Week 10: Theory Solutions

1. The required probabilities may be computed as follows:

 $\Pr[\text{at least } k \text{ bits get flipped}] = 1 - \Pr[\text{at most } k \text{ bits get flipped}]$ $= 1 - \sum_{i} \Pr[i \text{ bits get flipped}]$ $= 1 - \sum_{i} p^{i} (1 - p)^{n - i}$

$$=1-\sum_{i}^{n}p^{i}(1-p)^{n-1}$$

2. Since three-qubit code for bit-flip errors can correct at most 1 bit, the required probability is

Pr[at most 1 bit flipped for n operations].

Since each of the operations are independent this equals,

$$\sum_{i=1}^{n} \Pr[\text{at most 1 bit for operation i}].$$

This in turn equals,

$$\sum_{i=1}^{n} \sum_{j=0}^{1} \Pr[j \text{ bits flipped in operation } i].$$