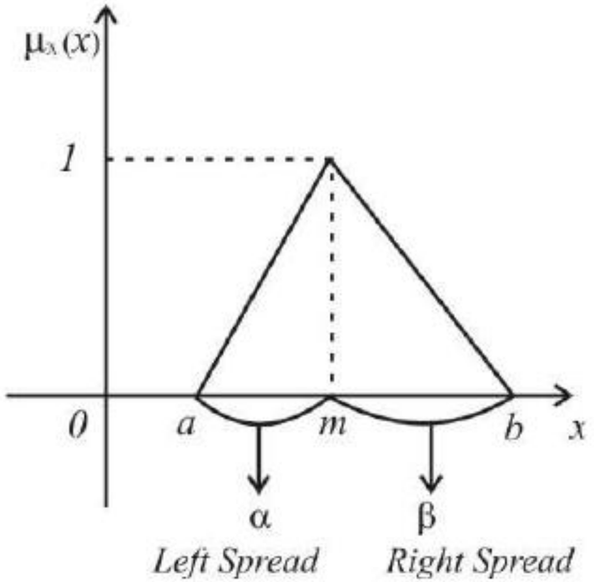
Defuzzification formulae for triangular fuzzy number

a) Mean Measure (MM) of triangular fuzzy number $\tilde{A} = (a, m, b; 1)$



$$MM(\tilde{A}) = \frac{(a+b)}{2} = R(\tilde{A}) \tag{6}$$

$$LRM(\tilde{A}) = \frac{1}{2}\left[m + \frac{(a+b)}{2}\right] = R(\tilde{A}) \tag{7}$$



b) Left-Right Measure (LRM) of triangular fuzzy number $\tilde{A} = (a, m, b; 1)$

CHAPTER - 2

FUZZY TRANSPORTATION PROBLEM

Abstract:

This chapter focus on FTP, UFTP where the transportation cost, supply and demand are in the form of triangular fuzzy numbers. Here the main aim is to transform a problem with fuzzy parameters to a defuzzified version applying the ranking function Mean Measure (MM) / Left Right Measure (LRM) and to solve it by using fuzzy one point method and fuzzy zero point method. The solution procedure is illustrated with numerical example.

INTRODUCTION

- The transportation problem (TP) is a problem where a product is to be transported from ‘m’ sources to ‘n’ destinations.
- There are cases that the cost coefficients, the supply and demand quantities of a transportation problem may be uncertain due to some uncontrollable factors.
- Fuzzy transportation problem is a transportation problem in which the transportation cost, supply and demand quantities are fuzzy quantities.
- The objective of the fuzzy transportation cost problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying the fuzzy supply and demand limits.

MOTIVATION FACTOR

- **Pandian** and **Natarajan** proposed fuzzy zero point method for finding the solution for a FTP.
- **Sujatha** and **Elizabeth** proposed fuzzy one point method for finding the solution for FTP and FUAP in 3 stages.
- **Elizabeth et al.** developed fuzzy zero point method for finding the fuzzy objective value for UFTP.

Fuzzy Transportation Table

	D_1	$\dots$	D_n	FS
S_1	$\tilde{c}_{11}$	$\dots$	$\tilde{c}_{1n}$	$\tilde{a}_1$
$\vdots$	$\vdots$		$\vdots$	$\vdots$
S_m	$\tilde{c}_{m1}$	$\dots$	$\tilde{c}_{mn}$	$\tilde{a}_m$
FD	$\tilde{b}_1$	$\dots$	$\tilde{b}_n$	

Here $S_1, \dots S_m$ are m sources and $D_1, \dots D_n$ are n destinations. FS is the fuzzy supply points and FD is the fuzzy demand points.

LAYOUT OF THIS CHAPTER

- **Mathematical Formulation for FTP and UFTP**
- **Theoretical background for FTP**
  - ⟹ **under fuzzy one point method**
  - ⟹ **under fuzzy zero point method**
- **Procedure for FTP using fuzzy one point method**
  - ⟹ **Numerical Example -Balanced FTP**
  - ⟹ **Numerical Example -Unbalanced FTP**
- **Procedure for FTP using fuzzy zero point method**
  - ⟹ **Numerical Example -Balanced FTP**
  - ⟹ **Numerical Example -Unbalanced FTP**
- **Comparison**

Mathematical Formulation for FTP

$$Min \quad \tilde{z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij} \tag{2.1.1}$$

Subject to the contraints

$$\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i, i = 1 \quad to \quad m; \tag{2.1.2}$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j, j = 1 \quad to \quad n; \tag{2.1.3}$$

$$\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j; \tag{2.1.4}$$

$$\tilde{x}_{ij} \succeq \tilde{0}, \quad i = 1 \quad to \quad m \quad and \quad j = 1 \quad to \quad n \tag{2.1.5}$$

Conversion of FTP to CTP

$$Min \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} R(\tilde{c}_{ij}).x_{ij} \tag{2.1.6}$$

Subject to the contraints

$$\sum_{j=1}^{n} x_{ij} = R(\tilde{a}_i), i = 1 \quad to \quad m; \tag{2.1.7}$$

$$\sum_{i=1}^{m} x_{ij} = R(\tilde{b}_j), j = 1 \quad to \quad n; \tag{2.1.8}$$

$$\sum_{i=1}^{m} R(\tilde{a}_i) = \sum_{j=1}^{n} R(\tilde{b}_j); \tag{2.1.9}$$

$$x_{ij} \geq 0, i = 1 \quad to \quad m \quad and \quad j = 1 \quad to \quad n \tag{2.1.10}$$

Here $R(\tilde{c}_{ij}) = c_{ij}, R(\tilde{a}_i) = a_i, R(\tilde{b}_j) = b_j$.

$c_{ij} \geq 0$ (in the case of fuzzy zero point method);

$c_{ij} \geq 1$ (in the case of fuzzy one point method)

Mathematical Formulation for UFTP in the case of fuzzy one point method

$$Min \quad z = (\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}.x_{ij}) - (\sum_{i=1}^{m} x_{ij}) \;,$$ where j=n is a dummy column **8**

$$Min \quad z = (\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}.x_{ij}) - (\sum_{j=1}^{n} x_{ij}) \;,$$ where i=m is a dummy row **9**

Subject to the constraints given in the equations (2.1.7) to (2.1.10)

Theoretical background for FTP under fuzzy one point method

Theorem 2.3.1 *Any optimal solution to the problem* (P_1) *where* (P_1) *is* $Min\ z^* = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}/u_i/v_j).x_{ij}$, *subject to the constraints given in the equations* (2.1.12) *to* (2.1.15) *are satisfied, where* u_i *and* v_j *are some positive real crisp numbers, is an optimal solution to the problem*(P) *where* (P) *is* $Min\ z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}.x_{ij}$, *subject to the constraints given in the equations* (2.1.12) *to* (2.1.15) *are satisfied.*

Proof.

$$\text{Min } z^* = \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{c_{ij}.x_{ij}}{u_i v_j}$$

As $u_i \geq 1$ and $v_j \geq 1, \forall \quad i = 1$ to m and $j = 1$ to n.

$\frac{c_{ij}.x_{ij}}{u_i v_j} \leq c_{ij}.x_{ij}, \forall \quad i = 1$ to m and $j = 1$ to n.

i.e, $\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{c_{ij}.x_{ij}}{u_i v_j} \leq \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}.x_{ij}$

$\Rightarrow Min \quad z^* \leq Min \quad z$

Hence we can conclude that any optimal solution to the problem (P_1) is also an optimal solution to the problem (P).

Theoretical background for FTP under fuzzy zero point method

Theorem 2.3.3 *Any optimal solution to the FTP* (P_1^*) *where* (P_1^*) *is* $Min\ z^* = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij} - u_i - v_j).x_{ij} = Z^*$, *subject to the constraints given in the equations (2.1.12) to (2.1.15) are satisfied, where* u_i *and* v_j *are some positive real crisp numbers, is an optimal solution to the problem* (P^*) *where* (P^*) *is* $Min\ z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}.x_{ij} = Z$, *subject to the constraints given in the equations (2.1.12) to (2.1.15) are satisfied.*

Proof.

$$Min \quad z^* = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij} - u_i - v_j).x_{ij}$$

As $u_i \geq 0$ and $v_j \geq 0, \forall \quad i = 1$ to m and $j = 1$ to n.

$(c_{ij} - u_i - v_j).x_{ij} \leq c_{ij}.x_{ij}, \forall \quad i = 1$ to m and $j = 1$ to n.

$$\sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij} - u_i - v_j).x_{ij} \leq \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}.x_{ij}$$

$$\Rightarrow Min \quad z^* \leq Min \quad z$$

Hence we can conclude that any optimal solution to the problem (P_1^*) is also an optimal solution to the problem (P^*).

Procedure for FTP using fuzzy one point method

Step 2.4.1: Check whether the given FTP is a balanced one. If not, convert it into balanced FTP by introducing the dummy column or dummy row with cost entry as $(1, 1, 1)$.

Step 2.4.2: Fuzzy transportation cost, supply and demand interms of triangular fuzzy numbers are defuzzified using MM or LRM.

Step 2.4.3: Divide each row entries of the transportation table by row minimum that is if u_i is the minimum of the ith row of the table $[c_{ij}]$ then divide the i^{th} row entries by u_i, so that the resulting table is $[c_{ij}/u_i]$.

Procedure for FTP under fuzzy one point method

Step 2.4.4: Divide each column entries of the resulting transportation table after using the Step 2.4.3 by the column minimum that is if v_j is the minimum of j^{th} column of the resulting table $[c_{ij}/u_i]$ then divide j^{th} column entries by v_j so that the resulting table is $[(c_{ij}/u_i)/v_j]$. It may be noted that $(c_{ij}/u_i)/v_j \geq 1$ for all i, j. Each row and each column of the resulting table $[(c_{ij}/u_i)/v_j]$ has atleast one '1' entry.

Step 2.4.5: Choose the row or column with only one '1' and allot the minimum of source and demand corresponding to that cell. Check whether the supply points are fully used and all demand points are fully received. If so go to Step 2.4.7. If not, go to Step 2.4.6.

Procedure for FTP using fuzzy one point method

Step 2.4.6: Draw minimum number of lines horizontally and vertically to cover all the 1's. Then choose the least uncovered element and divide all the uncovered elements using it and multiply it at the intersection of lines, leaving other elements unchanged. Now, check whether each row and each column has atleast one '1' entry. If so, go to Step 2.4.5, else go to Step 2.4.3, Step 2.4.4 and then to Step 2.4.5.

Procedure for FTP using fuzzy one point method

Step 2.4.7: This allotment yields an optimal solution to the given FTP,

- with the objective function (2.1.11), if the FTP is balanced.

(or)

- with the objective function (2.2.1), if the FTP is balanced by introducing a dummy column $j = n$.

(or)

- with the objective function (2.2.2) , if the FTP is balanced by introducing a dummy row $i = m$.

All these objective functions are subject to the constraints (2.1.12) to (2.1.15).

Numerical Example (Balanced FTP using fuzzy one point method)

	D_1	D_2	D_3	FS $= \tilde{a}_i$
S_1	(8,10,16)	(8,9,10)	(6,8,14)	(6,8,14)
S_2	(8,10,16)	(5,7,13)	(8,10,16)	(5,7,13)
S_3	(9,11,13)	(8,9,10)	(5,7,13)	(8,9,10)
S_4	(10,12,14)	(12,14,16)	(8,10,16)	(3,4,9)
FD $= \tilde{b}_j$	(8,10,16)	(8,10,16)	(6,8,14)	**(22,28,46)**

	D_1	D_2	D_3	DS
S_1	11	9	9	9
S_2	11	8	11	8
S_3	11	9	8	9
S_4	12	14	11	5
DD	11	11	9	**31**

	D_1	D_2	D_3	DS
S_1	1.22	1	1	9
S_2	1.38	1	1.38	8
S_3	1.38	1.13	1	9
S_4	1.09	1.27	1	5
DD	11	11	9	**31**

	D_1	D_2	D_3	DS
S_1	1.12	1	1	9
S_2	1.27	1	1.38	8
S_3	1.27	1.13	1	9
S_4	1	1.27	1	5
DD	11	11	9	**31**

	D_1	D_2	D_3	DS
S_1		* 3	*	9
S_2		* 8		8
S_3			* 9	9
S_4	* 5		*	5
DD	11	11	9	**31**

Final optimal table

	D_1	D_2	D_3	DS
S_1	*6	* 3	*	9
S_2		* 8		8
S_3			* 9	9
S_4	* 5		*	5
DD	11	11	9	**31**

Numerical Example (Balanced FTP using fuzzy one point method)

The optimal solution is $x_{11}^0 = 6, x_{12}^0 = 3, x_{22}^0 = 8, x_{33}^0 = 9, x_{41}^0 = 5$ with an optimal objective value 289. Whereas the crisp transportation problem is 240.

Numerical Example (UFTP using fuzzy one point method)

	D_1	D_2	D_3	D_4	FS=$\tilde{a}_i$
S_1	(5,7,9)	(3,4,5)	(2,3,4)	(3,4,6)	(13,15,17)
S_2	(2,3,6)	(1,2,3)	(6,7,8)	(4,5,6)	(18,25,28)
S_3	(3,5,8)	(2,4,5)	(2,3,7)	(6,7,9)	(18,20,22)
S_4	(8,9,10)	(5,7,8)	(4,5,7)	(2,3,5)	(39,40,45)
FD=$\tilde{b}_j$	(11,12,13)	(6,8,10)	(30,35,36)	(24,25,26)	(71,80,85) \(88,100,112)

	D_1	D_2	D_3	D_4	D_5	FS
S_1	(5,7,9)	(3,4,5)	(2,3,4)	(3,4,6)	(1,1,1)	(13,15,17)
S_2	(2,3,6)	(1,2,3)	(6,7,8)	(4,5,6)	(1,1,1)	(18,25,28)
S_3	(3,5,8)	(2,4,5)	(2,3,7)	(6,7,9)	(1,1,1)	(18,20,22)
S_4	(8,9,10)	(5,7,8)	(4,5,7)	(2,3,5)	(1,1,1)	(39,40,45)
FD	(11,12,13)	(6,8,10)	(30,35,36)	(24,25,26)	(17,20,27)	**(88,100,112)**

Numerical Example (UFTP using fuzzy one point method)

Final optimal table

	D_1	D_2	D_3	D_4	D_5	DS
S_1			$*$ 15			15
S_2	$*$ 12	$*$ 8			$*$ 4	24
S_3			$*$ 19		$*$ 1	20
S_4				$*$ 25	$*$ 16	41
DD	12	8	34	25	21	**100**

The optimal solution is

$$x^0_{13} = 15, x^0_{21} = 12, x^0_{22} = 8, x^0_{32} = 4, \ x^0_{33} = 19, x^0_{35} = 1, x^0_{44} = 25, x^0_{45} = 16$$

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with an optimal objective value 255.5. Whereas the unbalanced crisp transportation problem (UCTP) is 232.

Procedure for FTP using fuzzy zero point method

Step 2.5.1: Check whether the given FTP is a balanced one. If not, convert it into balanced FTP by introducing the dummy column or dummy row with cost entry as $(0, 0, 0)$.

Step 2.5.2: Fuzzy transportation cost, supply and demand interms of triangular fuzzy numbers are defuzzified using MM or LRM.

Step 2.5.3: Subtract each row entries of the transportation table by row minimum that is if u_i is the minimum of the ith row of the table $[c_{ij}]$ then subtract the i^{th} row entries by u_i , so that the resulting table is $[c_{ij} - u_i]$.

Procedure for FTP using fuzzy zero point method

Step 2.5.4: Subtract each column entries of the resulting transportation table after using the Step 2.5.3 by the column minimum that is if v_j is the minimum of j^{th} column of the resulting table $[c_{ij} - u_i]$ then subtract j^{th} column entries by v_j so that the resulting table is $[(c_{ij} - u_i) - v_j]$. It may be noted that $(c_{ij} - u_i) - v_j \geq 0$ for all i, j. Each row and each column of the resulting table $[(c_{ij} - u_i) - v_j]$ has atleast one zero entry.

Step 2.5.5: Choose the row or column with only one '0' and allot the minimum of source and demand corresponding to that cell. Check whether the supply points are fully used and all demand points are fully received. If so go to Step 2.5.7. If not, go to Step 2.5.6.

Procedure for FTP using fuzzy zero point method

Step 2.5.6: Draw minimum number of lines horizontally and vertically to cover all the zeros. Then choose the least uncovered element and subtract it from all the uncovered elements and add at the intersection of lines, leaving other elements unchanged. Now, check whether each row and each column has atleast one zero entry. If so, go to Step 2.5.5, else go to Step 2.5.3, Step 2.5.4 and then to Step 2.5.5.

Step 2.5.7: This allotment yields an optimal solution to the given FTP with the objective function (2.1.11) subject to the constraints (2.1.12) to (2.1.15).

Numerical Example (Balanced FTP using fuzzy zero point method)

	D_1	D_2	D_3	$\text{FS} = \tilde{a}_i$
S_1	(8,10,16)	(8,9,10)	(6,8,14)	(6,8,14)
S_2	(8,10,16)	(5,7,13)	(8,10,16)	(5,7,13)
S_3	(9,11,13)	(8,9,10)	(5,7,13)	(8,9,10)
S_4	(10,12,14)	(12,14,16)	(8,10,16)	(3,4,9)
$\text{FD} = \tilde{b}_j$	(8,10,16)	(8,10,16)	(6,8,14)	**(22,28,46)**

$\Rightarrow$

	D_1	D_2	D_3	DS
S_1	11	9	9	9
S_2	11	8	11	8
S_3	11	9	8	9
S_4	12	14	11	5
DD	11	11	9	**31**

	D_1	D_2	D_3	DS
S_1	2	0	0	9
S_2	3	0	3	8
S_3	3	1	0	9
S_4	1	3	0	5
DD	11	11	9	**31**

	D_1	D_2	D_3	DS
S_1	1	0	0	9
S_2	2	0	3	8
S_3	2	1	0	9
S_4	0	3	0	5
DD	11	11	9	**31**

	D_1	D_2	D_3	DS
S_1		* 3	*	9
S_2		* 8		8
S_3			* 9	9
S_4	* 5		*	5
DD	11	11	9	**31**

Final optimal table

	D_1	D_2	D_3	DS
S_1	*6	* 3	*	9
S_2		* 8		8
S_3			* 9	9
S_4	* 5		*	5
DD	11	11	9	**31**

Numerical Example (Balanced FTP using fuzzy point method)

The optimal solution is $x_{11}^0 = 6, x_{12}^0 = 3, x_{22}^0 = 8, x_{33}^0 = 9, x_{41}^0 = 5$ 12

with an optimal objective value 289. Whereas the crisp transportation problem is 240.

Comparison of fuzzy one point method with fuzzy zero point method and crisp solution

<table>
<tr><th></th><th>FTP</th><th>CTP</th></tr>
<tr><td>Solution under fuzzy one point method</td><td>289</td><td rowspan="2">240</td></tr>
<tr><td>Solution under fuzzy zero point method</td><td>289</td></tr>
</table>

Numerical Example (UFTP using fuzzy zero point method)

	D_1	D_2	D_3	D_4	FS=$\tilde{a}_i$
S_1	(5,7,9)	(3,4,5)	(2,3,4)	(3,4,6)	(13,15,17)
S_2	(2,3,6)	(1,2,3)	(6,7,8)	(4,5,6)	(18,25,28)
S_3	(3,5,8)	(2,4,5)	(2,3,7)	(6,7,9)	(18,20,22)
S_4	(8,9,10)	(5,7,8)	(4,5,7)	(2,3,5)	(39,40,45)
FD=$\tilde{b}_j$	(11,12,13)	(6,8,10)	(30,35,36)	(24,25,26)	(71,80,85) $\backslash$ (88,100,112)

	D_1	D_2	D_3	D_4	D_5	FS
S_1	(5,7,9)	(3,4,5)	(2,3,4)	(3,4,6)	(0,0,0)	(13,15,17)
S_2	(2,3,6)	(1,2,3)	(6,7,8)	(4,5,6)	(0,0,0)	(18,25,28)
S_3	(3,5,8)	(2,4,5)	(2,3,7)	(6,7,9)	(0,0,0)	(18,20,22)
S_4	(8,9,10)	(5,7,8)	(4,5,7)	(2,3,5)	(0,0,0)	(39,40,45)
FD	(11,12,13)	(6,8,10)	(30,35,36)	(24,25,26)	(17,20,27)	**(88,100,112)**

Numerical Example (UFTP using fuzzy zero point method)

Fuzzy optimal table

	D_1	D_2	D_3	D_4	D_5	DS
S_1			* 15			15
S_2	* 12	* 8			* 4	24
S_3			* 19		* 1	20
S_4				* 25	* 16	41
DD	12	8	34	25	21	**100**

The optimal solution is

$$x^0_{13} = 15, x^0_{21} = 12, x^0_{22} = 8, x^0_{32} = 4,\ x^0_{33} = 19, x^0_{35} = 1, x^0_{44} = 25, x^0_{45} = 16$$

with an optimal objective value 255.5. Whereas the unbalanced crisp transportation problem is 232.

Comparison of fuzzy one point method with fuzzy zero point method and crisp solution

	UFTP	UCTP
Solution under fuzzy one point method	255.5	232
Solution under fuzzy zero point method	255.5	

CHAPTER - 3

FUZZY ASSIGNMENT PROBLEM

Abstract:

In this chapter a detail discussion is made on FAP and FUAP under fuzzy one point and fuzzy zero point method, where the assignment cost is in the form of triangular fuzzy numbers. Here the objective of the problem is to minimize the total cost or to maximize the total profit subject to some crisp constrains. Numerical example is included to demonstrate the proposed approach.

INTRODUCTION

- An assignment problem (AP) which is special type of linear programming problem plays an important role in industry and other applications.
- In an assignment problem, the main task is to assign exactly one job to one person, so that for performing all the jobs, the total cost is minimum or the total profit is maximum.
- A fuzzy assignment problem is an assignment problem in which the assignment costs are fuzzy quantities. It is a special case of FTP.

LAYOUT OF THIS CHAPTER

- **Mathematical Formulation for FAP and FUAP**
- **Theoretical background for FAP**
  - ⟹ **under fuzzy one point method**
  - ⟹ **under fuzzy zero point method**
- **Procedure for FAP (Minimization)**
  - ⟹ **Numerical Example -Balanced FAP**
  - ⟹ **Numerical Example -Unbalanced FAP**
- **Procedure for FAP (Maximization)**
  - ⟹ **Numerical Example -Balanced FAP**
- **Comparison**

Mathematical Formulation for FAP

$$Min \quad \tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij} \tag{3.1.1}$$

Subject to the contraints

$\tilde{c}_{11}$	$\tilde{c}_{12}$	$\ldots$	$\tilde{c}_{1n}$
$\tilde{c}_{21}$	$\tilde{c}_{22}$	$\ldots$	$\tilde{c}_{2n}$
$\vdots$	$\vdots$		$\vdots$
$\tilde{c}_{n1}$	$\tilde{c}_{n2}$	$\ldots$	$\tilde{c}_{nn}$

$$\sum_{i=1}^{n} \tilde{x}_{ij} \approx \tilde{1}, j = 1 \quad to \quad n; \tag{3.1.2}$$

$$\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{1}, i = 1 \quad to \quad n; \tag{3.1.3}$$

$$\tilde{x}_{ij} \approx \tilde{0} \quad or \quad \tilde{1}, \quad i = 1 \quad to \quad n \quad and \quad j = 1 \quad to \quad n \tag{3.1.4}$$

Conversion of FAP to CAP

$$Min \quad z = \sum_{i=1}^{n}\sum_{j=1}^{n} R(c_{ij}).x_{ij} \tag{3.1.5}$$

Subject to the contraints

$$\sum_{i=1}^{n} x_{ij} = R(\tilde{1}), j = 1 \quad to \quad n; \tag{3.1.6}$$

$$\sum_{j=1}^{n} x_{ij} = R(\tilde{1}), i = 1 \quad to \quad n; \tag{3.1.7}$$

$$x_{ij} = 0 \quad or \quad 1, i = 1 \quad to \quad n \quad and \quad j = 1 \quad to \quad n \tag{3.1.8}$$

Here $R(\tilde{c}_{ij}) = c_{ij}, R(\tilde{1}) = 1$

$c_{ij} \geq 0$ (in the case of fuzzy zero point method);

$c_{ij} \geq 1$ (in the case of fuzzy one point method)

Mathematical Formulation for FUAP in the case of fuzzy one point method

$$Min \quad z = \left(\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}.x_{ij}\right) - 1$$

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Subject to the constraints in the equation (3.1.6) to (3.1.8).

Theoretical background for FAP under fuzzy one point method

Theorem 3.3.1 *Any optimal solution to the problem* (P_3) *where* (P_3) *is* $Min\ z^* = \sum_{i=1}^{n}\sum_{j=1}^{n}(c_{ij}/u_i/v_j).x_{ij}$, *subject to the constraints given in the equations (3.1.10) to (3.1.12) are satisfied, where* u_i *and* v_j *are some positive real crisp numbers, is an optimal solution to the problem* (P_2) *where* (P_2) *is* $Min\ z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}.x_{ij}$, *subject to the constraints given in the equations (3.1.10) to (3.1.12) are satisfied.*

Proof.

$$\text{Min } z^* = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{c_{ij}.x_{ij}}{u_i v_j}$$

As $u_i \geq 1$ and $v_j \geq 1$, $\forall \quad i = 1$ to n and $j = 1$ to n.

$\frac{c_{ij}.x_{ij}}{u_i v_j} \leq c_{ij}.x_{ij}, \forall \quad i = 1$ to n and $j = 1$ to n.

i.e, $\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{c_{ij}.x_{ij}}{u_i v_j} \leq \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}.x_{ij}$

$\Rightarrow Min \quad z^* \leq Min \quad z$

Hence we can conclude that any optimal solution to the problem (P_3) is also an optimal solution to the problem(P_2).

Theoretical background for FAP under fuzzy zero point method

Theorem 3.3.3 *Any optimal solution to the FTP* (P_3^*) *where* (P_3^*) *is* $Min\ z^* = \sum_{i=1}^{n}\sum_{j=1}^{n}(c_{ij} - u_i - v_j).x_{ij} = Z^*$ *subject to the constraints given in the equations (3.1.10) to (3.1.12) are satisfied, where* u_i *and* v_j *are some positive real crisp numbers, is an optimal solution to the problem* (P_2^*) *where* (P_2^*) *is* $Min\ z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}.x_{ij} = Z$*, subject to the constraints given in the equations (3.1.10) to (3.1.12) are satisfied.*

Proof.

$$Min \quad z^* = \sum_{i=1}^{n}\sum_{j=1}^{n}(c_{ij} - u_i - v_j).x_{ij}$$

As $u_i \geq 0$ and $v_j \geq 0, \forall \quad i = 1$ to n and $j = 1$ to n.

$(c_{ij} - u_i - v_j).x_{ij} \leq c_{ij}.x_{ij}, \forall \quad i = 1$ to n and $j = 1$ to n.

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(c_{ij} - u_i - v_j).x_{ij} \leq \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}.x_{ij}$$

$$\Rightarrow Min \quad z^* \leq Min \quad z$$

Hence we can conclude that any optimal solution to the problem (P_3^*) is also an optimal solution to the problem (P_2^*).

Procedure for FAP (Minimization) using fuzzy one point method

Step 3.4.1: Check whether the given fuzzy assignment problem is a balanced one. If not, convert FUAP into balanced fuzzy assignment problem by introducing the dummy column or dummy row with cost (1, 1, 1).

Step 3.4.2: Fuzzy Assignment cost interms of triangular fuzzy numbers are defuzzified using MM or LRM.

Step 3.4.3 and Step 3.4.4: These steps are same as Step 2.4.3 and Step 2.4.4 respectively.

Step 3.4.5: Test whether we can choose only one 1's in each column and in each row. If so go to Step 3.4.7. If not, go to Step 3.4.6.

Procedure for FAP (Minimization) using fuzzy one point method

Step 3.4.6: Draw minimum number of lines horizontally and vertically to cover all the 1's. Then choose the least uncovered element and divide all the uncovered elements using it and multiply it at the intersection of lines, leaving other elements unchanged. Now, check whether each row and each column has atleast one '1' entry. If so go to Step 3.4.5, else go to Step 3.4.3, Step 3.4.4 and then to Step 3.4.5.

Step 3.4.7: This allotment yields an optimal solution to the given FAP

- with the objective function (3.1.9), if the FAP is balanced.
- with the objective function (3.2.1), if the FAP is balanced by introducing a dummy row or a dummy column.

All these objective functions are subject to the constaints (3.1.10) to (3.1.12).

Note: Procedure for FAP (Minimization problem) using fuzzy zero point method is similar to that of traditional Hungarian method.

Numerical Example (Balanced FAP using fuzzy one point method)

Jobs

Operators

	J_1	J_2	J_3	J_4
O_1	(9,11,15)	(16,17,20)	(6,8,10)	(12,16,18)
O_2	(5,9,11)	(6,7,8)	(8,12,14)	(4,6,9)
O_3	(10,13,16)	(12,16,18)	(10,15,20)	(8,12,14)
O_4	(12,14,18)	(6,10,13)	(8,12,14)	(9,11,15)

$\Rightarrow$

	J_1	J_2	J_3	J_4
O_1	11.5	17.5	8	15.5
O_2	8.5	7	11.5	6.25
O_3	13	15.5	15	11.5
O_4	14.5	9.75	11.5	11.5

	J_1	J_2	J_3	J_4
O_1	1.44	2.19	**1**	1.94
O_2	1.36	1.12	1.84	**1**
O_3	1.13	1.35	1.30	**1**
O_4	1.49	**1**	1.18	1.18

	J_1	J_2	J_3	J_4
O_1	1.27	2.19	**1**	1.19
O_2	1.20	1.12	1.84	**1**
O_3	**1**	1.35	1.30	**1**
O_4	1.32	**1**	1.18	1.18

Fuzzy optimal table

	J_1	J_2	J_3	J_4
O_1			**1** $\star$	
O_2				**1** $\star$
O_3	**1** $\star$			**1** $\bullet$
O_4		**1** $\star$		

The optimal solution is

$$x^0_{13} = x^0_{24} = x^0_{31} = x^0_{42} = 1$$

with an optimal objective value 37.

Verification using fuzzy zero point method

	J_1	J_2	J_3	J_4
O_1	11.5	17.5	8	15.5
O_2	8.5	7	11.5	6.25
O_3	13	15.5	15	11.5
O_4	14.5	9.75	11.5	11.5

$\Rightarrow$

	J_1	J_2	J_3	J_4
O_1	3.5	9.5	**0**	7.5
O_2	2.25	0.75	5.25	**0**
O_3	1.5	4	3.5	**0**
O_4	4.75	**0**	1.75	1.75

$\Downarrow$

Fuzzy optimal table

	J_1	J_2	J_3	J_4
O_1	2	9.5	$\mathbf{0}\star$	7.5
O_2	0.75	0.75	5.25	$\mathbf{0}\star$
O_3	$\mathbf{0}\star$	4	3.5	$\mathbf{0}\bullet$
O_4	3.25	$\mathbf{0}\star$	1.75	1.75

The optimal solution is

$$x^0_{13} = x^0_{24} = x^0_{31} = x^0_{42} = 1$$

with an objective value 37.

Comparison of fuzzy one point method with fuzzy zero point method and crisp solution

	FAP (Min)	CAP (Min)
Solution under fuzzy one point method	37	37
Solution under fuzzy zero point method	37	

Numerical Example (FUAP using fuzzy one point method)

Roads

Contractors

	R_1	R_2	R_3	R_4
C_1	(8,9,10)	(8,14,20)	(16,19,22)	(10,15,20)
C_2	(5,7,9)	(12,17,22)	(15,20,25)	(16,19,20)
C_3	(8,9,10)	(16,18,20)	(18,21,24)	(16,18,20)
C_4	(5,10,15)	(9,12,15)	(16,18,20)	(16,19,22)
C_5	(5,10,15)	(10,15,20)	(18,21,24)	(10,16,22)

	R_1	R_2	R_3	R_4	R_5
C_1	(8,9,10)	(8,14,20)	(16,19,22)	(10,15,20)	(1,1,1)
C_2	(5,7,9)	(12,17,22)	(15,20,25)	(16,19,20)	(1,1,1)
C_3	(8,9,10)	(16,18,20)	(18,21,24)	(16,18,20)	(1,1,1)
C_4	(5,10,15)	(9,12,15)	(16,18,20)	(16,19,22)	(1,1,1)
C_5	(5,10,15)	(10,15,20)	(18,21,24)	(10,16,22)	(1,1,1)

	R_1	R_2	R_3	R_4	R_5
C_1	9	14	19	15	1
C_2	7	17	20	19	1
C_3	9	18	21	18	1
C_4	10	12	18	19	1
C_5	10	15	21	16	1

Fuzzy optimal table

	R_1	R_2	R_3	R_4	R_5
C_1			$\mathbf{1}\ \star$	$\mathbf{1}\ \bullet$	
C_2	$\mathbf{1}\ \star$				
C_3					$\mathbf{1}\ \star$
C_4		$\mathbf{1}\ \star$	$1\ \bullet$		
C_5				$\mathbf{1}\star$	$1\ \bullet$

The optimal solution is

$$x^0_{13} = x^0_{21} = x^0_{35} = x^0_{42} = x^0_{54} = 1$$

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with an optimal objective value 54.

Verification using fuzzy zero point method

Fuzzy optimal table

	R_1	R_2	R_3	R_4	R_5
C_1	2	1	$\mathbf{0}\star$	$\mathbf{0}\bullet$	1
C_2	$\mathbf{0}\star$	4	1	4	1
C_3	2	4	1	2	$\mathbf{0}\star$
C_4	4	$\mathbf{0}\star$	$\mathbf{0}\bullet$	5	2
C_5	2	1	1	$\mathbf{0}\star$	$\mathbf{0}\bullet$

The optimal solution is $x^0_{13} = x^0_{21} = x^0_{35} = x^0_{42} = x^0_{54} = 1$

with an optimal objective value 54.

Comparison of fuzzy one point method with fuzzy zero point method and crisp solution

	FUAP (Min)	CUAP (Min)
Solution under fuzzy one point method	54	54
Solution under fuzzy zero point method	54	

Procedure for FAP (Maximization) using fuzzy one point method

Step 3.5.1 and Step 3.5.2: These steps are same as Step 3.4.1 and Step 3.4.2.

Step 3.5.3: Choose the largest element (cost) and divide it by the other elements. This converts the profit matrix to the loss matrix. The optimal solution corresponding to the maximum profit is the same as the solution corresponding to minimum loss. Therefore we adopt the algorithm for minimization for the loss matrix.

Rest of the steps are the same as in minimization problem given in section 3.4

Note: Procedure for FAP (Maximization problem) using fuzzy zero point method is similar to that of traditional Hungarian method.

Numerical Example (Balanced FAP using fuzzy one point method)

Jobs

Machines

	1	2	3	4	5
A	(4,5,6)	(10,11,13)	(9,10,11)	(11,12,15)	(3,4,5)
B	(1,2,3)	(3,4,5)	(5,6,7)	(2,3,4)	(4,5,6)
C	(2,3,4)	(11,12,15)	(4,5,6)	(13,14,16)	(5,6,7)
D	(5,6,7)	(13,14,16)	(3,4,5)	(10,11,13)	(6,7,8)
E	(6,7,8)	(8,9,10)	(7,8,9)	(11,12,15)	(4,5,6)

Profit matrix

	1	2	3	4	5
A	5	11.25	10	12.5	4
B	2	4	6	3	5
C	3	12.5	5	14.25	6
D	6	14.25	4	11.25	7
E	7	9	8	12.5	5

$\Rightarrow$

Loss matrix

	1	2	3	4	5
A	2.85	1.27	1.43	1.14	3.56
B	7.13	3.56	2.38	4.75	2.85
C	4.75	1.14	2.85	**1**	2.38
D	2.38	**1**	3.56	1.27	2.04
E	2.04	1.58	1.78	1.14	2.85

$\Downarrow$

Fuzzy optimal table

	1	2	3	4	5
A		**1** $\bullet$	**1** $\star$	**1** $\bullet$	2.07
B			**1** $\bullet$		**1** $\star$
C				**1** $\star$	
D		**1** $\star$			
E	**1** $\star$				

The optimal solution is

$$x_{13}^{0} = x_{25}^{0} = x_{34}^{0} = x_{42}^{0} = x_{51}^{0} = 1$$

with an optimal objective value 50.5.

Verification using fuzzy zero point method

Fuzzy optimal table

	1	2	3	4	5
A	1	0.25	$\mathbf{0}\star$	$\mathbf{0}\bullet$	5
B	$\mathbf{0}\bullet$	3.5	$\mathbf{0}\bullet$	5.5	$\mathbf{0}\star$
C	4.75	0.75	6.75	$\mathbf{0}\star$	4.75
D	2.75	$\mathbf{0}\star$	8.75	4	4.75
E	$\mathbf{0}\star$	3.5	3	1	5

The optimal solution is $x_{13}^0 = x_{25}^0 = x_{34}^0 = x_{42}^0 = x_{51}^0 = 1$

with an optimal objective value 50.5.

Comparison of fuzzy one point method with fuzzy zero point method and crisp solution

	FAP (Max)	CAP (Max)
Solution under fuzzy one point method	50.5	50
Solution under fuzzy zero point method	50.5	

CHAPTER - 4

FUZZY TRAVELLING SALESMAN PROBLEM

Abstract:

In this chapter, a new procedure based on fuzzy one point and fuzzy zero point method is introduced for solving FTSP. The proposed procedure is effective and easy to understand because of its natural similarity to classical method of solving TSP. The example shown in this paper, guaranties the correctness and effectiveness of the working rule of the procedure. The result match with the existing technique and also satisfy the regular TSP conditions.

INTRODUCTION

- The objective of the problem is to find the shortest route of salesman starting from a given city, visiting all other cities only once and finally come to the same city where he started.
- A fuzzy travelling salesman problem is a travelling salesman problem in which the travelling cost or distance are fuzzy quantities. It is a special case of FAP.

LAYOUT OF THIS CHAPTER

- **Mathematical Formulation for FTSP**
- **Procedure for FTSP using fuzzy one point method**

  Numerical Example -Balanced FTSP

  ➡ **using one point method**

  ➡ **using one point method**

- **Comparison**

Mathematical Formulation for FTSP

	To					
	City	A_1	A_2	A_3	$\dots$	A_n
	A_1	∞	$\tilde{c}_{12}$	$\tilde{c}_{13}$	$\dots$	$\tilde{c}_{1n}$
	A_2	$\tilde{c}_{21}$	∞	$\tilde{c}_{23}$	$\dots$	$\tilde{c}_{2n}$
From	A_3	$\tilde{c}_{31}$	$\tilde{c}_{32}$	∞	$\dots$	$\tilde{c}_{3n}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$
	A_n	$\tilde{c}_{n_1}$	$\tilde{c}_{n_2}$	$\tilde{c}_{n_3}$	$\dots$	∞

Let $\tilde{c}_{ij} = (c_{1ij}, c_{2ij}, c_{3ij})$ be the distance from city i to city j .

Mathematical Formulation for FTSP

Here we have two restrictions.

1) The salesman cannot visit from city i to city i .To avoid such visits we take $\tilde{c}_{ij} = (\infty, \infty, \infty) = \infty$, for i=j.

2) No city is visited more than once until all the cities are visited.

Let $\tilde{x}_{ij} = \tilde{1}$ denote that the salesman makes a visit directly from the city i to city j and $\tilde{x}_{ij} = \tilde{0}$ denote that the salesman does not go directly from city i to city j.

Mathematical Formulation for FTSP

$$Min \quad z = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij} \tag{4.1.1}$$

Subject to the constraints

$$\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{1}, i = 1, 2, \ldots n \tag{4.1.2}$$

$$\sum_{i=1}^{n} \tilde{x}_{ij} = \tilde{1}, j = 1, 2, \ldots n \tag{4.1.3}$$

$$\tilde{x}_{ij} = \tilde{0} \quad or \quad \tilde{1}, \quad i = 1 \quad to \quad n \quad and \quad j = 1 \quad to \quad n \tag{4.1.4}$$

Conversion of FTSP to CTSP

$$Min \quad z = \sum_{i=1}^{n}\sum_{j=1}^{n} R(\tilde{c}_{ij}).x_{ij} \tag{4.1.5}$$

Subject to the contraints

$$\sum_{i=1}^{n} x_{ij} = R(\tilde{1}), j = 1 \quad to \quad n; \tag{4.1.6}$$

$$\sum_{j=1}^{n} x_{ij} = R(\tilde{1}), i = 1 \quad to \quad n; \tag{4.1.7}$$

$$x_{ij} = 0 \quad or \quad 1, \quad i = 1 \quad to \quad n \quad and \quad j = 1 \quad to \quad n \tag{4.1.8}$$

Here $R(\tilde{c}_{ij}) = c_{ij}, R(\tilde{1}) = 1$.

$c_{ij} \geq 0$ (in the case of fuzy zero point method);

$c_{ij} \geq 1$ (in the case of fuzzy one point method)

Procedure for FTSP using fuzzy one point method

Construct the fuzzy travelling salesman table $[\tilde{c}_{ij}]$. Here $\tilde{c}_{ij} = (c_{1ij}, c_{2ij}, c_{3ij})$ for i $\neq$ j; $\tilde{c}_{ij} = (\infty, \infty, \infty)$ for i=j is in the form of triangular fuzzy numbers.

Step 4.2.1 to Step 4.2.7: Same as Step 3.4.1 to Stap 3.4.7 of section 3.4

Step 4.2.8: Scrutinizing the solution obtained, to see if the route conditions are satisfied. If it satisfies then that is the solution of FTSP. If not, making adjustments in assignments to satisfy the condition with minimum increase in total cost (i.e) to satisfy route condition where "next best solution" may require considering.

Numerical Example (Balanced FTSP using one point method)

To

From

City	A	B	C	D
A	∞	(15,20,24)	(10,15,20)	(8,11,13)
B	(15,20,24)	∞	(25,30,33)	(7,10,13)
C	(10,15,20)	(25,30,33)	∞	(10,20,32)
D	(8,11,13)	(7,10,13)	(10,20,22)	∞

	A	B	C	D
A	∞	19.75	15	10.75
B	19.75	∞	29.5	10
C	15	29.5	∞	20.5
D	10.75	10	18	∞

$\Rightarrow$

	A	B	C	D
A	∞	1.84	1.40	1
B	1.98	∞	2.95	1
C	1	1.97	∞	1.37
D	1.08	1	1.8	∞

$\Downarrow$

	A	B	C	D
A	∞	1.84	1	1
B	1.98	∞	2.12	1
C	1	1.97	∞	1.37
D	1.08	1	1.29	∞

$\Leftarrow$

	A	B	C	D
A	∞	1.88	**1**	1
B	2.06	∞	2.52	**1**
C	**1**	2.29	∞	1.16
D	1.10	**1**	1.02	∞

A $\rightarrow$ C $\rightarrow$ A and B $\rightarrow$ D $\rightarrow$B.

Numerical Example (Balanced FTSP using fuzzy one point method)

Case (1):

	A	B	C	D
A	∞	1.84	1	**1**
B	1.98	∞	**2.12**	1
C	**1**	1.97	∞	1.37
D	1.08	**1**	1.29	∞

$A \to D \to B \to C \to A$

The optimal total cost is 65.25

Case (2):

	A	B	C	D
A	∞	1.84	**1**	1
B	**1.98**	∞	2.12	1
C	1	1.97	∞	**1.37**
D	1.08	**1**	1.29	∞

$A \to C \to D \to B \to A$

The optimal total cost is 65.25

Case (3):

	A	B	C	D
A	∞	1.84	**1**	1
B	1.98	∞	2.12	**1**
C	1	**1.97**	∞	1.37
D	**1.08**	1	1.29	∞

$A \to C \to B \to D \to A$

The optimal total cost is 65.25

Verification using fuzzy zero point method

	A	B	C	D
A	∞	9	**0**	0
B	9.75	∞	15.25	**0**
C	**0**	14.5	∞	5.5
D	0.75	**0**	3.75	∞

$A \rightarrow C \rightarrow A$ and $B \rightarrow D \rightarrow B$.

Verification for fuzzy zero point method

Case (1):

	A	B	C	D
A	∞	9	0	**0**
B	9.75	∞	**15.25**	0
C	**0**	14.5	∞	5.5
D	0.75	**0**	3.75	∞

$A \to D \to B \to C \to A$

The optimal total cost is 65.25

Case (2):

	A	B	C	D
A	∞	9	**0**	0
B	**9.75**	∞	15.25	0
C	0	14.5	∞	**5.5**
D	0.75	**0**	3.75	∞

$A \to C \to D \to B \to A$

The optimal total cost is 65.25

Case (3):

	A	B	C	D
A	∞	9	**0**	0
B	9.75	∞	15.25	**0**
C	0	**14.5**	∞	5.5
D	**0.75**	0	3.75	∞

$A \to C \to B \to D \to A$

The optimal total cost is 65.25

Comparison of fuzzy one point method with fuzzy zero point method and crisp solution

	FTSP	CTSP
Solution under fuzzy one point method	65.25	65
Solution under fuzzy zero point method	65.25	

CHAPTER - 5

CONCLUSION

"So far as the laws of Mathematics refer to reality, they are not certain and so far as they are certain, they do not refer to reality."

-Albert Einstein

- Fuzzy set theory is an excellent mathematical tool to handle the uncertainty arising due to vagueness. It has a rich potential for applications in several directions.

CONCLUSION

- **Chapter 2** deals with FTP. The economic growth of a country depends on the increase of the capacity of transport. Therefore the study of FTP is essential.
- In the conventional procedures in order to obtain the initial basic feasible solution, the tabular methods such as the North West corner rule, the Least cost method and the Vogel's approximation method are used. Then the MODI method provides the optimal solution.
- Here the methods used for FTP, namely fuzzy one point method and fuzzy zero point method are independent of the above conventional procedures.

CONCLUSION

- **Chapter 3** deals with FAP. Here the objective is to minimize the total cost or maximize the total profit while assigning people to projects or jobs to machines or workers to jobs etc. Hence a deeper analysis of this FAP becomes necessary.
- In the same way as in the case of FTP, the procedure is given for FAP using fuzzy one point method which is independent of the traditional Hungarian method.

CONCLUSION

- Sujatha and Elizabeth proposed the procedure for FTP and FAP in 3 stages by applying the ranking function MM/LRM. Whereas we have proposed the procedure for FTP and FAP in a single stage using the same ranking function which is the simplified and easy method to solve the problem.

CONCLUSION

- **Chapter 4** deals with the FTSP whose objective is to find the cheapest way of visiting all the cities and returning to the starting point. Here we have proposed the procedure for FTSP using fuzzy one point method which is an alternative method to get the solution for TSP under fuzzy environment.
- For the sake of verification the solutions obtained for FTP, FAP and FTSP are compared with the existing results. Suitable numerical examples with detailed explanation is illustrated in all the chapters adding clarity to the work done.

THANK YOU