

Binary Search Trees: Balance

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Data Structures
Data Structures and Algorithms

Learning Objectives

- Think about the runtime of basic binary tree operations.
- Understand the motivation behind binary search tree balance.
- Implement a rotation.

Outline

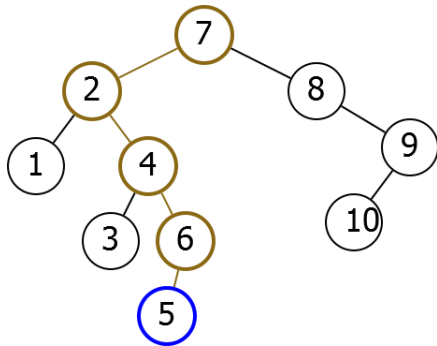
- 1 Runtime
- 2 Balanced Trees
- 3 Rotations

Runtime

How long do Binary Search Tree operations take?

Find

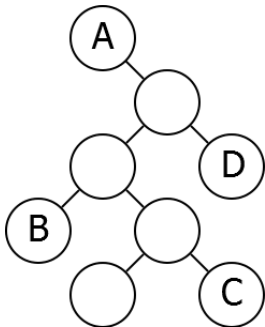
Find(5)



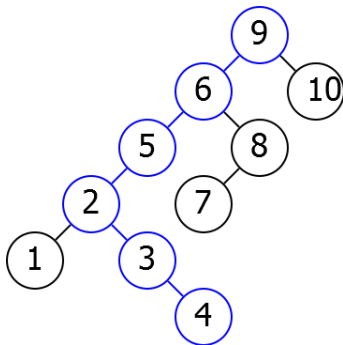
Number of operations = $O(\text{Depth})$

Problem

Which nodes will be faster to search for in the following tree?



Example I

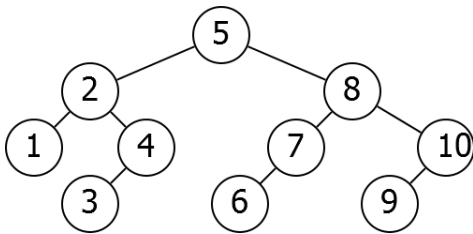


Depth can be as bad as n .

Outline

- ① Runtime
- ② **Balanced Trees**
- ③ Rotations

Example II



Depth can be much smaller.

Balance

- Want left and right subtrees to have approximately the same size.

Balance

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- Suppose perfectly balanced:

Balance

- Want left and right subtrees to have approximately the same size.
- Suppose perfectly balanced:
 - Each subtree half the size of its parent.
 - After $\log_2(n)$ levels, subtree of size 1.
 - Operations run in $O(\log(n))$ time.

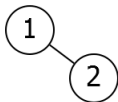
Problem

Insertions and deletions can destroy balance!

1

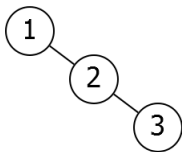
Problem

Insertions and deletions can destroy balance!



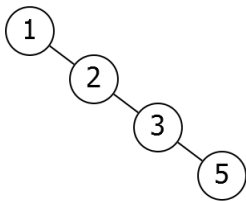
Problem

Insertions and deletions can destroy balance!



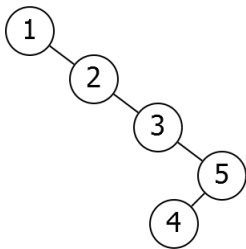
Problem

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Outline

- ① Runtime
- ② Balanced Trees
- ③ Rotations

Rebalancing

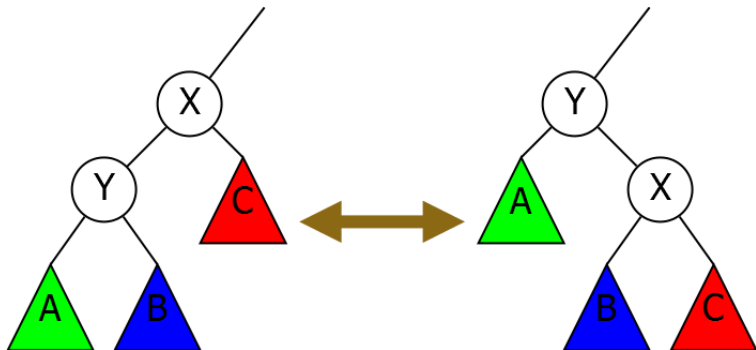
Idea: Rearrange tree to maintain balance.

Rebalancing

Idea: Rearrange tree to maintain balance.

Problem: How do we rearrange tree while maintaining order?

Rotations



$$A < Y < B < X < C$$

Implementation

RotateRight(X)

$P \leftarrow X.\text{Parent}$

$Y \leftarrow X.\text{Left}$

$B \leftarrow Y.\text{Right}$

$Y.\text{Parent} \leftarrow P$

$P.\text{AppropriateChild} \leftarrow Y$

$X.\text{Parent} \leftarrow Y, Y.\text{Right} \leftarrow X$

$B.\text{Parent} \leftarrow X, X.\text{Left} \leftarrow B$

Next Time

How to keep a tree balanced. AVL trees.