MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.
   (a) Printing more money may cause inflation and therefore is not an ideal way for governments to raise capital to pay off their debts or fund activities.

   (b) When the economy is transitioning from expansion to slower development and even recession, yields on longer-maturity bonds tend to fall and yields on shorter-term securities likely rise, make a normal yield curve into a flat yield curve.

   (c) Quantitative easing (QE) is a form of unconventional monetary policy in which a central bank purchases longer-term securities from the open market in order to increase the money supply and encourage lending and investment. On March 15, 2020, the U.S. Federal Reserve announced a quantitative easing plan of over 700 billion dollars and then, on June 10, 2020, the Fed extended its program, committing to buy at least 80 billion a month in Treasuries and 40 billion in mortgage-backed securities, until further notice [1].

2. The following 10 bonds will be used to create the 0-5 year yield & spot curves: *1: CAN 1.5 Aug 21, 2: CAN 1.5 Feb 22, 3: CAN 0.25 Aug 22, 4: CAN 0.25 Feb 23, 5: CAN 1.5 Jun 23, 6: CAN 2.25 Mar 24, 7: CAN 1.5 Sep 24, 8: CAN 1.25 Mar 25, 9: CAN 0.5 Sep 25, 10: CAN 0.25 Mar 26.* Ideally the selected 10 bonds with semi-annual coupon will cover the 5 year span of the yield curve perfectly. However, in this case, there is no exact bond portfolio that have exact half year increments. So I decided to include bond 5 as a connecting point between bonds paying coupon in February & August, and bonds paying coupon in March & September. In this case, the 10 bonds still effectively covers the 5 year span. It is worth mentioning that as the objective for this project is to construct a 5-year yield curve from February 1st 2021, the closest maturity bond chosen is matured 6 month from February 1st. The most important characteristic in choosing the bonds are their maturity date must cover most of the span of 5 years after Feb 01 2021, and therefore more accurate for later interpolation and bootstrapping. It is worth mentioning that while selecting the bonds, on-the-run bonds are chosen prior to off-the-run bonds since they are more liquid and therefore better describes the actual underlying yield, so I choose not to use the extremely high coupon bonds issued from last century.

3. The eigenvector with the largest eigenvalue is called the first principal component in PCA. The principle components represent the direction of the maximum variance of the data. If the unique point follows a stochastic processes, eigenvectors or principal components in this case will represent the direction where the points have largest variability among the directions of the movement. And the eigenvalues will be the sum of squared distance between each point and the principal component, which represents the intensity of variance in this eigenvector (principal component) direction.

Empirical Questions - 75 points

4.

(a) Figure 1 below shows the YTM from year 0 to year 5 using data collected from Jan 18 to Feb 01. The following formula (equation 1) is used to calculate the YTM for plotting this figure:

$$P + \frac{c \times t}{365} = (\frac{c}{yield} \times (1 - \frac{1}{(1+\frac{\lambda}{2})^n}) + \frac{100}{(1+\frac{\lambda}{2})^n}) \times (1 + \frac{\lambda}{2})^{(t/182.5)} \tag{1}$$

where λ is the yield, c is the coupon rate, t is the number of days between coupon payment date and the settlement date to account for accrued interests. The yield λ is computed using Python with a numerical method (Newton's Method: trials and errors). A linear interpolation is chosen as the interpolation tool to calculate the yield, as it is a simple and effective way to estimate yield λ between two time points.

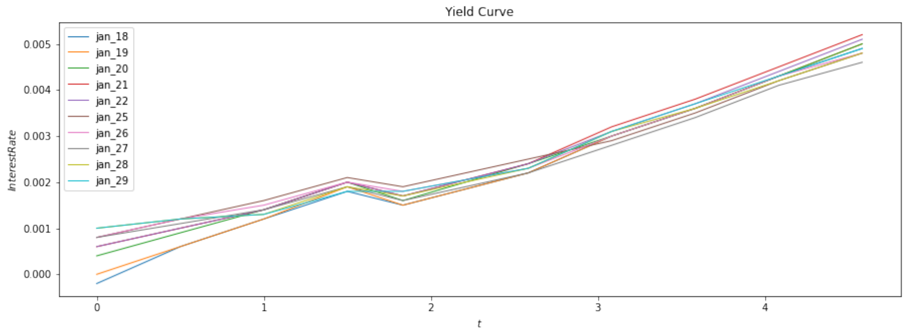

Figure 1: 0-5 year yield curve calculated using numerical methods and linear interpolation

(b) **def: spotForEachday(YTMlist, date) $\rightarrow$ spot rate**. The input YTMlist is the data set used to plot figure 1. The list is constructed using 10 bonds' data using equation 1. With this list, I performed linear interpolation followed by bootstrapping. I first performed a linear interpolation to obtain a list of yields on dates with 0.5 increments, Y. Then the following algorithm is used for each recording day to plot figure 2. :

$$\begin{cases} s_j = Y_j = Y_1 & \text{if } j = 1 \\ s_j = (\frac{1+Y_j}{1 - \sum_{k=1}^{j-1} \frac{Y_j}{(1+s_k)^k}})^{\frac{1}{j}} & \text{if } j = 2...10 \end{cases}$$

where Y_j is the yield at period j, which is the coupon rate for par bonds (assume notional = 1).

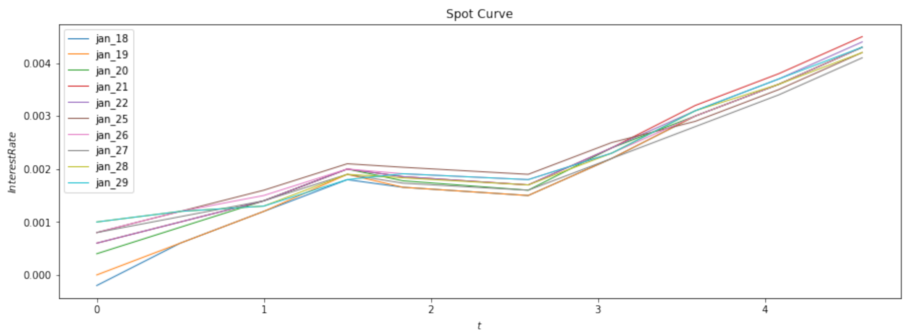

Figure 2: 0-5 year spot curve calculated using numerical methods and linear interpolation

(c) With the spot rate at time j, s_j, the forward rate for each date is calculated iteratively using formula 2. Using this algorithm, the forward curve is plotted as shown in figure 3.

$$\begin{cases} f_{1\to 0} = s_1 & \text{if } j = 1 \\ f_{1\to j} = \left(\frac{(1+s_{j+1})^{j+1}}{(1+s_1)\times\prod_{k=1}^{j-1}(1+f_{1\to k})}\right) - 1 & \text{if } j = 1...4 \end{cases} \tag{2}$$

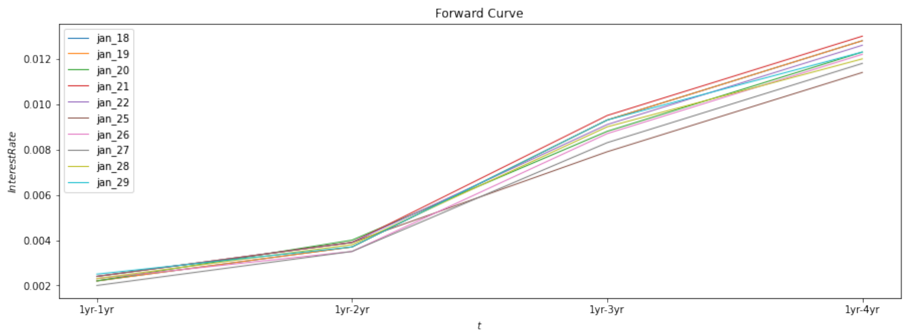

Figure 3: 1-year forward curve with terms from 2-5 years calculated by numerical methods

Note here only the non-base case is used for plotting, which contains 4 data points per recording date.

5. Two co-variances matrices for log return of yield and forward rates are calculated using numpy's cov function:

$$X^{(1)} = \begin{pmatrix} 0.00691 & 0.00379 & -0.00177 & -0.0008 & -0.0010 \\ 0.00379 & 0.00503 & 0.00104 & 0.00059 & 0.00055 \\ -0.00177 & 0.00104 & 0.00322 & 0.00166 & 0.00181 \\ -0.0008 & 0.00059 & 0.00063 & 0.00109 & 0.00107 \\ -0.0010 & 0.00055 & 0.00181 & 0.00107 & 0.00126 \end{pmatrix} X^{(2)} = \begin{pmatrix} 0.00670 & -0.0006 & 0.00331 & 0.00021 \\ -0.0006 & 0.00481 & -0.0051 & -0.0009 \\ 0.00331 & -0.0051 & 0.00966 & 0.00123 \\ 0.00021 & -0.0009 & 0.00123 & 0.00058 \end{pmatrix}$$

6. The eigen value and eigen vectors can be calculated using *np.linalg.eig* as:

$$\lambda^{(1)} = [0.01008, 0.00063, 0.00008, 0.00002, 0.00001], \lambda^{(2)} = [0.01434, 0.00576, 0.00126, 0.00037]$$

$$\nu^{(1)} = \begin{pmatrix} -0.8070 & -0.1199 & -0.5672 & -0.1042 & 0.04195 \\ -0.5495 & 0.51046 & 0.65172 & 0.10963 & -0.0264 \\ 0.17200 & 0.66154 & -0.2572 & -0.6825 & 0.02619 \\ 0.07812 & 0.36503 & -0.3285 & 0.46927 & -0.7297 \\ 0.10545 & 0.39262 & -0.2818 & 0.53942 & 0.68143 \end{pmatrix} \nu^{(2)} = \begin{pmatrix} -0.3828 & 0.8840 & 0.2680 & 0.0070 \\ 0.4576 & 0.4284 & -0.7634 & 0.1555 \\ -0.7951 & -0.1679 & -0.5807 & -0.0480 \\ -0.1081 & -0.0820 & 0.0901 & 0.9866 \end{pmatrix}$$

These yield return first-eigenvalue accounts for $\frac{0.01008}{0.01008+0.00063+0.00008+0.00002+0.00001} = 93\%$ in magnitude and the forward first-eigenvalue accounts for $\frac{0.01434}{0.01434+0.00576+0.00126+0.00002+0.00037} = 66\%$ magnitude, which means that the first eigenvectors could explain 93% and 66% of the total variance of the log return of yield & forward rate, respectively.

References and GitHub Link to Code

[1]https://www.investopedia.com/terms/q/quantitative-easing.asp

Github: https://github.com/ThatManRong/APM466_A1