Chaos Theory Explored Through Double Pendulums

The double pendulum is the simplest example of a chaotic system. It consists of two simple pendulums attached end to end. Studying this offers insight into other more advanced chaotic systems like earth’s weather. To study this, I manipulated the difference between initial conditions (Δθ). I then manipulated the position and potential energy of the pendulums’ masses using the θ1 and θ2 values. Finally, I manipulated other parameters like mass and length. In my research, I found that there are two different types of mechanical systems, linear and chaotic. In chaotic motion, double pendulums are the simplest systems. I predicted that no matter how small the Δθ value was, it would be chaotic. I believed that angles lower than a certain threshold will not be chaotic. Finally, I predicted that mass and length wouldn’t affect the system. To do the experiment, I used code from <https://matplotlib.org/examples/animation/double_pendulum_animated.html> plus <https://scipython.com/blog/the-double-pendulum/> and adapted it for my own purposes. First, I modified the value called “sensitivity” on line 64 while keeping all other variables constant. Next, I modified the values marked th1 and th2 on lines 60 and 62, respectively. Finally, I modified mass and length on lines 23-27. When sensitivity was modified, the time it took for the pendulums to diverge increased as the value on the line decreased. Up until 10­­-18 after 10­­-18, my computer considered the number too small to compute. Unfortunately, this is the limit for a 64-bit Windows® machine. When 1 angle was kept constant while the other was manipulated to find the minimum value needed for chaotic motion (critical angle), a sharp border was found. When it was plotted, it showed the border between chaotic and normal motion. Finally, critical angle for the systems was found and plotted against the mass/length difference, it was it was symmetric across the line y=x+79.15. In conclusion, on my first hypothesis, I was incorrect, however, I am theoretically correct. This is because it should still diverge according to chaos theory however, my system treated the number as 0 meaning it didn’t show any difference. When it came to angles, I was correct and, on top of that, I found the borderline between chaotic and linear systems. Finally, when it came to mass and length, I was incorrect because they both showed some very interesting correlations. Doing this project took a lot of effort and I would like to thank Dr. Han Htoon of Los Alamos National Labs for making this project possible for me.

Works Cited

“Animation Example Code: double\_pendulum\_animated.Py¶.” *Animation Example Code: double\_pendulum\_animated.Py - Matplotlib 2.0.2 Documentation*, matplotlib.org/examples/animation/double\_pendulum\_animated.html. Accessed 12 Jan. 2020.

Asylum, The Science. “Lagrangian Mechanics: How Powerful Is It?” YouTube, YouTube, 9 July 2019, <https://www.youtube.com/watch?v=MIBfKJHMWHU>

The Editors of Encyclopaedia Britannica. “Chaos Theory.” *Encyclopædia Britannica*, Encyclopædia Britannica, Inc., 26 Dec. 2019, www.britannica.com/science/chaos-theory. Accessed 12 Jan. 2020.

*The Double Pendulum*, scipython.com/blog/the-double-pendulum/. Accessed 12 Jan. 2020.