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057504 - ADVANCED METHODS FOR THE OPTIMAL MANAGEMENT OF THE ELECTRICAL GRID

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Case study 1 – Manual Frequency Restoration Reserve service via Prosumers Aggregation

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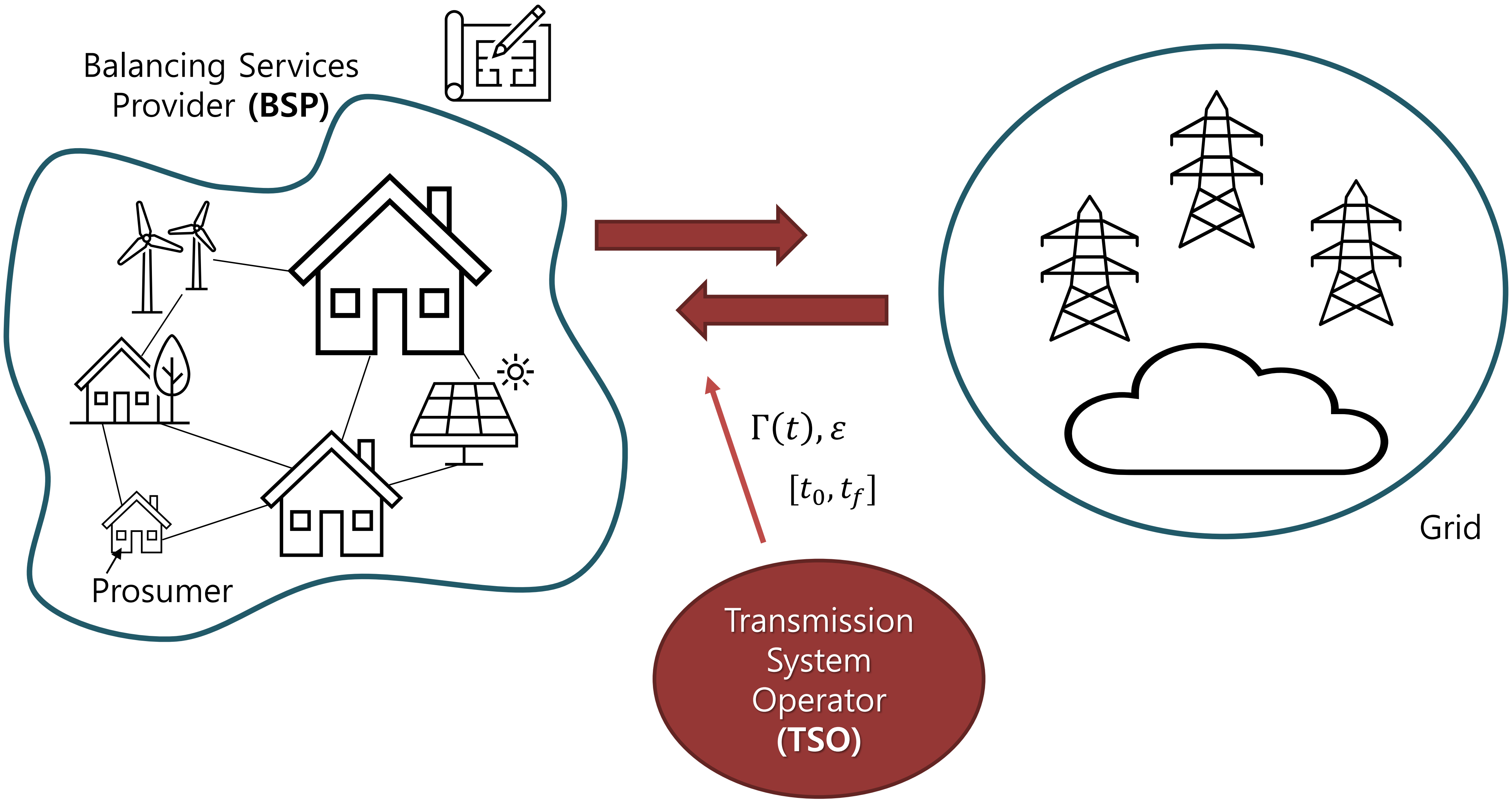
CASE STUDY DESCRIPTION

BALANCING SERVICES PROVISION VIA PROSUMERS AGGREGATION



$$s(t+1) = A_t s(t) + B_{1t} u(t) + B_{2t} \eta(t) + B_{3t} z(t)$$

$$y(t) = C_t s(t) + D_{1t}(t) + D_{2t} \eta(t) + D_{3t} z(t)$$

$$E_{2t} \eta(t) + E_{3t} z(t) \leq E_{1t} u(t) + E_{4t} s(t) + E_{5t}$$

$s = \begin{bmatrix} s_c \\ s_l \end{bmatrix}$, $s_c \in \mathbb{R}^{n_c}$, $s_l \in \{0,1\}^{n_l}$ $n = n_c + n_l$ **State**

$y = \begin{bmatrix} y_c \\ y_l \end{bmatrix}$, $y_c \in \mathbb{R}^{n_c}$, $y_l \in \{0,1\}^{n_l}$ $\mathrm{p} = p_c + p_l$ **Output**

$u = \begin{bmatrix} u_c \\ u_l \end{bmatrix}$, $u_c \in \mathbb{R}^{n_c}$, $u_l \in \{0,1\}^{n_l}$ $m = m_c + m_l$ **Input**

$\eta \in \{0,1\}^{r_l}$, $\mathrm{z} \in \{0,1\}^{r_c}$ **Auxiliary Variables**

Mixed-Integer LP

$$\min_{x} \; c^\top x$$

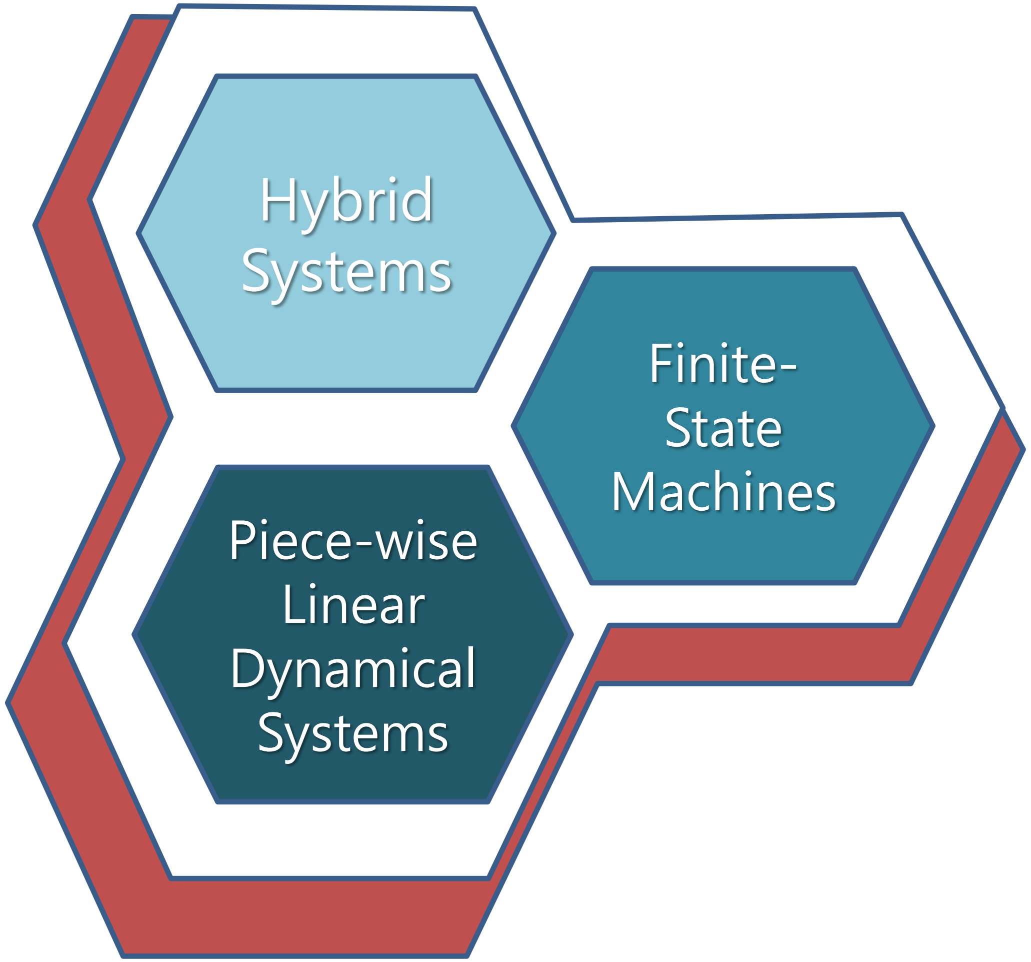
subcject to: $\quad Ax \le b$

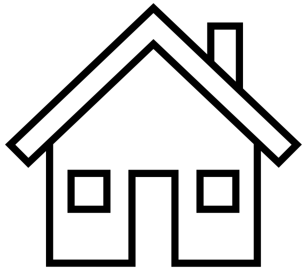
$$x \in \mathbb{R}^{n_c} \times \mathbb{Z}^{n_d}$$

continuous

discrete

Mixed Logical-Dynamical Systems

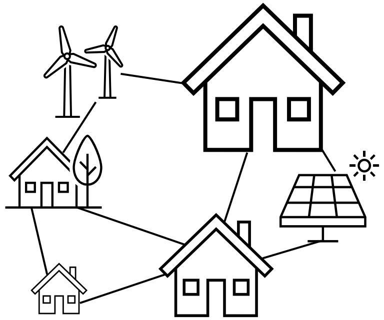




Prosumer

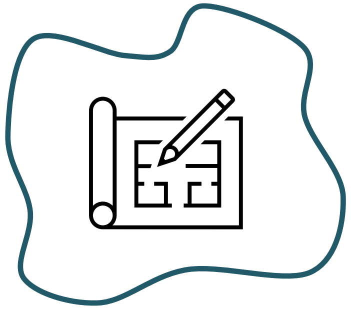
- Controllable generator G : $P_i^G > 0$
- Programmable load L : $P_i^L < 0$

  Assumed to work on n_i^L levels
- Battery Storage Device B : $P_i^B \lesseqgtr 0$
- Reference daily profile $\tilde{P}_i$



Pool

- m prosumers
- Power exchanged with the grid $P = \sum_{i=1}^{m}(P_i^G + P_i^B + P_i^L)$



BSP

One-day time horizon → M slots of duration τ_s

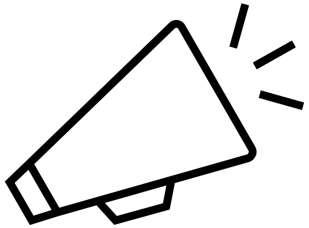
- t → $(t\tau_s, (t+1)\tau_s)$

  $t \in \{0, \dots, M-1\}$

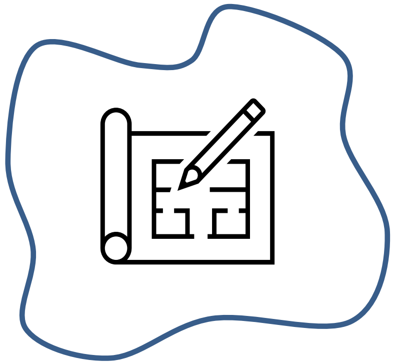
TSO

Variation of the power profile: $\Gamma(t)(1 \pm \varepsilon)$ $\varepsilon \in (0,1)$

In the time-interval $t_0, \dots, t_f$



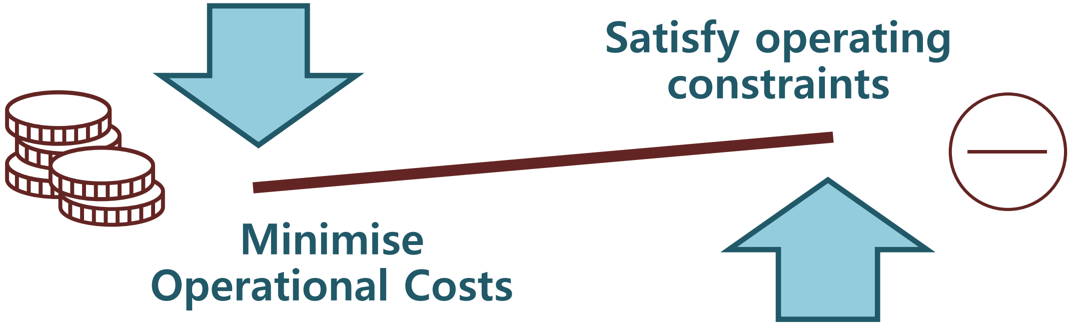
BSP



Re-distributes the request among all prosumers

$$\Gamma(t)(1-\varepsilon) \leq P(t) - \tilde{P}(t) \leq \Gamma(t)(1+\varepsilon) \qquad \forall t \in [t_0, t_f]$$

$$P_i(t) = \tilde{P}_i \qquad \forall t \in [t_f + 1, M-1]$$



MLD model

Variables (of the i^{th} prosumer)

$P_i^G > 0$

$\delta_i^G \in \{0,1\}$

$P_i^B \lessgtr 0$

$S_i(t)$

$P_i^L \in \left\{0,\ \frac{\bar{P}_i^L}{n_i^L},\ 2\frac{\bar{P}_i^L}{n_i^L}\ \dots\ \bar{P}_i^L\right\}$

Assumption:

$n_i^L = 2^{J_i^L} - 1\ , \quad J_i^L \in \mathbb{N}$

$\delta_{i,j}^L(t) \in \{0,1\} \qquad j = 1, \dots, J_i^L$

$$P_i^L = \sum_{j=1}^{J_i^L} \left(2_i^{j-1} \bar{P}_i^L \cdot \delta_{i,j}^L(t)\right) = \sigma^\top \cdot \delta_i^L(t)$$

$$\delta_i^L(t) = \begin{bmatrix} \delta_{i,1}^L \\ \vdots \\ \delta_{i,J_i^L}^L \end{bmatrix}$$

MLD model

Variables (of the i^{th} prosumer)

$P_i^G > 0$

$\delta_i^G \in \{0,1\}$

$P_i^B \lesseqgtr 0$

$S_i(t)$

$P_i^L \in \left\{ 0, \frac{\bar{P}_i^L}{n_i^L}, 2\frac{\bar{P}_i^L}{n_i^L} \ \dots \ \bar{P}_i^L \right\}$

$\delta_i^L(t) \in \{0,1\}^{J_i^L}$

State Vector

$$s(t) = \begin{bmatrix} S_1(t) \\ \vdots \\ S_m(t) \end{bmatrix}$$

Input Vector

$$u(t) = \begin{bmatrix} u_1(t) \\ \vdots \\ u_m(t) \end{bmatrix}$$

$$u_i(t) = \begin{bmatrix} u_{i,c}(t) \\ u_{i,d}(t) \end{bmatrix} = \begin{bmatrix} P_i^G(t) \\ P_i^B(t) \\ \delta_i^G(t) \\ \delta_i^L(t) \end{bmatrix}$$

MLD model

Operating Constraints (of the i^{th} prosumer)

Battery Storage Dynamics

$$S_i(t+1) = S_i(0) - \tau_s \sum_{s=0}^{t} P_i^B(s)$$
$$= S_i(t) - \tau_s P_i^B(t)$$

Energy Consumed by L

$$\sum_{t=t_0}^{M-1} \tau_s P_i^L(t) = \sum_{t=t_0}^{M-1} \tau_s \sigma^\top \delta_i^L(t) = E_i^L$$

Min/Max Energy Level

$$\underline{S}_i \leq S_i(t) \leq \overline{S}_i$$

Min/Max Power Produced by G

$$\delta_i^G(t)\underline{P}_i^G \leq P_i^G(t) \leq \delta_i^G(t)\overline{P}_i^G$$

Charging/Discharging rates

$$P_i^{B,c} \leq P_i^B(t) \leq P_i^{B,d}$$

MLD model

Rescheduling Problem Constraints

Flexibility Limitation of L

$$P_i^L(t) = \sigma^\top \delta_i^L(t) = \widetilde{P}_i^L(t)$$

$$t < t_i^{L,0} \vee t > t_i^{L,f}$$

Rebound Effect Avoidance

$$P_i(t) = P_i^G(t) + \sigma^\top \delta_i^L(t) + P_i^B(t) = \widetilde{P}_i(t)$$

$$t = t_f + 1, \dots, M-1$$

Power Variation (TSO request)

$$(1-\varepsilon)\Gamma(t) \leq \sum_{i=1}^{m} \left(P_i^G(t) + \sigma^\top \delta_i^L(t) + P_i^B(t)\right) - \widetilde{P}(t) \leq (1+\varepsilon)\Gamma(t),$$

$$t = t_0, \dots, t_f$$

MILP

Operational Costs

$C_i^G > 0$: Unitary cost of the energy produced by G

$C_i^B > 0$: Unitary cost of the aging of battery B

$C_i^L > 0$: Unitary cost for changes in the programmable load consumption profile

MILP

Operational Costs

! NON-LINEAR

$$J(\cdot) = \sum_{i=1}^{m}\sum_{t=t_0}^{M-1}\left(C_i^G P_i^G(t) + C_i^B \boxed{\left|P_i^B(t) - P_i^B(t-1)\right|} + C_i^L \boxed{\left|P_i^L(t) - \widetilde{P}_i^L(t)\right|}\right)$$

Re-formulation

$h_i^B(t)$ $h_i^L(t)$ auxiliary variables subject to:

$$\begin{cases} h_i^B(t) = \left|P_i^B(t) - P_i^B(t-1)\right| \\ h_i^L(t) = \left|P_i^L(t) - \widetilde{P}_i^L(t)\right| \end{cases}$$

$$\forall\, t = t_0, \ldots, M-1$$

$$J(\cdot) = \sum_{i=1}^{m}\sum_{t=t_0}^{M-1}\left(C_i^G P_i^G(t) + C_i^B h_i^B(t) + C_i^L h_i^L(t)\right)$$

! NON-LINEAR

! LINEAR

MILP

Re- Formulation

$$J(\cdot) = \sum_{i=1}^{m} \sum_{t=t_0}^{M-1} \left(C_i^G P_i^G(t) + C_i^B h_i^B(t) + C_i^L h_i^L(t)\right)$$

with $h_i^B(t)$ $h_i^L(t)$ subject to:

$$h_i^B(t) = \left|P_i^B(t) - P_i^B(t-1)\right| \qquad \begin{cases} P_i^B(t) - P_i^B(t-1) \leq h_i^B(t) \\ -P_i^B(t) + P_i^B(t-1) \leq h_i^B(t) \end{cases}$$

$$h_i^L(t) = \left|P_i^L(t) - \tilde{P}_i^L(t)\right| \qquad \begin{cases} \sigma^\top \delta_i^L(t) - \tilde{P}_i^L(t) \leq h_i^L(t) \\ -\sigma^\top \delta_i^L(t) + \tilde{P}_i^L(t) \leq h_i^L(t) \end{cases}$$

$$\forall\ t = t_0, \ldots, M-1$$

MILP

Decision Variables

$$x_i^\top = \left[\boxed{u_i(t_0) \quad h_i^B(t_0) \quad h_i^L(t_0)} \quad \cdots \quad u_i(M-1) \quad h_i^B(M-1) \quad h_i^L(M-1)\right]$$

$$\parallel$$

$$\left[P_i^G(t) \quad P_i^B(t) \quad \delta_i^G(t) \quad \delta_i^L(t) \quad h_i^B(t) \quad h_i^L(t)\right]$$

Cost Coefficients

$$J(\cdot) = \sum_{i=1}^{m} \sum_{t=t_0}^{M-1} \left(C_i^G P_i^G(t) + C_i^B h_i^B(t) + C_i^L h_i^L(t) \right)$$

$$\begin{array}{ccc} \left[\underline{P_i^G(t) \quad P_i^B(t) \quad \delta_i^G(t) \quad \delta_i^L(t)} \right. & h_i^B(t) & \left. h_i^L(t)\right] \\ c_i^u = \left[C_i^G \quad 0 \quad 0 \quad 0_{1\times J_i^L}\right] & C_i^B & C_i^L \end{array}$$

$$c_i^\top = \left[c_i^u \quad C_i^b \quad C_i^L \quad \cdots \quad c_i^u \quad C_i^b \quad C_i^L\right]$$

$$\Rightarrow \quad \boxed{J(\cdot) = \sum_{i=1}^{m} c_i^\top x_i}$$

Separable across the agents!

MILP

Local decision vector

$$\min_{x_1,\dots,x_m} \sum_{i=1}^{m} c_i^\top x_i$$

subject to:

Power Variation (TSO request)

Operating Constraints of the i^{th} prosumer

Flexibility Limitation of L

Rebound-Effect Avoidance

Coupling Constraint

$$\sum_{i=1}^{m} A_i x_i \leq b$$

Local Constraints

X_i local constraint set $\forall\ i = 1, \dots, m$

MULTI-AGENT MILPS

m agents

$$\min_{x_1,\ldots,x_m} \sum_{i=1}^{m} c_i^\top x_i$$

Separable cost function

subject to:

$$\sum_{i=1}^{m} A_i x_i \leq b$$

Coupling Constraint

$A_i \in \mathbb{R}^{p \times n_i}$

$b \in \mathbb{R}^p$

$$x_i \in X_i, \; i = 1, \ldots, m,$$

Local Constraints

$$X_i = \{x_i \in \mathbb{R}^{n_{c,i}} \times \mathbb{Z}^{n_{d,i}} : D_i x_i \leq d_i\},$$

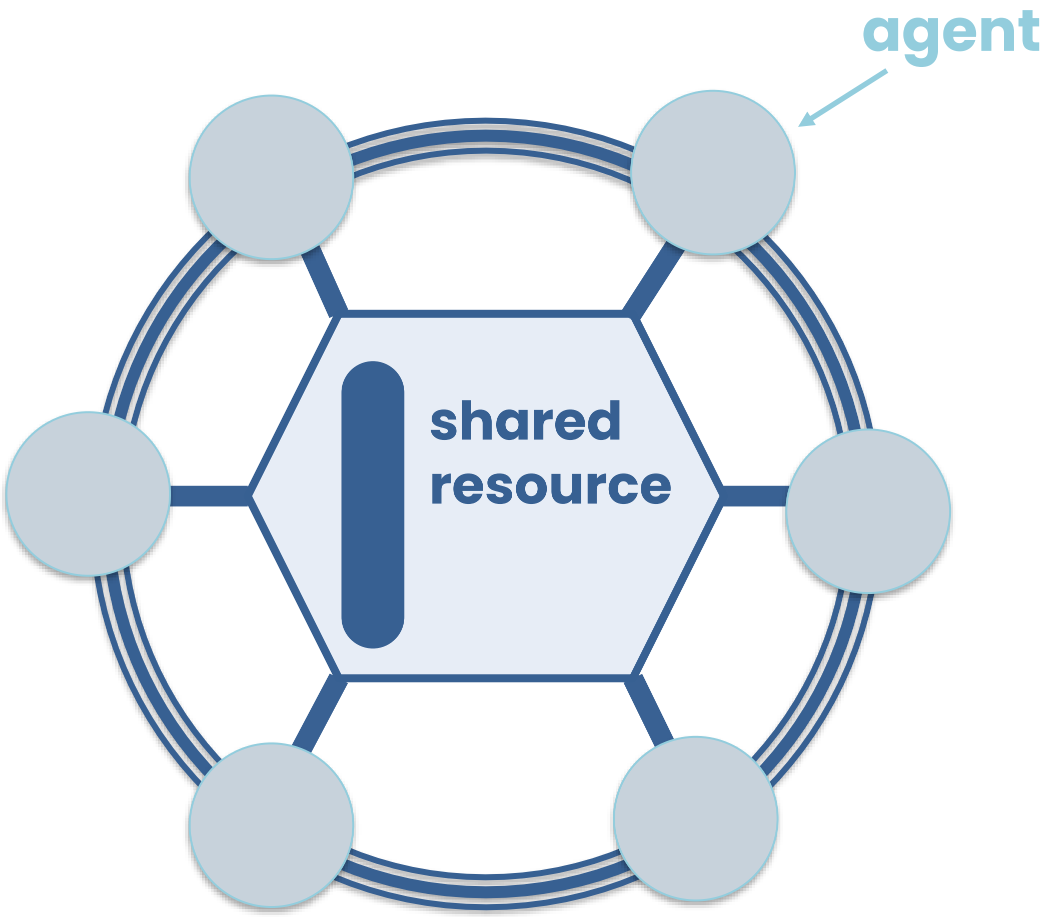
$n_{d,i}$ **Discrete** decision variables

$n_{c,i}$ **Continuous** decision variables

! **Combinatorial Complexity**

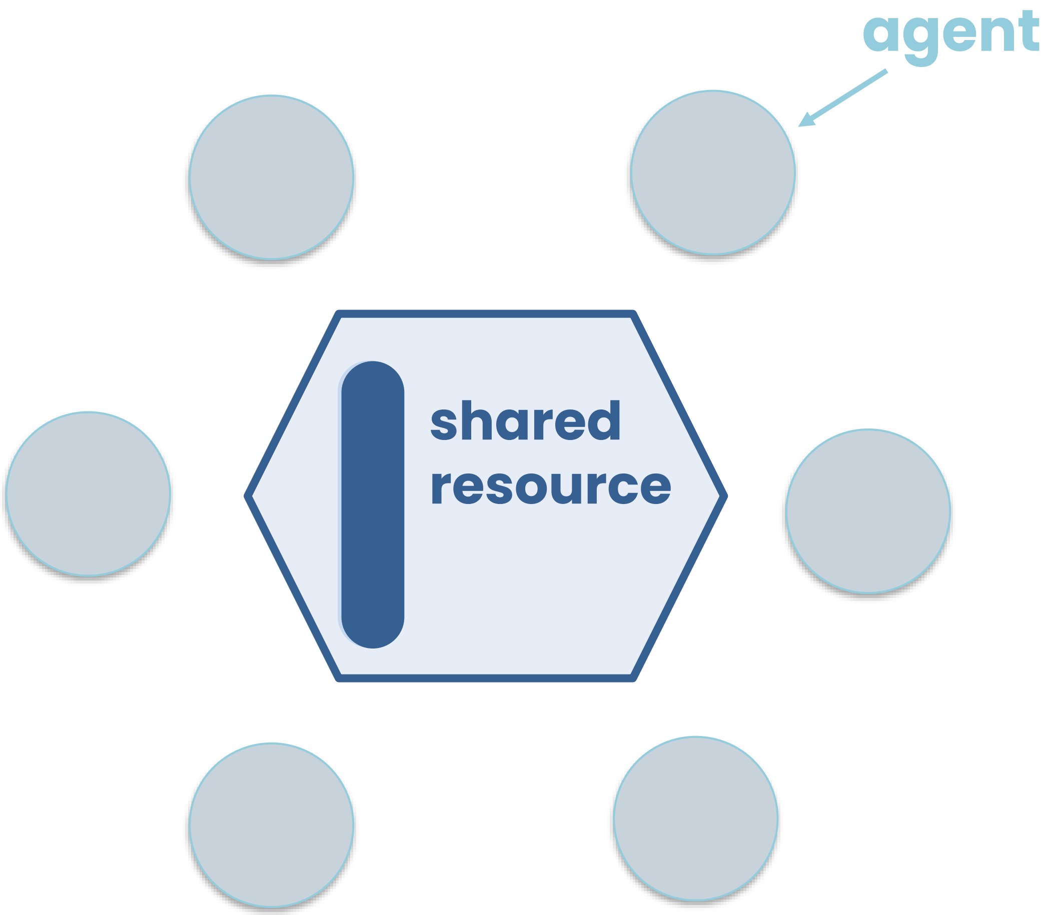
Decentralized iterative resolution scheme combining

Dual decomposition



Decentralized iterative resolution scheme combining

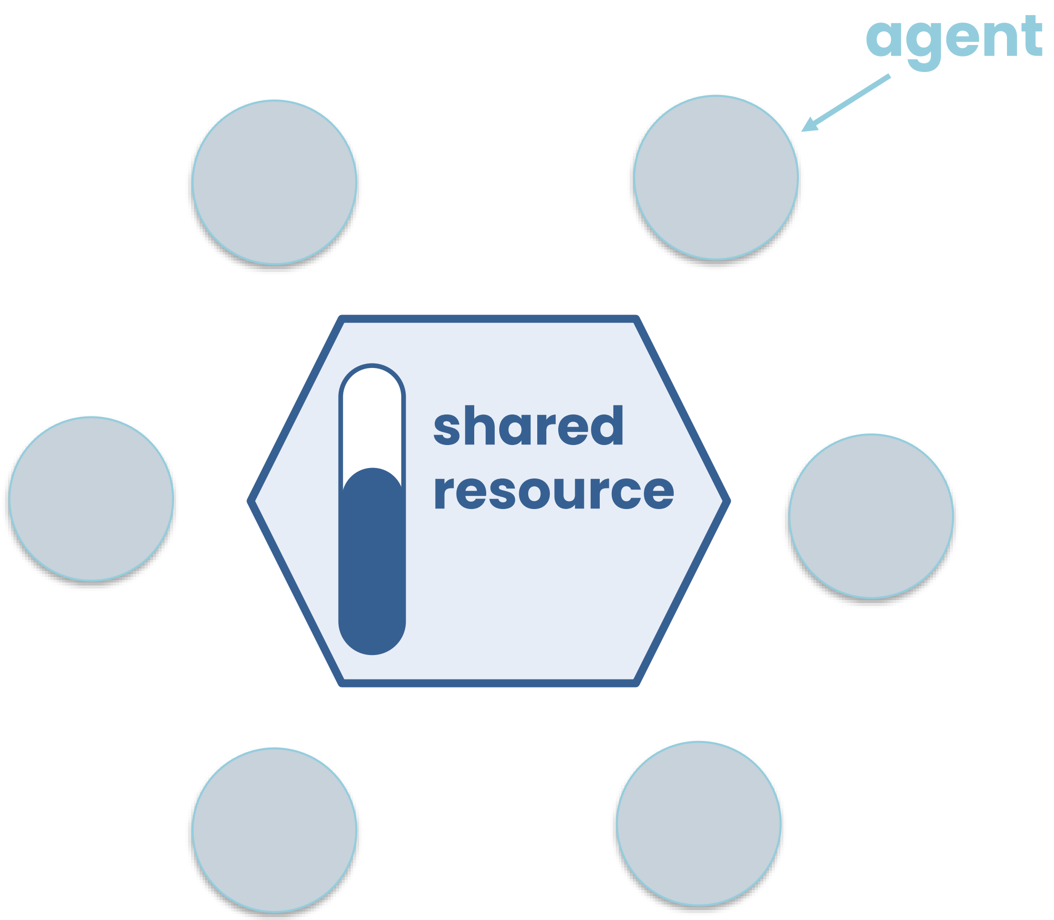
Dual decomposition



Decentralized iterative resolution scheme combining

Dual decomposition

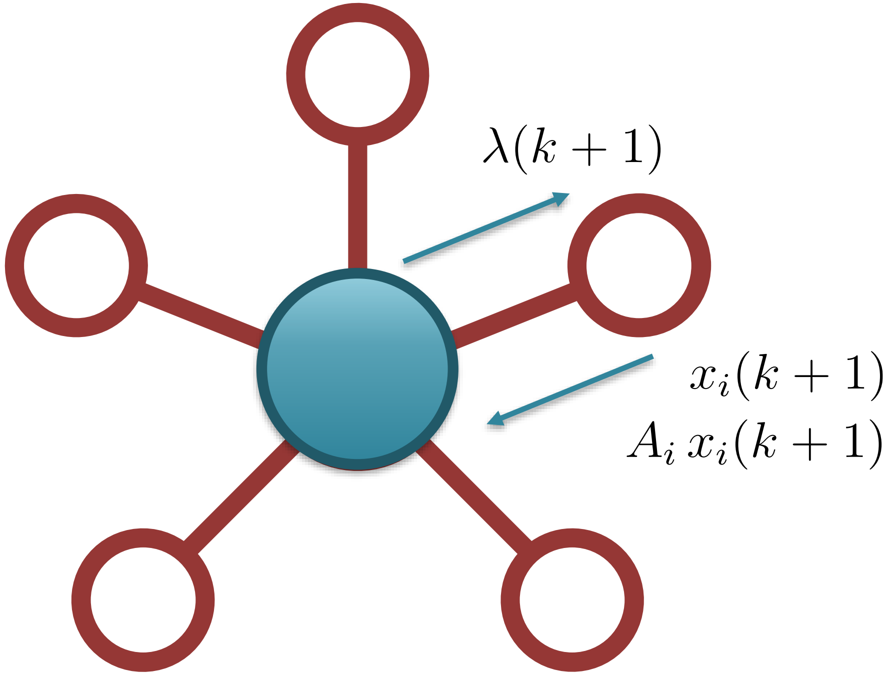
Resource tightening (feasibility)



Adaptive tightening + Lagrangian Duality

Properties

Decentralised scheme



Tightening Update

- Only based on **explored tentative solutions**
- Ensures **convergence** in finite number of iterations
- Provides a **final $\boldsymbol{\rho}$** no worse than the worst-case tightening $\tilde{\rho}$

Reference material

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