UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: January 2024

Module Number: EE5351 **Module Name: Control Systems Design**

[Three Hours]

[Answer **all questions**, each question carries 15.0 marks]

Q1 a) i) If the damping ratio and undamped natural frequency of a system are denoted by ξ and ω_n, respectively, locate a pair of complex conjugate poles in the s-plane and express the rectangular coordinates of the poles in terms of and ξ and ω_n.

ii) Write the general form of state-space model of a system. Name all the matrices in your model.

iii) Derive the transfer function of system using the state-space model you have written in Part ii).

[5.0 Marks]

b) In a cascade electrical system, the circuit diagram of the process circuit and the locations of poles and zeroes of the actuator circuit in S-Plane are shown in Figure Q1(b)-1 and Figure Q1(b)-2, respectively.

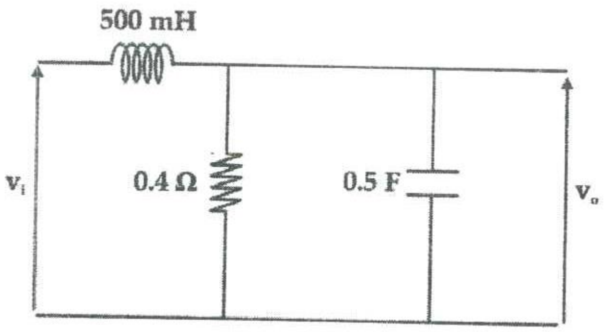

Figure Q1(b)-1

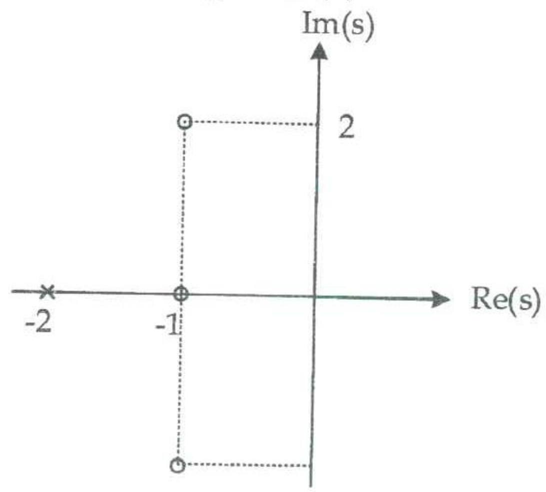

Figure Q1(b)-2

i) Consider the circuit shown in Figure Q1(b)-1. The input voltage is $\mathbf{v_i}$ and the output voltage ($\mathbf{v_o}$) is the voltage across the capacitor. Find a suitable state vector to express the system in state-variable form.

ii) Obtain the state-space model of the circuit given in Q1(b)-1.

iii) Hence show that the transfer function of the process circuit is,

$$G(s) = \frac{4}{(s+1)(s+4)}$$

iv) Obtain the overall transfer function of cascade electrical system.

v) Find the system response for the applied DC voltage of 3 V. Hence, obtain the steady state value of system response.

vi) Discuss the stability of the overall system.

[10.0 Marks]

Q2 a) i) Write the transfer functions for controller type P, PI, PD and PID. Here, P, I and D denote proportional, integral and derivative, respectively.

ii) What is the main objective of adding the integral control to a system?

iii) Briefly explain the method of manual tuning the PID parameters for a bounded response with a steady state error.

[4.5 Marks]

b) Show that the ***damping*** of the system shown in Figure Q1 (b) can be improved by adding a PD type controller. All the notations have their usual meanings.

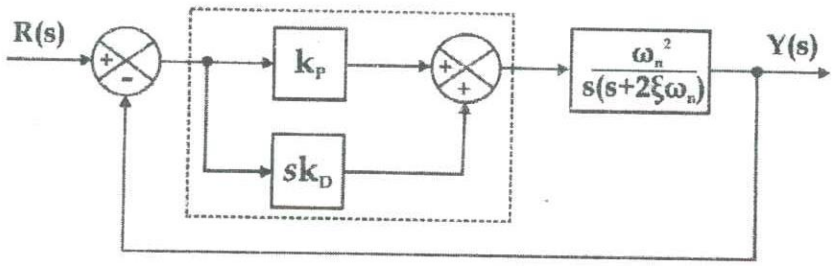

Figure Q1(b)

[2.5 Marks]

c) You are required to design a simple speed control system for a DC motor as shown in Figure Q1 (c). All the notations have their usual meanings.

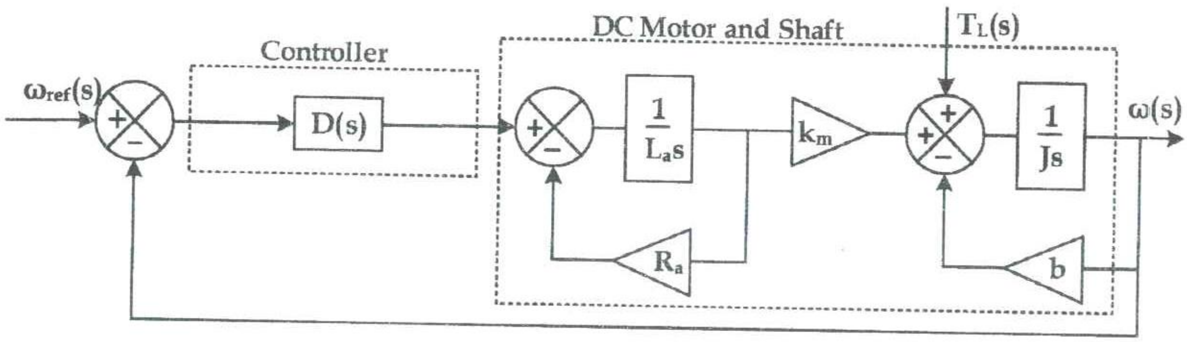

Figure Q1(c)

i) Show that the motor speed control system undergoes critically damped stage at $k_p = \frac{(bL_a+JR_a)^2}{4Jk_mL_a} - \frac{bR_a}{k_m}$, if a proportional controller is used.

ii) The electrical resistance (R_a), inductance (L_a), and the moment of inertia of the DC motor are 1 Ω, 0.5 H and 0.2 kgm^2, respectively. The torque constant (k_m) and viscous friction constant (b) are 5 NmA^{-1} and 1 Nms. Calculate the ***overshoot*** of the system time response.

iii) A PD type controller is used to reduce the present overshoot by 40%. Calculate the required derivative constant (k_D) of the controller.

iv) If the DC motor runs a load with 3 Nm load torque, find the range of (k_p) to keep the steady state error due to the load torque less than 0.01 rads^{-1}.

[8.0 Marks]

Q3 a) i) Define the root locus considering a negative feedback system.

ii) Explain the magnitude and the phase conditions to be satisfied at a point on the root locus.

[4.0 Marks]

b) You are required to design a proportional controller for the system shown in Figure Q3 (b) using the root locus design method.

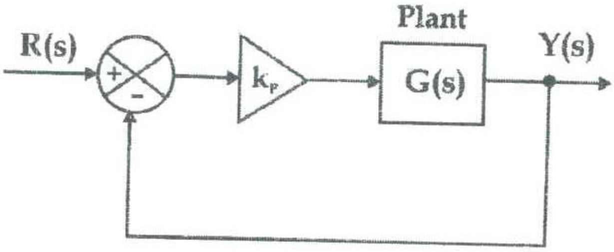

Figure Q3 (b)

Here,

$$G(s) = \frac{s+3}{s(s^2+6s+8)}$$

i) Following necessary rules for drawing the root locus, sketch the root locus of the system shown in Figure Q3 (b).

ii) Determine the range of the controller gain (k_p) so that the system is stable.

iii) For a stable system, it is required to have the damping factor ξ=0.5. Determine the required controller gain (k_p).

[7.0 Marks]

c) i) Roughly sketch the root locus of the system shown in Figure Q3 (b) when k_p varies from 0 to ∞. Here,

$$G(s) = \frac{1}{s(s+2)}$$

Note : Detail step by step calculations are not required.

ii) In order to increase the system speed to $\omega_n = 3$ rads^{-1} while maintaining the damping ratio (ξ) at 0.5, it is decided to use a lead compensator [i.e. $D(s) = k(s+z)/(s+p)$]. The noise suppression requirements require that the lead pole to be at -1. Using necessary geometrical constructions in the s-plane, determine the values for the zero (z) and the gain (k) of the lead compensator.

[4.0 Marks]

Q4 a) i) What is the frequency response of a system?

ii) Define amplitude ratio (M) and phase (ϕ) related to the frequency response of a system whose transfer function is $G(s)$.

iii) Define the terms; phase margin and gain margin, associated with the Bode plots.

iv) How do we examine the system stability using the stability margins?

[6.0 Marks]

b) It is required to control the angular displacement of a DC motor as a unity feedback system. The transfer function of the DC motor is,

$$G(s) = \frac{\theta_L(s)}{V_T(s)} = \frac{K_M}{s(T_f s + 1)(T_M s + 1)}$$

Where, motor gain constant (K_M), Time constant (T_f) and the mechanical time constant (T_M) are 1, 1 and 0.1, respectively.

i) Obtain the transfer function of the DC motor.

ii) Obtain the steady-state output of the DC motor, when it is subjected to the input $V_T = 2\,sin(4t - 30^0)$.

iii) Draw the approximate Bode plots for the system.

iv) Determine the phase margin and gain margin of the system.

v) Hence, discuss the stability of the system.

[9.0 Marks]