Mathematics Methods Unit 1: Chapter 7

Ex 7A.

1. Find y intercept

$x = 0$

a) $y = x^3 + x^2 + x + 1$

$y = 0^3 + 0^2 + 0 + 1$

$y = 1$

ie $(0,1)$ ✓

b) $y = 3x^2 - 5x^2 - 2x - 5$

$y = 3(0)^2 - 5(0)^2 - 2(0) - 5$

$y = -5$

ie $(0,-5)$ ✓

c) $y = x^3 + 8$

$y = 0^3 + 8$

$y = 8$

ie $(0,8)$ ✓

d) $y = 2x^3 + 3x^2 + 6$

$y = 2(0)^3 + 3(0)^2 + 6$

$y = 6$

ie $(0,6)$ ✓

e) $y = 2 + 3x + 7x^2 - x^3$

$y = 2 + 3(0) + 7(0)^2 - (0)^3$

$y = 2$

ie $(0,2)$ ✓

f) $y = 5x + 3 + 2x^3$

$y = 5(0) + 3 + 2(0)^3$

$y = 3$

ie $(0,3)$ ✓

2. Find roots (x intercepts)

* use null factor law

& make brackets $= 0$

a) $y = (x-2)(x-3)(x-4)$

$x = 2 \quad x = 3 \quad x = 4$

ie $(2,0)$, $(3,0)$ and $(4,0)$ ✓

b) $y = (x+7)(x-1)(x-5)$

$x = -7 \quad x = 1 \quad x = 5$

ie $(-7,0)$, $(1,0)$ and $(5,0)$ ✓

c) $y = (2x-5)(x+1)(5x-3)$

$x = 5/2 \quad x = -1 \quad x = 3/5$

ie $(5/2, 0)$, $(-1,0)$ and $(3/5, 0)$ ✓

d) $y = (1-x)(1+x)(x-7)$

$x = 1 \quad x = -1 \quad x = 7$

ie $(1,0)$, $(-1,0)$ and $(7,0)$ ✓

e) $y = x(4x-1)(2x-7)$

$x = 0$, $x = 1/4$, $x = 7/2$

ie $(0,0)$, $(1/4, 0)$ and $(7/2, 0)$ ✓

f) $y = (x+1)^2(x-5)$

$x = -1 \quad x = 5$

$(-1,0)$ and $(5,0)$ ✓

g) $y = x^3 - 9x$

$y = x(x^2 - 9)$

$y = x(x-3)(x+3)$

$x = 0$, $x = 3$ and $x = -3$

$(0,0)$, $(3,0)$ and $(-3,0)$ ✓

h) $y = x^3 + 2x^2 - 15x$

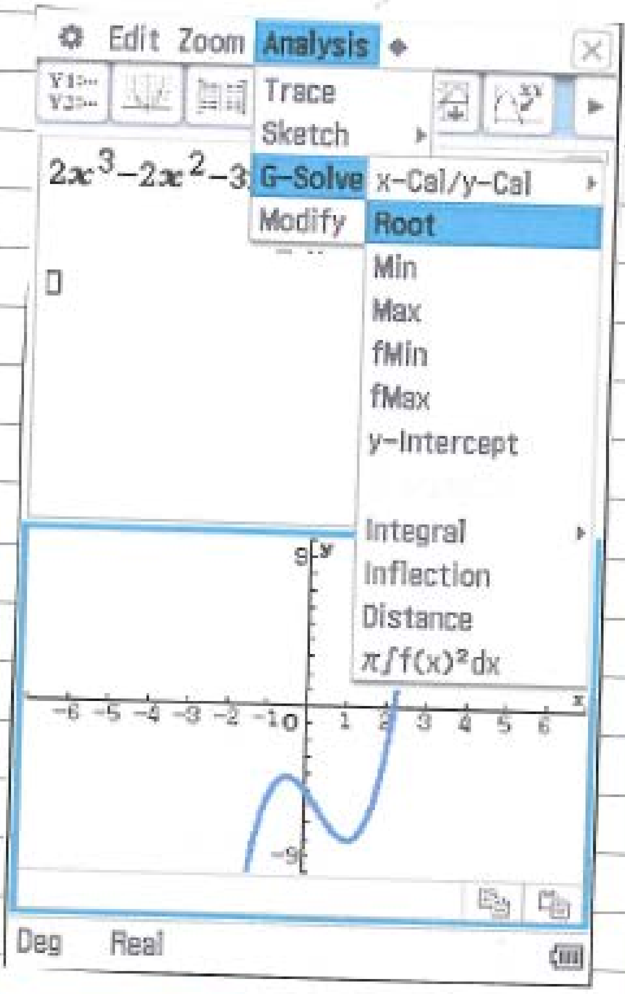
$y = x(x^2 + 2x - 15)$

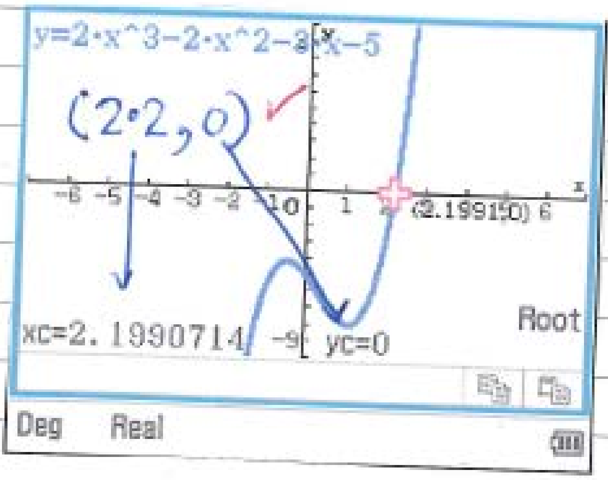
$y = x(x+5)(x-3)$

$x = 0,\ x = -5,\ x = 3$

$(0,0)$, $(-5,0)$ and $(3,0)$ ✓

3. $y = 2x^3 - 2x^2 - 3x - 5$ ✓





4 a) $x^3 + 5x^2 - 12x - 36$

$= (x+2)(x-3)(x-k)$

$(2)(-3)(-k) = -36$

$6k = -36$

$k = -6$ ✓

b) cuts x-axis ⇒ Roots

ie $(x+2)(x-3)(x+6) = 0$

$x = -2 \quad x = 3 \quad x = -6$ ✓

ie $(-2,0)$, $(3,0)$ and $(-6,0)$ ✓

5 $f(x) = x^3 - 6x^2 - x + 6$

a) $f(-1) = (-1)^3 - 6(-1)^2 - (-1) + 6$

$= 0$ ✓

b) $f(1) = 1^3 - 6(1)^2 - 1 + 6$

$= 0$ ✓

c) $f(2) = 2^3 - 6(2)^2 - 2 + 6$

$= -12$ ✓

d) $f(6) = 6^3 - 6(6)^2 - 6 + 6$ ✓

$= 0$

if $f(x) = 0$ then x is a root

$\therefore x = -1,\ x = 1\ x = 6$

so $(x+1)(x-1)(x-6)$ ✓

7. $f(x) = 3x^3 - 14x^2 - 7x + 10$

$= (3x-2)(ax^2+bx+c)$

$= 3ax^3 + 3bx^2 + 3cx - 2ax^2 - 2bx - 2c$

$= 3ax^3 + x^2(3b-2a) + x(3c-2b) - 2c$

So $3ax^3 = 3x^3$

$\therefore a = 1$

So $(3b-2a)x^2 = -14x^2$

$3b - 2a = -14$

$3b = -14 + 2(1)$

$3b = -12$

$b = -4$

So $-2c = 10$

$c = -5$

$\therefore a=1 \quad b=-4 \quad c=-5$

ie $x^2 - 4x - 5$

$\therefore 3x^3 - 14x^2 - 7x + 10$

$= (3x-2)(x^2-4x-5)$

$= (3x-2)(x-5)(x+1)$

$x = 2/3 \quad x = 5 \quad x = -1$

ie $(2/3, 0)$, $(5, 0)$ and $(-1, 0)$

6. $f(x) = x^3 - 10x^2 + 31x - 30$

a) $f(1) = 1^3 - 10(1)^2 + 31(1) - 30$

$= -8$

b) $f(2) = 2^3 - 10(2)^2 + 31(2) - 30$

$= 0$

c) $f(3) = 3^3 - 10(3)^2 + 31(3) - 30$

$= 0$

because $f(x)=0$ means x is a root.

$(x-2)(x-3)(x-k)$ are factors of $x^3 - 10x^2 + 31x - 30$

but $(-2)(-3)(-k) = -30$

$-6k = -30$

$k = 5$

$\therefore (x-2)(x-3)(x-5)$ are factors of

$x^3 - 10x^2 + 31x - 30$.

3a) $y = (x+2)(x-2)(x-5)$

roots are $x = -2, 2, 5$ ✓

Yint is $\Rightarrow (0+2)(0-2)(0-5)$

$= (2)(-2)(-5)$

$= 20$ ie $(0, 20)$ ✓

as $x \to +\infty$

$y = (\infty+2)(\infty-2)(\infty-5)$

$= (\infty)(\infty)(\infty)$

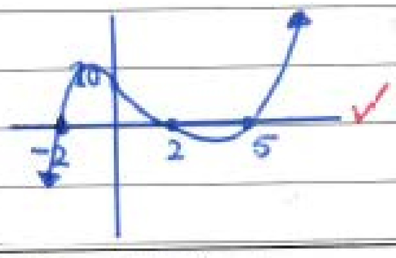
$= +\text{ve } \infty$ ✓

as $x \to -\infty$

$y = (-\infty+2)(-\infty-2)(-\infty-5)$

$= (-\infty)(-\infty)(-\infty)$

$= -\text{ve } \infty$ ✓



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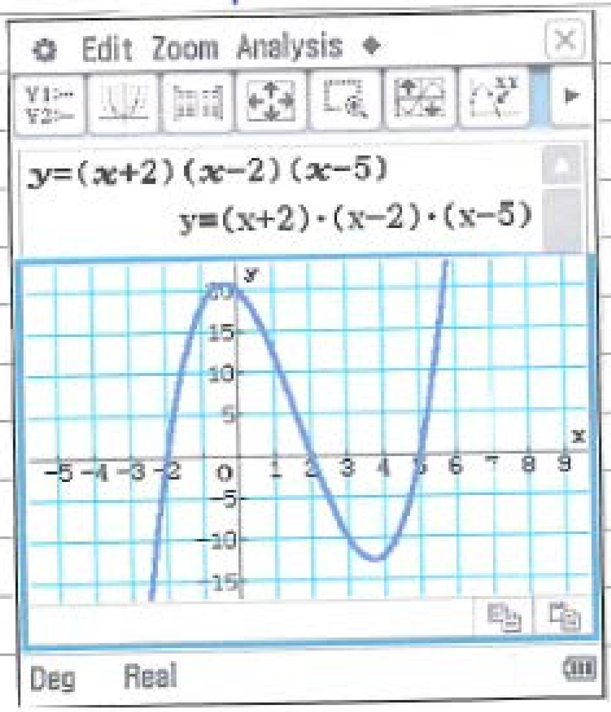


b) $y = (x+4)(x+1)(x-5)$

roots are $x = -4, -1, 5$ ✓

Yint is $= (0+4)(0+1)(0-5)$

$= (4)(1)(-5)$

$= -20$ ie $(0, -20)$ ✓

as $x \to +\infty$

$y = (\infty+4)(\infty+1)(\infty-5)$

$= (\infty)(\infty)(\infty)$

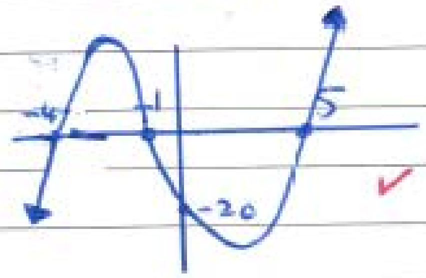
$= +\text{ve } \infty$ ✓

as $x \to -\infty$

$y = (-\infty+4)(-\infty+1)(-\infty-5)$

$= (-\infty)(-\infty)(-\infty)$

$= -\text{ve } \infty$ ✓



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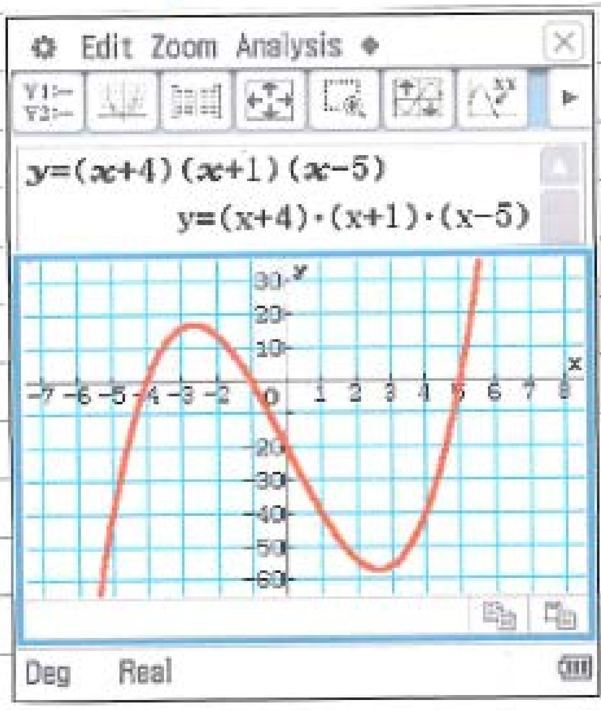


c) $y = 2(x+4)(x+1)(x-5)$

Roots $\Rightarrow$ $x=-4,\ x=-1,\ x=5$ ✓

$Y_{int} \Rightarrow 2(0+4)(0+1)(0-5)$

$2(4)(1)(-5)$

$= -40 \quad (0,-40)$ ✓

as $x \to +\infty$

$y = 2(\infty+4)(\infty+1)(\infty-5)$

$= 2(\infty)(\infty)(\infty)$

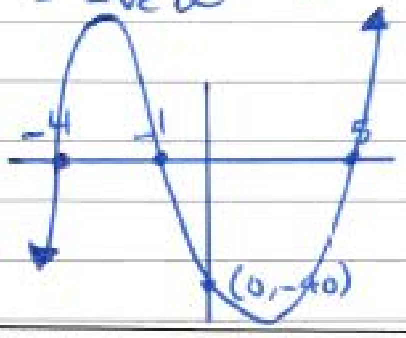
$= +ve\ \infty$ ✓

as $x \to -\infty$

$y = 2(-\infty+4)(-\infty+1)(-\infty-5)$

$= 2(-\infty)(-\infty)(-\infty)$

$= -ve\ \infty$ ✓



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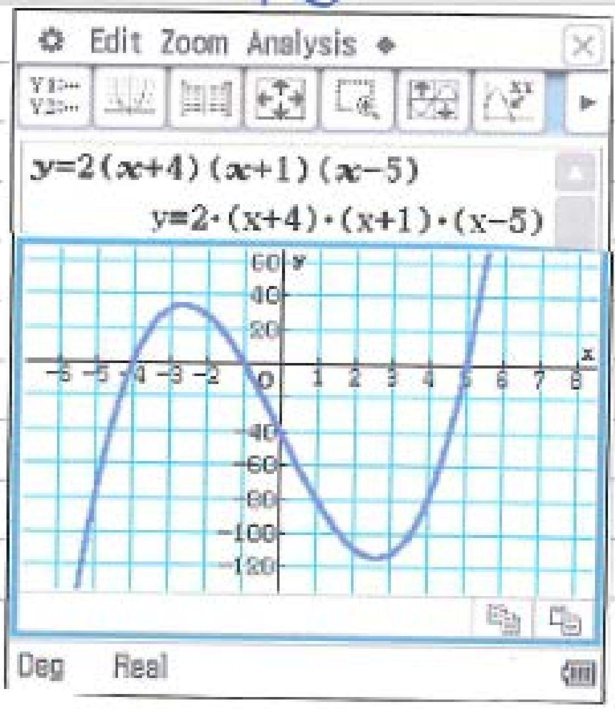


d) $y = x(3-x)(x-7)$

Roots $\Rightarrow$ $x=0,\ x=3,\ x=7$ ✓

$Y_{int} \Rightarrow 0(3-0)(0-7)$

$=(0)(3)(-7)$

$= 0 \quad$ ie $(0,0)$ ✓

as $x \to +\infty$

$y = \infty(3-\infty)(\infty-7)$

$= (\infty)(-\infty)(\infty)$

$= -ve\ \infty$ ✓

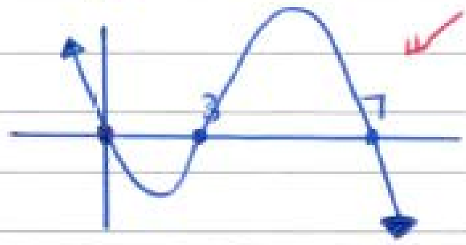
as $x \to -\infty$

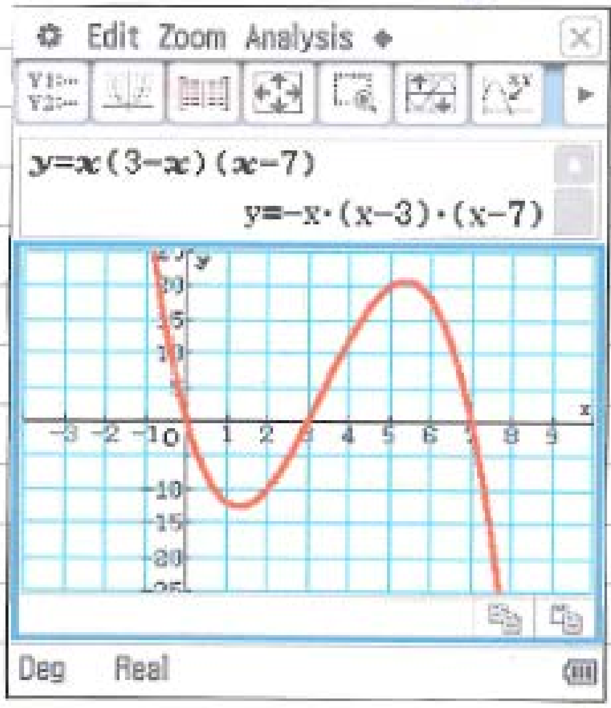
$y = -\infty(3--\infty)(-\infty-7)$

$= (-\infty)(\infty)(-\infty)$ ✓

$= +ve\ \infty$

✓





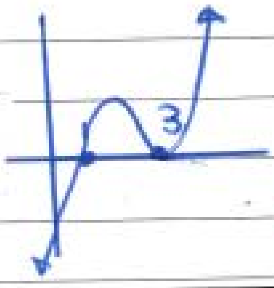
e) $y = (x-1)(x-3)^2$ ← repeated root

roots $x = 1 \quad x = 3$ ✓

Yint $x = 0 \quad y = (0-1)^2(0-3)^2$
$= -9$ ie $(0,-9)$ ✓

as $x \to +\infty$
$y = (\infty - 1)(\infty - 3)^2$
$= (\infty)(\infty)^2$
$= \infty$ ✓

as $x \to -\infty$
$y = (-\infty - 1)(-\infty - 3)^2$
$= (-\infty)(-\infty)^2$
$= -\infty$ ✓



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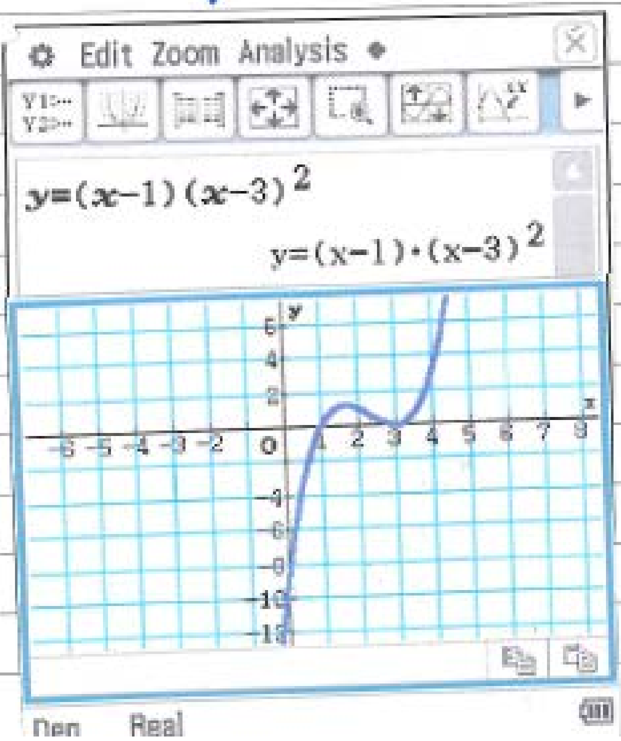
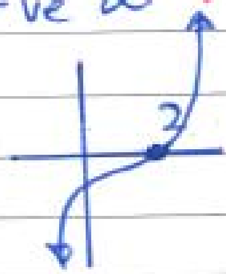


f) $y = (x-2)^3$
only 1 root at $x = 2$ ✓

Yint $x = 0 \quad y = (0-2)^3$
$= -8$ ie $(0,-8)$ ✓

as $x \to \infty$
$y = (\infty - 2)^3$
$= (\infty)^3$
$= +ve\ \infty$ ✓

as $x \to -\infty$
$y = (-\infty - 2)^3$
$= (-\infty)^3$
$= -ve\ \infty$ ✓



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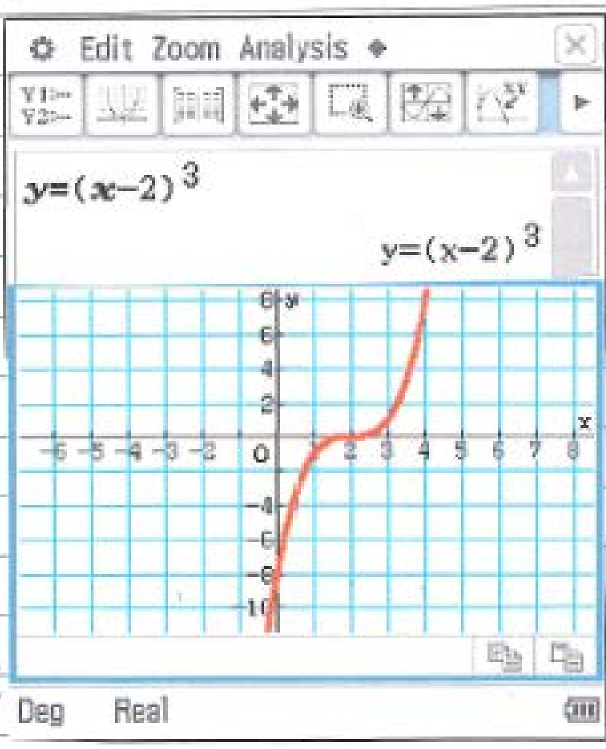
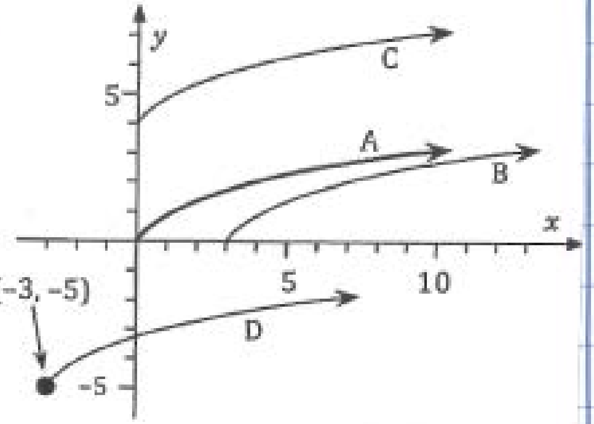


Ex 7B.

1.



$y=\sqrt{x}$ A is original

B has moved right 3

$B=\sqrt{x-3}$ ✓

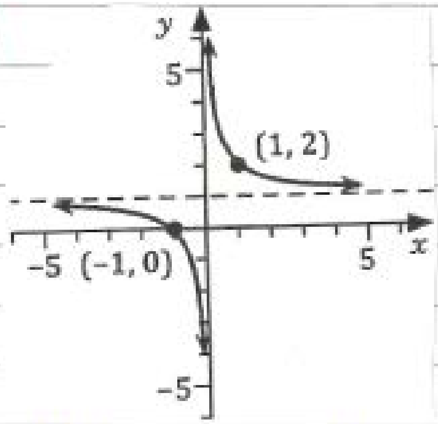
C has moved up 4

$C=\sqrt{x}+4$ ✓

D has moved left 3 and down 5

$D=\sqrt{x+3}-5$ ✓

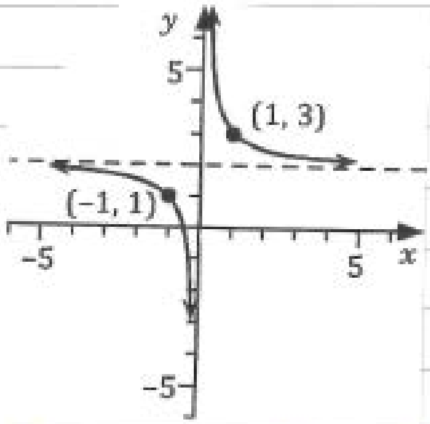
2a)



Graph has moved up 1 unit

$\therefore\ y=\frac{1}{x}+1$ ✓

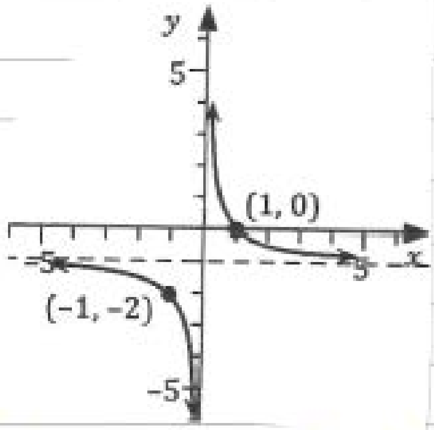
2b.



Graph has moved up 2 units

$y=\frac{1}{x}+2$ ✓

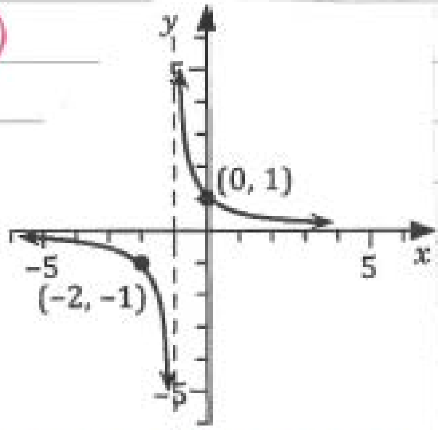
2c.



Graph has moved down 1 unit

$y=\frac{1}{x}-1$ ✓

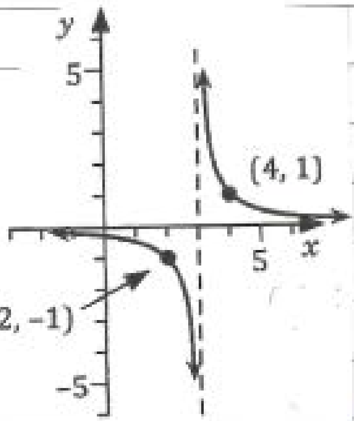
3a)



Graph has moved left 1 unit

$y=\frac{1}{x+1}$ ✓

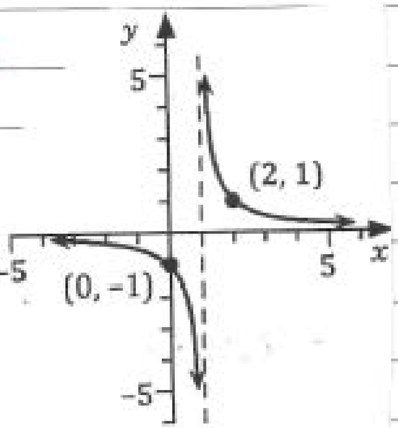
3b.



Graph has moved 3 right

$y = \frac{1}{x-3}$ ✓

3c.



Graph has moved 1 unit right

$y = \frac{1}{x-1}$ ✓

4. $y = x^3$

$y = x^3 + 1$

has moved 1 unit upwards ✓

5. $y = \frac{1}{x}$

$y = \frac{1}{x-1}$

graph has moved 1 unit to the right ✓

6. $y = \sqrt{x}$

$y = 2\sqrt{x}$

graph has dilated vertically by a scale factor of 2. ✓

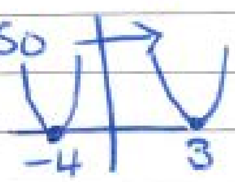
7. $y = (x+4)^2$ orginal

new $y = (x-3)^2$

this is 3 units to the right

this is already 4 units left

so



move 7 units to the right. ✓

9. $y = \frac{1}{x}$ to $y = \frac{3}{x-1}$

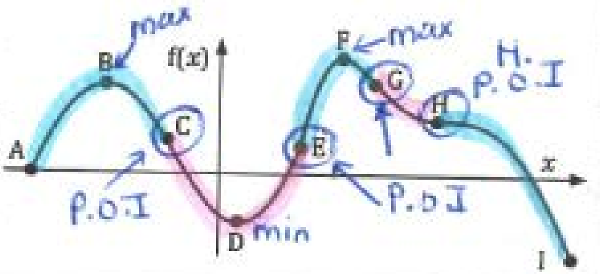
move 1 unit right, then dilate vertically by a scale factor of 3. ✓

8. $y = \sqrt{x}$

$y = \sqrt{x-2} + 1$

graph has moved 2 units to the right, then move 1 unit upwards.

10.



a) maximum turning point = B and F

b) minimum turning point = D

c) Points of inflection = C, E, G, H

*P.O.I is where graph changes from concave up to concave down.

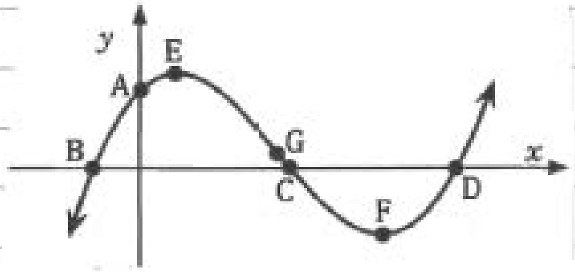
d) Horizontal Point of inflection = H

*H.P.O.I is where graph is also flat & has gradient of zero.

e) concave up from $C<x<E$ and $G<x<H$

f) concave down from $A<x<C$, $E<x<G$, $H<x<I$

11. graph on calc also!



B, C and D = roots
A is Y int
E is max F is min
G is P.O.I

A (0,10)
B (-0.51,0)
C (3.08,0)
D (6.42,0)
E (1,17)
F (5,-15)
G (3,1)

*Graph in main menu
G solve to find all required points

12. $P = \frac{k}{V}$

$P = \frac{400}{V}$

a) $P = 40 \quad V = ?$

$40 = \frac{400}{V}$ solve on calc

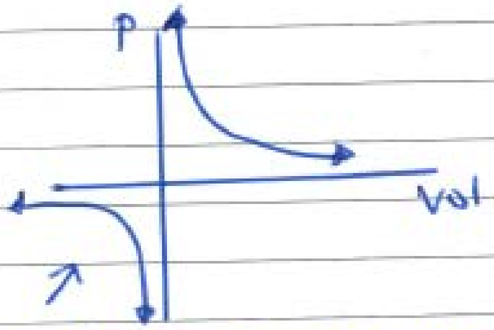
$V = 10$ ✓

b) $P = 20 \quad V = ?$

$20 = \frac{400}{V}$ solve on calc

$V = 20$

$P = \frac{400}{V}$ is a reciprocal graph



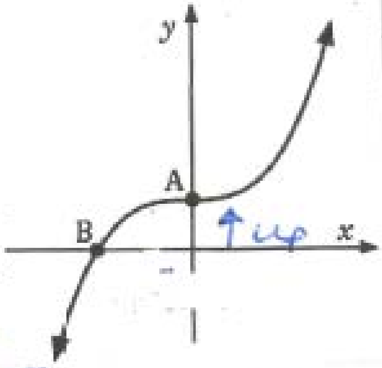
cannot have negative volume

so $V > 0$

Cannot have volume = 0 as then there will be no pressure to measure.

13.

Cubic graph



∴ from given table in ques

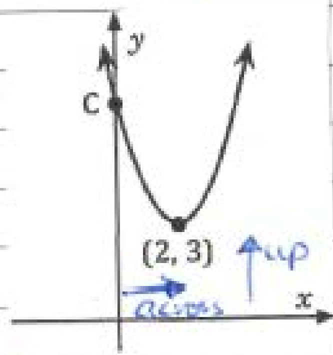
$y = x^3 + 8$ ✓ up 8

graph on calc

Yint A = 8 Root B = −2

(0,8) ✓ (−2,0) ✓

quadratic moved up 3 move right 2 units



$y = (x - d)^2 + e$

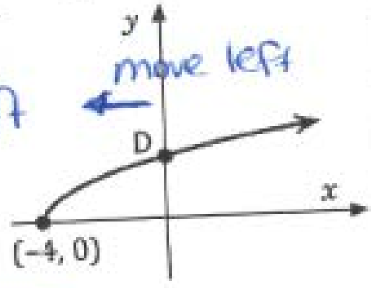
$y = (x - 2)^2 + 3$

$d = 2$ ✓ $e = 3$ ✓

graph on calc

C ⇒ Yint ⇒ (0,7) ✓

square root graph move 4 left

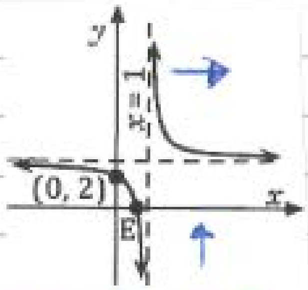


$y = \sqrt{x + 4}$ ✓ graph on calc

D = Yint D = (0,2) ✓

reciprocal graph

move right 1
move up 3



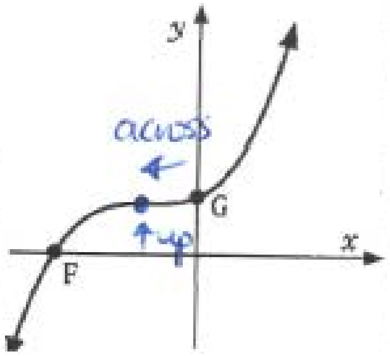
graph on calc to check.

$y = \frac{1}{x-1} + 3$

$f = 1$ ✓ $g = 3$ ✓

E is Yint (Root) $= (2/3, 0)$ ✓

cubic
move 1 unit left
3 units up



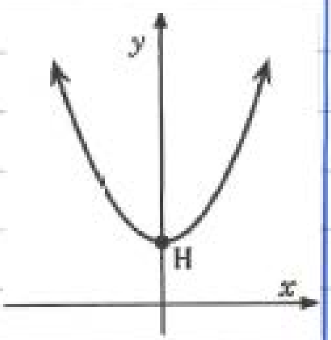
$y = (x+1)^3 + 8$

F is Root $F = (-3, 0)$ ✓
G is Yint $G = (0, 9)$ ✓

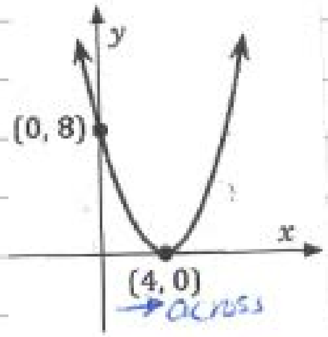
Quadratic moved up

$y = x^2 + 4$ ✓

H is Yint
$H = (0, 4)$ ✓



quadratic moved right & dilated



$y = b(x-c)^2$

$y = b(x-4)^2$ $c = 4$ ✓

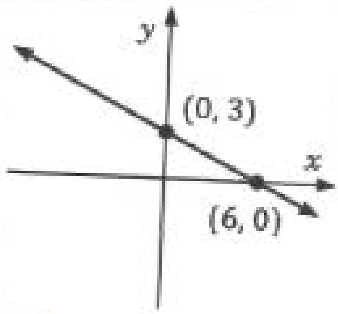
yint is $(0, 8)$

$8 = b(0-4)^2$

$8 = 16b \therefore b = 1/2$ ✓

Linear

$y = hx + i$



Yint is $(0, 3)$

$\therefore y = hx + 3$

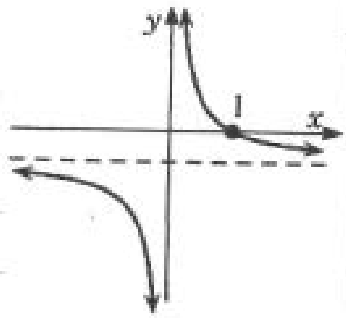
grad $= \frac{3-0}{0-6} = -\frac{1}{2}$

$\therefore y = -\frac{1}{2}x + 3$

$h = -\frac{1}{2}$ ✓ $i = 3$ ✓

reciprocal graph

graph moved down



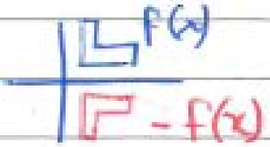
$y = \frac{8}{x} - 2$

graph on calc

I = root $I = (4, 0)$ ✓

* Draw a sketch, dont draw on calc.

Ex 7C.



1a) $y = f(x)$

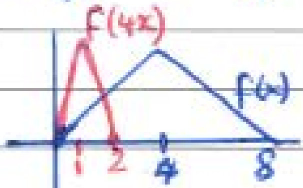
$y = -f(x)$

reflection on the x axis

b) $y = f(x)$

$y = f(4x)$

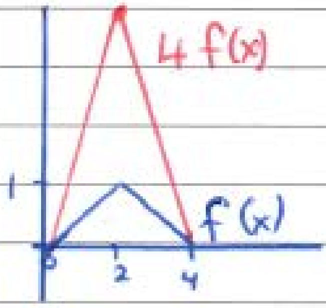
dilate horizontally by a Scale factor of $1/4$ ✓



* all "x" values ÷ by 4

c) $y = f(x)$ $y = 4f(x)$

dilate vertically by a s.f of 4 ✓



* all the "y" values has been x by 4 ✓

2a) $y = x^2 + 3x$

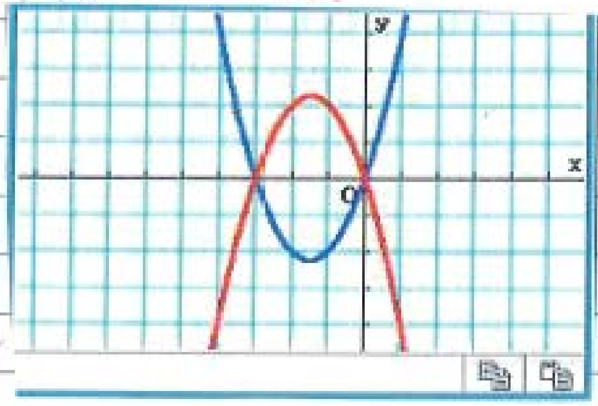
$y = -x^2 - 3x$

$y = -(x^2 + 3x)$

$y = -f(x)$

graph has been reflected on x axis ✓

f(x) -f(x)



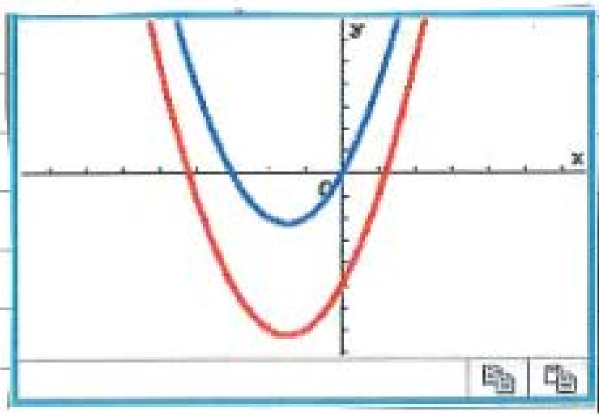
b) $y = x^2 + 3x - 5$

$y = (x^2 + 3x) - 5$

$y = f(x) - 5$

graph has moved down 5 units ✓

f(x) f(x) -5



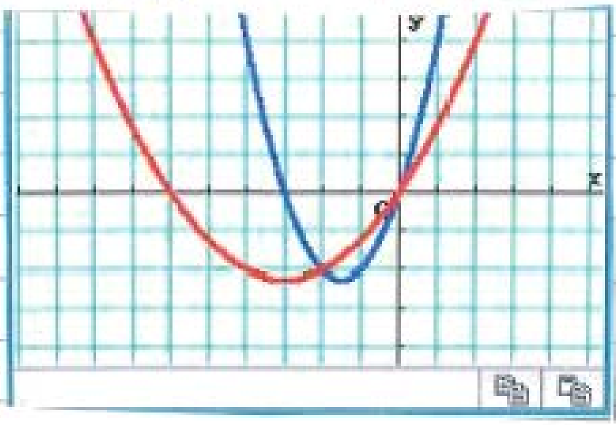
2c. $y = \frac{x^2}{4} + \frac{3x}{2}$

$y = (\frac{1}{2}x)^2 + \frac{1}{2}(3x)$

$= f(\frac{1}{2}x)$

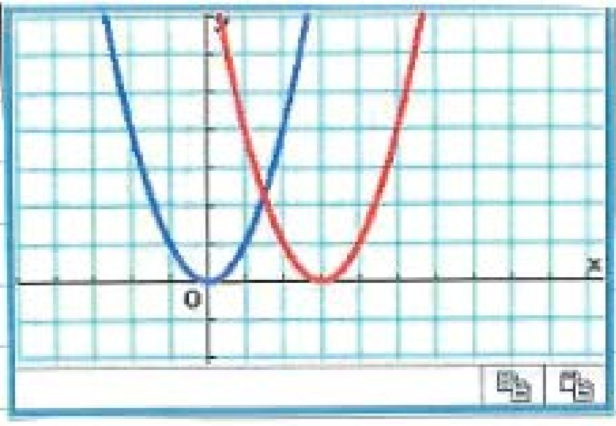
dilate horizontally by s.f of 2. ✓
(graph becomes fatter).

$f(x)$ $f(\frac{1}{2}x)$



3a) $y = x^2$
$y = (x-3)^2$
graph has moved 3 units to the right. ✓

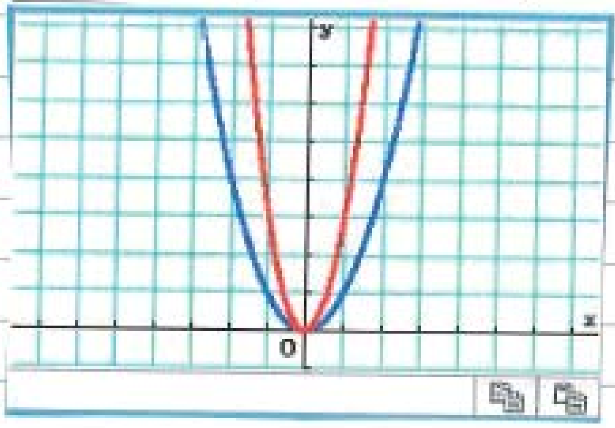
$f(x)$ $f(x-3)$



3b) $y = 3x^2$

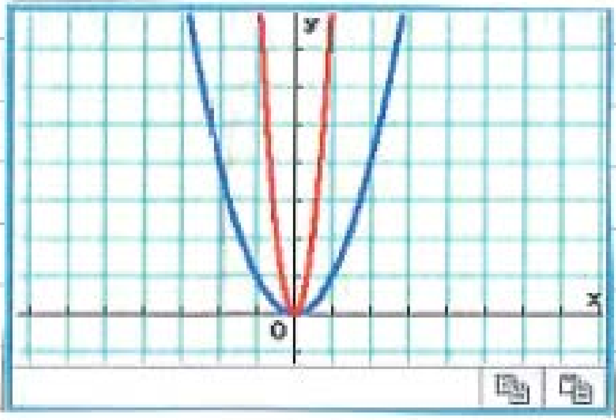
dilate vertically by a scale factor of 3
(all "y" values are x by 3) ✓

$f(x)$ $3f(x)$

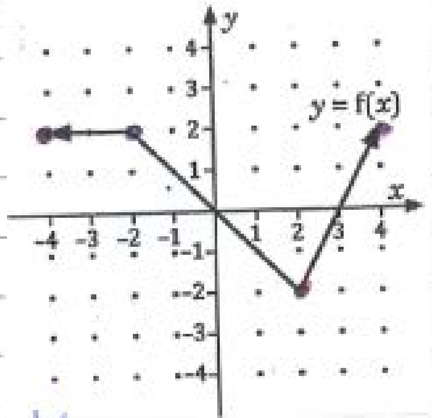


3c) $y = (3x)^2$
dilate horizontally by a scale factor of 1/3. ✓
(all "x" values are x by 1/3)

$f(x)$ $f(3x)$



4.



pick key points to move

a) $f(x-2)$

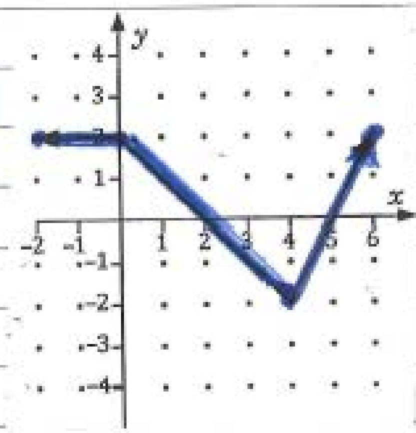
move right 2 units

$(-4,2) = (-2,2)$

$(-2,2) = (0,2)$

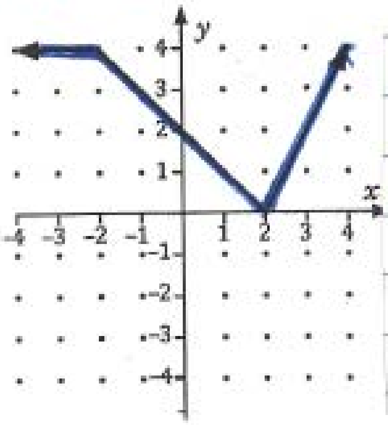
$(2,-2) = (4,-2)$

$(4,2) = (6,2)$



✓

b) $f(x)+2$ move up 2



$(-4,2) = (-4,4)$

$(-2,2) = (-2,4)$

$(2,-2) = (2,0)$

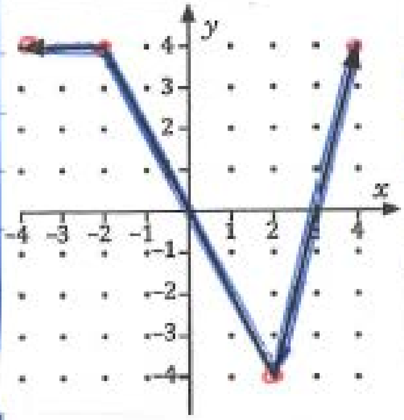
$(4,2) = (4,4)$

✓

c) $y = 2f(x)$

dilate vertically s.f 2

so all "y" are x by 2.



$(-4,2) = (-4,4)$

$(-2,2) = (-2,4)$

$(2,-2) = (2,-4)$

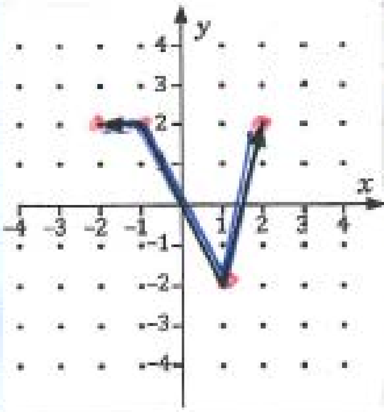
$(4,2) = (4,4)$

✓

d) $f(2x)$

dilate horizontally by a scale factor of 1/2

(all "x" values are 1/2)



$(-4,2) = (-2,2)$

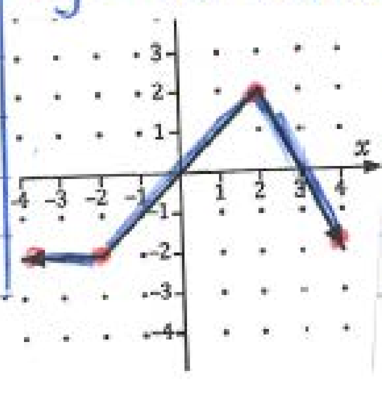
$(-2,2) = (-1,2)$

$(2,-2) = (1,-2)$

$(4,2) = (2,2)$

✓

e) $y = -f(x)$ reflect on x axis



$(-4,2) = (-4,-2)$

$(-2,2) = (-2,-2)$

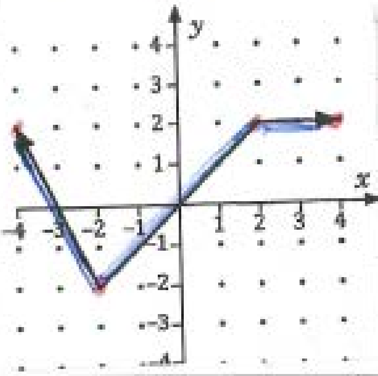
$(2,-2) = (2,2)$

$(4,2) = (4,-2)$

✓

4f. $y = f(-x)$ reflect on the y axis

so all "x" → "−x"



$(-4,2) = (4,2)$

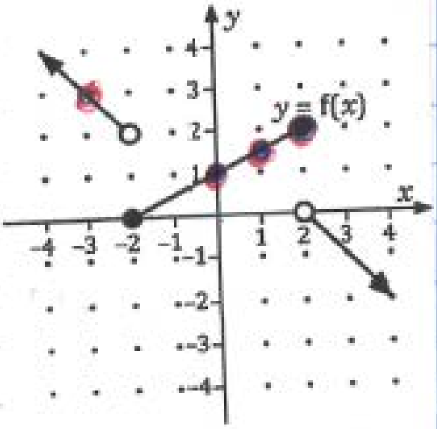
$(-2,2) = (2,2)$

$(2,-2) = (-2,-2)$

$(4,2) = (-4,2)$

✓

5.



a) $f(0)$

$= 1$ ✓

b) $f(1)$

$= 1\frac{1}{2}$ ✓

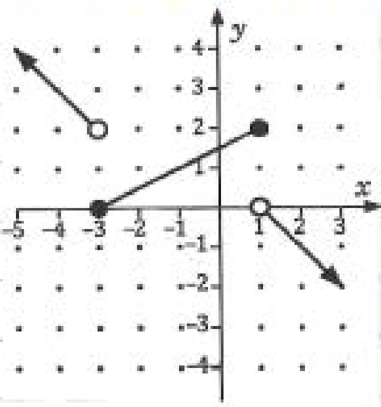
c) $f(2)$

$= 2$ ✓ (there is a hollow circle at (2,0) so use (2,2))

✓

d) $f(-3) = 3$ ✓

e) $f(x+1)$ move left 1 unit



$(-4,4) = (-5,4)$

$(-2,2) = (-3,2)$

$(-2,0) = (-3,0)$

$(2,2) = (1,2)$

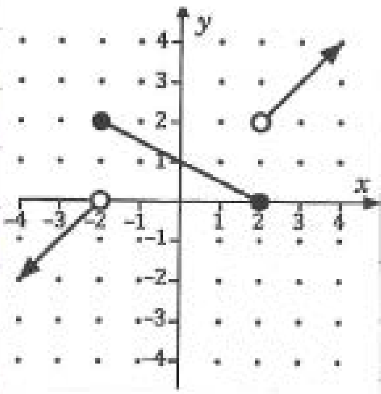
$(2,0) = (1,0)$

$(4,-2) = (3,-2)$

✓

f). $f(-x)$ reflect on y axis

all "x" become "−x"



$(-4,4) = (4,4)$

$(-2,2) = (2,2)$

$(-2,0) = (2,0)$

$(2,2) = (-2,2)$

$(2,0) = (-2,0)$

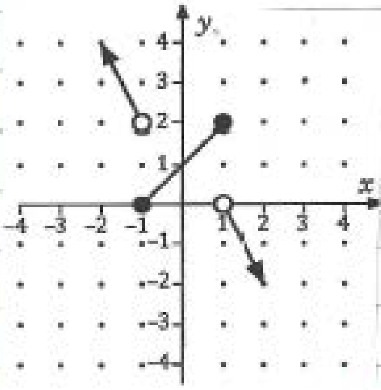
$(4,-2) = (-4,2)$

✓

g). $f(2x)$

dilate horizontally by s.f $\frac{1}{2}$

so all "x" become "$\frac{1}{2}x$"



$(-4,4) = (-2,4)$

$(-2,2) = (-1,2)$

$(-2,0) = (-1,0)$

$(2,2) = (1,2)$

$(2,0) = (1,0)$

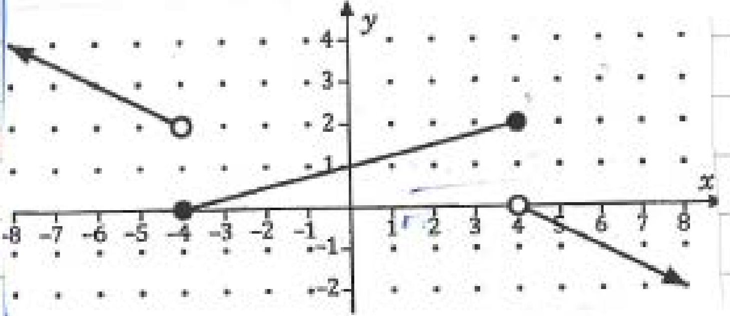
$(4,-2) = (2,-2)$

✓

h). $y = f(0.5x)$

dilate horizontally by s.f 2

all "x" become "2x" ✓



$(-4,4) = (-8,4)$ $(-2,0) = (-4,0)$

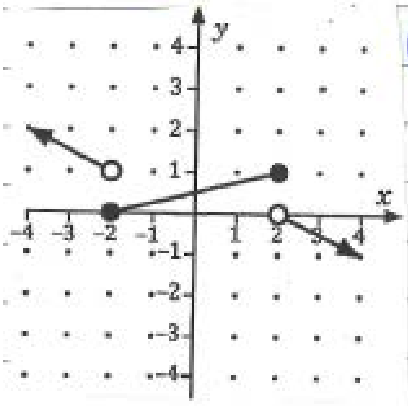
$(-2,2) = (-4,2)$ $(2,2) = (4,2)$

$(2,0) = (4,0)$ $(4,-2) = (8,-2)$

5i) $y = 0 \cdot 5f(x)$

dilate vertically by s.f ½

all "y" become "½y"



$(-4,4) = (-4,2)$

$(-2,2) = (-2,1)$

$(-2,0) = (-2,0)$

$(2,2) = (2,1)$

$(2,0) = (2,0)$

$(4,2) = (4,-1)$

6.

Key points

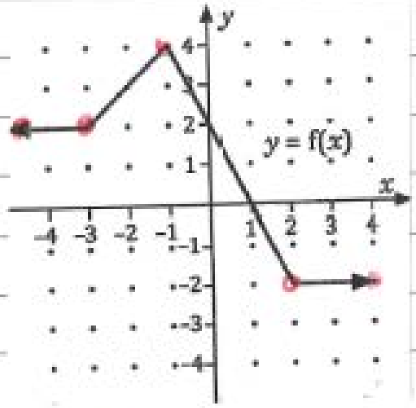
$(-5,2)$

$(-3,2)$

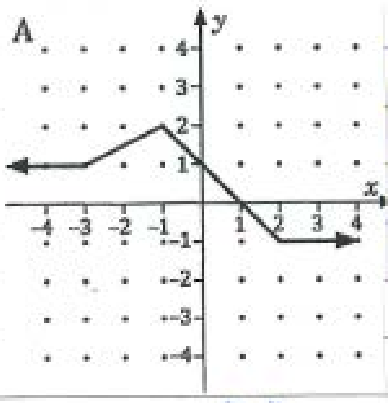
$(-1,4)$

$(2,-2)$

$(4,-2)$



A



$(-5,2) = (-5,1)$

$(-3,2) = (3,1)$

$(-1,4) = (-1,2)$

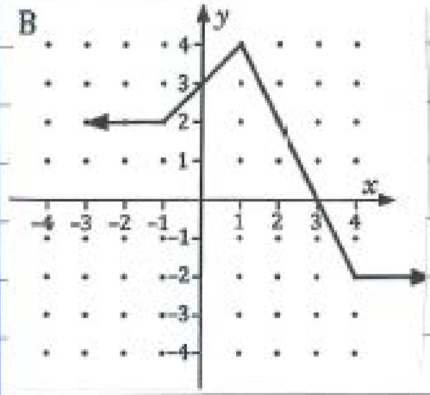
$(2,-2) = (2,-1)$

$(4,-2) = (4,-1)$

all the "y" values have halved

$\therefore\ y = 0 \cdot 5f(x)$

B



$(-5,2) = (-3,2)$

$(-3,2) = (-1,2)$

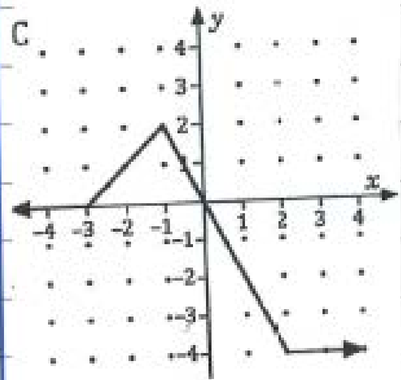
$(-1,4) = (1,4)$

$(2,-2) = (4,-2)$

all the x values have moved 2 units to the right

$\therefore\ y = f(x-2)$

C



$(-5,2) = (-5,0)$

$(-3,2) = (-3,0)$

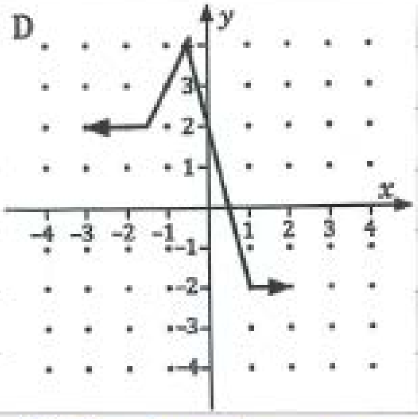
$(-1,4) = (-1,2)$

$(2,-2) = (2,-4)$

$(4,-2) = (4,-4)$

all the y values have moved down 2 units

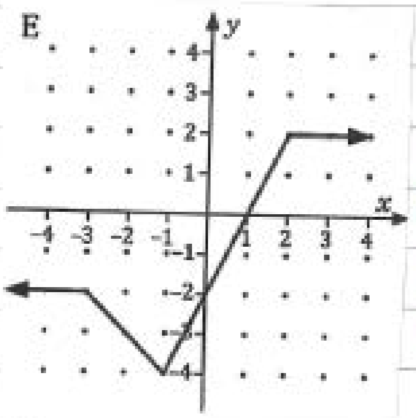
$\therefore\ y = f(x) - 2$



(-5,2) = (*, 2)
(-3,2) = (-1.5, 2)
(-1,4) = (-½, 4)
(2,-2) = (1,-2)
(4,-2) = (2,-2)

all the x values have halved ie. Sf = ½

∴ $y = f(2x)$ ✓

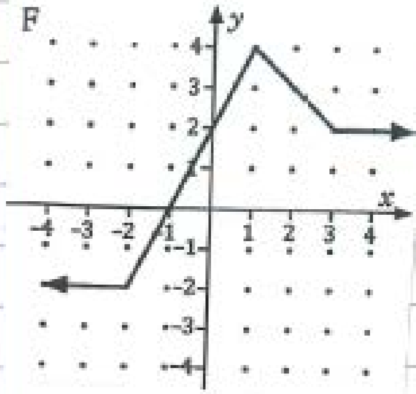


all y values have become "−y"

(-5,2) = (-5,2)
(-3,2) = (-3,-2)
(-1,4) = (-1,-4)
(2,-2) = (2,2)
(4,-2) = (4,2)

∴ reflect on the x axis

∴ $y = -f(x)$ ✓

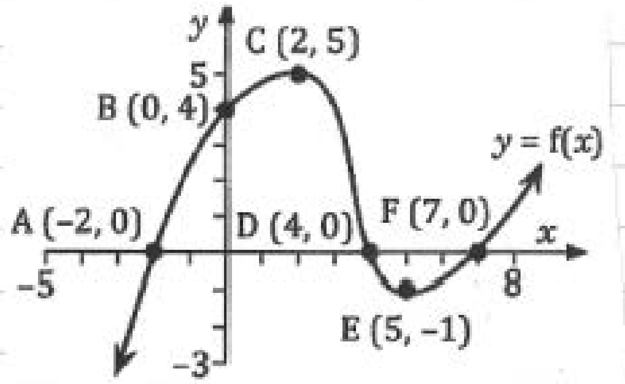


(-5,2) = (5,2)
(-3,2) = (3,2)
(-1,4) = (1,4)
(2,-2) = (-2,-2)
(4,-2) = (-4,-2)

all the x values have become "−x" so reflect on the y axis

∴ $y = f(-x)$ ✓

7.



a) $y = f(x-3)$ move right 3
(-2,0) = (1,0) A ✓
(4,0) = (7,0) D ✓
(7,0) = (10,0) F ✓

b) $y = f(2x)$ all "x" = "½x"
(-2,0) = (-1,0) A ✓
(4,0) = (2,0) D ✓
(7,0) = (3.5,0) F ✓

c) $y = -f(x)$ reflect on x axis
(-2,0) = (-2,0) A ✓
(4,0) = (4,0) D ✓
(7,0) = (7,0) F ✓

d) $y = f(-x)$

reflect on y axis

all "x" = "-x"

$(-2,0) = (2,0)$ ✓

$(4,0) = (-4,0)$ ✓

$(7,0) = (-7,0)$ ✓

e) $y = f(x)+3$

has moved up 3 units

$C(2,5) = (2,8)$ ✓

f) $y = -f(x)$

graph reflect on x axis

So min ⇒ max

max ⇒ min

So E (min) ⇒ E (max)

$E(5,-1) = (5,1)$ ✓

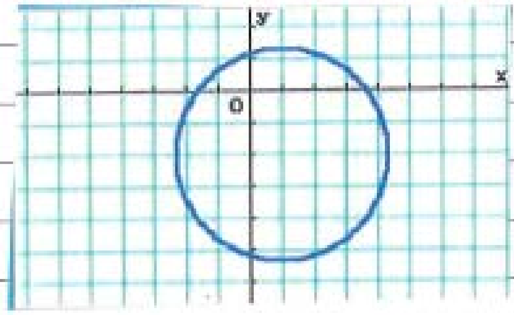
Ex 7D

1.a) $x^2+y^2-2x+4y=6$

yes because

✓ $1x^2 + 1y^2$

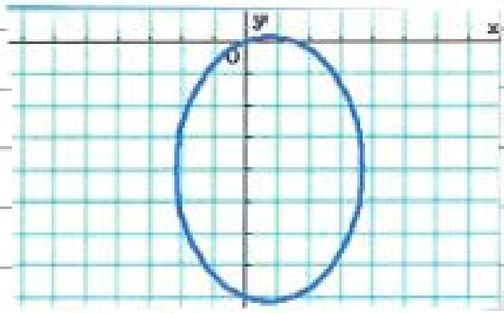
(draw on calc also)



b) $2x^2+y^2-3x+8y+10=0$

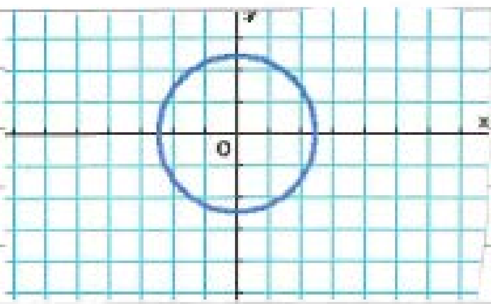
No! ✓

$2x^2 + y^2$ not same.



c) $x^2+y^2=6$

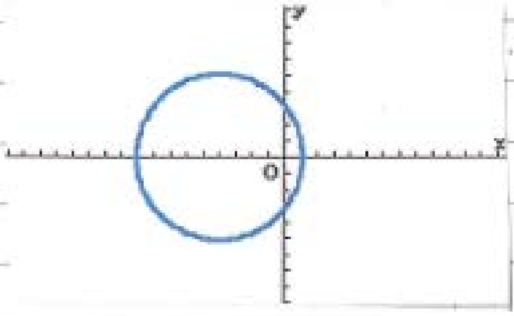
yes $1x^2 + 1y^2$



d) $x^2+y^2+8x=10$

Yes

$\underline{1}x^2+\underline{1}y^2$



e) $x^2-y^2+2x+10y=10$

NO! $x^2 - y^2$

no, must be +

f) $x^2+6xy+y^2+15y=20$

NO! $\underline{6xy}$ not allowed

2 centre (0,0)
radius = 10 units

$\therefore \quad x^2+y^2=10^2$

A(-6, a)

$(-6)^2+(a)^2=100$

$a^2=100-36$

$a^2=64$

$a=\pm 8$

$\therefore a=8$ ✓

ques says a is +ve

B(3, b)

$(3)^2+(b)^2=10^2$

$b^2=100-9$

$b^2=\sqrt{91}$

$b=\pm\sqrt{91}$

$\therefore b=\sqrt{91}$ as b is +ve

C (0, c)

$0^2+c^2=10^2$

$c^2=10^2$

$c=\pm\sqrt{100}$

$\therefore c=-10$ c is -ve

D(d, 5) $d^2+5^2=10^2$

$d^2=100-25$

$d^2=75$

$d=\pm\sqrt{75}$

$\therefore d=-\sqrt{75}$ as D is -ve.

$=-5\sqrt{3}$ ← simplify on calc

3. a) centre (2, -3) radius = 5

$(x-2)^2+(y+3)^2=25$ ✓

b) Centre (3, 2) radius = 7

$(x-3)^2+(y-2)^2=49$ ✓

c) centre (-10, 2) radius = $3\sqrt{5}$

$(x+10)^2+(y-2)^2=45$ ✓ square

d) centre (-1, -1) radius = 6

$(x+1)^2+(y+1)^2=36$ ✓

4. a) centre $(3,5)$ $r=5$

$(x-3)^2+(y-5)^2=25$

$x^2-6x+9+y^2-10y+25=25$

$x^2+y^2-6x-10y=-9$ ✓

b) centre $(-2,1)$ $r=\sqrt{7}$

$(x+2)^2+(y-1)^2=7$

$x^2+2x+4+y^2-2y+1=7$

$x^2+y^2+2x-2y=2$ ✓

c) centre $(-3,-1)$ $r=2$

$(x+3)^2+(y+1)^2=4$

$x^2+6x+9+y^2+2y+1=4$

$x^2+y^2+6x+2y=-6$ ✓

d) centre $(3,8)$ $r=2\sqrt{7}$

$(x-3)^2+(y-8)^2=(2\sqrt{7})^2$

$x^2-6x+9+y^2-16y+64=28$

$x^2+y^2-6x-16y=-45$ ✓

5a) $x^2+y^2=25$

$x^2+y^2=5^2$

centre $(0,0)$

radius $=5$ units ✓

b) $25x^2+25y^2=9$

$25(x^2+y^2)=9$

$x^2+y^2=\frac{9}{25}$

centre $(0,0)$

radius $=\sqrt{\frac{9}{25}}=\frac{3}{5}$ ✓

c) $(x-3)^2+(y+4)^2=25$

centre $(3,-4)$ ✓

radius $=\sqrt{25}=5$ units

d) $(x+7)^2+(y-1)^2=100$

centre $(-7,1)$ ✓

radius $=\sqrt{100}=10$ units

e) $x^2+y^2-6x+4y+4=0$

$x^2-6x+y^2+4y+4=0$

$(x-3)^2+(y+2)^2=+9$

gives extra $+9$

centre $(3,-2)$

radius $=\sqrt{9}=3$ units ✓

f) $x^2+y^2+2x-6y=15$

$x^2+2x+y^2-6y=15$

$(x+1)^2+(y-3)^2=\underbrace{15+1+9}_{25}$

gives extra +1   gives extra +9

centre $(-1,3)$

radius $=\sqrt{25}=5$ ✓

g) $x^2+y^2+2x=14y+50$

$x^2+2x+y^2-14y=50$

$(x+1)^2+(y-7)^2=\underbrace{50+1+49}_{100}$

gives extra +1   gives extra +49

centre $(-1,7)$

radius $=\sqrt{100}=10$ units ✓

h) $x^2+10x+y^2=151+14y$

$x^2+10x+y^2-14y=151$

$(x+5)^2+(y-7)^2=\underbrace{151+25+49}_{225}$

gives extra +25   gives extra +49

centre $(-5,7)$

radius $=\sqrt{225}=15$ units

i) $x^2+y^2=20x+10y+19$

$x^2-20x+y^2-10y=19$

$(x-10)^2+(y-5)^2=\underbrace{19+100+25}_{144}$

extra +100   extra +25

centre $(10,5)$

radius $=\sqrt{144}=12$ units

j) $2x^2-2x+2y^2+10y=-5$

$2(x^2-x+y^2+5y)=-5$

$x^2-x+y^2+5y=-2.5$

$(x-\frac{1}{2})^2+(y+2.5)^2=\underbrace{-2.5+\frac{1}{4}+6.25}_{4}$

gives an extra $\frac{1}{4}$ gives an extra 6.25

centre $(\frac{1}{2},-2.5)$

radius $=\sqrt{4}=2$ units ✓

6. $(x-3)^2+(y-7)=36$

centre $=(3,7)$

$(x-2)^2+(y-9)^2=49$

centre $=(2,9)$

distance $=\sqrt{(3-2)^2+(9-7)^2}$

$=\sqrt{1^2+2^2}$

$=\sqrt{5}$ units

7. $(x-3)^2+(y+4)^2=25$

centre $=(3;-4)$

$(x-2)^2+(y-7)^2=9$

centre $=(2,7)$

grad $= \frac{7--4}{2-3} = \frac{11}{-1} = -11$

$y-y_1=m(x-x_1)$

$y-7=-11(x-2)$

$y=-11x+22+7$

$y=-11x+29$ ✓

8. $(x+1)^2+(y-7)^2=36$

current centre $=(-1,7)$

move 4 right & 3 down

$(-1+4, 7-3)=(3,4)$

$\therefore (x-3)^2+(y-4)^2=36$ ✓

9. $x^2+y^2-6x+10y=-25$

$x^2-6x+y^2+10y=-25$

$(x-3)^2+(y+5)^2=\underbrace{-25+9+25}_{9}$

extra +9 extra +25

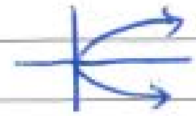
centre $(3,-5)$ radius $=3$

move 7 left 2 up

$(3-7, -5+2)=(-4;-3)$

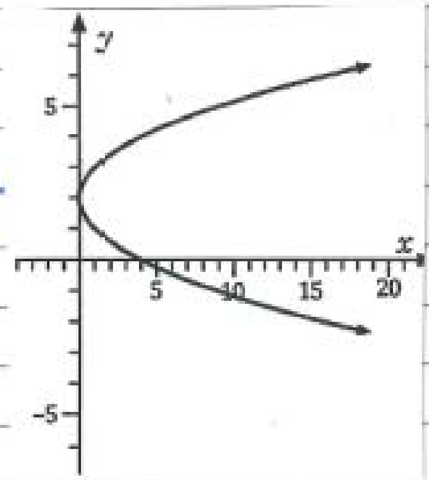
$\therefore (x+4)^2+(y+3)^2=9$ ✓

10. $y^2=x$

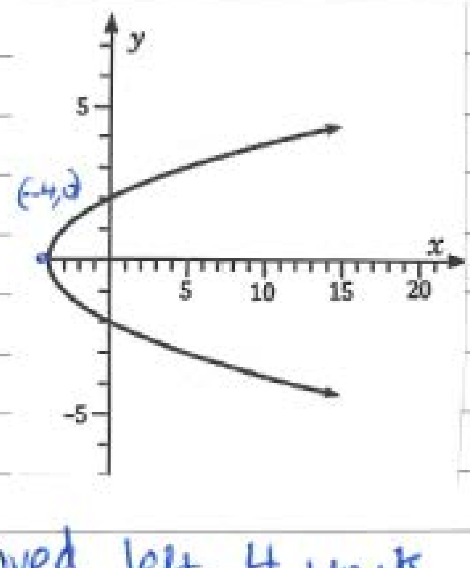


a) up 2 units

$\therefore (y-2)^2=x^2$



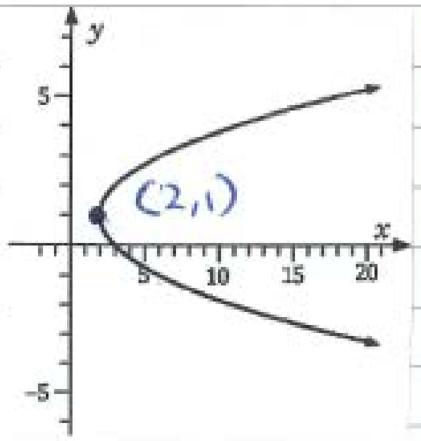
b)



moved left 4 units

$y^2=(x+4)$

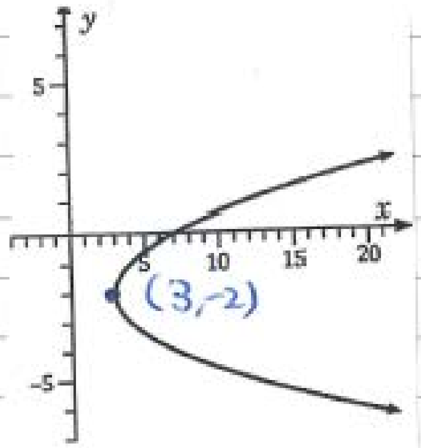
c)



right 2units up 1 unit

$(y-1)^2=(x-2)$ ✓

d)



right 3 units, down 2 units

$(y+2)^2=(x-3)$ ✓

11. $(x-3)^2+(y-11)^2=144$

centre $(3,11)$

radius $=\sqrt{144}=12$

$(x-12)^2+(y+1)^2=9$

centre $(12,-1)$

radius $=\sqrt{9}=3$

a) $AB=\sqrt{(12-3)^2+(-1-11)^2}$

$=\sqrt{9^2+12^2}$

$=\sqrt{225}=15$ units ✓

b)

radius of circles is 12 & 3

so if distance from centres is 15 $12+3=15$

so they touch once only ✓

13. $(x-4)^2+(y-2)^2=25$

line $y=x-3$ sub

$(x-4)^2+(x-3-2)^2=25$

$(x-4)^2+(x-5)^2=25$

$x^2-8x+16+x^2-10x+25=25$

$2x^2-18x-16=0$

$x^2-9x-8=0$

$(x-8)(x-1)=0$

$x=8$ or $x=1$

so $y=8-3=5$ $(8,5)$ ✓

so $y=1-3=-2$ $(1,-2)$ ✓

14. $4y = x + 30$

$4y - 30 = x$ Sub into eqn

$(x+5)^2 + (y-2)^2 = 34$

$(4y-30+5)^2 + (y-2)^2 = 34$

$(4y-25)^2 + (y-2)^2 = 34$

$16y^2 - 200y + 625 + y^2 - 4y + 4 = 34$

$17y^2 - 204y + 595 = 0$

Solve on calc $y=5$ $y=7$

$\therefore x = 4(5) - 30 = -10$ $(-10, 5)$ ✓

$\therefore x = 4(7) - 30 = -2$ $(-2, 7)$ ✓

15. $3y = x + 25 \Rightarrow x = 3y - 25$

$(x-7)^2 + (y-4)^2 = 40$ Sub into eqn

$(3y-25-7)^2 + (y-4)^2 = 40$

$(3y-32)^2 + (y-4)^2 = 40$

$9y^2 - 192y + 1024 + y^2 - 8y + 16 = 40$

$10y^2 - 200y + 1000 = 0$

$10(y^2 - 20y + 100) = 0$

$y^2 - 20y + 100 = 0$

$(y-10)^2 = 0$

$y = 10$ only 1 solution

$\therefore y = 10$

$\therefore x = 3(10) - 25$

$= 5$

$\therefore (5, 10)$ ✓

16. $x^2 + 2x + y^2 - 10y = -a$

$(x+1)^2 + (y-5)^2 = -a + 1 + 25$

extra +1 extra +25

$(x+1)^2 + (y-5)^2 = 26 - a$

radius must be bigger than 0

$\therefore 26 - a > 0$

$\therefore a < 26$ ✓