

# School of Isolated and Distance Education Practice Test 2



## MATHEMATICS METHODS Year 11 Calculator-FREE

### TIME ALLOWED

Working time: 40 minutes

Marks allocation: 36 marks

### PERMISSIBLE ITEMS

*Standard Items:* Pens, pencils, pencil sharpener, highlighter, eraser, ruler

*Special Items:* None

**STANDARD FORMULAE SHEET IS PROVIDED. NO OTHER ITEMS MAY BE USED.**

### INSTRUCTIONS FOR CANDIDATES

All work must be done in the space provided. Should you need extra working area you may use the blank pages at the end. Show all your working clearly. Incorrect answers given without supporting reasoning cannot be awarded marks.

Student's name

Ugish

SIDE Teacher's name

### SUPERVISOR'S DECLARATION

I declare that this test paper has been completed without assistance by the student named above. The time and resource restrictions have been observed and the student has NOT accessed notes, texts, reference books, the internet, a computer, a mobile phone or other electronic devices. I understand that this paper will not be counted for assessment if these conditions have not been met and that notifications will occur.

Supervisors name

Supervisor's signature

Date

**Question 1 (2 + 2 = 4 marks)**

- a) Solve the following equation

$$9x^2 - 16 = 0$$

(2 marks)

$$\begin{aligned} 9x^2 - 16 &= 0 \\ 9x^2 &= 16 \\ \sqrt{9x^2} &= \sqrt{\frac{16}{9}} \\ x &= \pm \sqrt{\frac{16}{9}} \\ &= \pm \frac{4}{3} \end{aligned}$$

- b) Consider the equation  $ax^2 - 2x + 3 = 0$

Using the discriminant, determine the value(s) of  $a$  if there is just one real solution to the equation.

(2 marks)

$$\begin{aligned} \Delta &= b^2 - 4ac = 0 \leftarrow \text{one real solution} \\ \Delta &= (-2)^2 - 4a(3) \\ &= 4 - 12a \\ 4 &= 12a \\ a &= \frac{1}{3} \end{aligned}$$

**Question 2 (2 + 2 = 4 marks)**

Consider the function  $f(x) = x^2 + 6x + 10$

- a) Express  $f(x)$  in the form  $(x + a)^2 + b$ , where  $a$  and  $b$  are integers.

(2 marks)

$$\begin{aligned} f(x) &= x^2 + 6x + 10 \\ &= (x + 3)^2 - 9 + 10 \end{aligned}$$

$$f(x) = (x + 3)^2 + 1$$

- b) Describe geometrically the transformations which map the graph of  $x^2$  onto the graph of  $f(x)$ .

(2 marks)

horizontally - 3 units left  
vertically - 1 unit up

**Question 3 [1 + 3 + 2 + 2 = 8 marks]**

A quadratic function is given by the formula  $y = x^2 + 5x + \frac{21}{4}$ . For the graph of the function:

- (a) Determine the equation of the line of symmetry (1 mark)

$$\frac{-b}{2a} = \frac{-5}{2} = -2.5$$

- (b) Determine the equation of the quadratic function in turning point form, i.e. in the form  $y = a(x - p)^2 + q$ . (3 marks)

$$y = x^2 + 5x + \frac{21}{4} = 0$$

$$(x + 2.5)^2 - 6.25 + 5.25 = 0$$

$$y = (x + 2.5)^2 - 1$$

$$\begin{array}{r} 2.5 \\ \times 2.5 \\ \hline 12.5 \\ 50.0 \\ \hline 62.5 \end{array}$$

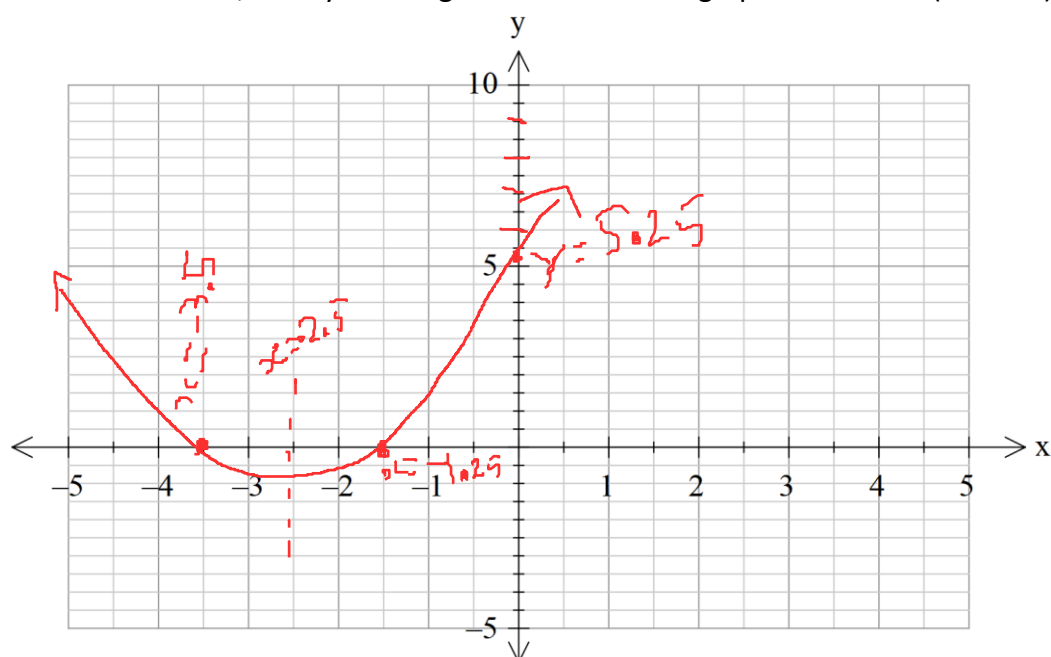
- (c) Determine the coordinates of the x intercepts. (2 marks)

$$\sqrt{(x + 2.5)^2 - 1} = 0$$

$$x + 2.5 = \pm 1$$

$$x = -1.5 \text{ or } x = -3.5$$

- (d) Hence sketch the curve, clearly showing all features of the graph (2 marks)



**Question 4 (3 + 3 = 6 marks)**

- (a) Line  $q$  is perpendicular to the line with the equation  $y = 2x - 3$ . Line  $q$  has the same  $x$ -intercept as the line with the equation  $x + 2y = 6$ . Determine the equation of line  $q$ .

(3 marks)

$$\frac{2}{1}x - \frac{1}{2} = -1 \quad \text{q - } y = -\frac{1}{2}x + c$$

$$2x + 2y = 6$$

$$x + 2(0) = 6$$

$$x = 6$$

$$(6, 0)$$

$$y = -\frac{1}{2}x + c$$

$$0 = -\frac{1}{2}(6) + c$$

$$0 + 3 = -3 + c$$

$$c = 3$$

$\therefore$  2 equations is  $y = -\frac{1}{2}x + 3$

- (b) For what value(s) of  $k$  is the line through the points  $(3, 2k+1)$  and  $(8, 4k-5)$  parallel to the  $x$ -axis. (3 marks)

(y values should be equal)

$$2k+1 = 4k-5$$

$$\frac{2k}{2} = \frac{6}{2}$$

$$k = 3$$

**Question 5 (1 + 1 + 1 + 1 + 2 + 4 + 4 = 14 marks)**

The manager of a private bus company has worked out a model for the relation between the number of passengers carried each week and the profit, (in tens of \$), the company makes. If  $n$

is the number of passengers carried, the profit is given by  $P(n) = 2n - 2000 - \left(\frac{n}{100}\right)^2$ .

- (a) Can the relation  $P(n)$  be described as a function. Justify your answer. (1 mark)

yes, passes vertical line test

- (b) Describe the concavity of the relation. (1 mark)

negative concavity - concave down

- (c)

- (i) Calculate the profit for 1000 people. (1 mark)

$$\begin{aligned} 2n - 2000 - \left(\frac{n}{100}\right)^2 \\ 2(1000) - 2000 - \left(\frac{1000}{100}\right)^2 \\ = -\$100 \Rightarrow -\$1000 \end{aligned}$$

- (ii) Calculate the profit for 1100 people. (1 mark)

$$\begin{aligned} 2(1100) - 2000 - \left(\frac{1100}{100}\right)^2 \\ 2200 - 2000 - 121 \\ = 79 \Rightarrow \$790 \end{aligned}$$

- (iii) Comment on the profit you determined in (i) and (ii) (2 mark)

1000 people makes a loss in part (i)  
1100 people make a profit in part (ii)

- (d) Determine the number of passengers needed to be carried each week that would give the maximum profit. State the maximum profit for this number of passengers.

$$\begin{aligned}
 & 2n - 2000 - \left(\frac{n}{100}\right)^2 \\
 & - \left(\frac{n}{100}\right)^2 + 2n - 2000 \\
 & = \left(\left(\frac{n}{100}\right)^2 - 2n + 2000\right) \\
 & \left(\left(\frac{n}{100}\right)^2\right)
 \end{aligned}$$

(4 marks)

- (e) Determine the simplified expression for  $P(n+1) - P(n)$ , and explain what this expression represents.

(4 marks)

# School of Isolated and Distance Education

## Practice Test 2

### MATHEMATICS METHODS Year 11



#### Calculator Allowed

#### TIME ALLOWED

Working time: 35 minutes

Marks allocation: 28 marks

#### PERMISSIBLE ITEMS

*Standard Items:* Pens, pencils, pencil sharpener, highlighter, eraser, ruler

*Special Items:* drawing instruments, templates, **ONE** A4 page of notes printed one side only, up to three calculators – CAS, graphic or scientific.

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#### INSTRUCTIONS FOR CANDIDATES

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Student's name \_\_\_\_\_

SIDE Teacher's name \_\_\_\_\_

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Supervisors name \_\_\_\_\_

Supervisor's signature \_\_\_\_\_ Date \_\_\_\_\_

**Question 6 (3 marks)**

(a) The function  $f(x) = (x - 2)^2$  has a restricted domain of  $\{x: x \in \mathbf{R}, 0 < x \leq 3\}$ . State the range.

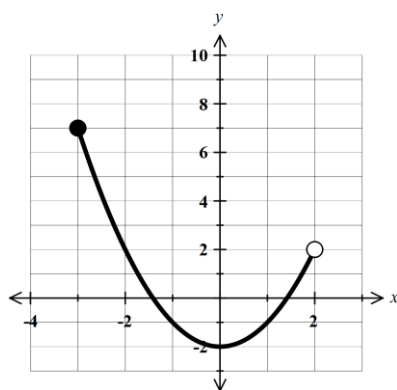
(b) If  $g(x) = 2^x$ , state the value of  $g(3)$ .

(c) State the range of  $y = \sqrt{x + 1}$ .

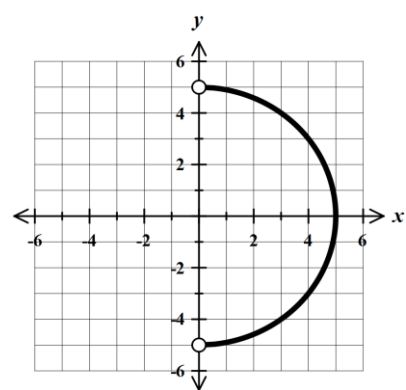
**Question 7 (3, 3 = 6 marks)**

For each of the following, state the domain and range:

a)



b)





**Question 8 (2, 2, 2 = 6 marks)**

(a) State the range produced when the domain for  $f(x) = (x-2)^2$  is restricted to  $0 \leq x < 6$

(b) If  $g(x) = \cos x$ , find the range of the function if the domain is  $\{0, \frac{\pi}{4}, \pi, \frac{5\pi}{3}\}$

(c) Given that:  $P: \{(0,8), (1,5), (3,5), (5,7), (6,7)\}$  and  $Q: \{(0,8), (1,5), (1,7), (3,7), (5,5)\}$ , state which of P or Q is a function. Justify your answer.

**Question 9 (2, 2, 2 = 6 marks)**

State the natural domain and the corresponding range for each of the following:

(Hint draw a sketch)

(a)  $f(x) = 2x + 7$

(b)  $f(x) = \sqrt{x - 9}$

(c)  $f(x) = \frac{1}{x - 5}$

**Question 10 (2 + 4 = 6 marks)**

Functions  $f$  and  $g$  are defined by  $f(x) = 4x^2 - 4x + 5$  and  $g(x) = 2x^2 - 8x + 6$ .

(a) Determine the discriminant of  $f$  and the discriminant of  $g$ .

(b) State, with justification, the roots of both functions  $f$  and  $g$ .