UNIVERSITY OF PERADENIYA

DEPARTMENT OF STATISTICS & COMPUTER SCIENCE

ST203 - THEORY OF STATISTICS

Tutorial # 03

1. Suppose that Y is normally distributed with mean 0 and unknown variance σ^2. Then $\frac{Y^2}{\sigma^2}$ has a χ^2 distribution with 1 df. Use the pivotal quantity $\frac{Y^2}{\sigma^2}$ to find a
   (a) 95% confidence interval for σ^2.
   (b) 95% upper confidence limit for σ^2.
   (c) 95% lower confidence limit for σ^2.

2. Scholastic Assessment Test (SAT) scores, which have fallen slowly since the inception of the test, have now begun to rise. Originally, a score of 500 was intended to be average. The mean scores for 2015 were approximately 508 for the verbal test and 520 for the mathematics test. A random sample of the test scores of 20 seniors from a large urban high school produced the means and standard deviations listed in the accompanying table.

	Verbal	**Mathematics**
Sample mean	505	495
Sample standard deviation	57	69

   (a) Find a 90% confidence interval for the mean verbal SAT scores for high school seniors from the urban high school.
   (b) Does the interval that you found in part (a) include the value 508, the true mean verbal SAT score for 2015? What can you conclude?
   (c) Construct a 90% confidence interval for the mean mathematics SAT score for the urban high school seniors. Does the interval include 520, the true mean mathematics score for 2015? What can you conclude?

3. A study was conducted to check whether the SAT scores for high school students differ depending on the students' intended field of study. Fifteen students who intended to major in engineering were compared with 15 students who intended to major in language and literature. Given in the accompanying table are the means and standard deviations of the scores on the verbal and mathematics portion of the SAT for the two groups of students.

	Verbal		**Math**	
	Mean	**Standard deviation**	**Mean**	**Standard deviation**
Engineering	446	42	548	57
Language/Literature	534	45	517	52

   (a) Construct a 95% confidence interval for the difference in average verbal scores of students majoring in engineering and of those majoring in language/literature.
   (b) Construct a 95% confidence interval for the difference in average math scores of students majoring in engineering and of those majoring in language/literature.
   (c) Interpret the results obtained in parts (a) and (b).
   (d) What assumptions are necessary for the methods used previously to be valid?

4. The operations manager of a large production plant would like to estimate the mean amount of time a worker takes to assemble a new electronic component. Assume that the standard deviation of this assembly time is 3.6 minutes.
   (a) After observing 120 workers assembling similar devices, the manager noticed that their average time was 16.2 minutes. Construct a 92% confidence interval for the mean assembly time.
   (b) How many workers should be involved in this study in order to have the mean assembly time estimated up to ± 15 seconds with 92% confidence?

5. Suppose a consumer advocacy group would like to conduct a survey to find the proportion p of consumers who bought the newest generation of an MP3 player were happy with their purchase.
   (a) How large a sample n should they take to estimate p with 2% margin of error and 90% confidence?
   (b) The advocacy group took a random sample of 1000 consumers who recently purchased this MP3 player and found that 400 were happy with their purchase. Find a 95% confidence interval for p.

6. Recently, 50 residents of a particular community were asked "how many minutes do you spend daily on some form of rigorous physical exercise?" The mean response was half an hour, with a standard deviation of 4.2 minutes. Find a 95% confidence interval for the mean time spent on exercise by all residents.

7. The HSE wants to estimate the proportion of patients who contract the winter vomiting bug while in a particular hospital. From a random sample of 250 patients 13.4% are reported to have contracted the bug. Find a 99% confidence interval for the proportion of all patients in this hospital who contract the bug.

8. A sample of 457 pet owners was asked to indicate the number of times they visit a vet each year. The sample mean was 3.59 visits with a sample standard deviation of 1.045. Based on these results a confidence interval from 3.49 to 3.69 was calculated. What confidence level had been used in constructing this confidence interval?