MIC Assignment-2

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1 Question 1: X-Ray Computed Tomography – Radon Transform

A 128×128 Shepp–Logan phantom was generated and treated as the image $f(x, y)$. The coordinate origin was logically placed at the center pixel.

(a) Implementation of `myXrayIntegration()`

The function `myXrayIntegration(f, t, theta_deg, delta_s)` computes the line integral of image intensities along a line parameterized by t and θ.

Implementation details:

- The angle `theta_deg` is converted to radians.
- A sampling variable `s` is created from $-N$ to N with step size `delta_s`.
- For each value of `s`, corresponding (x, y) coordinates are computed.
- `map_coordinates()` is used to interpolate image values.
- The interpolated values are summed and multiplied by `delta_s`.

Interpolation Scheme: Bilinear interpolation (`order=1`) was used because:

- It provides smooth transitions between pixels.
- Nearest-neighbor interpolation produces jagged artifacts.
- Higher-order interpolation increases computation unnecessarily.

(b) Implementation of `myXrayCTRadonTransform()`

The function `myXrayCTRadonTransform()` computes the Radon transform over discrete values:

- `t = -90 to 90` with step size $\Delta t = 5$
- `theta = 0 to 175` with step size $\Delta\theta = 5$

For every (t, θ) pair, the function calls `myXrayIntegration()` and stores the result in a 2D array.

(c) Comparison of Different Δs Values

Choice of Δs: Different values were tested: $0.1, 0.5, 1, 3, 10$ pixel units.

Observations:

- Very small Δs increases computation with little improvement.
- Infact $\Delta s = 0.1$ performed worse than $\Delta s = 0.5$.
- This is due to overfitting by interpolation.
- $\Delta s = 0.5$ or 1 gives stable and smooth results.
- Large Δs (3 or 10) causes rough and blocky sinograms.

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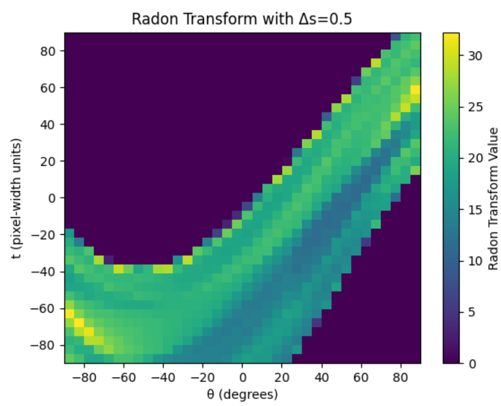
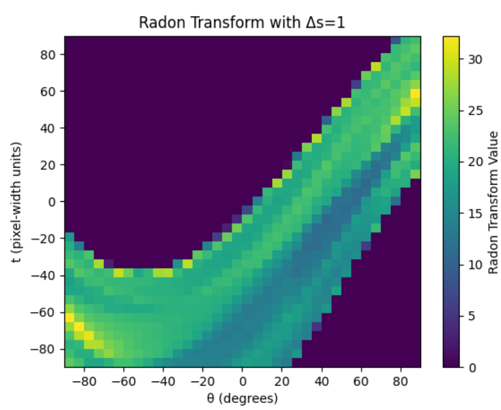
Δs	Smoothness Score
0.1	0.00766441
0.5	0.00766361
1	0.00766573
3	0.00792422
10	0.01310446

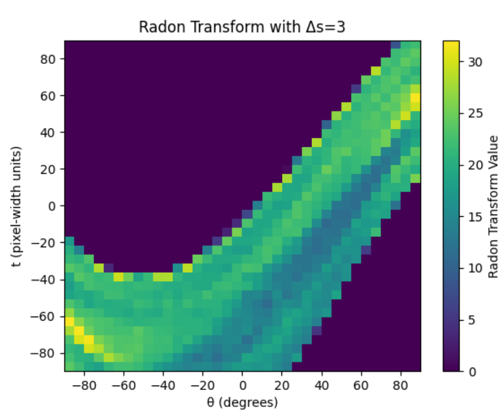
Table 1: Smoothness scores for different Δs values.



$\Delta s = 0.5$



$\Delta s = 1$



$\Delta s = 3$

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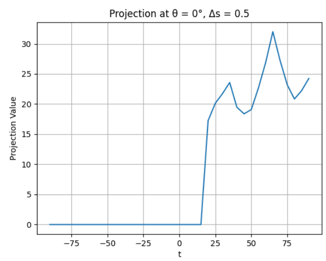
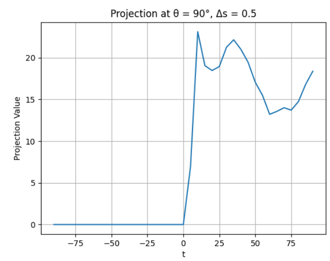
Additionally, 1D plots of the sinogram values were examined for:

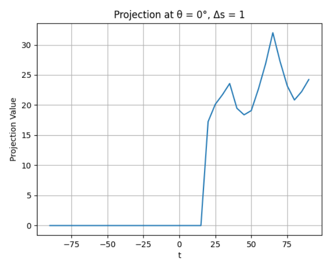
1D Projection Comparisons for Different Δs

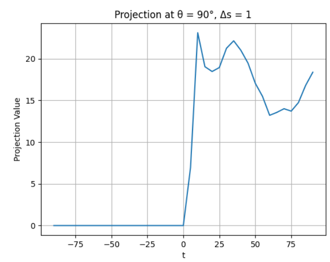
Δs — $\theta = 0^\circ$ — $\theta = 90^\circ$



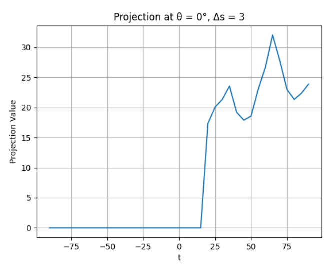


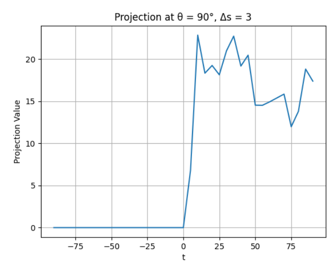
0.5





1





3

Observations:

- Smaller Δs produces smoother 1D curves.
- Larger Δs produces visible roughness due to coarse sampling.
- Among the tested values, $\Delta s = 0.5$ appears smoothest.

A smoothness score was computed using the function `apply_prior()`, which evaluates local intensity differences. Lower values indicate smoother sinograms.

Results show:

- $\Delta s = 0.5$ gives the lowest smoothness score.
- $\Delta s = 1$ and 3 give slightly higher scores, indicating more roughness.

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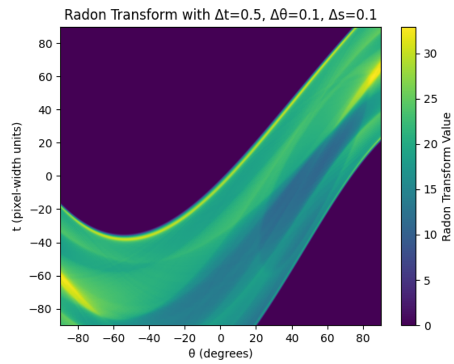
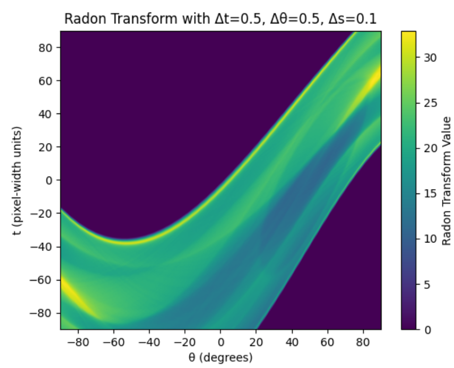
(d) Choice of Δt and $\Delta\theta$ in Scanner Design

$\Delta\theta$:

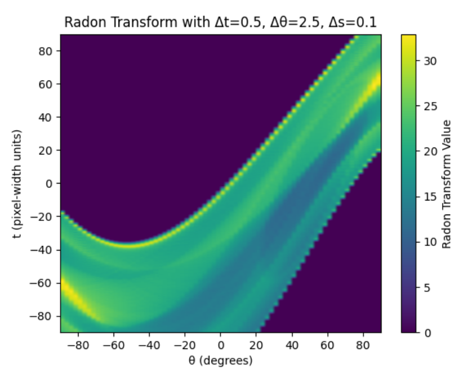
- Smaller $\Delta\theta$ improves angular resolution.
- Too small increases radiation exposure and scan time.
- A moderate small value is preferred.



$\Delta\theta = 0.1$



$\Delta\theta = 0.5$



$\Delta\theta = 2.5$

Δt:

- Smaller Δt improves spatial sampling.
- Very small Δt increases noise sensitivity.

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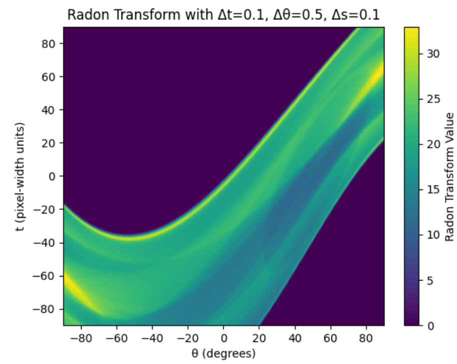
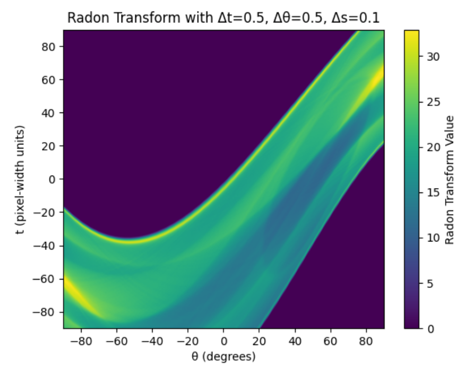
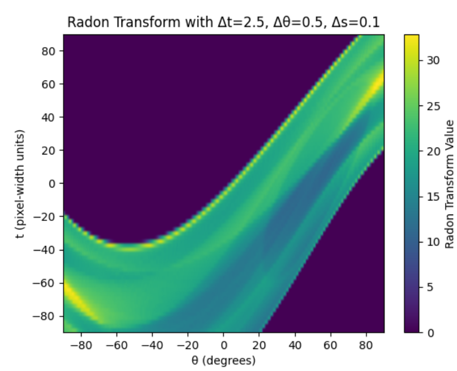
- Larger Δt introduces discretization artifacts.



$\Delta t = 0.1$



$\Delta t = 0.5$



$\Delta t = 2.5$

Thus, both parameters must balance resolution, noise, and acquisition cost.

(e) Design Considerations for ART Reconstruction

Number of Pixels and Pixel Size:

- More pixels improve spatial resolution.
- However, computational cost increases significantly.
- Smaller pixels receive fewer photons, increasing noise variance.

A moderate grid resolution is therefore preferred.

Effect of Δs:

- $\Delta s \ll 1$ pixel width:
  - More accurate integration.
  - Higher computational cost.
  - Less energy per pixel so noise has greater effect.
- $\Delta s \gg 1$ pixel width:
  - Underestimates line integrals.
  - Produces blocky reconstruction.
  - Slows ART convergence.

Summary

The experiments demonstrate that:

- Δs should be comparable to pixel width.
- Δt and $\Delta \theta$ should be small but not excessively small.
- There is a trade-off between resolution, noise sensitivity, radiation dose, and computational complexity.

2 Question 2: X-Ray Computed Tomography — Reconstruction by Filtered Backprojection (FBP)

(a) Filter Implementation and Reconstruction Results

Three filters were implemented in the Fourier domain using FFT/IFFT:

- Ram-Lak
- Shepp-Logan
- Cosine

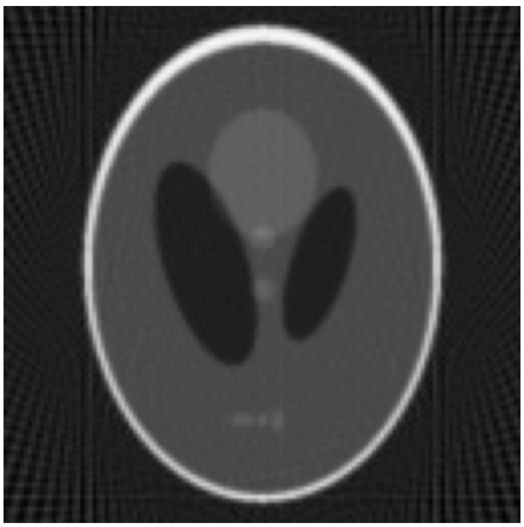
Each filter was evaluated at two cutoff frequencies:

$$L = \omega_{\max}, \qquad L = \frac{\omega_{\max}}{2},$$

where $\omega_{\max}$ is the highest discrete frequency determined by the sampling of the sinogram.

The unfiltered backprojection was also computed for comparison.

No Filtering



Ram-Lak ($L = \omega_{\max}$)

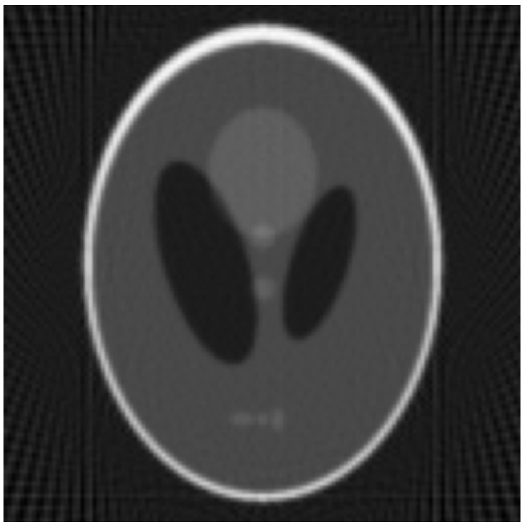
Ram-Lak ($L = \omega_{\max}/2$)

Observations and Discussion

- **Unfiltered Backprojection:** The unfiltered reconstruction appears overly bright and severely blurred. Edges are poorly defined and fine structures are lost. This behavior is expected because simple backprojection introduces a $1/|\omega|$ frequency weighting, which excessively emphasizes low frequencies while failing to properly compensate high-frequency attenuation. As a result, the image lacks sharpness and contrast.
- **Ram-Lak Filter:** The Ram-Lak filter corresponds to the ideal ramp $|\omega|$ and restores the correct frequency weighting required by the inversion formula.

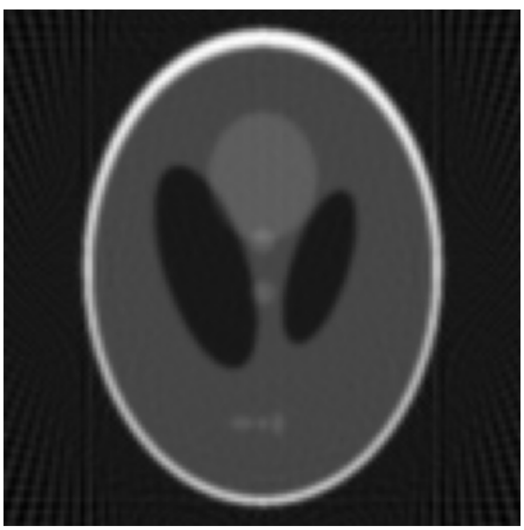
  For $L = \omega_{\max}$, edges are clearly delineated and the reconstruction is the sharpest among all cases. However, noticeable streak artifacts appear, particularly outside the circular support, due to high-frequency amplification combined with finite angular sampling.

  When $L = \omega_{\max}/2$, high frequencies are truncated. The reconstruction becomes smoother and streak artifacts are reduced, but edge definition is slightly degraded.
- **Shepp-Logan Filter:** The Shepp-Logan filter multiplies the ramp by a sinc term, which gradually attenuates higher frequencies.



Shepp-Logan ($L = \omega_{\max}$)

Shepp-Logan ($L = \omega_{\max}/2$)



Cosine ($L = \omega_{\max}$)

Cosine ($L = \omega_{\max}/2$)

Figure 1: Reconstruction results using different FBP filters and cutoff frequencies.

Compared to Ram-Lak at full bandwidth, the reconstruction shows reduced streaking while preserving most structural details. For the reduced cutoff, the image becomes smoother, with slightly diminished edge contrast but fewer high-frequency artifacts.

- **Cosine Filter:** The cosine filter applies a cosine apodization to the ramp, leading to stronger high-frequency suppression.

  For $L = \omega_{\max}$, the reconstruction is smoother than both Ram-Lak and Shepp-Logan, though mild ringing patterns are visible. When $L = \omega_{\max}/2$, the image becomes noticeably smoother, and edges appear softer due to the stronger attenuation of high frequencies.

Influence of the Cutoff Frequency Across all filters, increasing L improves edge sharpness by retaining more high-frequency components. However, it also enhances streak artifacts and oscillatory patterns caused by discretization and limited angular sampling. Reducing L suppresses these artifacts at the cost of spatial resolution.

Overall, the Ram-Lak filter with full bandwidth yields the sharpest reconstruction but exhibits the strongest artifacts. The Shepp-Logan filter provides a balanced compromise between resolution and stability, while the cosine filter produces the smoothest images with reduced edge contrast.

(b) Effect of Gaussian Blurring on Reconstruction

(c) RRMSE as a Function of Cutoff Frequency

3 Question 3

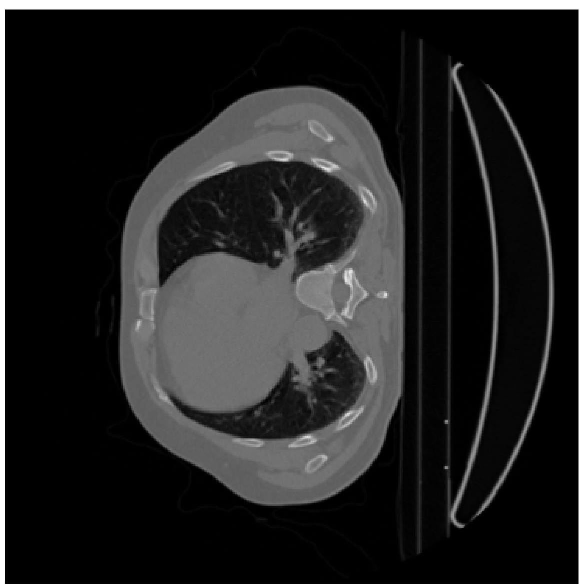


Figure 2: Original Image - Chest CT

Figure 3: Original Image - Phantom

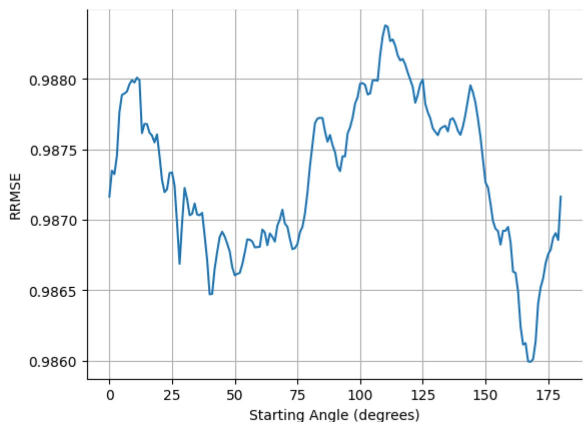


Figure 4: RRMSE Plot - Chest CT

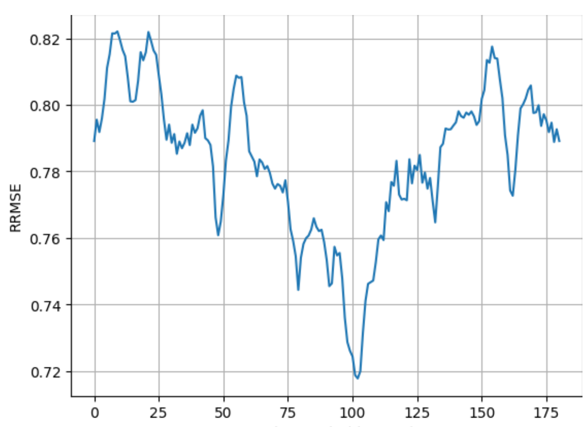


Figure 5: RRMSE Plot - Phantom

Minima of RRMSE for Chest CT is an RRMSE of 0.9859919140151212 at 168 degrees. For phantom, it is an RRMSE of 0.7176981638852817 at 102 degrees.

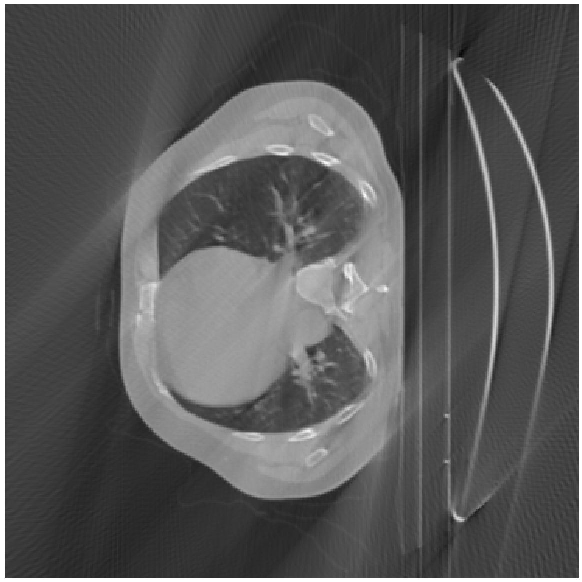


Figure 6: Optimal Reconstruction - Chest CT

Figure 7: Optimal Reconstruction - Phantom