EE5351:CONTROL SYSTEM DESIGN

ASSIGNMENT 03

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Q1)

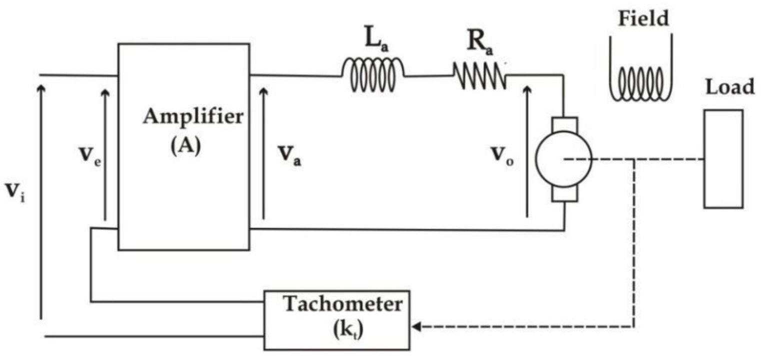

Figure 1: A simple and stable speed control system of a DC motor

Considering given the information:

- Voltage constant of the motor (*kb*) - 0.85V/rads-1
- Torque constant of the motor (*km*) - 0.9 Nm/A
- Tachometer constant (*kt*) - 0.015 V/rads-1
- Inertia of the rotating parts of the motor (*J*) - 0.85 kgm2
- Input DC voltage (*Vi*(t)) - 10.0 V
- Voltage gain of the amplifier (A) - 100
- Armature resistance and inductance -1.3 Ω, 0.5 H respectively
-

I. Get;
Transfer function for DC motor : G_m
Overall Transfer function : G

I. G_m =

- Assume that the armature circuit current as I_a

$V_a\ (t) = I_a(t)R_a + L_a\frac{di}{dt} + V_o$

$V_o\ (t) = k_b\omega_m$

$T_m = K_mI_a(t) = J\frac{d\omega}{dt}$

- Convert the equations into the laplase domain

$V_a(s) = I_a(s)R_a + L_aI_aS + V_o(s)_0$

$V_0(s) = k_b\omega_m(s)$

$T_m(s) = K_mI_a(s) = J\omega_m s$

- By use those equations

$$G_m = \frac{k_m}{jLs^2+JR_as+k_mk_b}$$

- Considering overall transfer function

$$G = \frac{AG_m}{1+k_tAG_m}$$

$$G = \frac{Ak_m}{JL_as^2+JR_as+k_m(k_b+Ak_t)}$$

- By substituting the values

$$G = \frac{100\times0.9}{0.85\times0..5s^2+0.85\times1.3S+0.9(0.85+100\times0.015)}$$

$$G = \frac{90}{0.425s^2+1.10S+2.115}$$

II. Mathlab code

```
% Define the transfer function
numerator = 90; %numerator
denominator = [0.425, 1.105, 2.115]; %denominator

% transfer function
G = tf(numerator, denominator);

poles = pole(G); %Define poles
zeros = zero(G); %Define zeros

disp('Poles:');
disp(poles); %Display poles
disp('Zeros:');
disp(zeros); %Display zeros

% Plot the poles and zeros in the s-plane
pzmap(G);
title('Pole-Zero Plot for t/f');
grid on;
```

- In the command window

```
>> Q1_2
Poles:
```

-1.3000 + 1.8129i
-1.3000 - 1.8129i

Zeros:
>>

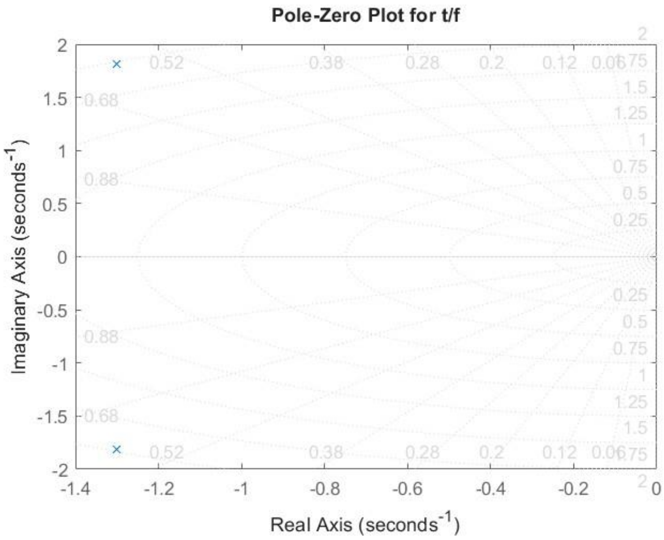

Figure 2: S plane with poles-Zero

III. Mathlab code

```
% Define the transfer function
numerator = 900; %numerator
denominator = [0.425 1.105 2.115]; %denominator

% transfer function
G = tf(numerator, denominator);

% Define the time vector
t = 0:0.01:10; % time vector from 0 to 10 seconds with a step of 0.01

% step response
[y, t] = step(G, t);

% Plot the step response
step(G, t);
grid on;
title('Step Response');

% Calculate steady-state value
steady_state_value = y(end);
disp(['Yss value of the step response : ', num2str(steady_state_value)]);
```

- Command Window
- >> Q1_3
- Yss value of the step response : 425.5316

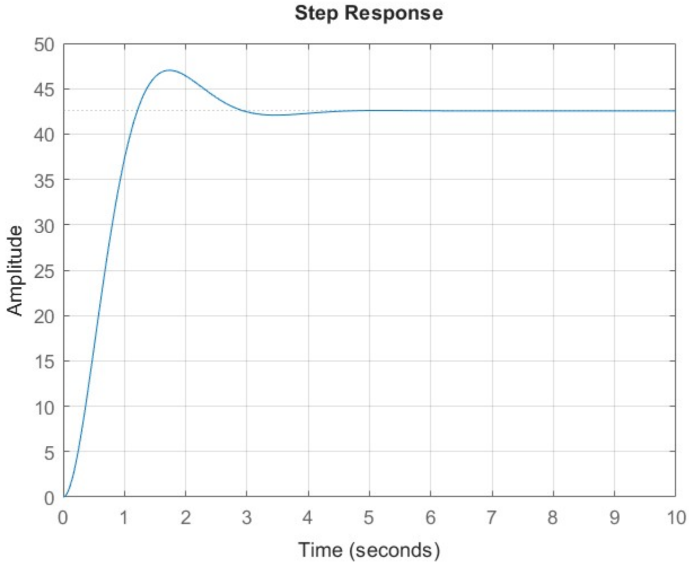

Figure 3:Step Response

IV. By simplifying the transfer function

$$G = \frac{211.756}{S^2+2.6S+4.976}$$

Considering that, characteristic equation :

$$S^2 + 2.6S + 4.976 = 0$$

$\omega^2{}_n$ = 4.976

ω_n =2.23

2€ ω_n =2.6

€ =0.583

M_p $=e^{\frac{-\pi\epsilon}{\sqrt{1-\epsilon}}}$

M_p =0.105

Now find the system output ϖ(t)

ϖ(s) =G(s)xV_i(s)

$$=\frac{90}{0.425S^2+1.105s+2.115} \text{x} \frac{10}{s}$$

$$=\frac{900}{s(0.425S^2+1.105s+2.115)}$$

- MATHLAB code

```
% Define the transfer function
numerator = 900; %numerator
denominator = [0.425 1.105 2.115 0]; % denominator

% Using residue to find the partial fraction expansion
[residues, poles, direct_terms] = residue(numerator, denominator);

% Display the results
disp('Residues:');
disp(residues);
disp('Poles:');
disp(poles);
disp('Direct Terms:');
disp(direct_terms);
```

- Command window

```
>> Q1_4
Residues:
   1.0e+02 *

  -2.1277 + 1.5257i
  -2.1277 - 1.5257i
   4.2553 + 0.0000i

Poles:
  -1.3000 + 1.8129i
  -1.3000 - 1.8129i
   0.0000 + 0.0000i
```

ω_s $=\left(\frac{4.2553}{s}+\frac{-2.1277+1.5257i}{s+1.3-1.8129i}+\frac{-2.1277-1.5257i}{s+1.3+1.8129i}\right)\times 10^2$

$=\left(\frac{4.2553}{s}-\frac{4.26s+11.034}{s^2+2.6s+4.976}\right)\times 10^2$

From the time domain

ω_t $=[4.2553u(t)-4.26e^{-1.3t}\cos(1.813t)-3.03e^{-1.3t}\sin(1.813t)]\times 10^2$

considering the example that shows that all the poles are located in the left hand side there fore this system is stable. Now apply the final value theorem to find the steady state ω_{ss}

ω_{ss} $=\lim_{t\to\alpha}[(4.2553u(t)-4.26e^{-1.3t}\cos(1.813t)-3.03e^{-1.3t}\sin(1.813t)]\times 10^2)$

$=\underline{\underline{4.2553}}\times 10^2\underline{\underline{\text{rad}s^{-1}}}$

By using the overshoot equation:

$$M_p = \frac{\omega_{mp} - \omega_{ss}}{\omega_{ss}}$$

$$0.105 = \frac{\omega_{mp} - 4.2553\times10^2}{4.2553\times10^2}$$

$$\omega_{mp} = \underline{\underline{4.702\times 10^2 \text{rad}s^{-1}}}$$; Maximum possible speed

V. $T_p = \frac{\pi}{\omega_{n\sqrt{1-\epsilon^2}}}$

$= \underline{\underline{1.73\text{s}}}$

VI.

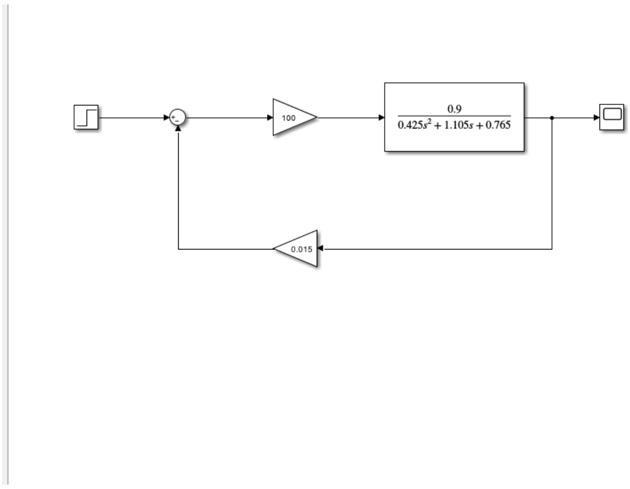

Figure 4:Simulinked system

VII.

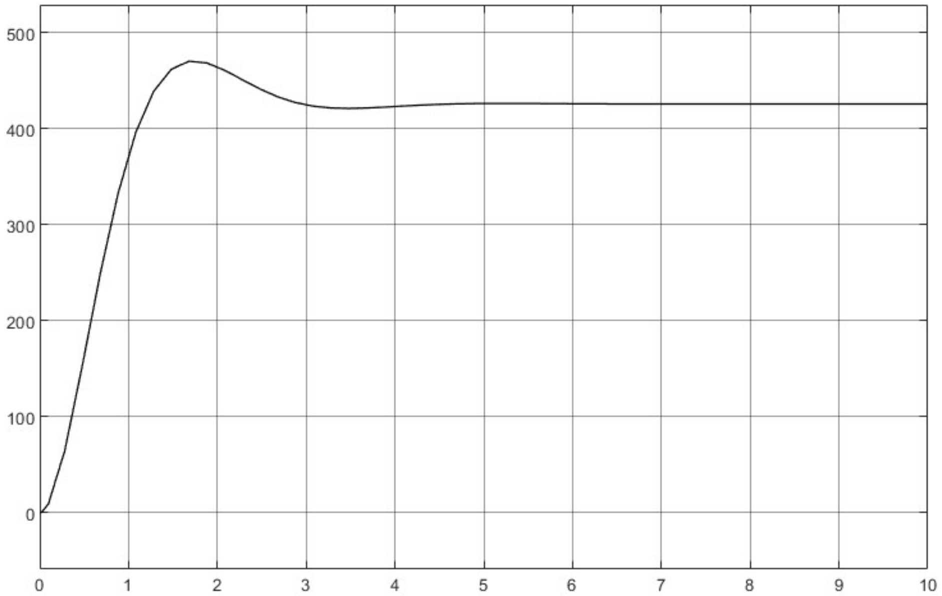

Figure 5:Output Function

VIII. Considering the s plane of the system all the poles are situated in the left hand side. Thus the system is stable.