

$$\left. \begin{array}{l} \text{i) profit} \propto \text{Investment} \\ \text{ii) Profit} \propto \text{Time} \end{array} \right\} \text{Profit} \propto \text{Investment} \times \text{Time}$$

$$\frac{\text{Profit}_A}{\text{Profit}_B} = \frac{\text{Investment}_A}{\text{Investment}_B} \times \frac{\text{Time}_A}{\text{Time}_B}$$

1. A & B started business investing ₹ 48000 & ₹ 64000 respectively but B left the business after 2 months. Find the difference between the share of profit of A if the total profit at the end of the year is ₹ 12,1

Soln:-

$$A = 48000/m \quad B = 64000/m$$

~~$\frac{2}{+5}$~~ ~~128000~~ ~~$\frac{9}{+8}$~~ ~~196000~~
 ~~$+5$~~ ~~$\frac{42}{+2}$~~ ~~148000~~
 ~~64~~ ~~$\frac{144}{288}$~~ ~~288~~ ~~576~~
 ~~128000~~ ~~576~~

$$B = 64000 \text{ m} \quad \text{64} \quad \text{SRC} \quad \frac{44}{288} \quad \underline{128000 : 576000}$$

$$9x + 2x = 12100$$

$$11x = 12100$$
$$x = \frac{11}{12100}$$

$$x = 1100$$

$$A's \ profit = 9(1100)$$

= 9900

$$B's \text{ profit} = 2(1100)$$

$$= 2200 \text{ GPa}$$

$$\text{difference} = 9900 - 2200$$

$$= 7700$$

(08)

$$\frac{P_A}{P_B} = \frac{T_A}{T_B} \times \frac{T_B}{T_A}$$

$$= \frac{\cancel{48000}^{\cancel{48}^3}}{\cancel{64000}^{\cancel{16}}} \times \frac{\cancel{12}^3}{2} = \frac{9}{2}$$

$$\frac{P_A}{P_B} = \frac{9}{2} \rightarrow \text{difference} = 7 \text{ parts}$$

2. A, B & C enter into a partnership. A contributes ₹360 for 4 months, B contributes ₹420 for 3 months, C contributes ₹180 for 5 months. If the total profit at the end of the year was ₹400. What is the share of A in the profit?

Soln:-

$$A \rightarrow 360 - 4 \text{ months} \Rightarrow 1440$$

$$B \rightarrow 420 - 3 \text{ months} \Rightarrow 1260$$

$$C \rightarrow 180 - 5 \text{ months} \Rightarrow 900$$

$$1440 : 1260 : 900$$

$$\begin{array}{r} 72 \\ 72 \\ \hline 8 \end{array} \quad \begin{array}{r} 63 \\ 63 \\ \hline 7 \end{array} \quad \begin{array}{r} 45 \\ 45 \\ \hline 5 \end{array}$$

$$A:B:C = 8:7:5 \text{ Total } 20$$

$$A's \text{ share} = \frac{8}{20} \times 400$$

$$= 160$$

3. A milkman rented a pasture. A grazed 16 cows for 3 months, B grazed 20 cows for 4 months, C grazed 18 cows for 6 months & D grazed 42 cows for 2 months. If A's share in the rent is ₹2400. Find the rent paid by C.

Soln:-

$$A \Rightarrow 16 \rightarrow 3m \quad C \Rightarrow 18 \rightarrow 6m$$

$$B \Rightarrow 20 \rightarrow 4m \quad D \Rightarrow 42 \rightarrow 2m$$

$$A:B:C:D = \frac{8^4}{18 \times 3} : \frac{10^5}{20 \times 4} : \frac{9^3}{18 \times 6} : \frac{21}{42 \times 2}$$

$$= 12 : 20 : 27 : 21$$

$$A's \text{ share} = 12 \text{ parts}$$

4. A started a business with a capital of ₹ 85000. B joins him with a capital of ₹ 42500 after some months. Find how much profit B has there in the business? If the profit at the end of the year is shared in the ratio 8:19, then what is A's share?

Soln:-

$$A \rightarrow 85000$$

$$B \rightarrow 42500$$

$$\frac{P_A}{P_B} = \frac{I_A}{I_B} \times \frac{T_A}{T_B}$$

$$\frac{3}{1} = \frac{850}{425} \times \frac{12}{T_B}$$

$$T_B = \frac{850 \times 12}{425 \times 3} = \frac{85}{17} = 5$$

$$\boxed{T_B = 5}$$

5. A & B invest in the ratio 3:5. After 6 months, C joined the business with investment which is same as B. At the end of year, find the ratio of their profits?

Soln:

$$A:B = 3:5$$

$$A:B:C = 3x \times 12 : 5x \times 12 : 5x \times 6 \\ = 6:10:5$$

6. A started a business by investing ₹ 25000, After 6 months B joined by investing ₹ 15000. After another 6 months B invested an additional amount of ₹ 15000. At the end of 3 years they earned a profit of ₹ 21471000. Find the share of B in the profit.

(A & B invested in the ratio of 12 : 8)

Soln: A invested for 36 months & B invested for 12 months

$$A \rightarrow 25000 \times 36$$

$$B \rightarrow 15000 \times 30$$

After 6 months $\frac{12}{36} \text{ of profit} = \frac{1}{3}$ & $\frac{25}{15} \text{ of capital} = \frac{5}{3}$

$$B \rightarrow 15000 \times 24$$

$$\frac{120000}{90000} = \frac{12}{9} = \frac{4}{3}$$

$$\frac{15 \times 6}{15 \times 6} = 1$$

$$\frac{25 \times 36}{15 \times 20 + 15 \times 24} = \frac{25}{72}$$

$$\frac{25 \times 3}{72} = \frac{25}{24}$$

$$P_A : P_B = 5 : 12 : 3 : 10 : 2 : 8 = 10 : 5 + 4$$

$$= 10 : 9$$

$$19P = 247000$$

$$B's \text{ share} = \frac{9}{19} \times 247000$$

$$\boxed{B's \text{ share} = 117000}$$

7. A & B started a business with investments in the ratio, 5:3 respectively. After 6 months from the start of business, C joined them with an investment ratio of between B:C as 2:3. If the annual profit earned by them is ₹ 12300, find the difference between the shares of B & C in the profit.

Soln: A:B = (5:3) × 2

$$A:B:C = 10:6:9$$

$$B:C = (2:3) \times 3$$

$$P_A : P_B : P_C = 10 \times 12 : 6 \times 12 : 9 \times 6 = 20:12:9 = 2:2:3$$

20: $\frac{1}{12} \times 4 \times IPR = \frac{1}{12} \times 4 \times 1200 = 400$ rupees per month
 i.e. $3P = P = \frac{1}{12} \times 1200 = 100$
 To sum up, $I = 100 + 100 = 200$
 So, $I = 200$

8. A started a business with a capital of ₹ 54000 and admitted joined by B & C after 4 months & 6 months respectively. At the end of the year if the profits are shared in the ratio 1:4:5. What is the difference between the investments of B & C?

Soln:-

$$1:4:5 = 54000 \times 12 : I_B \times 8 : I_C \times 6$$

$$I_B \times 8 = 4P$$

$$I_C \times 6 = 5P$$

$$\frac{54 \times 12}{108}$$

$$\frac{54}{54}$$

$$\frac{648000}{648}$$

$$648000 = 1P$$

$$I_B = \frac{4 \times 648000}{8} = 324000$$

$$I_B = 324000$$

$$\frac{108000}{824000}$$

$$\frac{5 \times 648000}{648}$$

$$I_C = 540000$$

$$\frac{540000}{324000}$$

$$I_C - I_B = 216000$$

$$\frac{216000}{216000}$$

9. Two friends Mohan & Rohan started a business by investing ₹ 1,50,000 & ₹ 2,00,000 respectively. After 6 months, Rohan withdraw some amount from his investment. At the same time, Gaurav joined the business by investing the same amount which Rohan has withdrawn. At the end of the year, Rohan & Gaurav has distributed their profits in the ratio 5:3. If the total profit for the year was ₹ 15400.

Find the share of Mohan in the profit.

Soln:-

$$\text{Mohan} - 1,50,000$$

$$\text{Rohan} - 2,00,000$$

After 6 month,

$$\text{Rohan} - 2,00,000 - x$$

$$\text{Gaurav} - x$$

$$P_A : P_B : P_C = 150,000 \times 12 : (200,000 \times 6) + (200,000 - x) \times 6 +$$

$$= 12 \times 150,000 : 6 \times (200,000 + 200,000 - x) +$$

$$x \times 6$$

$$\Rightarrow \frac{5}{12+6+6} = \frac{200,000 + 200,000 - x}{12 \times 150,000}$$

~~Let's calculate the total investment of Rohan~~

~~is Rs 6,00,000 + 6,00,000 = 12,00,000~~

~~so his profit is 12,00,000 * 3/10 = 3,60,000~~

~~so his profit is 12,00,000 * 3/10 = 3,60,000~~

~~so his profit is 12,00,000 * 3/10 = 3,60,000~~

$$P_A : P_B : P_C = 12 \times 150,000 : 200,000 + 200,000 - 150,000 : 150,000$$

$$= 200,000 : 250,000 : 150,000$$

$$= 6 : 5 : 3$$

$$P_A = \frac{6}{14} \times 77,000$$

$$= 66,000$$

$$\boxed{P_A = 66,000}$$

- Q. A, B & C started a business. A invested for 4 months, and claims $\frac{1}{8}$ th of total profit, B invested for 6 months, & claims $\frac{1}{3}$ rd of total profit while C invested for 8 months. Find the investment made by A & B together.

Soln:

$$A - 4 \text{ month } - \frac{1}{8}$$

$$B - 6 \text{ month } - \frac{1}{3}$$

$$C - ₹ 1560 - 8 \text{ month}$$

$$1 - \left[\frac{1}{8} + \frac{1}{3} \right] = C$$

$$1 - \left[\frac{8+3}{24} \right] = C$$

$$\frac{24-11}{24} = C$$

$$= \frac{13}{24} C$$

11. A, B & C started a business with A investing $33\frac{1}{3}\%$ of total capital, B investing $25\frac{1}{4}\%$ of remaining and C invested the rest. If the total profit at the end of the year was ₹ 1,62,000. Find the share of A in the profit.

Soln:-

$$P_A : P_B : P_C = \left(\frac{1}{3}, \frac{1}{6}, \frac{1}{2} \right) \times 6$$

$$\begin{array}{c} \text{Total} \\ \diagdown \quad \diagup \\ A \quad B \quad C \\ \frac{1}{3} \quad \frac{2}{3} \times \frac{25}{100} \quad \frac{1}{2} \\ = \frac{1}{6} \end{array}$$

$$P_A : P_B : P_C = 2 : 1 : 3$$

$$\frac{27000}{81000}$$

$$\text{A's share} = \frac{2}{6} \times 162000$$

$$1 - \left[\frac{1}{3} + \frac{1}{6} \right]$$

$$1 - \left[\frac{2+1}{6} \right]$$

$$= \frac{1}{2}$$

12. A, B & C started a business with a investment in the ratio $2:3:5$. A increased his capital by 50% after 4 months. B increased his capital $33\frac{1}{3}\%$ after 6 months & C withdraw his capital by 50% after 8 months. At the end of the year if the total profit was ₹ 86,800.

Find the difference between the shares of A & C in the profit.

Soln: $P_A : P_B : P_C = 2 \times 4 : 3 \times 8 : 3 \times 6 + 4 \times 6 : 5 \times 8 + 2.5 \times 4$

$$= 16 : 24 : 50 \text{ or } 16 : 24 : 50 \text{ or } 16 : 24 : 50$$

$$= 16 : 24 : 50 \text{ or } 16 : 24 : 50$$

$$62P \rightarrow 86800$$

$$A - C = 25P - 16P$$

$$= 9P$$

$$9P = \frac{9}{62} \times 86800$$

or

$$= 12600$$

13. A, B & C started a business each investing ₹ 20,000. After 5 months A withdraw ₹ 5000, B withdraw ₹ 4000, and C invests ₹ 6000 more. At the end of the year if the total profit is ₹ 69900. Find the share of A in the profit.

Soln:-

$$P_A : P_B : P_C = 20000 \times 5 + 15000 \times 7 : 20000 \times 5 + 16000 \times 7$$

$$: 20000 \times 5 + 26000 \times 7$$

$$= 100000 + 105000 : 100000 + 112000 : 100000 + 182000$$

$$= 205000 : 212000 : 282000$$

$$\text{Total profit} = 205 : 212 : 282$$

A's share = $\frac{205}{699} \times 69900$ as total sum of shares

$$699P = 69900$$

$$1P = 100$$

$$\boxed{A's share = 20500}$$

Q. A, B & C entered into partnership by investing in the ratio $\frac{2}{5} : \frac{3}{4} : \frac{5}{8}$. After 4 months A increased capital by 50% but B decreased capital by 20%. What is the share of B in the total profit of ₹ 282100 at the end of the year?

soln:

$$\frac{2}{8} \times 410 : \frac{3}{8} \times 410 : \frac{5}{8} \times 410$$

16:30:25

$$\begin{array}{r}
 2 \overline{)5,4,1,8} \\
 2 \overline{)5,2,4} \\
 2 \overline{)5,1,2} \\
 5 \overline{)5,1,1}
 \end{array}$$

$$P_A : P_B : P_C = 16 \times 2 + 24 \times 8 : 30 \times 4 + 24 \times 8 : 25 \times 12$$

LCM = 120

$$= 64 + 192 : 120 + 192 : 300$$

$$\frac{30}{100} \times 20$$

$$= 256 : 312 : 300$$

$$= 64 : 78 : 75$$

$$217 P = 282100$$

$$1800$$

$$\text{B's share} = \frac{78}{27} \times 282100$$
$$= 101400$$

$$\begin{array}{r} 18 \times 13 \\ \hline 0'34 \\ 78 \\ \hline 1014 \end{array}$$

15. A & B started a partnership. A's investment was 2 times the investment of B and his time period of investment is 2 times that of B. If B received a profit of ₹ 4000. Find a total profit.

Soln :-

$$P_n = 4000 \quad P_A = 2$$

15. If $\frac{P_A}{P_B} = \frac{3}{B} \times \frac{2}{B}$ and A & B invests in a business in the ratio of 3:2, then the profit will be shared in the ratio of 6:5. If the total profit is Rs 60000, then the share of A is

$$P_A = 6(4000)$$

$$P_A = 24000$$

$$\text{Total profit} = 24000 + 4000$$

$$= 28000$$

16. In a business partnership among A, B, C & D the profit is shared as follows:

$\frac{A}{B} = \frac{B}{C} = \frac{C}{D} = 1:3$. If the total profit is Rs 4,00,000. Find the share of C in the profit.

Soln:-

A	B	C	D
1	3	3	3

Let the total profit be $P = 1400000$ then A's share is $\frac{1}{3}P = 400000$. Now $\frac{A}{B} = \frac{B}{C} = \frac{C}{D} = 1:3$ so $B = 3A$, $C = 3B = 9A$ and $D = 3C = 27A$. So $P = A + B + C + D = A + 3A + 9A + 27A = 40A$. So $A = \frac{P}{40} = \frac{1400000}{40} = 35000$. So C's share is $9A = 90000$.

17. The time periods of A & B in a business are in the ratio 4:5. If they had to share the profit in the ratio 5:3. Find the ratio of their investment.

Soln: $\frac{P_A}{P_B} = \frac{I_A}{I_B} \times \frac{T_A}{T_B}$

$$\frac{I_A}{I_B} = \frac{P_A}{P_B} \times \frac{T_B}{T_A} = \frac{5}{3} \times \frac{5}{4}$$

$$\frac{I_A}{I_B} = \frac{25}{12}$$

$$I_A : I_B = 25 : 12$$

18. A, B & C started a business by investing the capital in the ratio 5:6:8. At the end of the business, if they receive the profit in the ratio 5:3:12. Find the ratio of time period for which they invested the capitals.

Soln:

$$P_A : P_B : P_C = I_A \times T_A : I_B \times T_B : I_C \times T_C$$

$\therefore 5:3:12 = 5 \times T_A : 6 \times T_B : 8 \times T_C$

$$\frac{T_A}{T_B} = \frac{5}{6} \quad \frac{T_B}{T_C} = \frac{3}{8}$$

$$\therefore \frac{T_A}{T_B} \times \frac{T_B}{T_C} = \frac{5}{6} \times \frac{3}{8} = \frac{5}{16}$$

$$\text{Time ratio } T_A : T_B : T_C = (5:16:3) \times 2 = 10:32:6$$

$$\boxed{T_A : T_B : T_C = 2 : 1 : 3}$$

19. A puts ₹ 600 more than B in a business but B has invested his capital for 5 months while A invested for only 4 months. If the share of A in the profit is ₹ 48 more than that of B out of total profit of ₹ 528.

Find the capital invested by A.

$$A = B + 600$$

$$\frac{528}{5+6} = 48 + 240$$

$$B = 240$$

$$P_A = 48 + B$$

$$P_B = 528 - P_A$$

$$P_A = 48 + 528 - P_A$$

$$2P_A = 48 + 528$$

$$2P_A = 576$$

$$\boxed{P_A = 288}$$

$$\boxed{P_B = 240}$$

$\frac{P_A}{P_B} = \frac{I_A}{I_B} \times \frac{T_A}{T_B}$ given profit constant out of
 i.e. $\frac{P_A}{P_B} = \frac{18}{20} \times \frac{12}{12}$ profit share will be
 $\frac{P_A}{P_B} = \frac{288}{360} = \frac{x+600}{5x}$, with P_A & P_B
 having $\frac{240}{480}$ with profit share proportion will
 be $\frac{60}{120}$ works 2/3rd part with profits
 difference $180 = \frac{30}{5} \times 60$ $\Rightarrow 1200$ hundred
 $\frac{12}{6} = \frac{42+2400}{5x}$ earned profit when
 $x = 1200$

$$6x - 4x = 2400$$

$$2x = 2400$$

$$x = 1200$$

$$A = 600 + x$$

$$= 600 + 1200$$

$$\boxed{A = 1800}$$

20. A & B entered into partnership with ₹60000 &
 ₹50000 respectively. C joins them before 'b' months
 contributing ₹70000 and A leaves them after 'a'
 months from the starting of the year. If they share
 the profits in the ratio of 18:20:21. Find the values of
 a & b.

soln: $P_A : P_B : P_C = 18 : 20 : 21$
 $18 : 20 : 21 = 6a : 5 \times 12 : 7b$

$$18 : 20 : 21 = 6a : 60 : 7b$$

$$\frac{18}{20} = \frac{6a}{60} \Rightarrow 6a = \frac{18 \times 60}{20}$$

$$6a = 54$$

$$\boxed{a = 9}$$

$$\frac{20}{21} = \frac{60}{7b}$$

$$7b = \frac{60 \times 21}{20}$$

$$7b = 63$$

$$\boxed{b = 9}$$

21. Two business man A & B invested in a business in the ratio 5:8. They decided to reinvest 30% of the profit they earned back into the business. The remaining profit they have distributed among themselves. If A's share in the profit distributed was ₹ 87500. Find the total profit made by the business.

Soln:

$$A's \text{ share} = 87500$$

$$\frac{5}{13} \times \frac{70}{100} \times x = 87500$$

$$\frac{1}{26} x = 87500$$

$$x = \frac{87500 \times 26}{1}$$

$$\frac{125 \times 26}{1} = 125 \times 26$$

22. A and B started a business by investing ₹ 3,50,000 and ₹ 1,40,000 respectively. B gets 20% of the yearly profit for managing the business. The remaining profit is divided among them in the ratio of their investment. If A receives ₹ 88,000 more than B at the end of the year. Find the total profit made by the business.

Soln:-

$$\frac{P_A}{P_B} = \frac{I_A}{I_B} = \frac{250000}{140000} = \frac{25}{14} = \frac{5}{2}$$

$$\begin{aligned} A's \text{ profit} &= \frac{20x}{100} + \frac{5}{7} \times \frac{80x}{100} = \frac{2x}{5} + \frac{4x}{7} \\ &= \frac{7x + 20x}{35} = \frac{27x}{35} \end{aligned}$$

$$B's \text{ profit} = \frac{2}{7} \left(\frac{80x}{100} \right) = \frac{80x}{35}$$

$$P_A : P_B = \frac{27x}{25} : \frac{80x}{35} \\ = 27 : 8$$

$$P_A = 38000 + P_B$$

$$P_A - P_B = 38000$$

$$19P = 38000$$

$$1P = 2000$$

$$\boxed{35P = 700000.}$$

Q3. A sum of ₹ 3115 is divided among A, B & C such that if ₹ 25, ₹ 28 & ₹ 52 are subtracted from their shares respectively. The amount ratio will be 8 : 15 : 20. Find the amount received by A, B & C.

Soln:-

$$A + B + C = 3115$$

$$A - 25 : B - 28 : C - 52 = 8 : 15 : 20$$

$$\frac{A-25}{8} : \frac{B-28}{15} : \frac{C-52}{20} = 1 : 1 : 1$$

$$\frac{A-25}{8} = \frac{B-28}{15} = \frac{C-52}{20} = x$$

$$A = 8x + 25$$

$$B = 15x + 28$$

$$C = 20x + 52$$

$$8x + 25 + 15x + 28 + 20x + 52 = 3115$$

$$43x + 105 = 3115$$

$$43x = 3010$$

$$\boxed{x = 70}$$

$$\begin{array}{r} 3115 \\ 105 \\ \hline 3010 \end{array}$$

Solved Ques

1. If $289 = 17^{\frac{x}{5}}$, Find the value of x .

Soln:

$$17^2 = 17^{\frac{x}{5}}$$

$$\frac{x}{5} = 2$$

$$\boxed{x = 10}$$

2. Find the value of $\frac{9^2 \times 18^4}{3^{16}}$

Soln:

$$= \frac{(3^2)^2 \times 18^4}{3^{16}}$$

$$= \frac{2^4 \times 3^8}{3^{12}} = \frac{2^4}{3^4} = \frac{16}{81}$$

3. If $10^x = \frac{1}{2}$, value of 10^{-8x}

Soln:

$$10^x = \frac{1}{2} \Rightarrow 10^{-8x} = (10^x)^{-8}$$

$$= (\frac{1}{2})^{-8}$$

$$= (2^{-1})^{-8}$$

$$= 2^8$$

$$= 256 //$$

4. If $2^{2n-1} = \frac{1}{8^{n-3}}$, $n=?$ for solving with hint 3

Soln:-

$$2^{2n-1} = \frac{1}{8^{n-3}}$$

$$2^{2n-1} = 8^{-(n-3)}$$

$$2^{2n-1} = 8^{3-n}$$

$$2^{2n-1} = (2^3)^{3-n}$$

$$2^{2n-1} = 2^{9-3n}$$

$$2n-1 = 9-3n$$

$$5n = 10$$

$$\boxed{n=2} \quad \text{for solving hint 3, } 2+2=6 \text{ is } 10 \text{ plus}$$

5. If $2^{n-1} + 2^{n+1} = 320$. Find the value of n ? (10)

Soln:-

$$2^{n-1} + 2^{n+1} = 320$$

$$\frac{2^n}{2} + 2^n(2) = 320$$

$$2^n\left(\frac{1}{2} + 2\right) = 320$$

$$2^n\left(\frac{5}{2}\right) = 320$$

$$2^n = \frac{320 \times 2}{5}$$

$$2^n = \frac{640}{5}$$
for solving with hint 3

$$2^n = 128$$

$$\boxed{n=7}$$

6. Find the value of $\frac{2^{n-1} - 2^n}{2^{n+4} + 2^{n+1}}$

$$\begin{aligned}\frac{2^{n-1} - 2^n}{2^{n+4} + 2^{n+1}} &= \frac{2^n(2^{-1}) - 2^n}{2^n(2^4) + 2^n(2)} \\&= \frac{2^n(1/2 - 1)}{2^n(2^4 + 2)} \\&= \frac{-1/2}{18} \\&= \frac{-1}{36}\end{aligned}$$

7. If $x = 5 + 2\sqrt{6}$, Find value of $\sqrt{x} - \frac{1}{\sqrt{x}}$

Soln:

$$x = 5 + 2\sqrt{6}$$

$$= 3 + 2 + 2\sqrt{3}\sqrt{2}$$

$$= (\sqrt{3})^2 + (\sqrt{2})^2 + 2\sqrt{3}\sqrt{2}$$

$$x = (\sqrt{3} + \sqrt{2})^2$$

$$\sqrt{x} = \sqrt{3} + \sqrt{2}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

$$= \frac{\sqrt{3} - \sqrt{2}}{3 - 2}$$

$$= \sqrt{3} - \sqrt{2}$$

$$\sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{3} + \sqrt{2} - \sqrt{3} + \sqrt{2}$$

$$= \sqrt{2} + \sqrt{2}$$

$$= 2\sqrt{2}$$

8 Find the value of $\sqrt{6 - 4\sqrt{3}} + \sqrt{16 - 8\sqrt{3}}$

Soln:

$$16 - 8\sqrt{3} =$$

$$10. \text{ If } 2^x = 4^y = 8^z \text{ and } \frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7}$$

Find the value of z.

Soln:

$$\left. \begin{array}{l} 2^x = (2^2)^y = (2^3)^z \\ 2^x = 2^{2y} = 2^{3z} \\ \underline{x = 2y = 3z} \end{array} \right\} \begin{array}{l} \frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7} \\ \frac{1}{2} \left[\frac{1}{x} + \frac{1}{2y} + \frac{1}{3z} \right] = \frac{24}{7} \\ \frac{1}{2} \left[\frac{1}{x} + \frac{1}{x} + \frac{1}{x} \right] = \frac{24}{7} \end{array}$$

$$\frac{7}{16} = 3z$$

$$\boxed{z = \frac{1}{48}}$$

$$\frac{3}{2x} = \frac{24}{7}$$

$$2x = \frac{2 \times 7}{24/8} = \frac{7}{16}$$

$$12. \text{ If } 3^{2x-y} = 3^{x+y} = \sqrt{27}. \text{ Find the value of } y?$$

Soln:

$$3^{2x-y} = 3^{x+y} = 3^{3/2}$$

$$\sqrt{27} = (3^3)^{1/2} = 3^{3/2}$$

$$2x-y = x+y = 3/2$$

$$2x-y = 3/2$$

$$2x-y = 3/2$$

$$x+y = 3/2$$

$$2(1)-y=3/2$$

$$\underline{3x=3}$$

$$2-3/2=y$$

$$\boxed{y=1/2}$$

$$13. \text{ If } x^y = y^x, \text{ then find value of } (xy)^{x/y}$$

$$(x/y)^{x/y} = (x/y)^x \cdot (x/y)^{y/y}$$

$$= \frac{x^x}{y^x} \cdot \frac{x^{y/y}}{y^{y/y}}$$

$$= \frac{x^x}{x^y} \cdot \frac{x^{y/y}}{y^{y/y}}$$

$$= x^{x-y} \cdot \frac{x^{y/y}}{y^{y/y}}$$

$$x^y = y^x$$

$$x^x = y^y$$

$$x^2 + 5^{\frac{y}{2}} = k \quad \text{and} \quad y^2 + 5^{\frac{x}{2}} = k$$

$$x = k^{\frac{y}{2}} \quad y = k^{\frac{x}{2}}$$

$$\left(\frac{k^{\frac{y}{2}}}{k^{\frac{x}{2}}}\right)^{\frac{x}{2}} = \left(k^{\left(\frac{1}{2} - \frac{1}{2}\right)}\right)^{\frac{x}{2}}$$

14. If $3^x = 5^y = 45^z$, then which of the following is correct?

Soln:- a) $\frac{2}{z} = \frac{1}{y} - \frac{1}{x}$ b) $x+y+z=0$

c) $\frac{2}{y} = \frac{1}{x} - \frac{1}{2}$ d) $\frac{2}{x} = \frac{1}{2} - \frac{1}{y}$

$$3^x = 5^y = 45^z = k$$

$$3 = k^{\frac{y}{2}} \quad 5 = k^{\frac{x}{2}} \quad 45 = k^{\frac{z}{2}}$$

$$45 = 3^2 \times 5$$

$$k^{\frac{z}{2}} = 3^{\frac{2}{2}} \times 5^{\frac{1}{2}}$$

$$k^{\frac{z}{2}} = k^{\frac{2}{2} + \frac{1}{2}}$$

$$\frac{1}{2} = \frac{2}{2} + \frac{1}{y}$$

$$\frac{2}{2} = \frac{1}{2} - \frac{1}{y}$$

5. Which of the following is greater?

- a) $\sqrt{2}$ b) $\sqrt[3]{3}$ c) $\sqrt[4]{4}$ d) $\sqrt[6]{6}$

Soln:

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$
$2^{\frac{1}{2}}$	$3^{\frac{1}{3}}$	$4^{\frac{1}{4}}$	$6^{\frac{1}{6}}$
$\frac{1}{2} \times 1^2$	$\frac{1}{3} \times 1^2$	$\frac{1}{4} \times 1^2$	$\frac{1}{6} \times 1^2$

2^6	3^4	4^3	6^2	64	81	64	36
-------	-------	-------	-------	------	------	------	------

16. If $x = 2 + 2^{\frac{2}{3}} + 2^{\frac{1}{3}}$, then find value of $x^3 - 6x^2 + 6x$

Soln:

$$x = 2 + 2^{\frac{2}{3}} + 2^{\frac{1}{3}}$$

$$(x-2) = 2^{\frac{2}{3}} + 2^{\frac{1}{3}}$$

Cube on b.s

$$(x-2)^3 = (2^{\frac{2}{3}} + 2^{\frac{1}{3}})^3$$

$$x^3 + 2^3 - 3x(2)(x-2) = (2^{\frac{2}{3}})^3 + (2^{\frac{1}{3}})^3 + 3(2^{\frac{2}{3}})(2^{\frac{1}{3}})(2^{\frac{2}{3}} + 2^{\frac{1}{3}})$$

$$x^3 + 8 - 6x(x-2) = 4 + 2 + 3(2)(2^{\frac{2}{3}} + 2^{\frac{1}{3}})$$

$$x^3 + 8 - 6x^2 + 12 = 6 + 6(2^{\frac{2}{3}} + 2^{\frac{1}{3}})$$

$$x^3 - 6x^2 + 12x + 8 = 6 + 6(x-2)$$

$$x^3 - 6x^2 + 12x - 6x = 6 - 12 - 8$$

$$\boxed{x^3 - 6x^2 + 6x = 2}$$

Simple Interest

→ In simple interest, the interest is common for all the years, and the interest is calculated only on the principal for every year.

→ Principal is always 100%.

→ Total amount = $P + SI$

$$\Rightarrow SI = \frac{PTR}{100}$$

T - should always be in years only,

$$T = 3 \text{ months} = \frac{3}{12} \text{ year} = \frac{1}{4} \text{ year}$$

. Find the simple interest on ₹ 5760 at 6% PA for
years.

Soln:-

$$SI = \frac{5760 \times \frac{3}{100} \times \frac{3}{5}}{5} = \frac{576 \times 9}{5} = 1036.8$$

$$100\% = 5760$$

$$1\% = 57.6$$

$$6\% \text{ PA} \Rightarrow 3\%$$

$$18\% = 576$$

$$18\% = \frac{576}{5000} \times 18 = 18.24$$

$$18\% = 1036.8$$

Raju took a loan from a bank at 12% pa simple interest. After 3 years he pays ₹ 5400 as interest. Find the principal taken by him.

$$\text{Soln: } 5400 = \frac{P \times 3 \times 12}{100} = \frac{5400 \times 25}{9} = 15000$$

3. Rajesh bought ₹850 for 3 years at 10% pa simple interest. What is the total amount he should pay at the end of 3 years?

Soln:

$$SI = \frac{850 \times 10 \times 3}{100}$$

$$SI = 255$$

$$\text{Amount} = 850 + 255$$

Simple Interest

total amount after 3 years

$$\frac{850}{100} = 85$$

What is the simple interest?

4. A farmer borrowed ₹3600 at 15% SI per annum. At the end of 4 years he cleared this debt by paying a ₹4000 and a cow. Find the cost of cow.

Soln:

$$SI = \frac{3600 \times 15 \times 4}{100}$$

$$= 2160$$

$$\frac{36 \times 60}{2160} = 3$$

$$\text{Amount} = 3600 + 2160$$

$$= 5760$$

$$\text{Cow's cost} = 5760 - 4000$$

$$= 1760$$

5. Raju borrowed ₹5000 at 12% pa simple interest. What amount should Raju pay to the bank after 6 months to clear the debt?

$$SI = \frac{5000 \times 12 \times 1}{100}$$

$$= \frac{50 \times 12}{100}$$

$$= 300$$

$$\text{amount} = 5000 + 300$$

$$= 5300$$

6. A bank interest decreased by ₹ 61.50 due to fall in annual rate of interest from 8% to $7\frac{3}{4}\%$. Find the principal.

Soln:-

$$8\% \rightarrow 7\frac{3}{4}\%, \downarrow$$

$$\frac{1}{4}\% = 61.50$$

$$\frac{1}{4}\% \text{ of } x = 61.50$$

$$\frac{1}{4 \times 100} \times x = 61.50$$

$$x = 61.50 \times 4$$

$$x = 246.00$$

$$P = 24,600$$

2. Two equal amounts are borrowed at 5% pa & 4% pa SI. The total interest after 4 years is ₹ 405. What is the total amount borrowed?

Soln:-

$$\frac{P \times 4 \times 5}{100} + \frac{P \times 4 \times 4}{100} = 405$$

$$\frac{5P}{25} + \frac{4P}{25} = 405$$

$$9P = 405 \times 25$$

$$9P = \frac{405 \times 25}{225}$$

$$P = \frac{405 \times 25}{9}$$

$$P = 1125$$

7. If the rate 8 1/2% pa is a sum of ₹ 4800 will earn how much interest in 2 years 3 months?

Soln:-

$$2 \text{ years } 3 \text{ months} = 2 + \frac{1}{4} = 2\frac{1}{4}$$

$$T \Rightarrow \frac{9}{4}$$

$$\begin{aligned} SI &= \frac{4800 \times 9/4 \times 17/2}{100} \\ &= \frac{126}{4} \times 9 \times 17 \\ &= 4 \times 21 \\ &= 54 \times 17 \\ &= 918 \text{ ₹.} \end{aligned}$$

8. Find the simple interest on Rs. 1800 from March 9 2012 to May 21, 2012 at 7 1/2% pa at SI duration being 45 days.

Soln:-

$$\begin{aligned} T &= \frac{73}{365} && \text{Semi-monthly towards last day} \\ & A - 30 \\ SI &= \frac{1800 \times 73 \times 3}{100 \times 365 \times 2} && M - \frac{20}{73} \\ & 73 \\ SI &= 27 \end{aligned}$$

9. A person borrows Rs. 5000 for 2 years at 4% pa SI. He immediately lends the amount to another at 6 1/4% pa for same 2 years. Find his gain in the transaction per year.

Soln:-

$$\begin{aligned} & \frac{5000 \times 2 \times 4}{100} = 400 & \frac{137.5}{275} & \frac{125}{275} \\ & \frac{500 \times 25 \times 2}{100 \times 4} & \frac{275}{275} & \underline{\underline{27.5}} \\ & 2 \frac{500 \times 25 \times 8}{100 \times 4} = 125 & = 137.5 \end{aligned}$$

The simple interest on a certain sum at 16.25% pa
rate of interest, for 4 years, is 12600 ~~less~~ on the
principal. Find the principal.

Soln: Let P be the principal and the rate of interest be 16.25% per annum.
 $P - 12600 = \frac{P \times 16.25 \times 4}{100}$

or $P - 12600 = \frac{P \times 162.5}{25}$

$$P - 12600 = \frac{P \times 162.5}{25} \quad \text{or } P - 12600 = \frac{P \times 13}{20}$$

$$P - \frac{13P}{20} = 12600$$

$$\frac{7P}{20} = 12600$$

$$7P = \frac{12600 \times 20}{7}$$

$$\boxed{P = 36000}$$

11. If a, b, c are sums of money such that b is SI on
a and c is simple interest on b for same time and same
rate of interest. Find the relationship between a, b & c.

Soln: b is SI on a $b^2 = ac$

c is SI on b

$$\frac{c}{100}$$

~~to find the relation between a, b & c~~

~~to find the relation between a, b & c~~

c

12. Ramesh invested amounts in two different schemes, A & B in the ratio 5:4 for 5 years. Scheme A offers 8% pa SI and bonus of 20%. On the interest earned in 5 years, scheme B offers 9% pa SI only. If the amount he invested in scheme A is ₹ 20,000. What is the total amount he will receive after 5 years from both the schemes?

Soln.

$$A : B$$

$$5 : 4$$

$$A = 20000 - 5P$$

$$B = \frac{4}{5}P = 16000$$

A	B
20000	16000
8% × 5	9% × 5
= 40%	= 45%

$$\text{Interest on } \frac{40}{100} \times 20000 = 8000 \quad \text{and on } \frac{45}{100} \times 16000 = 7200$$

$$\text{Bonus on } \frac{20}{100} \times 8000 = 1600$$

$$\text{Amount} = 8000 + 7200 + 1600$$

$$= 16800$$

Find

13. The sum of money that will give ₹ 1 as interest per day 5% pa. simple interest.

$$I = \frac{P \times 1 \times 5}{\frac{5}{100} \times 100} \Rightarrow 7300$$

73

14. Ushai invested ₹ 1000 at 5% pa SI. If interest is added to principal after 10 years, the amount will become ₹ 2000 after how many years?

Soln:-

$$10 \text{ years} \rightarrow P + SI \\ \text{new principal}$$

$$SI = 10 \times 5\% = 50$$

$$= 50\% \text{ of } 1000 = 500$$

$$P = 1500 \xrightarrow{+500} 2000$$

Interest for $\frac{500}{1500} = \frac{1}{3}$ of 10 years \Rightarrow more than
10 years \Rightarrow 5/3 years with new principal of

$$\frac{1}{5\%} \times \frac{100}{3} = \frac{20}{3}$$

$$= 6 \frac{2}{3}$$

$$10 \text{ years} + 6 \frac{2}{3}$$

$$= 16 \frac{2}{3} \text{ years}$$

15. A person lends 40% of his money at 15% pa SI.
50% of the remaining amount at 10% pa and the
remaining amount 18% pa. SI. What is the effective
rate of interest on the whole sum?

Soln:-

$$\begin{array}{ccccccc} & & 100\% & & & & \\ & & \swarrow & & \searrow & & \\ \frac{3}{10} \times 15\% & = & 45\% & & \frac{9}{10} \times 18\% & = & 162\% \\ \frac{4}{10} \times 10\% & = & 40\% & & \frac{5}{10} \times 18\% & = & 90\% \\ \frac{12}{6} = 6\% & & & & \frac{87}{8} = 108.75\% & & \end{array}$$

16. A sum was invested on SI at 6%. Had it been put at 3% higher rate it would have fetched ₹72 more. What is the principal?

Soln:-

$$2 \text{ years } 3\% \rightarrow 72 \text{ more}$$

$$6\% \rightarrow 72/6$$

$$1\% = 12$$

$$100\% = 1200$$

17. A sum of ₹2400 amounts to ₹3264 in 4 years at some rate of interest on SI. If the rate of interest is increased by 1%, the same sum in same time would amount to _____.

Soln:

$$864 = \frac{2400 \times 4 \times r}{100}$$

$$\frac{2400}{864}$$

$$r = \frac{2400}{24 \times 4}, \\ \therefore$$

$$\frac{2400 \times 4 \times 10}{100} = 960$$

$$\begin{array}{r} 2400 \\ + 960 \\ \hline 3360 \end{array}$$

18. Rajesh took a loan of ₹ 20000 from a bank for 3 years with 5%, 7%, 9% rate of interest for consecutive years respectively. What is the total amount he needs to pay at the end of 3 years?

Soln: 5%, 7%, 9%
on P on P

$$\text{Effective ROI} = 21\%$$

$$100\% = 20000$$

$$121\% ?$$

$$= \frac{20000}{100} \times 121$$

$$= 24200/-$$

19. Vignesh borrowed some money at 6% pa for first 3 years & 9% pa for next 5 years and 13% pa for a period beyond 8 years. If the total interest paid by him at the end of 11 years is ₹ 8160. Find the principal borrowed by him.

Soln:-

$$\frac{6}{100} \times 3 \times P + \frac{9}{100} \times 5 \times P + \frac{13}{100} \times 3 \times P = 8160$$

$$\frac{18}{100} P + \frac{45}{100} P + \frac{39}{100} P = 8160$$

$$\frac{P}{100} [18 + 45 + 39] = 8160$$

$$P = \frac{8160 \times 100}{102} = 8000$$

8000

20. The simple interest on a sum of money is $\frac{8}{25}$ times that of the sum. If the number of years is numerically half of the rate of percent paid. Find the rate of interest per annum.

Soln:-

$$SI = \frac{8}{25} P$$

$$T = R/2$$

$$\frac{8}{25}P = \frac{P(R/2)T}{100}$$

$$R^2 = \frac{16}{800}$$

$$R = 4$$

21. ₹ 6000 becomes ₹ 7200 at certain rate of interest. If the rate of interest becomes 1.5 times of itself, find the amount of the same principal in 5 years.

Soln:-

$$SI = 7200 - 6000 = 1200 \text{ per year}$$

$$\Rightarrow 1200 - 4 \text{ years}$$

$$1200 = 300$$

$$R = \frac{300 \times 100}{6000}$$

$$R = 5\%$$

$$R = 5 \times 1.5$$

$$= 7.5\%$$

$$\begin{aligned}
 & 100\% = 6000 \text{ rupees} \\
 & 100\% + 5\% = 105\% \\
 & 105\% = 137.5 \times 60 \\
 & = 8250
 \end{aligned}$$

22. The interest on a certain deposit at 4.5% pa
is ₹ 202.50 in 1 year. How much will the additional
interest in 1 year on the same deposit at 5% pa?

Soln:- 4.5% pa RS - 202.50

$$\frac{SI_1}{SI_2} = \frac{R_1}{R_2} \Rightarrow \frac{202.50}{SI_2} = \frac{4.5}{5}$$

$$SI_2 = \frac{202.50 \times 5}{4.5} = 225$$

$$\text{Additional } = SI_2 - SI_1$$

$$= 225 - 202.50$$

$$= 22.50$$

23. A person deposited ₹ 400 for 2 yrs & ₹ 550 for 4 yrs
₹ 1200 for 6 yrs. He received simple interest of
₹ 1020. Find the rate of interest per annum?

$$1020 = \frac{400 \times 2 \times R}{100} + \frac{550 \times 4 \times R}{100} + \frac{1200 \times 6 \times R}{100}$$

$$1020 = 8R + 22R + 72R$$

$$102R = 1020$$

$$R = 100\%$$

24. A sum of money at simple interest amounts to ₹ 815 in 3 years and to ₹ 854 in 4 years. Find the principal.

Soln:-

$$(39 \rightarrow 1 \text{ yr}) \\ 815 \longrightarrow 854$$

Diff. w.r.t. the interest period is $39 \times 4 = 156$
 Consider unit period as 1 yr.
 $3 \text{ yr} = 3 \times 39 \\ = 117 \text{ rs}$

$$A = 815$$

$$P + SI = 815$$

$$P + 117 = 815$$

$$\boxed{P = 698 \text{ rs}}$$

25. A sum of money amounts to ₹ 5200 in 5 years and to ₹ 5680 in 7 years at simple interest. Find the rate of interest p.a?

Soln:-

$$480 - 2 \text{ yr} \\ 5680 \rightarrow 5200$$

$$2 \text{ yrs} \rightarrow 480$$

$$480 \text{ per year} = 240 \text{ per year} \\ 5 \text{ yrs} = 240 \times 5 \\ 1200 = 5680 - 5200$$

$$5680 - 1200 = 4480$$

$$R = \frac{1200}{4480} \times 100$$

$$130\% = 5 \text{ yrs}$$

$$\boxed{1 \text{ yr} = 6 \cdot 1 \text{ yrs}}$$

26. A sum of money doubles in 12 years at SI. In how many years it will become 3 times?

Soln:-

$$P \rightarrow x$$

$$SI = \frac{PTR}{100}$$

$$SI \propto T$$

$$SI_1 = x$$

$$T_1 = 12 \text{ yrs}$$

$$T_2 = ?$$

$$SI_2 = 2x$$

$$\frac{SI_1}{SI_2} = \frac{T_1}{T_2}$$

$$\frac{x}{2x} = \frac{12}{T_2}$$

or ~~double at 200% p.a. to double again after 12 years~~
~~so~~ $T_2 = 24$

27. If a sum of money becomes 21 times of itself in 100 years, in how many years it becomes 13 times of itself?

Soln:-

$$21 \text{ times} \rightarrow 100 \text{ yrs}$$

$$20P \rightarrow 100 \text{ yrs} \quad 1P = 5 \text{ yrs}$$

$$12P = 12 \times 5$$

therefore with 1P takes 5 years to grow. When
 12P will take 12 times 5 years
 i.e. 60 years to grow 13 times.

23. In how many years will a sum of money double itself at 18.75% pa simple interest?

Soln:-

$$SI = P$$

$$\frac{P \times T \times 18.75}{100} \times P$$

$$T = \frac{100}{18.75}$$

$$T = \frac{10000}{1875}$$

$$T = 5 \text{ years } 4 \text{ months}$$

24. If the simple interest of 6 years is equal to 30% of principal. Then in how many years it become equal.

~~If P is constant then P/6 is constant because if P/6 is constant then P/12 is constant which will give 6 years = 30%.~~

$$1 \text{ year} = 5\% \times 20$$

$$100\% = 20 \text{ years}$$

$$20 \text{ years}$$

25. If the ratio of principal and amount in the current year at simple interest SI is 9:17. After 22 years the ratio will become 15:43. Find the rate of interest pa.

Soln:-

$$P:A = 9:17$$

After 22 yrs

$$P:A = 15:43$$

$$\begin{matrix} 9 & : & 17 \\ \swarrow & & \searrow \\ 15 & : & 43 \end{matrix}$$

$$P \quad SI \quad A$$

$$\begin{aligned}
 & \text{P} \rightarrow 9 \text{ (part 8)} \quad P \rightarrow 17 \text{ (part 15)} \text{ invested in different years} \\
 & \text{SI} \rightarrow 15 \text{ part 28} \cdot 15 \times 9 \text{ to make } \frac{3}{4} \text{ of P} \\
 & \text{part. 12} \rightarrow 12 \text{ part 28} \rightarrow 12 \text{ part 9} \text{ to make } \frac{9}{4} \text{ of P} \\
 & \text{P} \rightarrow 135 \text{ part 28} \text{ and SI } \rightarrow 132 \text{ part 28} \\
 & \text{P} \rightarrow 135 \text{ and SI } \rightarrow 132 \text{ and } T = 22 \text{ yrs}
 \end{aligned}$$

$$T=22 \quad SI = 132 \quad P = 135$$

$$132 = \frac{135 \times 22 \times R}{100}$$

$$R = \frac{132 \times 100}{135 \times 22}$$

$$\frac{21}{9} \quad \frac{10}{10}$$

$$= \frac{40}{9}$$

Q1. Ramesh invested ₹ 7500 at 11% pa SI. He further invested some amount at 15% pa SI. The total interest earned at the end of the year 12% pa. Find the amount he invested at the rate of 15% pa.

Soln:- 7500 — 11% pa

₹ — 15% pa

$$7500 + x = 12\% \text{ pa}$$

$$\frac{7500 \times 11 \times 11}{100} + \frac{2x \times 15 \times 1}{100} = \frac{(7000+2x) \times 12 \times 1}{100}$$

$$7500 \times 11 + 15x = 12 \times 7500 + 12x$$

$$2x = 7500$$

$$2x = 2500$$

Q2. A man invests a certain sum of money at 6% pa SI. and another sum at 7% pa SI. His income after 2 years is ₹ 354 and $\frac{1}{4}$ th of 1st sum = $\frac{1}{5}$ th of 2nd sum. Find the total sum invested in both.

Soln:-