Solutions

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Question 1 **(2 + 2 = 4 marks)**

a) Solve the following equation

$9x^2 - 16 = 0$ (2 marks)

$9x^2 = 16$

$= \pm\sqrt{\frac{16}{9}}$ ✓

$= \pm \frac{4}{3}$ ✓

b) Consider the equation $ax^2 - 2x + 3 = 0$

Using the discriminant, determine the value(s) of a if there is just one real solution to the equation. (2 marks)

$\Delta = b^2 - 4ac = 0$ ✓ ← one real solution

$0 = (-2)^2 - 4(a)(3)$

$0 = 4 - 12a$

$12a = 4$

$a = \frac{1}{3}$ ✓

Question 2 (2 + 2 + 4 = 8 marks)

Consider the function $f(x) = x^2 + 6x + 10$

a) Express $f(x)$ in the form $(x + a)^2 + b$, where a and b are integers. (2 marks)

$f(x) = (x + 3)^2 - 9 + 10$

$f(x) = (x + 3)^2 + 1$ ✓ ✓

b) Describe geometrically the transformations which map the graph of x^2 onto the graph of $f(x)$. (2 marks)

Horizontal — 3 unit left ✓

Vertical — 1 unit up. ✓

Question 3 [1 + 3 + 2 + 2 = 8 marks]

A quadratic function is given by the formula $y = x^2 + 5x + \frac{21}{4}$. For the graph of the function:

(a) Determine the equation of the line of symmetry (1 mark)

$$LOS = x = \frac{-b}{2a} = \frac{-5}{2}$$

$$= -2.5$$

(b) Determine the equation of the quadratic function in turning point form, i.e in the form $y = a(x-p)^2 + q$.

(3 marks)

$$y = x^2 + 5x + \frac{21}{4}$$

$$y = \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} + \frac{21}{4}$$

$$y = (x + 2.5)^2 - 1$$

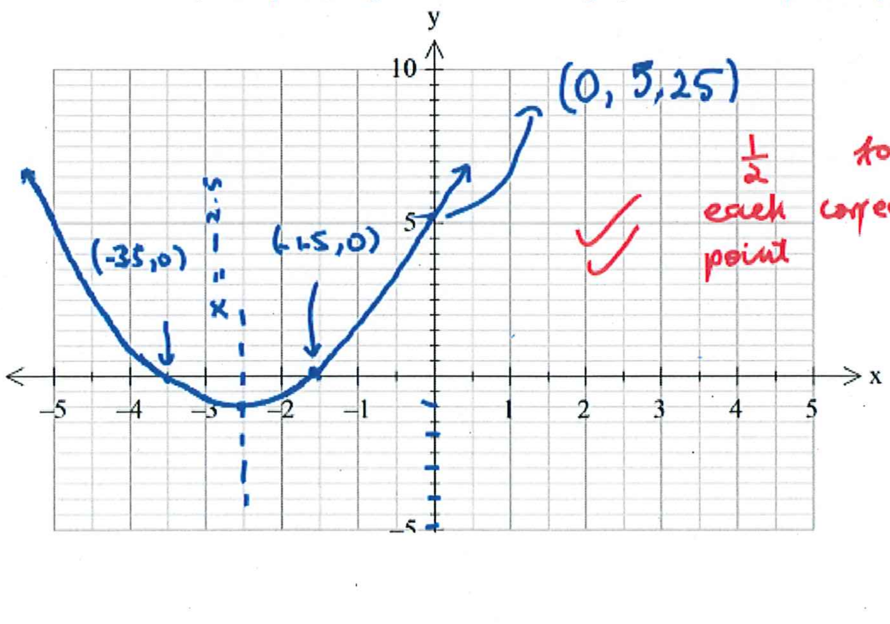
(c) Determine the coordinates of the x intercepts. (2 marks)

$$(x + 2.5)^2 = 1$$

$$x = \pm\sqrt{1} - 2.5$$

$$x = -1.5 \text{ or } -3.5$$

(d) Hence sketch the curve, clearly showing all features of the graph (2 marks)



$\frac{1}{2}$ for each correct point

Question 4 **(3 + 3 = 6 marks)**

(a) Line q is perpendicular to the line with the equation $y = 2x - 3$. Line q has the same x – intercept as the line with the equation $x + 2y = 6$. Determine the equation of line q.

(3 marks)

$y = 2x - 3$

$m = 2$

$x + 2y = 6$

$x + 2(0) = 6$

$x = 6$

line q

$m_q = -\frac{1}{2}$

$0 = -\frac{1}{2}(6) + b$

$b = 3$

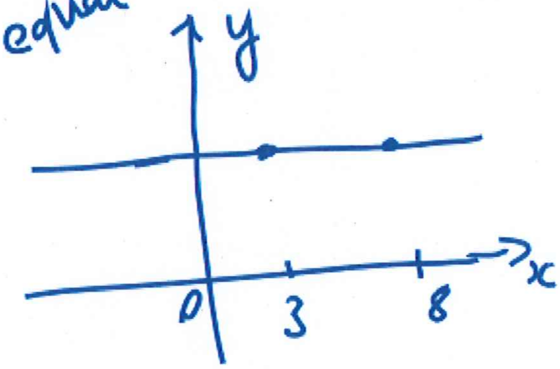
$y = -\frac{1}{2}x + 3$

gradient ✓
x-intercept ✓
rule ✓

(b) For what value(s) of k is the line through the points $(3, 2k+1)$ and $(8, 4k-5)$ parallel to the x – axis.

(3 marks)

$\left.\begin{matrix}(3, 2k+1) \\ (8, 4k-5)\end{matrix}\right]$ y values should be equal



$2k + 1 = 4k - 5$ ✓✓

$2k = 6$

$k = 3$ ✓

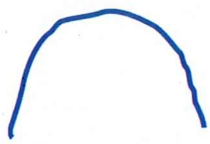
Question 5 (1 + 1 + 1 + 1 + 2 + 4 + 4 = 14 marks)

The manager of a private bus company has worked out a model for the relation between the number of passengers carried each week and the profit, (in tens of \$), the company makes. If n is the number of passengers carried, the profit is given by $P(n) = 2n - 2000 - \left(\frac{n}{100}\right)^2$.

(a) Can the relation $P(n)$ be described as a function. Justify your answer. (1 mark)

Yes — Vertical line test ✓

(b) Describe the concavity of the relation. (1 mark)



as $a = -$ value

Concave down ✓

(c)

(i) Calculate the profit for 1000 people. (1 mark)

$P(1000) = 2(1000) - 2000 - \left(\frac{1000}{100}\right)^2$

$P(1000) = \$ -100 \Rightarrow -\$1000$ ✓

(ii) Calculate the profit for 1100 people. (1 mark)

$P(1100) = 2(1100) - 2000 - \left(\frac{1100}{100}\right)^2$

$= \$79$

$= \$ \ 790$ ✓

(iii) Comment on the profit you determined in (i) and (ii) (2 mark)

1000 people - Loss in part (i) ✓

1100 people - profit in part (ii) ✓

$1000 < n \leq 1100$ — break even point.

(d) Determine the number of passengers needed to be carried each week that would give the maximum profit. State the maximum profit for this number of passengers.

(4 marks)

LOS $x = \dfrac{-2}{2\left(-\frac{1}{10000}\right)}$

$= 10000$ ✓

$P(10000) = 2(10000) - 2000 - \left(\dfrac{10000}{100}\right)^2$

$= 8000$ ✓ ✓

Maximum number of passengers 10000

Maximum profit = \$80000

(e) Determine the simplified expression for P(*n* + 1) – P(*n*), and explain what this expression represents.

(4 marks)

$P(n) = 2n - 2000 - \dfrac{n^2}{10000}$

$P(n+1) = 2(n+1) - 2000 - \dfrac{(n+1)^2}{10000}$ ✓

$\left(\cancel{2n} - \cancel{2000} - \dfrac{n^2}{10000}\right) - \left(\cancel{2n} + 2 - \cancel{2000} - \dfrac{(n^2+2n+1)}{10000}\right)$

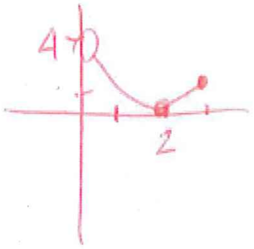
$= -\cancel{\dfrac{n^2}{10000}} - 2 + \cancel{\dfrac{n^2}{10000}} - \dfrac{2n}{10000} - \dfrac{1}{10000}$

$= \dfrac{2000 - 2n - 1}{10000}$ ✓✓

Change in profit when number of passenger increase by 1 ✓

Question 6 (3 marks)

(a) The function $f(x) = (x-2)^2$ has a restricted domain of $\{x: x \in \boldsymbol{R}, 0 < x \le 3\}$. State the range.

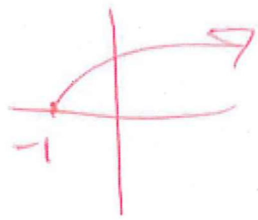


$0 \le f(x) < 4$ ✓ range with inequalities correct

(b) If g(x) = 2^x, state the value of g(3).

$g(3) = 2^3$
$= 8$ ✓ simple substitution

(c) State the range of $y = \sqrt{x+1}$.



$y \ge 0$ ✓ range

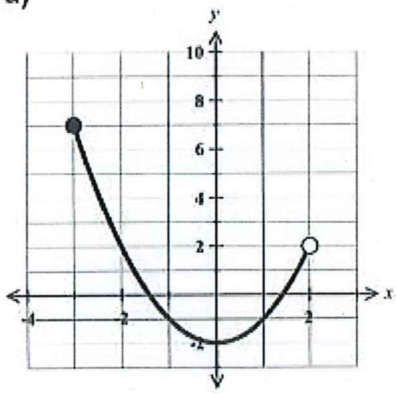
3

Question 7 (3, 3 = 6 marks)

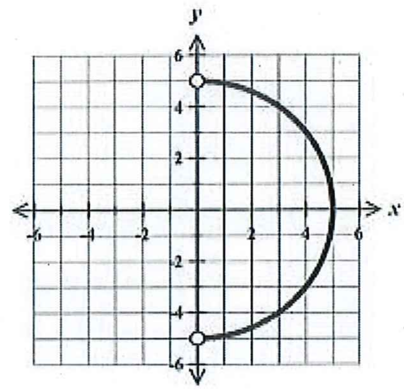
For each of the following, state the domain and range:

6

a)



b)

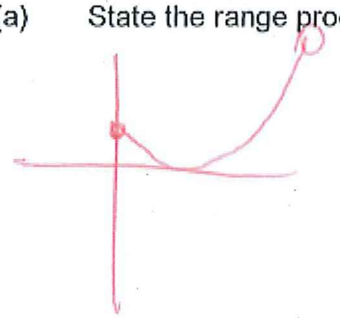


Domain $\{x: x \in R, -3 \le x < 2\}$ ✓
Range $\{y: y \in R, -2 \le y \le 7\}$ ✓
✓ inequalities correct in both

Domain $0 < x \le 5$ ✓
Range $-5 < y < 5$ ✓
✓ inequalities correct in both

Question 8 (2, 2, 2 = 6 marks)

(a) State the range produced when the domain for $f(x) = (x-2)^2$ is restricted to $0 \le x < 6$



$\{f(x): f(x) \in \mathbb{R}, 0 \le f(x) < 16\}$

2

✓ correct range
✓ correct inequalities

(b) If $g(x) = \cos x$, find the range of the function if the domain is $\{0, \frac{\pi}{4}, \pi, \frac{5\pi}{3}\}$

$\{g(x): g(x) \in \mathbb{R}, 1, \frac{\sqrt{2}}{2}, -1, \frac{1}{2}\}$ ✓ range ✓ values

2

(c) Given that: $P:\{(0,8),(1,5),(3,5),(5,7),(6,7)\}$ and $Q:\{(0,8),(1,5),(1,7),(3,7),(5,5)\}$, state which of P or Q is a function. Justify your answer.

P is a function ✓ correct function

2

Q $x = 1$ and $y = 5$ AND $x = 1$ and $y = 7$
(Is one to many)

✓ explain Q is 1 to many

Question 9 (2, 2, 2 = 6 marks)

State the natural domain and the corresponding range for each of the following:

(Hint draw a sketch)

(a) $f(x) = 2x + 7$

$x : x \in \mathbb{R}$ ✓ domain

$f(x) : f(x) \in \mathbb{R}$ ✓ range

(b) $f(x) = \sqrt{x-9}$

$x : x \in \mathbb{R}, x \geq 9$ ✓ domain

$y : y \in \mathbb{R}, y \geq 0$ ✓ range

(c) $f(x) = \dfrac{1}{x-5}$

$x : x \in \mathbb{R}, x \neq 5$ ✓ inequality domain

$f(x) : f(x) \in \mathbb{R}, f(x) \neq 0$ ✓ inequality range

6

Question 10 (2 + 4 = 6 marks)

Functions f and g are defined by $f(x) = 4x^2 - 4x + 5$ and $g(x) = 2x^2 - 8x + 6$.

(a) Determine the discriminant of f and the discriminant of g.

$\Delta f : (-4)^2 - 4(4)5 = -64$

$\Delta g : (-8)^2 - 4(2)(6) = 16$

(b) State, with justification, the roots of both functions f and g.

$\therefore f$ has no roots, $\Delta < 0$

g has 2 roots

$$x = \frac{8 \pm 4}{4}$$

$= 3$ or 1 $\therefore (1,0)$ and $(3,0)$