

EE5351:CONTROL SYSTEM DESIGN  
ASSIGNMENT 03

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Q1)

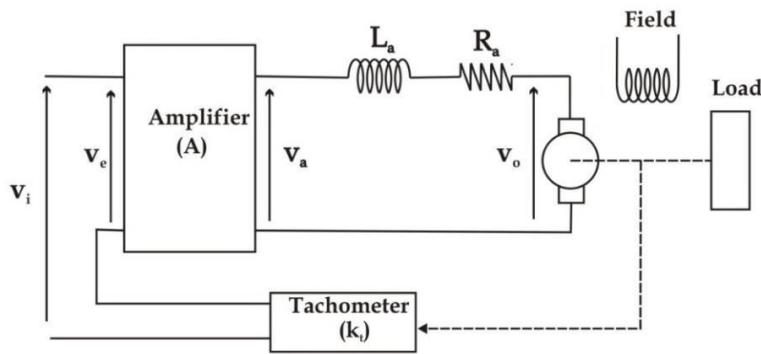


Figure 1: Sample DCMotor

### Sample Data Set

Voltage constant of the motor ( $k_b$ )	-	0.85V/rads-1
Torque constant of the motor ( $k_m$ )	-	0.9 Nm/A
Tachometer constant ( $k_t$ )	-	0.15 V/rads-1
Inertia of the rotating parts of the motor ( $J$ )	-	0.85kgm <sup>2</sup>
Input DC voltage ( $V_i(t)$ )	-	10.0V
Voltage gain of the amplifier (A)	-	100
Armature resistance and inductance	-	1.3 $\Omega$ , 0.5 H respectively

1. Assume that  $G_m$  as the transfer function of DC motor. Then assume the armature current as  $I_a$

$$V_a(t) = I_a R_a + L \frac{dI_a}{dt} + V_0$$

$$V_0 = k_b \omega_m$$

$$T_m = k_m I_a$$

Now convert the time equations into the laplace domain

$$V_a(s) = I_a R_a + L s I_a + V_0$$

$$V_0 = k_b \omega_m$$

$$T_m = k_m I_a$$

$$T_m = J \omega_m$$

$$G_m = \frac{K_m}{jls^2 + jR_a s + K_a k_b}$$

Assume the overall transfer function as  $G_s$

$$G_s = \frac{AG_m}{1+k_tAG_m}$$
$$= \frac{Ak_m}{JL_a s^2 + J R_a s + k_t A k_m + k_t k_m}$$

By substituting values

$$G_s = \frac{100 \times 0.9}{0.85 \times 0.5 s^2 + 0.85 \times 1.3 s + 0.9 \times (0.85 + 100 \times 0.015)}$$
$$= \frac{90}{0.425 s^2 + 1.1 s + 2.11}$$

## 2. Sample code

```
% Define the transfer function numerator and denominator
numerator = 90;
denominator = [0.425, 1.105, 2.115];
```

```
% Create transfer function
G = tf(numerator, denominator);
```

```
poles = pole(G); %Find poles
zeros = zero(G); %Find zeros
```

```
%Display poles and zeros
disp('Poles:');
disp(poles);
disp('Zeros:');
disp(zeros);
```

```
% Plot the poles and zeros in the s-plane
pzmap(G);
title('Pole-Zero Plot');
grid on;
```

```
output
>> Q1_2
Poles:
-1.3000 + 1.8129i
-1.3000 - 1.8129i
```

```
Zeros:
>>
```

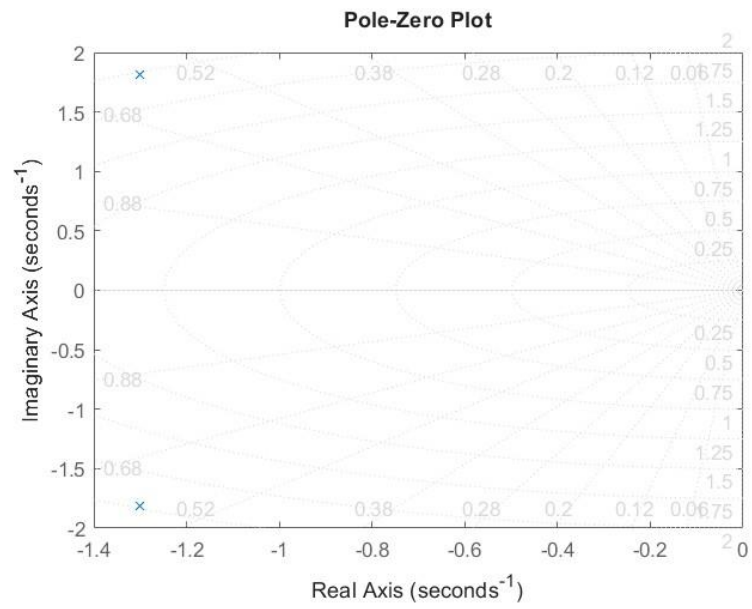


Figure 2:Pole zero plot of Splane

### 3. Sample code

```
% Define the transfer function numerator and denominator
numerator = 900;
denominator = [0.425, 1.105, 2.115];

% Create the transfer function
G = tf(numerator, denominator);

% Define the time vector
t = 0:0.01:10; % time vector from 0 to 10 seconds with a step of 0.01

% step response
[y, t] = step(G, t);

% Plot the step response
step(G, t);
grid on;
title('Step Response of the System');

% Calculate and display the steady-state value
steady_state_value = y(end);
disp(['The steady-state value of the step response : ', num2str(steady_state_value)]);
```

output

```
>> Q1_3
```

The steady-state value of the step response : 425.5316

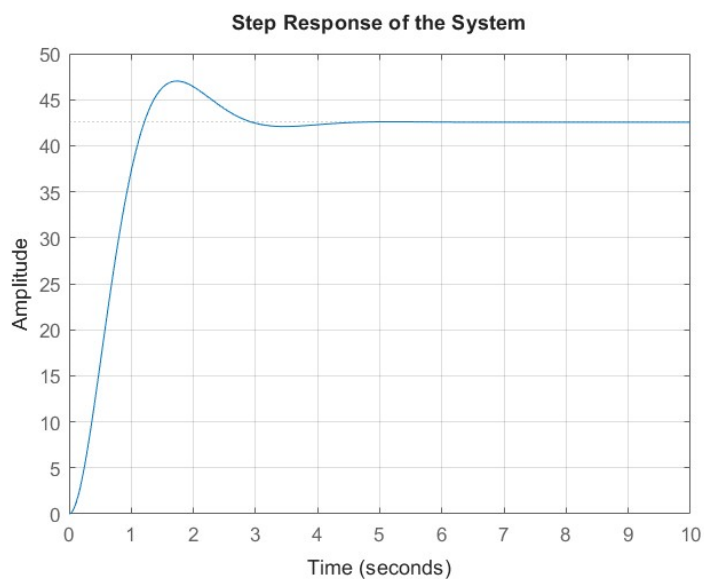


Figure 3:Step Response of the system

4. Considering the transfer function we can simplify it as follows.

$$G_s = \frac{211.756}{s^2 + 2.6s + 4.976}$$

So it shows that the characteristic equation as :

$$s^2 + 2.6s + 4.976 = 0$$

$$\omega_n^2 = 4.976$$

$$\omega_n = 2.23$$

$$2\varepsilon\omega_n = 2.6$$

$$\varepsilon = 0.583$$

$$M_p = e^{\frac{-\pi\varepsilon}{\sqrt{1-\varepsilon^2}}}$$

$$= 0.105$$

So it can be taken the o/p as follows

$$\begin{aligned}\omega(s) &= G(s) \times V(s) \\ &= \frac{90}{0.425s^2 + 1.1s + 2.11} \times \frac{10}{s} \\ &= \frac{900}{s(0.425s^2 + 1.1s + 2.11)}\end{aligned}$$

For that the sample matlab code

% Define the transfer function numerator and denominator

numerator = 900;

denominator = [0.425 1.105 2.115 0]; % Multiply by s

% Using residue to find the partial fraction expansion

[residues, poles, direct\_terms] = residue(numerator, denominator);

% Display the results

disp('Residues:');

disp(residues);

disp('Poles:');

disp(poles);

disp('Direct Terms:');

disp(direct\_terms);

from that it given the o/p as

>> Q1\_4

Residues:

$$1.0e+02 *$$

$$-2.1277 + 1.5257i$$

$$-2.1277 - 1.5257i$$

$$4.2553 + 0.0000i$$

Poles:

$$-1.3000 + 1.8129i$$

$$-1.3000 - 1.8129i$$

$$0.0000 + 0.0000i$$

$$\begin{aligned} \bullet \quad \omega_s &= \left[ \frac{4.2553}{s} + \frac{-2.1277+1.5257i}{s+1.3-1.8129i} + \frac{-2.1277-1.5257i}{s+1.3+1.8129i} \right] \times 10^2 \\ &= \left[ \frac{4.2553}{s} - \frac{4.26s+11.034}{s^2+2.6s+4.976} \right] \times 10^2 \end{aligned}$$

From inverse laplace domain

$$\bullet \quad \omega_t = [4.2553u(t) - 4.26e^{-1.3t} \cos(1.813t) - 3.03e^{-1.3t} \sin(1.813t)] \times 10^2$$

to find the  $\omega_{ss}$  it can be used final value theorem because all the poles are located in the left half of the s plane.

$$\begin{aligned} \omega_{ss} &= \lim_{t \rightarrow \infty} [4.2553u(t) - 4.26e^{-1.3t} \cos(1.813t) - 3.03e^{-1.3t} \sin(1.813t)] \times 10^2 \\ &= \underline{\underline{425.53 \text{ rads}^{-1}}} \end{aligned}$$



So the overshoot equation is given as

$$M_p = \frac{\omega_{MP} - \omega_{SS}}{\omega_{SS}} ; M_p = e^{\frac{-\pi \varepsilon}{\sqrt{1-\varepsilon^2}}} = 0.105$$

$$0.105 = \frac{\omega_{MP} - 4.2553 \times 10^2}{4.2553 \times 10^2}$$

$$\omega_{MP} = \underline{\underline{4.702 \times 10^2 \text{ rads}^{-1}}}$$

$$\begin{aligned} 5. \quad T_p &= \frac{\pi}{\omega \times \sqrt{1-\varepsilon^2}} \\ &= \frac{\pi}{2.23 \times \sqrt{1-0.583^2}} \\ &= \underline{\underline{1.73 \text{ s}}} \end{aligned}$$

6.

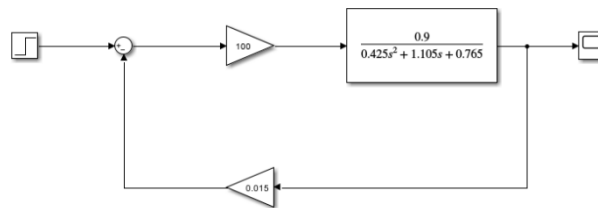


Figure 4:Figure of the Simulink system

7.

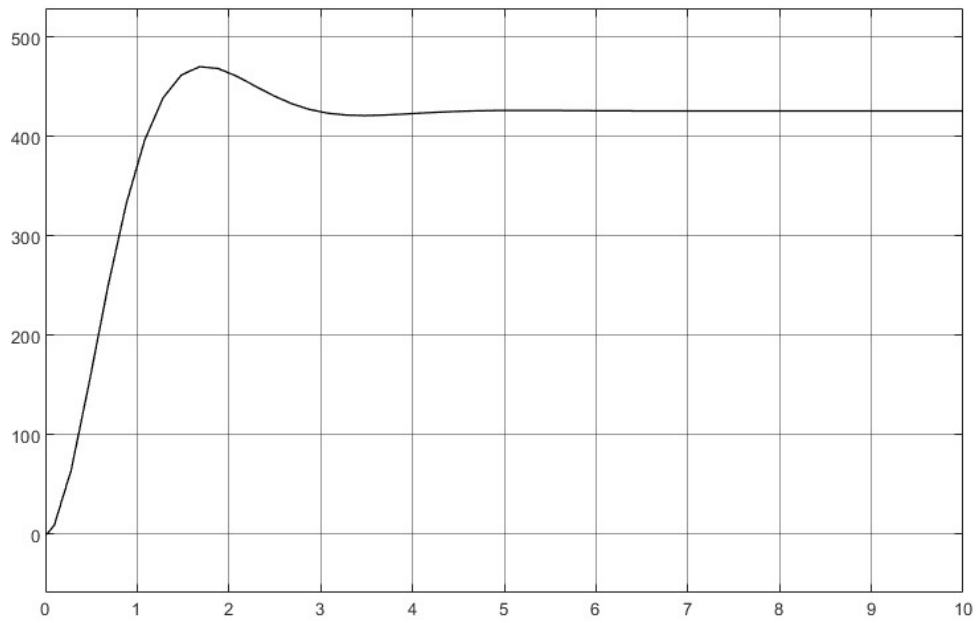


Figure 5:Final Output

8. By looking at the figure2 it is clear that ll the poles are located in the left hand side. Thus it is clear that the system is stable.