MIC Assignment-2

Aditya Neeraje (23B0940)
Chaitanyaa Maheshwari (23B0926)

February 14, 2026

Contents

Question 1: X-Ray Computed Tomography – Radon Transform

1 Question 1: X-Ray Computed Tomography – Radon Transform

A 128×128 Shepp–Logan phantom was generated and treated as the image $f(x, y)$. The coordinate origin was logically placed at the center pixel.

(a) Implementation of `myXrayIntegration()`

The function `myXrayIntegration(f, t, theta_deg, delta_s)` computes the line integral of image intensities along a line parameterized by t and θ.

Implementation details:

- The angle `theta_deg` is converted to radians.
- A sampling variable `s` is created from $-N$ to N with step size `delta_s`.
- For each value of `s`, corresponding (x, y) coordinates are computed.
- `map_coordinates()` is used to interpolate image values.
- The interpolated values are summed and multiplied by `delta_s`.

Interpolation Scheme: Bilinear interpolation (`order=1`) was used because:

- It provides smooth transitions between pixels.
- Nearest-neighbor interpolation produces jagged artifacts.
- Higher-order interpolation increases computation unnecessarily.

(b) Implementation of `myXrayCTRadonTransform()`

The function `myXrayCTRadonTransform()` computes the Radon transform over discrete values:

- `t = -90 to 90` with step size $\Delta t = 5$
- `theta = 0 to 175` with step size $\Delta\theta = 5$

For every (t, θ) pair, the function calls `myXrayIntegration()` and stores the result in a 2D array.

(c) Comparison of Different Δs Values

Choice of Δs: Different values were tested: $0.1, 0.5, 1, 3, 10$ pixel units.

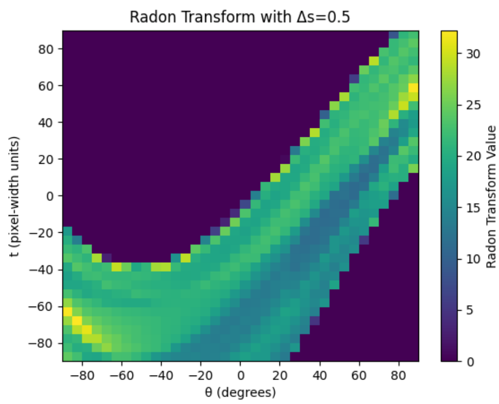
Observations:

- Very small Δs increases computation with little improvement.
- Infact $\Delta s = 0.1$ performed worse than $\Delta s = 0.5$.
- This is due to overfitting by interpolation.
- $\Delta s = 0.5$ or 1 gives stable and smooth results.
- Large Δs (3 or 10) causes rough and blocky sinograms.

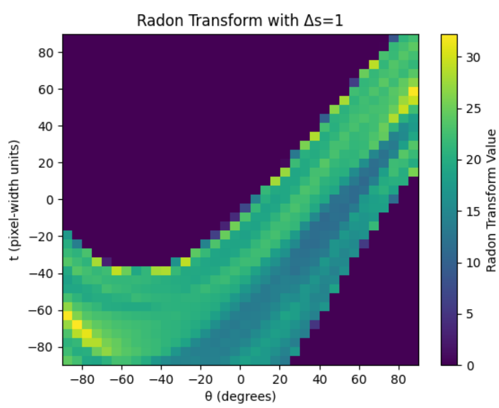
Question 1: X-Ray Computed Tomography – Radon Transform

Δs	Smoothness Score
0.1	0.00766441
0.5	0.00766361
1	0.00766573
3	0.00792422
10	0.01310446

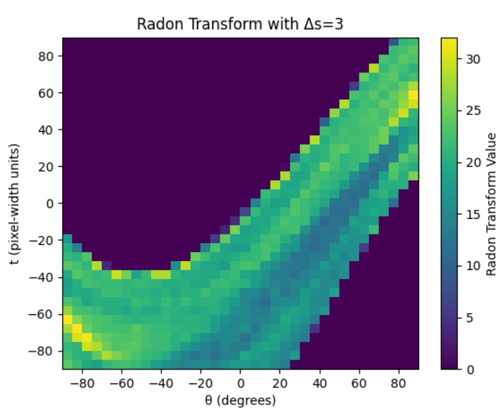
Table 1: Smoothness scores for different Δs values.



$\Delta s = 0.5$



$\Delta s = 1$

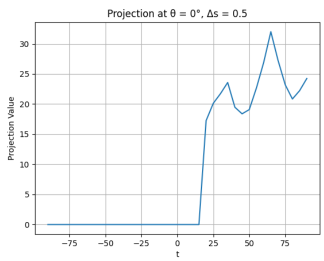
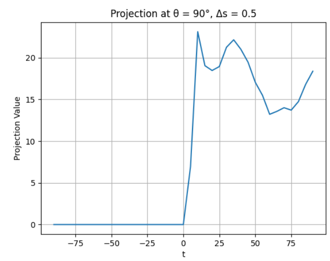
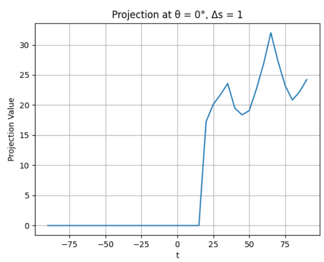
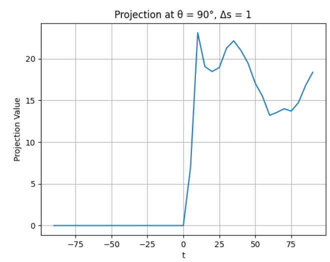
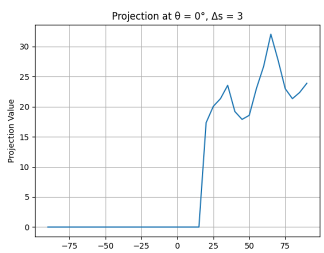
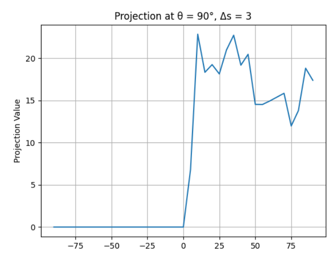


$\Delta s = 3$

Question 1: X-Ray Computed Tomography – Radon Transform

Additionally, 1D plots of the sinogram values were examined for:

1D Projection Comparisons for Different Δs

Δs	$\theta = 0°$	$\theta = 90°$
0.5		
1		
3		

Observations:

- Smaller Δs produces smoother 1D curves.
- Larger Δs produces visible roughness due to coarse sampling.
- Among the tested values, $\Delta s = 0.5$ appears smoothest.

A smoothness score was computed using the function `apply_prior()`, which evaluates local intensity differences. Lower values indicate smoother sinograms.

Results show:

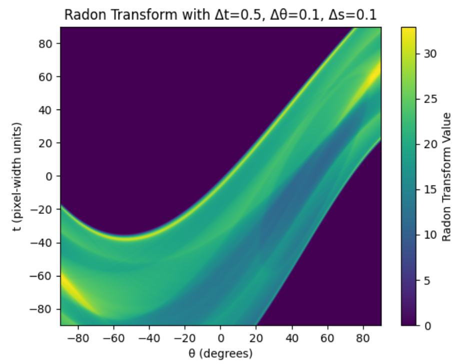
- $\Delta s = 0.5$ gives the lowest smoothness score.
- $\Delta s = 1$ and 3 give slightly higher scores, indicating more roughness.

Question 1: X-Ray Computed Tomography – Radon Transform

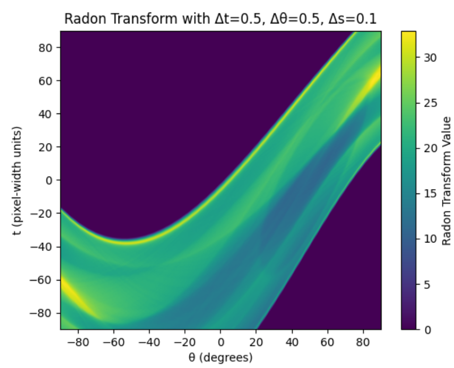
(d) Choice of Δt and $\Delta\theta$ in Scanner Design

$\Delta\theta$:

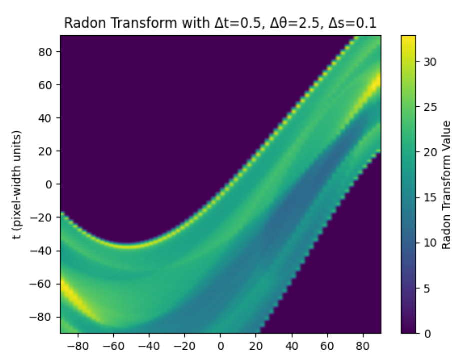
- Smaller $\Delta\theta$ improves angular resolution.
- Too small increases radiation exposure and scan time.
- A moderate small value is preferred.



$\Delta\theta = 0.1$



$\Delta\theta = 0.5$



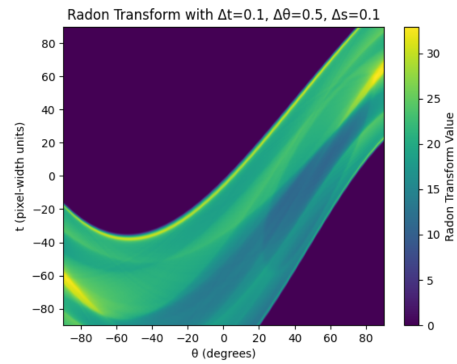
$\Delta\theta = 2.5$

Δt:

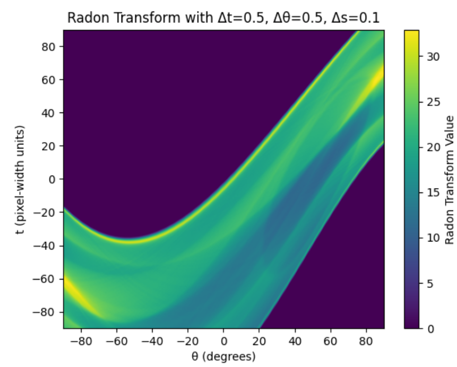
- Smaller Δt improves spatial sampling.
- Very small Δt increases noise sensitivity.

Question 1: X-Ray Computed Tomography – Radon Transform

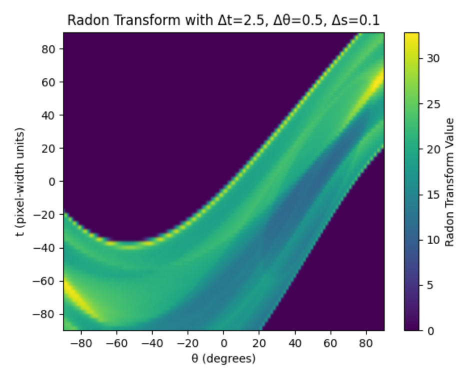
- Larger Δt introduces discretization artifacts.



$\Delta t = 0.1$



$\Delta t = 0.5$



$\Delta t = 2.5$

Thus, both parameters must balance resolution, noise, and acquisition cost.

(e) Design Considerations for ART Reconstruction

Number of Pixels and Pixel Size:

- More pixels improve spatial resolution.
- However, computational cost increases significantly.
- Smaller pixels receive fewer photons, increasing noise variance.

A moderate grid resolution is therefore preferred.

Effect of Δs:

- $\Delta s \ll 1$ pixel width:
  - More accurate integration.
  - Higher computational cost.
  - Less energy per pixel so noise has greater effect.
- $\Delta s \gg 1$ pixel width:
  - Underestimates line integrals.
  - Produces blocky reconstruction.
  - Slows ART convergence.

Summary

The experiments demonstrate that:

- Δs should be comparable to pixel width.
- Δt and $\Delta\theta$ should be small but not excessively small.
- There is a trade-off between resolution, noise sensitivity, radiation dose, and computational complexity.

2 Question 2

The observed RRMSES are as follows (this is after normalizing both the phantom and the reconstructions to be between 0 and 1):

- RRMSE (ram_lak, L=0.50000): 0.578613
- RRMSE (ram_lak, L=0.25000): 0.593879
- RRMSE (shepp_logan, L=0.50000): 0.601762
- RRMSE (shepp_logan, L=0.25000): 0.652877
- RRMSE (cosine, L=0.50000): 0.596825
- RRMSE (cosine, L=0.25000): 0.643436

The RRMSEs with $L_{\max}$ are slightly better, as we also observe in part c. This is because, with such a wide angular range, we can potentially overfit to the data, which is easier with a higher cutoff frequency. With a lower cutoff frequency, we are essentially applying a low-pass filter to the data, which can help to reduce noise and improve the stability of the reconstruction, but it may also lead to a loss of detail and an increase in RRMSE w.r.t the original phantom.

(a) Filter Implementation and Reconstruction Results

Three filters were implemented in the Fourier domain using FFT/IFFT:

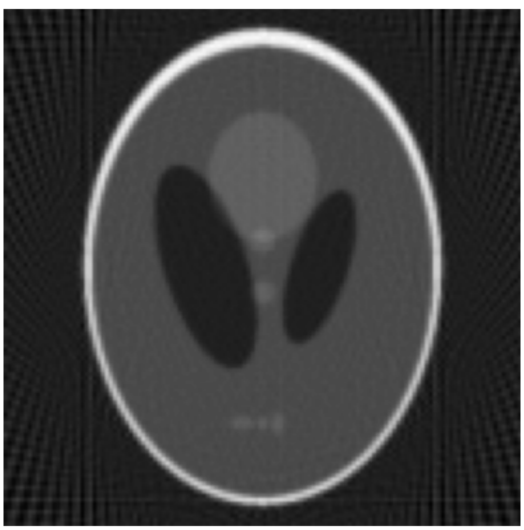
- Ram-Lak
- Shepp-Logan
- Cosine

Each filter was evaluated at two cutoff frequencies:

$$L = \omega_{\max}, \qquad L = \frac{\omega_{\max}}{2},$$

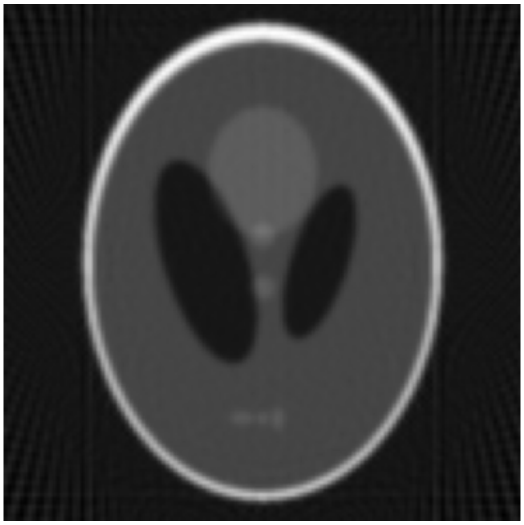
where $\omega_{\max}$ is the highest discrete frequency determined by the sampling of the sinogram.
The unfiltered backprojection was also computed for comparison.

No Filtering



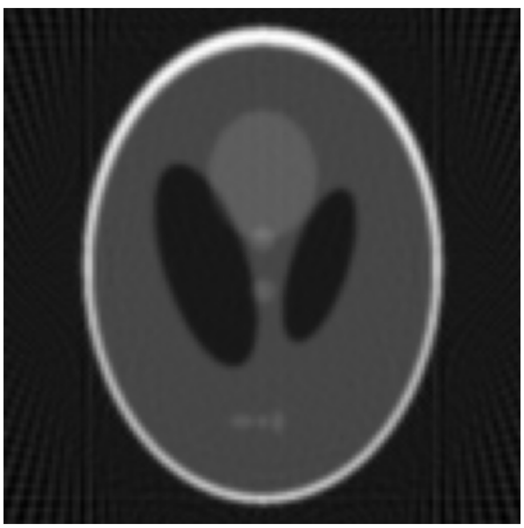
Ram-Lak ($L = \omega_{\max}$)

Ram-Lak ($L = \omega_{\max}/2$)



Shepp-Logan ($L = \omega_{\max}$)

Shepp-Logan ($L = \omega_{\max}/2$)



Cosine ($L = \omega_{\max}$)

Cosine ($L = \omega_{\max}/2$)

Figure 1: Reconstruction results using different FBP filters and cutoff frequencies.

Observations and Discussion

- **Unfiltered Backprojection:** The unfiltered reconstruction appears overly bright and severely blurred. Edges are poorly defined and fine structures are lost. This behavior is expected because simple backprojection introduces a $1/|\omega|$ frequency weighting, which excessively emphasizes low frequencies while failing to properly compensate high-frequency attenuation. As a result, the image lacks sharpness and contrast.

- **Ram-Lak Filter:** The Ram-Lak filter corresponds to the ideal ramp $|\omega|$ and restores the correct frequency weighting required by the inversion formula.

  For $L = \omega_{\max}$, edges are clearly delineated and the reconstruction is the sharpest among all cases. However, noticeable streak artifacts appear, particularly outside the circular support, due to high-frequency amplification combined with finite angular sampling.

  When $L = \omega_{\max}/2$, high frequencies are truncated. The reconstruction becomes smoother and streak artifacts are reduced, but edge definition is slightly degraded.

- **Shepp-Logan Filter:** The Shepp-Logan filter multiplies the ramp by a sinc term, which gradually attenuates higher frequencies.

  Compared to Ram-Lak at full bandwidth, the reconstruction shows reduced streaking while preserving most structural details. For the reduced cutoff, the image becomes smoother, with slightly diminished edge contrast but fewer high-frequency artifacts.

- **Cosine Filter:** The cosine filter applies a cosine apodization to the ramp, leading to stronger high-frequency suppression.

  For $L = \omega_{\max}$, the reconstruction is smoother than both Ram-Lak and Shepp-Logan, though mild ringing patterns are visible. When $L = \omega_{\max}/2$, the image becomes noticeably smoother, and edges appear softer due to the stronger attenuation of high frequencies.

Influence of the Cutoff Frequency Across all filters, increasing L improves edge sharpness by retaining more high-frequency components. However, it also enhances streak artifacts and oscillatory patterns caused by discretization and limited angular sampling. Reducing L suppresses these artifacts at the cost of spatial resolution.

Overall, the Ram-Lak filter with full bandwidth yields the sharpest reconstruction but exhibits the strongest artifacts. The Shepp-Logan filter provides a balanced compromise between resolution and stability, while the cosine filter produces the smoothest images with reduced edge contrast.

(b) Effect of Gaussian Blurring on Reconstruction

Figure 2: Images after Gaussian blurring with STD 0, 1 and 5

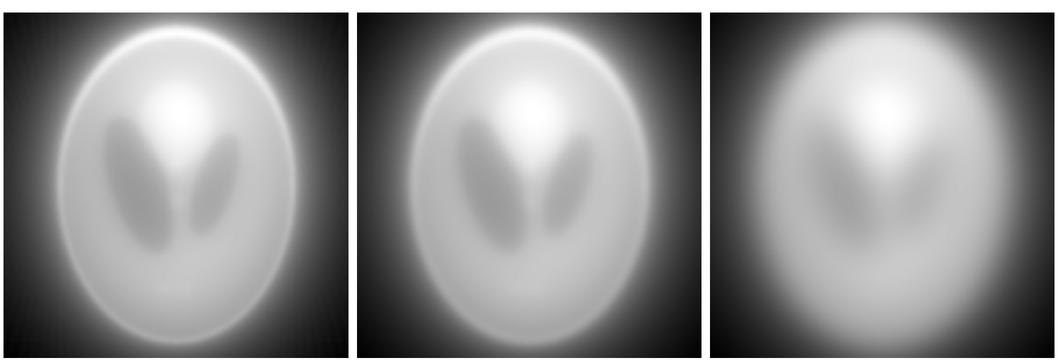
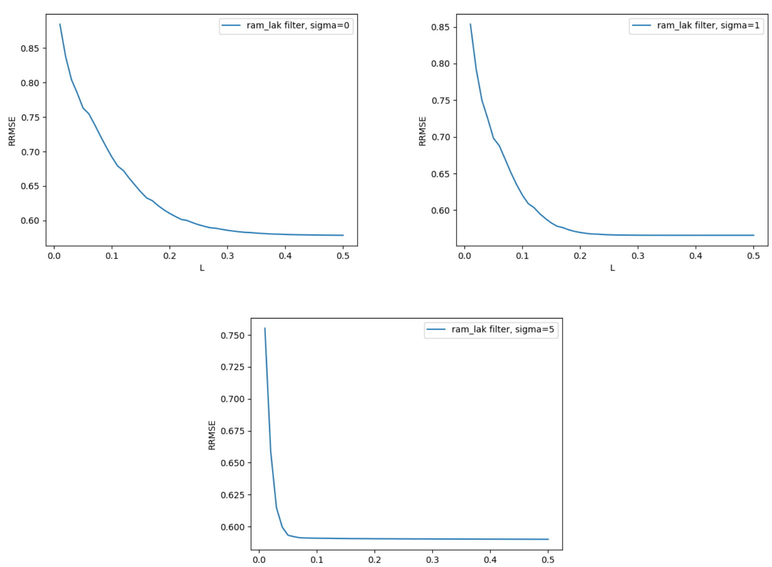


Figure 3: Images after reconstruction with no filter with STD 0, 1 and 5

Figure 4: Images after reconstruction with Ram-Lak filter with STD 0, 1 and 5

- RRMSE for ram_lak filter with L=0.500, sigma=0: 0.578613
- RRMSE for ram_lak filter with L=0.500, sigma=1: 0.565757
- RRMSE for ram_lak filter with L=0.500, sigma=5: 0.590054

The reconstruction error is lowest with the image convolved with Gaussian(51, 5) (S_5) and highest with the image convolved with Guassian(11, 1) (S_1). This is because the weak Gaussian blurring reduces the high-frequency noisy content in the sinogram, which makes it easier to reconstruct the original (noisy) image accurately. The extremely noisy image likely has very few learnable details because of the high noise, and hence reconstructing it from a sparse set of projections is difficult, leading to a higher RRMSE. The unblurred image suffers from having quite a bit of high-frequency content.



We observe that higher cutoff frequencies lead to lower RRMSE values. We expect the graph to flatten out because the number of high-frequency components in the sinogram will be limited, so beyond a certain cutoff we do not expect a significant improvement in reconstruction quality. The RRMSE values are higher for the unfiltered backprojection compared to the Ram-Lak filter, which is expected since the unfiltered backprojection does not compensate for the frequency response of the system, leading to a blurred reconstruction. The RRMSE values for sigma=0 and sigma=1 are roughly similar. They start off higher than the RRMSE values for sigma=5, but they decrease more rapidly as the cutoff frequency increases. This is because the unblurred and lightly blurred sinograms contain more high-frequency content, which can be better reconstructed with higher cutoff frequencies. In contrast, the strongly blurred sinogram (sigma=5) has already lost much of its high-frequency content, so increasing the cutoff frequency does not lead to as significant an improvement in reconstruction quality.

3 Question 3

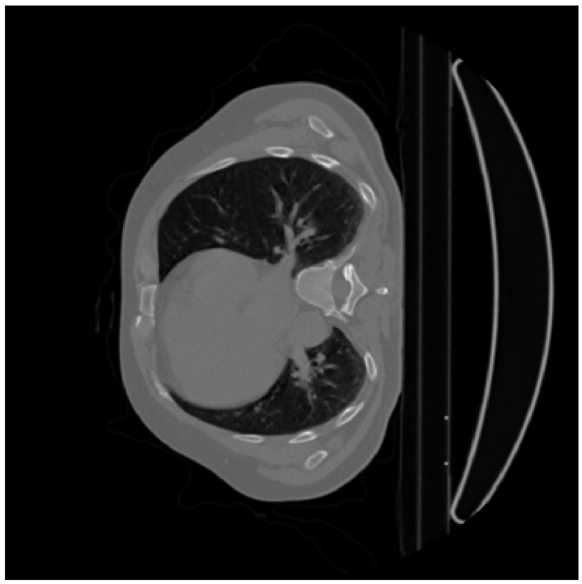


Figure 6: Original Image - Chest CT

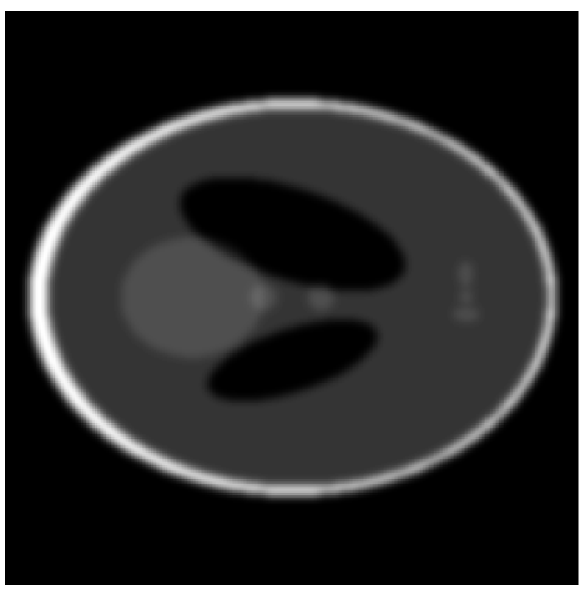


Figure 7: Original Image - Phantom

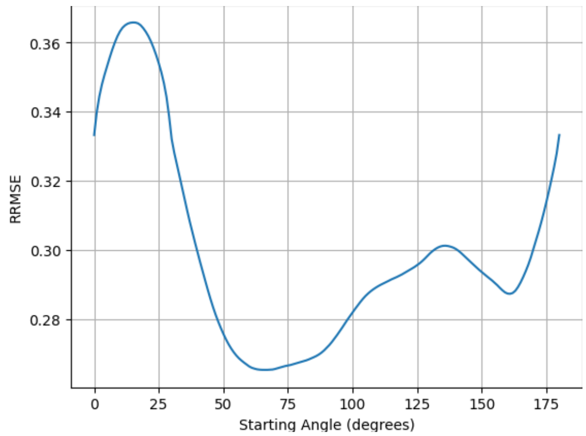


Figure 8: RRMSE Plot - Chest CT

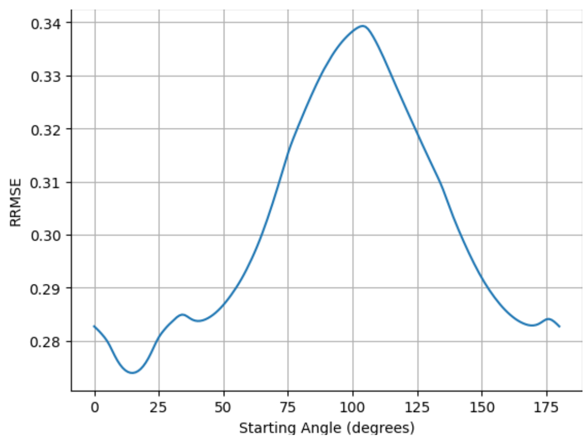


Figure 9: RRMSE Plot - Phantom

Minima of RRMSE for Chest CT is an RRMSE of 0.26530356761801244 at 67 degrees. For phantom, it is an RRMSE of 0.2739286037697438 at 15 degrees (a set of 2 cones involving the 2 ends of the major axis of the image are blurry).

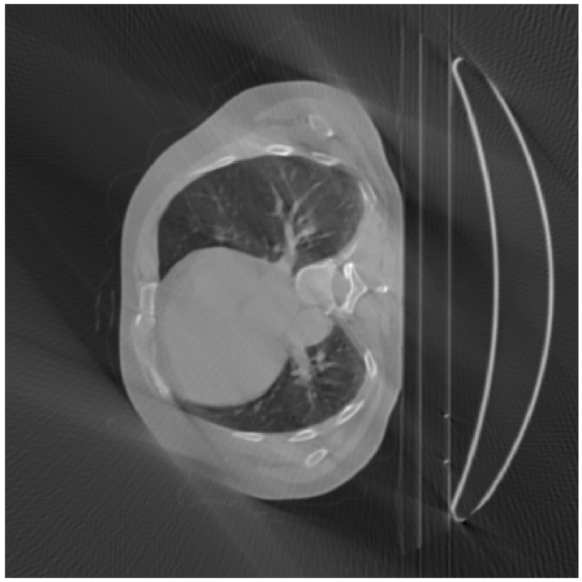


Figure 10: Optimal Reconstruction - Chest CT

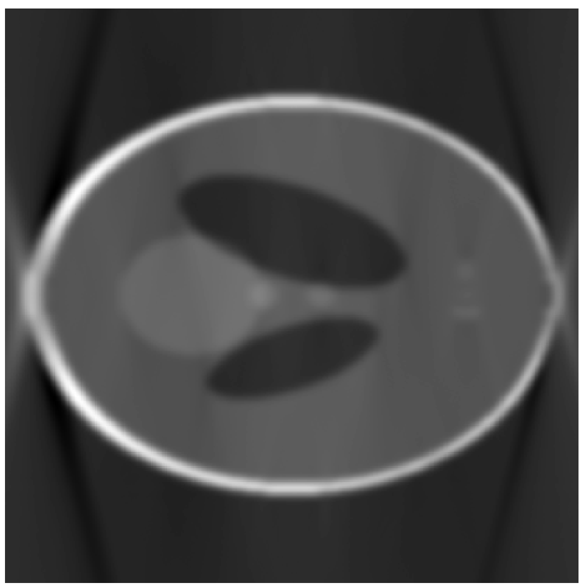
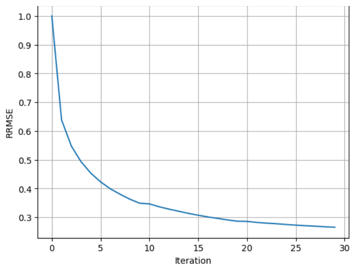
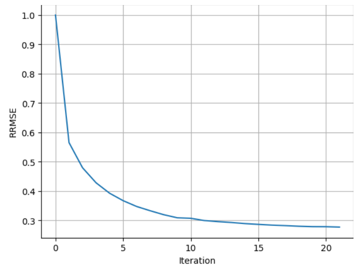


Figure 11: Optimal Reconstruction - Phantom

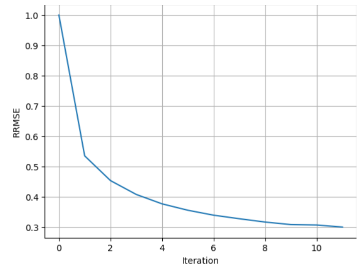
4 Question 4



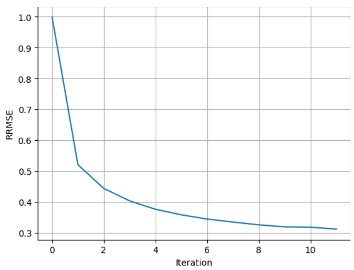
(a) $\lambda = 0.1$



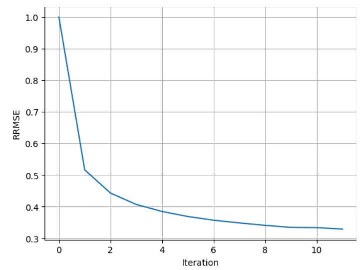
(b) $\lambda = 0.2$



(c) $\lambda = 0.3$

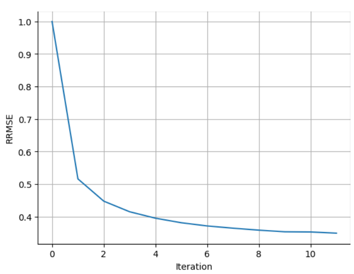


(d) $\lambda = 0.4$

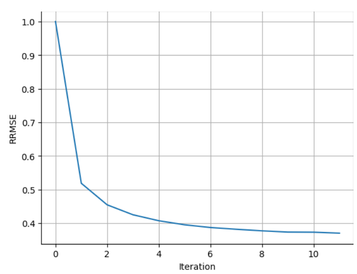


(e) $\lambda = 0.5$

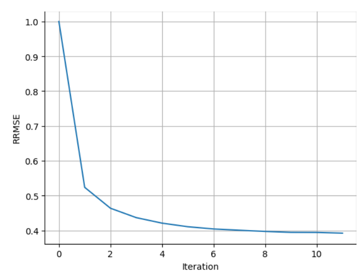
Figure 12: RRMSE plots for λ from 0.1 to 0.5.



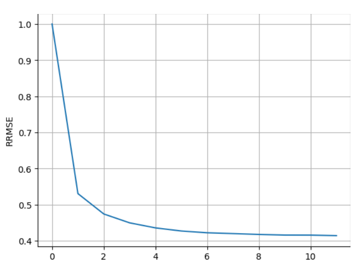
(a) $\lambda = 0.6$



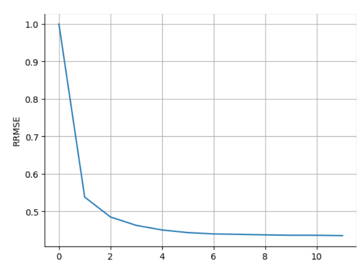
(b) $\lambda = 0.7$



(c) $\lambda = 0.8$

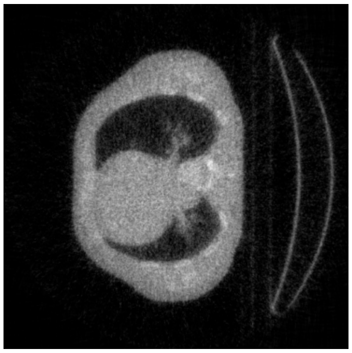


(d) $\lambda = 0.9$

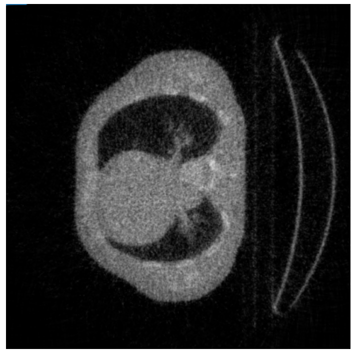


(e) $\lambda = 1.0$

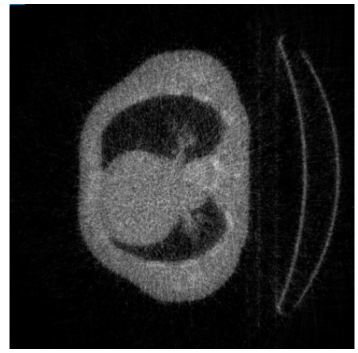
Figure 13: RRMSE plots for λ from 0.6 to 1.0.



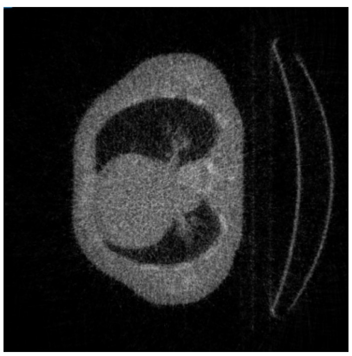
(a) $\lambda = 0.1$



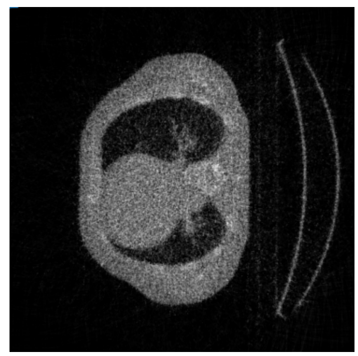
(b) $\lambda = 0.2$



(c) $\lambda = 0.3$

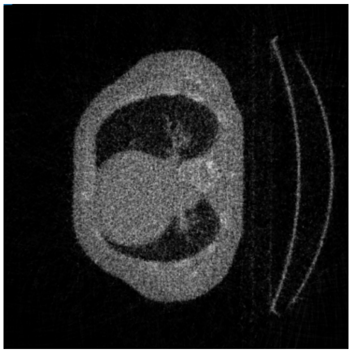


(d) $\lambda = 0.4$

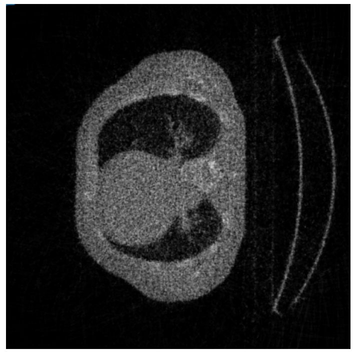


(e) $\lambda = 0.5$

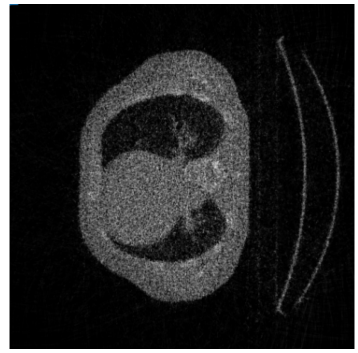
Figure 14: Reconstructed images for λ from 0.1 to 0.5.



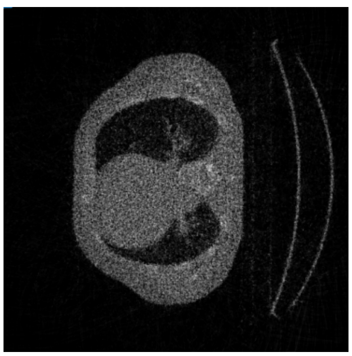
(a) $\lambda = 0.6$



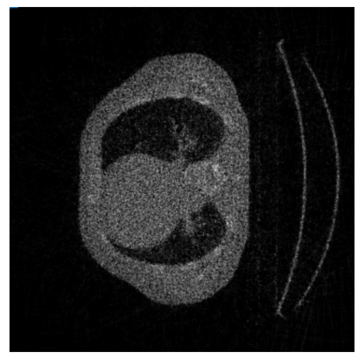
(b) $\lambda = 0.7$



(c) $\lambda = 0.8$



(d) $\lambda = 0.9$



(e) $\lambda = 1.0$

Figure 15: Reconstructed images for λ from 0.6 to 1.0.