Week 11: Quantum Hardware

Contents

1 Metrics for comparison of various QC technologies

1. **Coherence Time**: Coherence Time is a standard measure of how long the quantum states which represent the qubits remain coherent i.e. there exists a definite phase relationship between states. Longer time is obviously preferable. This gives one more time to complete a given quantum operation, allowing a deeper quantum circuit to take place within a given algorithm. The error correction will have to be applied very less, thus creating an overall lower overhead. When there is a loss of definite phase relationship between states, one witnesses decoherence. That is when things go ugly and measurements on the circuits give wrong answers. There is an external interference with the quantum states. Hence, a qubit's coherence decays with ease.

   Decoherence can be seen as the loss of quantum information from a quantum system into its surrounding environment (heat bath), since each such system is loosely combined to the energetic conditions of its environment. If one sees a system in segregation, its dynamics are non-unitary (the combined system+environment evolves in a unitary manner). In this way, the system alone evolves in an irreversible manner. Similarly as with any coupling, entanglements are created between the system and its environment. These have the impact of sharing quantum data with or moving it to the environment which results in the inaccuracies and inconsistencies during the measurement. This necessitates the need of quantum error-correcting codes. This is also one of the primary reasons that hybrid algorithms were introduced in 2013 with the sole aim to spend limited time on a quantum computer. However, hybrid algorithms face a communication bottleneck if the classical and quantum computers are not situated locally.

   Decoherence is calculated through the off-diagonal elements of the density matrices. With more qubits into the system, it is hard to achieve the goldilocks state of the right quantity of interference to achieve entanglement. Thus, one needs to keep qubits in isolation. In superconducting qubits, one needs refrigeration to lower the system's temperature. In ion traps, we use a vacuum chamber. This leads to an increase in cost. However, a diamond's interior is a potential candidate for the provision of an isolated environment. Thus, NV centre is an interesting approach to building quantum computers.

2. **Gate Latency**: It measures how long it takes to perform a single operation on a quantum computer. It is technology dependent and implementation dependent. If the gate operations are shorter, then we can execute the circuit with the limited coherence time.

3. **Gate Fidelity**: It measured how we can execute a gate without the introduction of unnecessary errors. Fidelity is a measure of how much an output state-vector is closer to the actual state-vector.

4. **Mobility**: It informs whether we can physically move the qubits. Superconducting qubits can not be moved, however in trapped ion quantum computing, one can move the ions through vacuum. This is an important metric in evaluation of the quantum entanglement.

2 DiVincenzo Criteria

DiVincenzo has given five criteria or fundamental requirements that must be kept in mind while designing a quantum computer.

- The designed physical system must be **scalable** with a framework of well-characterized qubits. We need to make two computational bases and using them, we can form composite systems.

- One must be able to **initialize** the qubit state to a simple reference or **fiducial state** like $|00...0\rangle$.

- We must have a **universal set of quantum gates** just like we have in classical computing. We can achieve this using U3 gate and CNOT gate as every one-qubit gate can be written in terms of U3 gate. We also know HXH=Z, hence Z-gate can be written using U3 and C-NOT gates. We should be able to control the qubits which means interacting with the qubits which can introduce noise.

- The system must be resistant to external environment such that it possesses **longer decoherence times which are longer than the gate operation times**. Qubit is very delicate and starts losing information once the program starts running. One needs to perform computations within the limited coherence time that we have. We measure the quality of a qubit using **readout and gate errors, T1 (relaxation time), T2 (coherence time)**. T1 is the time it takes the excited state $|1\rangle$ to decay to ground state $|0\rangle$. T2 is essentially the duration of dephasing effects which damage the superposition and we lose the quantum properties and get wrong results.

- We must possess a **qubit-specific measurement capability** which is quantified using readout error.

3 NMR Quantum Computing

Nuclear magnetic resonance quantum computing (NMRQC) is one of the proposed techniques for developing a quantum computer, that makes use of the spin states of the molecular nuclei as qubits. The quantum states are scrutinized through the nuclear magnetic resonances, enabling the NMRQC framework to be realized as a variation of nuclear magnetic resonance spectroscopy. NMR contrasts from other different frameworks of quantum computers in the sense that it utilizes an ensemble of quantum systems, for this situation molecules, instead of one pure state. At first the methodology was to utilize the spin properties of atoms of specific molecules in a liquid sample as qubits which came to be known as liquid state NMR (LSNMR). This methodology has since been supplanted by solid state NMR (SSNMR).

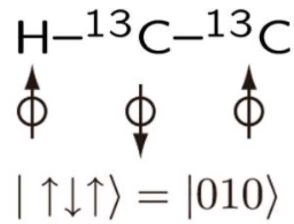


Figure 1: Spin Configuration

The perfect image of LSNMR depends on a molecule in which its atomic nuclei act as spin 1/2 systems. Contingent upon which nuclei, we are considering as different nuclei will have several energy levels and each nucleus will have different interactions with its surrounding nuclei. Thus, we can regard them as discernable qubits as the spectrum is non-degenerate (not always). In this framework we consider the inter-atomic bonding as the source of interactions which we can use to achieve quantum entanglement and use these spin-spin interactions to perform 2-qubit unitaries, for example, CNOTs, Toffoli gates, controlled rotation unitaries etc. An external magnetic field is applied to achieve single qubit unitaries by making use of the fact that locally different spins will experience different magnetic fields and will respond in a different way and in this way, we control the behavior of qubits.

LSNMRQC: The image depicted above is a long way from reasonable since we are treating a solitary molecule. NMR is performed on a gathering of molecules (10^{15}). This acquaints intricacies with the model, one of which is visible in form of decoherence. Specifically we have the issue of an open quantum framework communicating with the external environment. This has driven the advancement of methods for suppressing. The other noteworthy issue concerning working near thermal equilibrium is the mixedness of the state. As we bring increasingly logical qubits into our framework, we require huge sampling (shots in the implementation scenario) so as to achieve discernable signals while measuring. Even though the first quantum computer was developed using NMR technology, it was realized soon enough that this framework of achieving a universal quantum computation won't work since it has a very poor SNR ratio and is not scalable in terms of qubits.

SSNMRQC: Strong state NMR (SSNMR) contrasts from LSNMR in that we have a solid state sample, for instance a NV (nitrogen vacancy) diamon lattice instead of a fluid sample.

This has numerous favorable advantages, for example, absence of decoherence (not complete), lower temperatures can be accomplished to the point of reducing phonon decoherence and a more noteworthy assortment of control tasks that enable us to defeat one of the serious issues of LSNMR which is initialisation. Besides, as in a crystal structure we can localize the qubits, we can measure each qubit exclusively, rather than having an ensemble measurement as in LSNMR.

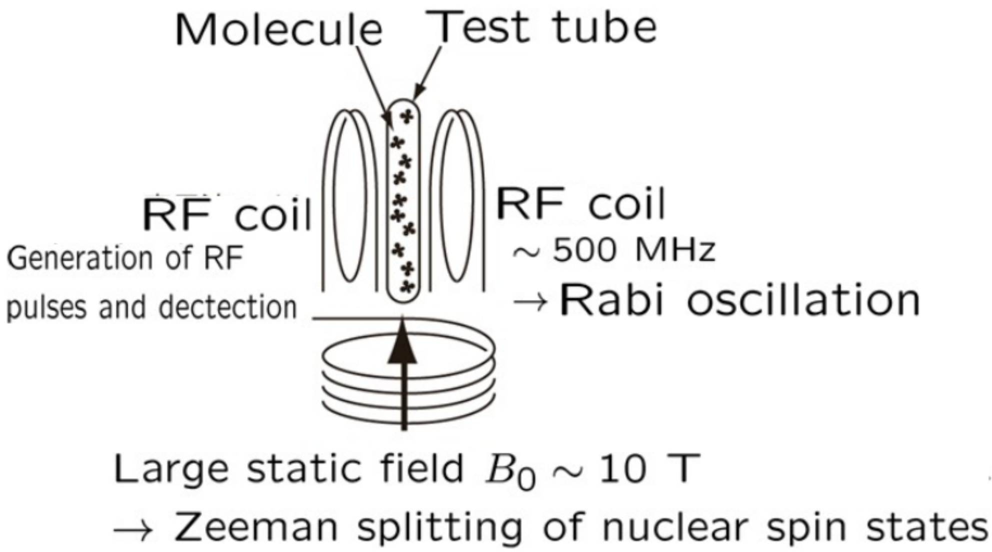


Figure 2: NMR Schematic

3.1 Mathematical Model

The initial state is a mixed state which is a thermal quilibrium state. That is why we don't have pure states in LSNMRQC. The state ρ represented in terms of density matrix is given by

$$\rho = \frac{e^{-\frac{\hat{H}}{K_B T}}}{Tr(e^{-\frac{\hat{H}}{K_B T}})} \tag{1}$$

where $\hat{H}$ is the Hamiltonian of a single molecule. To achieve unitaries, we apply RF pulses perpendicular to a strong external magnetic field. In presence of magnetic field in the z-direction, $\hat{H}$ for a single spin becomes

$$\hat{H} = \mu B_z = \omega I_z \tag{2}$$

where μ, $I_z = \frac{\sigma_z}{2}$ and ω represent the magnetic moment, the z^{th} component of nuclear angular momentum and spin resonance frequency respectively. For a two-spin 1/2 nuclei system, we have an additional dipole cpuling term. The $\hat{H}$ for a two-spin system becomes

$$\hat{H} = \omega_1 I_{z_1} + \omega_2 I_{z_2} + C_{12} I_{z_1} I_{z_2} \tag{3}$$

where C_{12} is the dipole coupling strength term. For an n-qubit linear chain, the hamiltonian can be generalized to

$$\hat{H} = \sum_{k=1}^{n-1} C_{k,k+1} I_{z_k} \otimes I_{z_{k+1}} + \sum_{k=1}^{n} \omega_{1k}(\cos\phi_k I_{x_k} + \sin\phi_k I_{y_k}) \tag{4}$$

where ϕ is the phase lag in the time-dependent oscillating magnetic field in the x direction. This is the basic mathematical model of NMRQC.We now study how to implement single qubit and two qubit unitaries by Rabi oscillations and C-coupling respectively in this framework.

3.2 Quantum Gates

3.2.1 Single-Qubit Gates

A general one-qubit rotation gate specified by 3 angles is given by

$$U(\theta, \Phi, \phi) = e^{[-\iota\theta(I_x \sin\Phi\cos\phi + I_y \sin\Phi\sin\phi + I_z\cos\Phi)]} \tag{5}$$

where $\theta = \omega_1 t$, Φ and ϕ are pulse nutation, co-latitude and phase angle respectively. We consider resonant RF fields such that $\Phi = \pi/2$ and we obtain the conventional rotation unitary

$$U(\theta, \phi) = e^{[-\iota\theta(I_x\cos\phi + I_y\sin\phi)]} \tag{6}$$

Thus, such pulses help to easily implement X and Y gates as 180°_x and 180°_y rotations respectively. Rotations about the z-axis can be constructed as

$$\begin{aligned} \theta_z &= 90^\circ_y \theta_x 90^\circ_{-y} \\ e^{(-\iota\theta I_z)} &= e^{(\frac{\iota\pi I_y}{2})} e^{(-\iota\theta I_x)} e^{(-\frac{\iota\pi I_y}{2})} \end{aligned} \tag{7}$$

where we first apply the leftmost pulse. We do selective addressing for each spin. Off-resonance RF pulses do not affect a given spin provided $\omega_1 << \omega_0$. Another important point to remember is that we can have chemical shift i.e. two spins belonging to the same spin species can possess different ω_0 due to difference in locations.

3.2.2 Two-Qubit Gates

The two-qubit gate in this framework is implemented as a sequence of pulses. For example, the C-NOT gate is implemented as

$$U_{C-NOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = Z_1 \bar{Z}_2 X_2 U_J(\frac{\pi}{J}) Y_2 \tag{8}$$

where the pulses are given as

$$
\begin{aligned}
Z_1 &= e^{(-\frac{\iota\pi I_z}{2})} \otimes I \\
\bar{Z}_2 &= I \otimes e^{(\frac{\iota\pi I_z}{2})} \\
X_2 &= I \otimes e^{(-\frac{\iota\pi I_x}{2})} \\
Y_2 &= I \otimes e^{(-\frac{\iota\pi I_x}{2})} \\
U_J(\frac{\pi}{J}) &= e^{(-\iota\pi I_z \otimes I_z)}
\end{aligned} \tag{9}
$$

3.3 Summary

We use **quantum state tomography** for measurement. The goal is to measure the density matrix but not enough equations are obtained. Observables like magnetizations are measured to obtain linear combinations of the elements of the density matrix. To obtain the desired number of equations, density matrix is deformed using pulses.

With the increase in qubit size, selective addressing becomes difficult to achieve. There are limited number of nuclei spins and one sees overlap of resonant frequencies quite frequently. SNR ratio also becomes poor and the system becomes highly sensitive to the external surroundings. There is no real entanglement and it becomes very hard to prepare a pseudopure initial state as exponential number of steps are required. We can not reduce interactions very much and thus, the refocusing technique becomes quite complex to achieve. That is why LSNMRQC is not scalable to use in future in this NISQ era.

Decoherence Time	$10^2 - 10^3$ s
Single qubit operation time	10^{-5}s
Two qubit operation time	10^{-2}s

Table 1: Metrics

4 Superconducting Quantum Computing

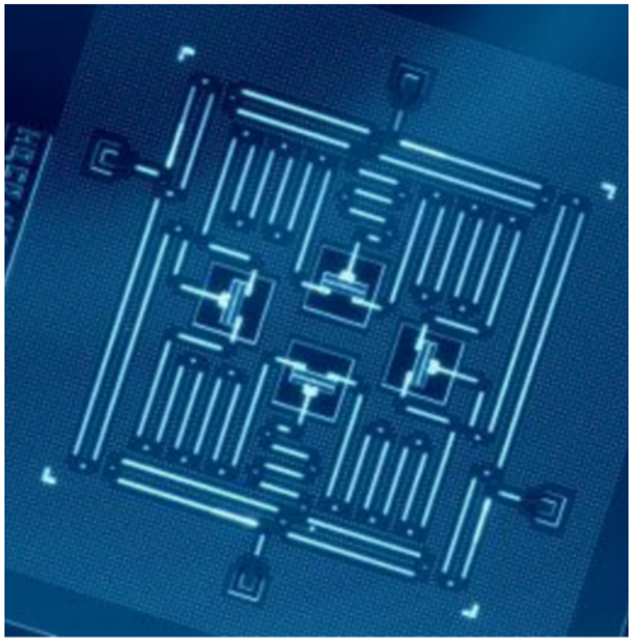
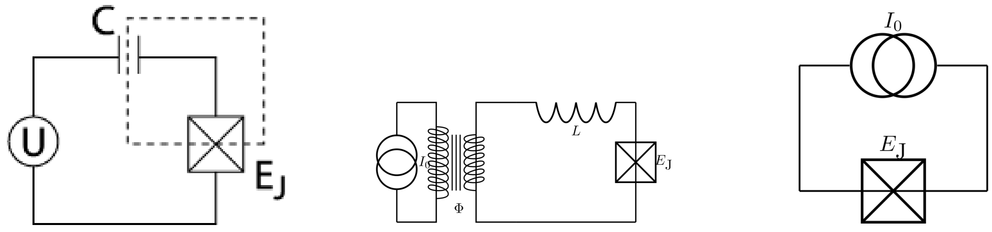


Figure 3: 4 Qubit System [1]

In Fig. 3, one can see 4 qubits in the square. Quantum Processor is the heart of a superconducting quantum computer. The superconducting quantum computers are designed, fabricated and manufactured in the RF range, chilled off in a diluted refrigeration underneath 100 mK and tended to with regular electronic instruments like spectrum analyzers. Commonplace measurements on the size of micrometers, with sub-micrometer resolution, permits an easy design of a quantum Hamiltonian with the entrenched incorporated circuit innovation.

A distinctive element of superconducting quantum devices is the utilization of a Josephson junction - an electrical component which is not present in ordinary conductors. In a superconductor, there is no magnetic field lines inside. It becomes a perfect dia-magnet. The Josephson junction is a weak connection between two leads of a superconducting wire, generally executed as a slim layer of insulator. The condensate wave works on the two different sides of the junction - they are permitted to have distinctive superconducting phases, as opposed to the instance of a consistent superconducting wire, where the superconducting wave work must be continuous. The current through the junction happens by quantum tunneling. This is utilized to make a non-linear inductance which is basic for qubit plan, as it permits a structure of anharmonic oscillators. A quantum harmonic oscillator can't be utilized as a qubit, as there is no real way to address just two of its states. There are 3 qubit architectures that we can use namely phase, flux and charge qubits (Fig. 4).



(a) Charge Qubit (b) Flux Qubit (c) Phase Qubit

Figure 4: Superconducting qubit architectures [2]

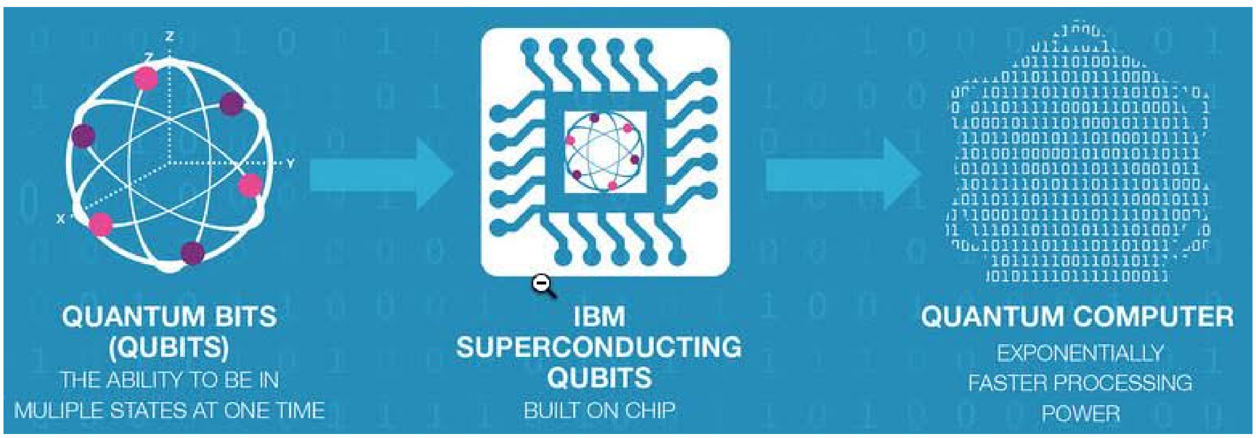


Figure 5: IBMQ Superconducting Quantum Computer [3]

The IBM Superconducting Flowchart is given in Fig. 5

4.1 Quantum Gates

4.1.1 Single Qubit Gates

We use microwave pulses to the transmission line coupled to the qubit with a frequency equivalent to corresponding to that of the energy difference between the levels. To implement the NOT-gate, we have the following unitary

$$U_X = -\iota X = e^{-\frac{\iota}{2}\int_0^t E_x(t')dt'.X} \tag{10}$$

where E_x is the component of the microwave in the x-direction.

4.1.2 Two-Qubit Gates

We use a DC squid. We first decouple the qubits by switching off the resonances between them. Through this, one can achieve near-neighbor coupling.

4.2 Summary

The plus points of this technology include easy fabrication, ease of coupling, electronic control over the qubits and mature technology. This technology has been used by many QC companies. However, it faces from the problem of a large footprint.

5 TIQC: Trapped Ions Quantum Computing

5.1 Experiment

1. Heat the material to a high temperature such that it transitions to vapor phase.

2. Bombard it with high-energy electrons to knock out the outer-most electrons which also becomes easier due to Step 1. This forms an ion.

3. Hold these ions in a vacuum chamber using some strong electric field.

4. Laser-cool these ions to the ground state to form the stationary state $|0\rangle$. Now, we can perform operations on these qubits using lasers or RF fields which are nothing but quantum gate operations.

5. During measurement, shine these ions with a laser of specific frequency. If the qubit is in $|0\rangle$, it will remain dark else it will floresce.

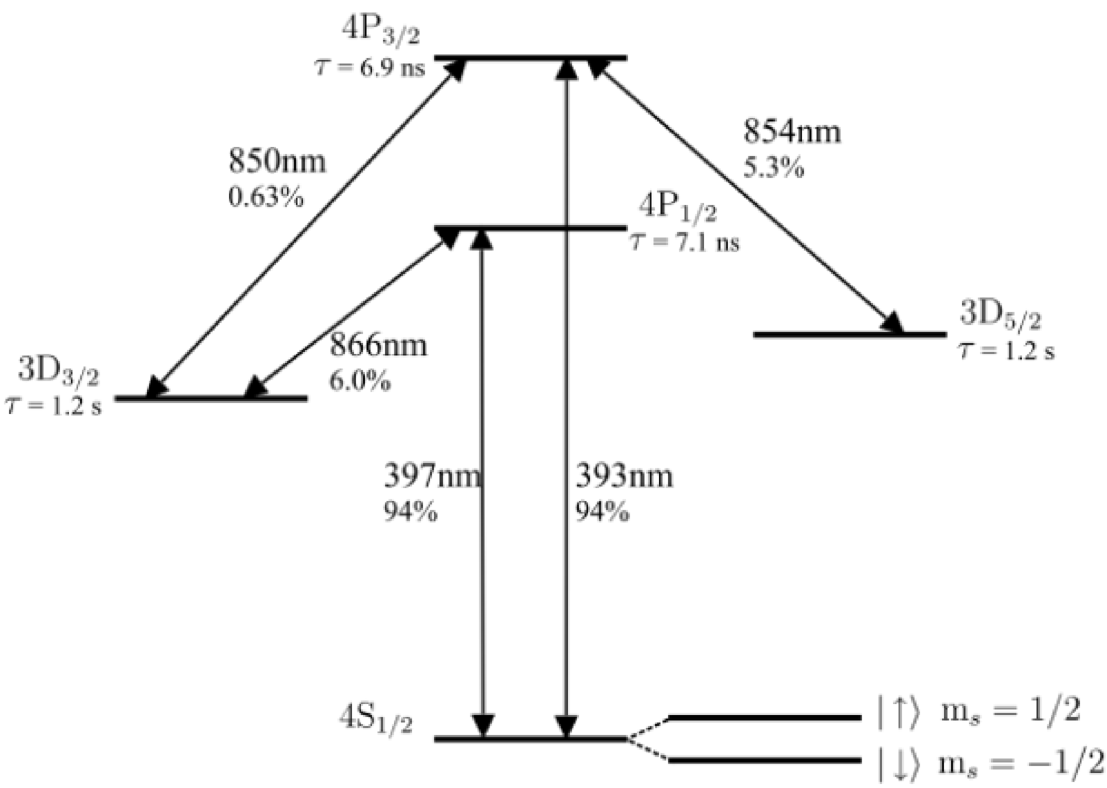


Figure 6: Transitions along with their lifetimes from 4P and 3D to 4S in Calcium [4]

In Fig. 6, we can see that the transition from 3D (5/2) to 4S has a longer lifetime than all its counterparts. Thus, it can be a potential candidate for $|1\rangle$. The transition between 4P(3/2) to 4S is an allowed electric dipole transition. Thus, we have fulfilled the first criterion of DiVincenzo. 4S and metastable state 3D (5/2) act as two computational bases required for computation. We then apply lasers of certain frequencies to perform operations on them. These are known as optical qubits. One can also have hyperfine qubits in which nuclei spin interacts with electronic spin creating a hyperfine structure. Hyperfine qubits are better than optical qubits in the sense that they are insensitive to magnetic perturbations, stable and the superposition lasts for minutes in certain cases. However, ion traps in general face controlling issues and are not that scalable since using a string of ions to act as qubits and performing high-fidelity operations on them is very hard. Switching is difficult to implement. Hyperfine qubits have longer lifetimes but the energy difference bwteen two such qubits is very close, hence controlling qubits becomes difficult, thus in contradiction to criterion 3.

5.2 Paul Trapping

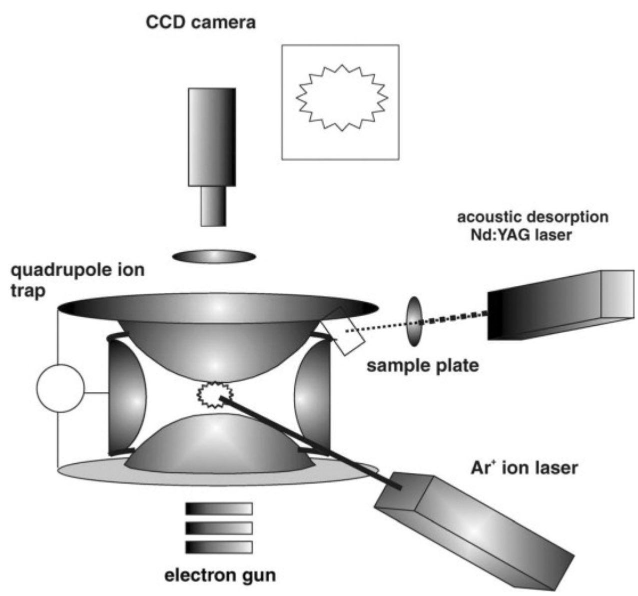


Figure 7: Paul Trap [5]

In Paul trap (Fig.7), we do not use the concept of gravitation to hold the ions. Instead, electrodes with high amplitude RF electric fields are used to trap ions. It enables us to create the same potential well as is done using gravitational pull.It is a quadrupole ion trap, also called as RF traps and named after Wolfgang Paul. The force on an ion is given by

$$m\frac{d^2\epsilon}{dt^2} + m\frac{\Omega^2}{4}\left[a_\epsilon - 2q_\epsilon \cos(\Omega t)\right]\epsilon = 0 \tag{11}$$

where ϵ represents the 3 coordinates, Ω is the radial frequency and $a_\epsilon \& q_\epsilon$ are the dimensionless trapping parameters. This equation is solved by Floquet Theorem and in Paul trap, we use the concept of ponderomotive force to calculate the particle trajectories.

5.3 Quantum Gates

5.3.1 Single Qubit Gate

The rotation unitaries can be implemented using Raman transitions for hyperfine qubits. We expose the ions to external EM fields for certain fixed times.

5.3.2 Two-Qubit Gate

C-NOT gate is implemented with the control qubit being the center of mass phonons and ion's atomic spin state is the target qubit. The target qubit is flipped if the phonon is in the state $|1\rangle$.

5.4 Summary

Even though they have large coherence times, but fluctuating EM fields can easily cause decoherence.

Exercises

1. To make multi-qubit operation possible, we can by tailoring photon spin through light-matter interactions, use photons as ideal spin-carriers in a liquid-crystal based twisting. True/False

2. Write Liouville Equation. Do density operators obey this equation?

3. Why is LSNMRQC not viable in the future?

4. What are pseudo-pure states which are used for initialization in NMRQC where it is hard to produce pure states?
5. What is the spectral bandwidth of a transform-limited pulsed laser which is operating at a 800 nm central bandwidth and is outputting a full-width Gaussian pulse half maximum of 10^{-14}s?

6. Consider a Ti: Sapphire laser in Kerr Lens locked mode whose pulse train's intensity is given by
$$I \sim E_0^2 \frac{\sin^2(N\delta\omega t/2)}{\sin^2(\delta\omega t/2)} \tag{12}$$
where E_0, N & $\delta\omega$ represent the electric field amplitude, number of modes and frequency spacing between modes respectively. What is the peak intensity, pulse width and time separation of pulses?

7. Consider a Yb:YAG crystal inside a laser cavity showing optical pumping at 940 nm. Yb has lasing wavelength of 1030 nm. A maximum power output of 912 mW is achieved if all emitted photons are absorbed by the crystal and used for the lasing. Determine the pump power required to attain this.

8. Polarization-encoded qubits are difficult to manipulate and less resistant to certain experimental errors than path-encoded qubits. True/False

9. Do you need a non-linear interaction between two single photons to implement a C-NOT gate?

10. In LOQC, Walsh-Hadamard transforms are replaced by Optical Fourier Transforms. True/False

11. Why is it that even if the optical implementation can use the phase information in the data register, it ends up showing different enhancements in computation in contrast to its classical analog?

12. Why has quantum mechanical control over atoms and molecules remained an elusive issue?

13. How can we control quantum interferences in Anthracene molecules in its vibrational state?

14. Discuss the behavior of a two-level quantum system if a laser pulse is shaped in such a way that its instantaneous frequency within itself changes from a) far-above resonance to far-below resonance b) far-below resonance to resonance and back to far-below resonance smoothly.

15. How many electrodes are there in ion traps?

16. Why is scaling of ion traps QC difficult?

17. What are the most common ions used in TIQC?

18. Wavefunctions which correspond to single ion eignevalues in trapped ions are non-degenerate. True/False

19. How do ions trapped in a radio-frequency trap interact?

20. In case of a laser interacting with atoms acting as qubits, one obtains an additional state. What is that state?

21. In the implementation of DJ Algorithm using TIQC, one uses only a single ion trap. How is coupling achieved in such a scenario?

22. What is the qubit for D-Wave computers?

23. Spintronics modern approach and NMR technology both use spin states to act as qubits. Then, what is the difference between the two approaches?

24. How does QC benefit from using GMR (Giant magnetoresistance)?

25. Identify one problem in quantum computation with neural atoms in optical nano-structures?

26. How is the lattice wavefuntion of the Bose-Einstein condensate atoms (used as qubits) projected into free space?

27. How does one measure the spin of a single electron on a solid surface experimentally?

28. List some advantages of superconducting QC?

29. What is the interaction of the two states with the imaginary part of an interaction term in NMRQC?

30. In a Mach-Zander Interferometer, beam pair spans a 2-D Hilbert space with $|0\rangle$ and $|1\rangle$. The input density matrix is given by $\rho_{input} = |0\rangle\langle 0|$. The mirror, beam-splitter and phase-shifter unitaries are U_m, U_b and U_p respectively. Determine $\rho_{output} = U_b U_m U_p U_b \rho_{input} U_b^\dagger U_p^\dagger U_m^\dagger U_b^\dagger$.

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